COMPANION TO THE AISC *STEEL CONSTRUCTION MANUAL*

Volume 1: Design Examples

Version 15.1

AMERICAN INSTITUTE
OF
STEEL CONSTRUCTION

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PREFACE

The primary objective of this Companion is to provide guidance and additional resources of the use of the 2016 AISC *Specification for Structural Steel Buildings* (ANSI/AISC 360-16) and the 15th Edition AISC *Steel Construction Manual*.

The Companion consists of design examples in Parts I, II and III. The design examples provide coverage of all applicable limit states, whether or not a particular limit state controls the design of the member or connection. In addition to the examples that demonstrate the use of the AISC *Manual* tables, design examples are provided for connection designs beyond the scope of the tables in the AISC *Manual*. These design examples are intended to demonstrate an approach to the design, and are not intended to suggest that the approach presented is the only approach. The committee responsible for the development of these design examples recognizes that designers have alternate approaches that work best for them and their projects. Design approaches that differ from those presented in these examples are considered viable as long as the AISC *Specification*, sound engineering, and project specific requirements are satisfied.

Part I of these examples is organized to correspond with the organization of the AISC *Specification*. The Chapter titles match the corresponding chapters in the AISC *Specification*.

Part II is devoted primarily to connection examples that draw on the tables from the AISC *Manual*, recommended design procedures, and the breadth of the AISC *Specification*. The chapters of Part II are labeled II-A, II-B, II-C, etc.

Part III addresses aspects of design that are linked to the performance of a building as a whole. This includes coverage of lateral stability and second-order analysis, illustrated through a four-story braced-frame and moment-frame building.

The Design Examples are arranged with LRFD and ASD designs presented side-by-side, for consistency with the AISC *Manual*. Design with ASD and LRFD are based on the same nominal strength for each element so that the only differences between the approaches are the set of load combinations from ASCE/SEI 7-16 used for design, and whether the resistance factor for LRFD or the safety factor for ASD is used.

CONVENTIONS

The following conventions are used throughout these examples:

1. The 2016 AISC *Specification for Structural Steel Buildings* is referred to as the AISC *Specification* and the 15th Edition AISC *Steel Construction Manual,* is referred to as the AISC *Manual*.

2. The 2016 ASCE *Minimum Design Loads and Associated Criteria for Buildings and Other Structures* is referred to as ASCE/SEI 7.

3. The source of equations or tabulated values taken from the AISC *Specification* or AISC *Manual* is noted along the right-hand edge of the page.

4. When the design process differs between LRFD and ASD, the designs equations are presented side-by-side. This rarely occurs, except when the resistance factor, ϕ, and the safety factor, Ω, are applied.

5. The results of design equations are presented to three significant figures throughout these calculations.

ACKNOWLEDGMENTS

The AISC Committee on Manuals reviewed and approved V15.1 of the AISC *Design Examples*:

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TABLE OF CONTENTS

Part I
Examples Based on the AISC *Specification*

This part contains design examples demonstrating select provisions of the AISC *Specification for Structural Steel Buildings.*

Chapter A
General Provisions

A1. SCOPE

These design examples are intended to illustrate the application of the 2016 AISC *Specification for Structural Steel Buildings*, ANSI/AISC 360-16 (AISC, 2016a), and the AISC *Steel Construction Manual*, 15th Edition (AISC, 2017) in low-seismic applications. For information on design applications requiring seismic detailing, see the 2016 AISC *Seismic Provisions for Structural Steel Buildings*, ANSI/AISC 341-16 (AISC, 2016b) and the AISC *Seismic Design Manual*, 2nd Edition (AISC, 2012).

A2. REFERENCED SPECIFICATIONS, CODES AND STANDARDS

Section A2 includes a detailed list of the specifications, codes and standards referenced throughout the AISC *Specification*.

A3. MATERIAL

Section A3 includes a list of the steel materials that are approved for use with the AISC *Specification*. The complete ASTM standards for the most commonly used steel materials can be found in *Selected ASTM Standards for Structural Steel Fabrication* (ASTM, 2016).

A4. STRUCTURAL DESIGN DRAWINGS AND SPECIFICATIONS

Section A4 requires that structural design drawings and specifications meet the requirements in the AISC *Code of Standard Practice for Steel Buildings and Bridges*, ANSI/AISC 303-16 (AISC, 2016c).

CHAPTER A REFERENCES

AISC (2012), *Seismic Design Manual*, 2nd Ed., American Institute of Steel Construction, Chicago, IL.

AISC (2016a), *Specification for Structural Steel Buildings*, ANSI/AISC 360-16, American Institute of Steel Construction, Chicago, IL.

AISC (2016b), *Seismic Provisions for Structural Steel Buildings*, ANSI/AISC 341-16, American Institute of Steel Construction, Chicago, IL.

AISC (2016c), *Code of Standard Practice for Steel Buildings and Bridges*, ANSI/AISC 303-16, American Institute of Steel Construction, Chicago, IL.

AISC (2017), *Steel Construction Manual*, 15th Ed., American Institute of Steel Construction, Chicago, IL.

ASTM (2016), *Selected ASTM Standards for Structural Steel Fabrication*, ASTM International, West Conshohocken, PA.

Chapter B
Design Requirements

B1. GENERAL PROVISIONS

The AISC *Specification* requires that the design of members and connections shall be consistent with the intended behavior of the framing system and the assumptions made in the structural analysis.

B2. LOADS AND LOAD COMBINATIONS

In the absence of an applicable building code, the default load combinations to be used with the AISC *Specification* are those from *Minimum Design Loads and Associated Criteria for Buildings and Other Structures*, ASCE/SEI 7-16 (ASCE, 2016).

B3. DESIGN BASIS

Chapter B of the AISC *Specification* and Part 2 of the AISC *Manual* describe the basis of design, for both load and resistance factor design (LRFD) and allowable strength design (ASD).

AISC *Specification* Section B3.4 describes three basic types of connections: simple connections, fully restrained (FR) moment connections, and partially restrained (PR) moment connections. Several examples of the design of each of these types of connections are given in Part II of these *Design Examples*.

Information on the application of serviceability and ponding provisions may be found in AISC *Specification* Chapter L and AISC *Specification* Appendix 2, respectively, and their associated commentaries. Design examples and other useful information on this topic are given in AISC Design Guide 3, *Serviceability Design Considerations for Steel Buildings*, Second Edition (West et al., 2003).

Information on the application of fire design provisions may be found in AISC *Specification* Appendix 4 and its associated commentary. Design examples and other useful information on this topic are presented in AISC Design Guide 19, *Fire Resistance of Structural Steel Framing* (Ruddy et al., 2003).

Corrosion protection and fastener compatibility are discussed in Part 2 of the AISC *Manual*.

B4. MEMBER PROPERTIES

AISC *Specification* Tables B4.1a and B4.1b give the complete list of limiting width-to-thickness ratios for all compression and flexural members defined by the AISC *Specification*.

Except for one section, the W-shapes presented in the compression member selection tables as column sections meet the criteria as nonslender element sections. The W-shapes with a nominal depth of 8 in. or larger presented in the flexural member selection tables as beam sections meet the criteria for compact sections, except for seven specific shapes. When noncompact or slender-element sections are tabulated in the design aids, local buckling criteria are accounted for in the tabulated design values.

The shapes listing and other member design tables in the AISC *Manual* also include footnoting to highlight sections that exceed local buckling limits in their most commonly available material grades. These footnotes include the following notations for W-shapes:

[c] Shape is slender for compression with F_y = 50 ksi.
[f] Shape exceeds compact limit for flexure with F_y = 50 ksi.
[g] The actual size, combination and orientation of fastener components should be compared with the geometry of the cross section to ensure compatibility.

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.
[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 50 ksi.

CHAPTER B REFERENCES

ASCE (2016), *Minimum Design Loads and Associated Criteria for Buildings and Other Structures*, ASCE/SEI 7-16, American Society of Civil Engineers, Reston, VA.

West, M.A., Fisher, J.M. and Griffis, L.G. (2003), *Serviceability Design Considerations for Steel Buildings*, Design Guide 3, 2nd Ed., AISC, Chicago, IL.

Ruddy, J.L., Marlo, J.P., Ioannides, S.A. and Alfawakhiri, F. (2003), *Fire Resistance of Structural Steel Framing*, Design Guide 19, AISC, Chicago, IL.

Chapter C
Design for Stability

C1. GENERAL STABILITY REQUIREMENTS

The AISC *Specification* requires that the designer account for both the stability of the structural system as a whole and the stability of individual elements. Thus, the lateral analysis used to assess stability must include consideration of the combined effect of gravity and lateral loads, as well as member inelasticity, out-of-plumbness, out-of-straightness, and the resulting second-order effects, P-Δ and P-δ. The effects of "leaning columns" must also be considered, as illustrated in the examples in this chapter and in the four-story building design example in Part III of these *Design Examples*.

P-Δ and P-δ effects are illustrated in AISC *Specification* Commentary Figure C-C2.1. Methods for addressing stability, including P-Δ and P-δ effects, are provided in AISC *Specification* Section C2 and Appendix 7.

C2. CALCULATION OF REQUIRED STRENGTHS

The calculation of required strengths is illustrated in the examples in this chapter and in the four-story building design example in Part III of these *Design Examples*.

C3. CALCULATION OF AVAILABLE STRENGTHS

The calculation of available strengths is illustrated in the four-story building design example in Part III of these *Design Examples*.

EXAMPLE C.1A DESIGN OF A MOMENT FRAME BY THE DIRECT ANALYSIS METHOD

Given:

Determine the required strengths and effective length factors for the columns in the moment frame shown in Figure C.1A-1 for the maximum gravity load combination, using LRFD and ASD. The uniform load, w_D, includes beam self-weight and an allowance for column self-weight. Use the direct analysis method. All members are ASTM A992 material.

Columns are unbraced between the footings and roof in the *x*- and *y*-axes and have pinned bases.

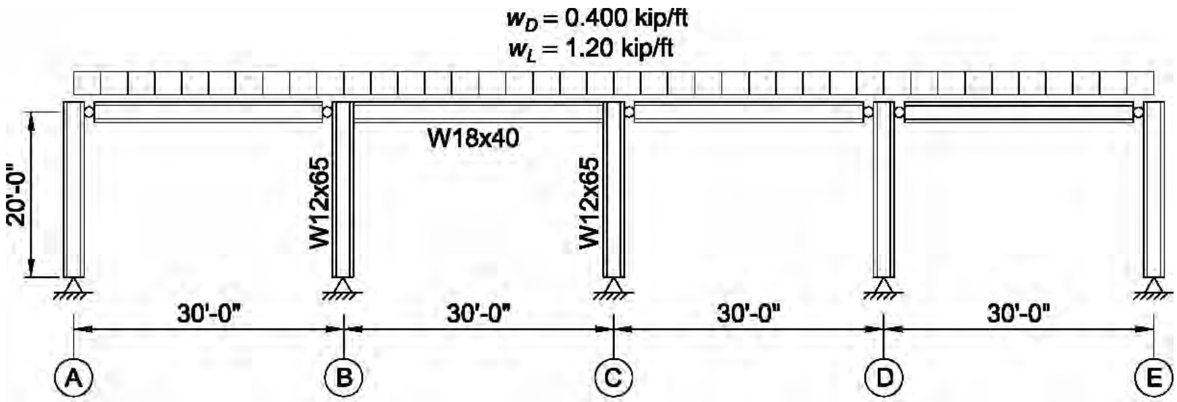


Fig. C.1A-1. Example C.1A moment frame elevation.

Solution:

From AISC *Manual* Table 1-1, the W12×65 has A = 19.1 in.^{2}

The beams from grid lines A to B and C to E and the columns at A, D and E are pinned at both ends and do not contribute to the lateral stability of the frame. There are no P-Δ effects to consider in these members and they may be designed using $L_c = L$.

The moment frame between grid lines B and C is the source of lateral stability and therefore will be evaluated using the provisions of Chapter C of the AISC *Specification*. Although the columns at grid lines A, D and E do not contribute to lateral stability, the forces required to stabilize them must be considered in the moment-frame analysis. The entire frame from grid line A to E could be modeled, but in this case the model is simplified as shown in Figure C.1A-2, in which the stability loads from the three “leaning” columns are combined into a single representative column.

From Chapter 2 of ASCE/SEI 7, the maximum gravity load combinations are:

LRFD	ASD
$w_u = 1.2D + 1.6L$ $= 1.2(0.400 \text{ kip/ft}) + 1.6(1.20 \text{ kip/ft})$ $= 2.40 \text{ kip/ft}$	$w_u = D + L$ $= 0.400 \text{ kip/ft} + 1.20 \text{ kip/ft}$ $= 1.60 \text{ kip/ft}$

Per AISC *Specification* Section C2.1(d), for LRFD, perform a second-order analysis and member strength checks using the LRFD load combinations. For ASD, perform a second-order analysis using 1.6 times the ASD load combinations and divide the analysis results by 1.6 for the ASD member strength checks.

Frame analysis gravity loads

The uniform gravity loads to be considered in a second-order analysis on the beam from B to C are:

LRFD	ASD
$w'_u = 2.40 \text{ kip/ft}$	$w'_a = 1.6(1.60 \text{ kip/ft})$ $= 2.56 \text{ kip/ft}$

Concentrated gravity loads to be considered in a second-order analysis on the columns at B and C contributed by adjacent beams are:

LRFD	ASD
$P'_u = \dfrac{w'_u l}{2}$ $= \dfrac{(2.40 \text{ kip/ft})(30.0 \text{ ft})}{2}$ $= 36.0 \text{ kips}$	$P'_a = \dfrac{w'_a l}{2}$ $= \dfrac{(2.56 \text{ kip/ft})(30.0 \text{ ft})}{2}$ $= 38.4 \text{ kips}$

Concentrated gravity loads on the representative "leaning" column

The load in this column accounts for all gravity loading that is stabilized by the moment frame, but is not directly applied to it.

LRFD	ASD
$P'_{uL} = (60.0 \text{ ft})(2.40 \text{ kip/ft})$ $= 144 \text{ kips}$	$P'_{aL} = (60.0 \text{ ft})(2.56 \text{ kip/ft})$ $= 154 \text{ kips}$

Frame analysis notional loads

Per AISC *Specification* Section C2.2, frame out-of-plumbness must be accounted for either by explicit modeling of the assumed out-of-plumbness or by the application of notional loads. Use notional loads.

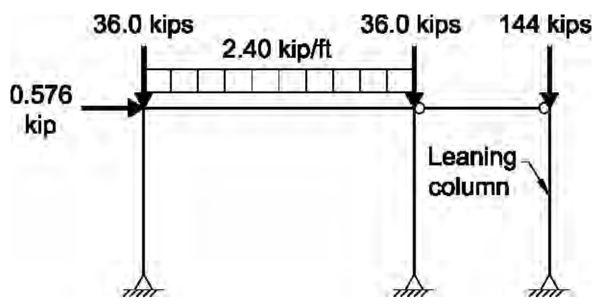
From AISC *Specification* Equation C2-1, the notional loads are:

LRFD	ASD
$\alpha = 1.0$ $Y_i = (120 \text{ ft})(2.40 \text{ kip/ft})$ $= 288 \text{ kips}$ $N_i = 0.002\alpha Y_i$ (*Spec.* Eq. C2-1) $= 0.002(1.0)(288 \text{ kips})$ $= 0.576 \text{ kip}$	$\alpha = 1.6$ $Y_i = (120 \text{ ft})(1.60 \text{ kip/ft})$ $= 192 \text{ kips}$ $N_i = 0.002\alpha Y_i$ (*Spec.* Eq. C2-1) $= 0.002(1.6)(192 \text{ kips})$ $= 0.614 \text{ kip}$

Summary of applied frame loads

The applied loads are shown in Figure C.1A-2.

LRFD



ASD

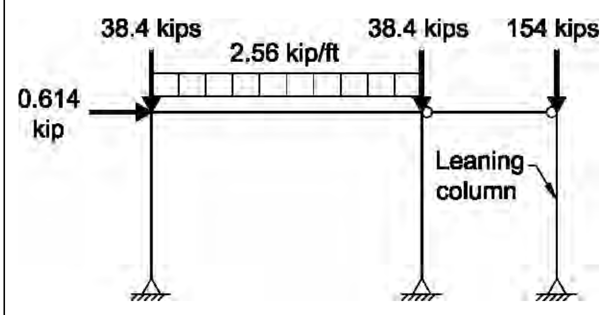


Fig. C.1A-2. Applied loads on the analysis model.

Per AISC *Specification* Section C2.3, conduct the analysis using 80% of the nominal stiffnesses to account for the effects of inelasticity. Assume, subject to verification, that $\alpha P_r / P_{ns}$ is not greater than 0.5; therefore, no additional stiffness reduction is required (τ_b = 1.0).

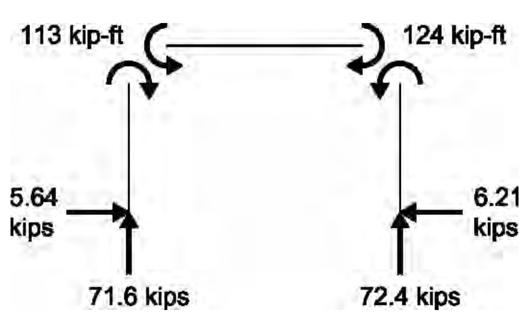
Half of the gravity load is carried by the columns of the moment-resisting frame. Because the gravity load supported by the moment-resisting frame columns exceeds one-third of the total gravity load tributary to the frame, per AISC *Specification* Section C2.1, the effects of P-δ and P-Δ must be considered in the frame analysis. This example uses analysis software that accounts for both P-Δ and P-δ effects. (If the software used does not account for P-δ effects this may be accomplished by subdividing the columns between the footing and beam.)

Figures C.1A-3 and C.1A-4 show results from a first-order and a second-order analysis. (The first-order analysis is shown for reference only.) In each case, the drift is the average of drifts at grid lines B and C.

First-order results

LRFD

Δ_{1st} = 0.181 in.



ASD
(Reactions and moments divided by 1.6)

Δ_{1st} = 0.193 in. (prior to dividing by 1.6)

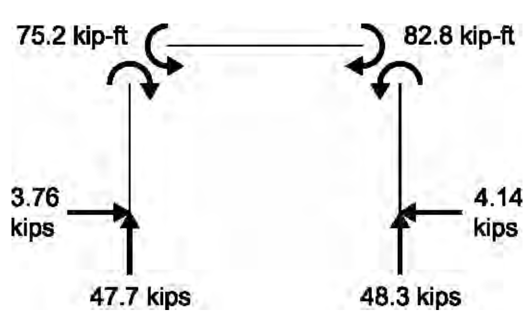


Fig. C.1A-3. Results of first-order analysis.

Second-order results

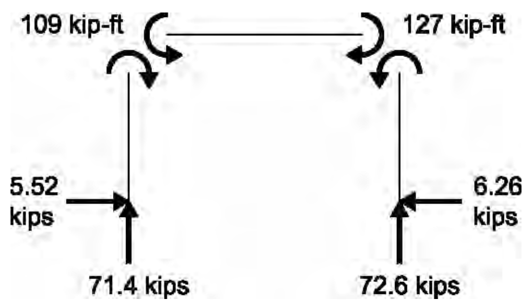
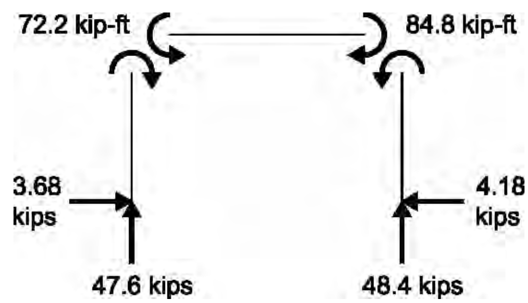
LRFD	ASD (Reactions and moments divided by 1.6)
$\Delta_{2nd} = 0.290$ in. Drift ratio: $\dfrac{\Delta_{2nd}}{\Delta_{1st}} = \dfrac{0.290 \text{ in.}}{0.181 \text{ in.}}$ $= 1.60$  	$\Delta_{2nd} = 0.321$ in. (prior to dividing by 1.6) Drift ratio: $\dfrac{\Delta_{2nd}}{\Delta_{1st}} = \dfrac{0.321 \text{ in.}}{0.193 \text{ in.}}$ $= 1.66$ 

Fig. C.1A-4. Results of second-order analysis.

Check the assumption that $\alpha P_r / P_{ns} \leq 0.5$ on the column on grid line C.

Because a W12×65 column contains no elements that are slender for uniform compression,

$$P_{ns} = F_y A_g$$
$$= (50 \text{ ksi})(19.1 \text{ in.}^2)$$
$$= 955 \text{ kips}$$

LRFD	ASD
$\dfrac{\alpha P_r}{P_{ns}} = \dfrac{1.0(72.6 \text{ kips})}{955 \text{ kips}}$ $= 0.0760 \leq 0.5$ **o.k.**	$\dfrac{\alpha P_r}{P_{ns}} = \dfrac{1.6(48.4 \text{ kips})}{955 \text{ kips}}$ $= 0.0811 \leq 0.5$ **o.k.**

The stiffness assumption used in the analysis, $\tau_b = 1.0$, is verified.

Note that the drift ratio, 1.60 (LRFD) or 1.66 (ASD), does not exceed the recommended limit of 2.5 from AISC *Specification* Commentary Section C1.

The required axial compressive strength in the columns is 72.6 kips (LRFD) or 48.4 kips (ASD). The required bending moment diagram is linear, varying from zero at the bottom to 127 kip-ft (LRFD) or 84.8 kip-ft (ASD) at the top. These required strengths apply to both columns because the notional load must be applied in each direction.

Although the second-order sway multiplier (drift ratio) is fairly large at 1.60 (LRFD) or 1.66 (ASD), the change in bending moment is small because the only sway moments are those produced by the small notional loads. For load combinations with significant gravity and lateral loadings, the increase in bending moments is larger.

Per AISC *Specification* Section C3, the effective length for flexural buckling of all members is taken as the unbraced length (K = 1.0):

L_{cx} = 20.0 ft
L_{cy} = 20.0 ft

EXAMPLE C.1B DESIGN OF A MOMENT FRAME BY THE EFFECTIVE LENGTH METHOD

Given:

Repeat Example C.1A using the effective length method.

Determine the required strengths and effective length factors for the columns in the moment frame shown in Figure C.1B-1 for the maximum gravity load combination, using LRFD and ASD. Use the effective length method.

Columns are unbraced between the footings and roof in the *x*- and *y*-axes and have pinned bases.

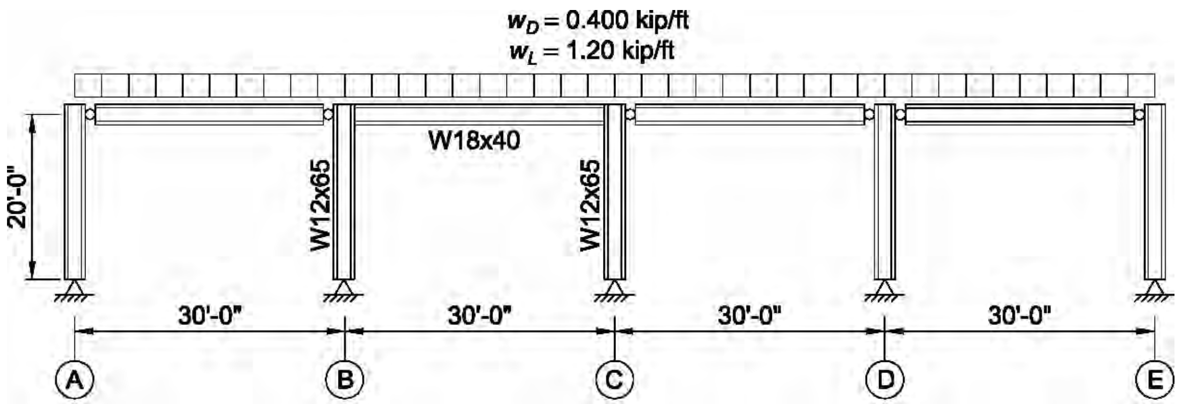


Fig. C.1B-1. Example C.1B moment frame elevation.

Solution:

From AISC *Manual* Table 1-1, the W12×65 has $I_x = 533$ in.^{4}

The beams from grid lines A to B and C to E and the columns at A, D and E are pinned at both ends and do not contribute to the lateral stability of the frame. There are no *P*-Δ effects to consider in these members and they may be designed using $L_c = L$.

The moment frame between grid lines B and C is the source of lateral stability and therefore will be evaluated using the provisions of Chapter C of the AISC *Specification*. Although the columns at grid lines A, D and E do not contribute to lateral stability, the forces required to stabilize them must be considered in the moment-frame analysis. The entire frame from grid line A to E could be modeled, but in this case the model is simplified as shown in Figure C.1B-2, in which the stability loads from the three "leaning" columns are combined into a single representative column.

Check the limitations for the use of the effective length method given in AISC *Specification* Appendix 7, Section 7.2.1:

(a) The structure supports gravity loads primarily through nominally vertical columns, walls or frames.
(b) The ratio of maximum second-order drift to the maximum first-order drift (both determined for LRFD load combinations or 1.6 times ASD load combinations, with stiffness not adjusted as specified in AISC *Specification* Section C2.3) in all stories will be assumed to be no greater than 1.5, subject to verification in the following.

From Chapter 2 of ASCE/SEI 7, the maximum gravity load combinations are:

LRFD	ASD
$w_u = 1.2D + 1.6L$ $= 1.2(0.400 \text{ kip/ft}) + 1.6(1.20 \text{ kip/ft})$ $= 2.40 \text{ kip/ft}$	$w_u = D + L$ $= 0.400 \text{ kip/ft} + 1.20 \text{ kip/ft}$ $= 1.60 \text{ kip/ft}$

Per AISC *Specification* Appendix 7, Section 7.2.2, the analysis must conform to the requirements of AISC *Specification* Section C2.1, with the exception of the stiffness reduction required by the provisions of Section C2.1(a).

Per AISC *Specification* Section C2.1(d), for LRFD perform a second-order analysis and member strength checks using the LRFD load combinations. For ASD, perform a second-order analysis at 1.6 times the ASD load combinations and divide the analysis results by 1.6 for the ASD member strength checks.

Frame analysis gravity loads

The uniform gravity loads to be considered in a second-order analysis on the beam from B to C are:

LRFD	ASD
$w'_u = 2.40 \text{ kip/ft}$	$w'_a = 1.6(1.60 \text{ kip/ft})$ $= 2.56 \text{ kip/ft}$

Concentrated gravity loads to be considered in a second-order analysis on the columns at B and C contributed by adjacent beams are:

LRFD	ASD
$P'_u = \frac{w'_u l}{2}$ $= \frac{(2.40 \text{ kip/ft})(30.0 \text{ ft})}{2}$ $= 36.0 \text{ kips}$	$P'_a = \frac{w'_a l}{2}$ $= \frac{(2.56 \text{ kip/ft})(30.0 \text{ ft})}{2}$ $= 38.4 \text{ kips}$

Concentrated gravity loads on the representative "leaning" column

The load in this column accounts for all gravity loads that is stabilized by the moment frame, but not directly applied to it.

LRFD	ASD
$P'_{uL} = (60.0 \text{ ft})(2.40 \text{ kip/ft})$ $= 144 \text{ kips}$	$P'_{aL} = (60.0 \text{ ft})(2.56 \text{ kip/ft})$ $= 154 \text{ kips}$

Frame analysis notional loads

Per AISC *Specification* Appendix 7, Section 7.2.2, frame out-of-plumbness must be accounted for by the application of notional loads in accordance with AISC *Specification* Section C2.2b. Note that notional loads need to only be applied to the gravity load combinations per AISC *Specification* Section C2.2b(d) when the requirement that $\Delta_{2nd} / \Delta_{1st} \le 1.7$ (using stiffness adjusted as specified in Section C2.3) is satisfied. Per the User Note in AISC *Specification* Appendix 7, Section 7.2.2, Section C2.2b(d) will be satisfied in all cases where the effective length method is applicable, and therefore the notional load need only be applied in gravity-only load cases.
From AISC *Specification* Equation C2-1, the notional loads are:

LRFD	ASD
$\alpha = 1.0$ $Y_i = (120 \text{ ft})(2.40 \text{ kip/ft})$ $= 288 \text{ kips}$ $N_i = 0.002\alpha Y_i$ (*Spec*. Eq. C2-1) $= 0.002(1.0)(288 \text{ kips})$ $= 0.576 \text{ kip}$	$\alpha = 1.6$ $Y_i = (120 \text{ ft})(1.60 \text{ kip/ft})$ $= 192 \text{ kips}$ $N_i = 0.002\alpha Y_i$ (*Spec*. Eq. C2-1) $= 0.002(1.6)(192 \text{ kips})$ $= 0.614 \text{ kip}$

Summary of applied frame loads

The applied loads are shown in Figure C.1B-2.

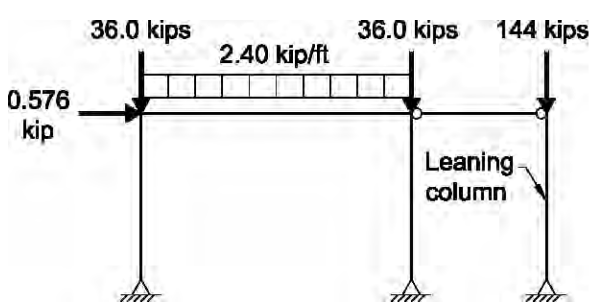
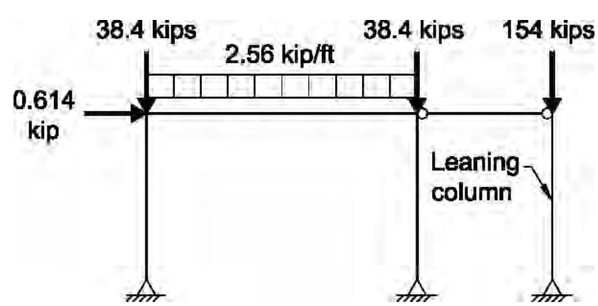
LRFD	ASD
	

Fig. C.1B-2. Applied loads on the analysis model.

Per AISC *Specification* Appendix 7, Section 7.2.2, conduct the analysis using the full nominal stiffnesses.

Half of the gravity load is carried by the columns of the moment-resisting frame. Because the gravity load supported by the moment-resisting frame columns exceeds one-third of the total gravity load tributary to the frame, per AISC *Specification* Section C2.1(b), the effects of P-δ on the response of the structure must be considered in the frame analysis. This example uses analysis software that accounts for both P-Δ and P-δ effects. When using software that does not account for P-δ effects, this could be accomplished by subdividing columns between the footing and beam.

Figures C.1B-3 and C.1B-4 show results from a first-order and second-order analysis. In each case, the drift is the average of drifts at grid lines B and C.

First-order results

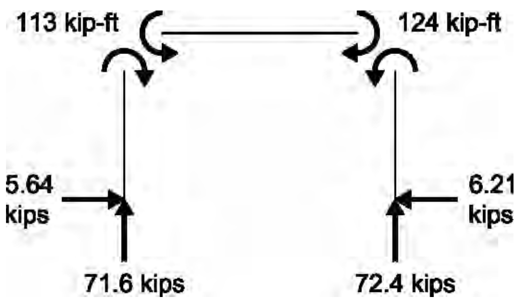
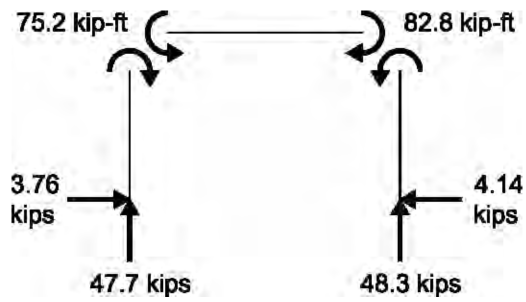
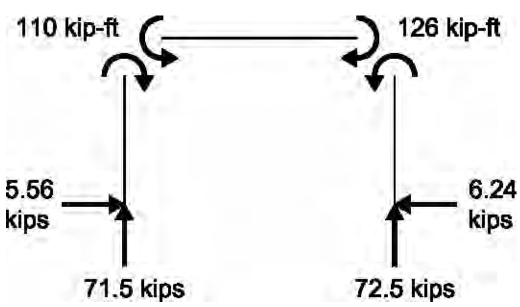
<table>
<tr><th>LRFD</th><th>ASD
(Reactions and moments divided by 1.6)</th></tr>
<tr><td>$\Delta_{1st} = 0.145$ in.

</td><td>$\Delta_{1st} = 0.155$ in. (prior to dividing by 1.6)

</td></tr>
</table>

Fig. C.1B-3. Results of first-order analysis.

Second-order results

<table>
<tr><th>LRFD</th><th>ASD</th></tr>
<tr><td>$\Delta_{2nd} = 0.204$ in.

Drift ratio:

$\frac{\Delta_{2nd}}{\Delta_{1st}} = \frac{0.204 \text{ in.}}{0.145 \text{ in.}}$
$= 1.41$

</td><td>$\Delta_{2nd} = 0.223$ in. (prior to dividing by 1.6)

Drift ratio:

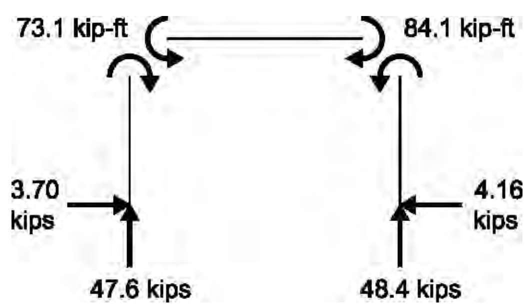
$\frac{\Delta_{2nd}}{\Delta_{1st}} = \frac{0.223 \text{ in.}}{0.155 \text{ in.}}$
$= 1.44$

</td></tr>
</table>

Fig. C-1B-4. Results of second-order analysis.

The assumption that the ratio of the maximum second-order drift to the maximum first-order drift is no greater than 1.5 is verified; therefore, the effective length method is permitted.

Although the second-order sway multiplier is fairly large at approximately 1.41 (LRFD) or 1.44 (ASD), the change in bending moment is small because the only sway moments for this load combination are those produced by the small notional loads. For load combinations with significant gravity and lateral loadings, the increase in bending moments is larger.

Calculate the in-plane effective length factor, K_x, using the "story stiffness approach" and Equation C-A-7-5 presented in AISC *Specification* Commentary Appendix 7, Section 7.2. With $K_x = K_2$:

$$K_x = \sqrt{\frac{P_{story}}{R_M P_r}\left(\frac{\pi^2 EI}{L^2}\right)\left(\frac{\Delta_H}{HL}\right)} \geq \sqrt{\frac{\pi^2 EI}{L^2}\left(\frac{\Delta_H}{1.7H_{col}L}\right)} \qquad \text{(\textit{Spec.} Eq. C-A-7-5)}$$

Calculate the total load in all columns, P_{story}, as follows:

LRFD	ASD
$P_{story} = (2.40 \text{ kip/ft})(120 \text{ ft})$ $= 288 \text{ kips}$	$P_{story} = (1.60 \text{ kip/ft})(120 \text{ ft})$ $= 192 \text{ kips}$

Calculate the coefficient to account for the influence of P-δ on P-Δ, R_M, as follows, using AISC *Specification* Commentary Appendix 7, Equation C-A-7-6:

LRFD	ASD
$P_{mf} = 71.5 \text{ kips} + 72.5 \text{ kips}$ $= 144 \text{ kips}$ $R_M = 1 - 0.15(P_{mf}/P_{story})$ (*Spec*. Eq. C-A-7-6) $= 1 - 0.15\left(\frac{144 \text{ kips}}{288 \text{ kips}}\right)$ $= 0.925$	$P_{mf} = 47.6 \text{ kips} + 48.4 \text{ kips}$ $= 96.0 \text{ kips}$ $R_M = 1 - 0.15(P_{mf}/P_{story})$ (*Spec*. Eq. C-A-7-6) $= 1 - 0.15\left(\frac{96.0 \text{ kips}}{192 \text{ kips}}\right)$ $= 0.925$

Calculate the Euler buckling strength of one moment frame.

$$\frac{\pi^2 EI}{L^2} = \frac{\pi^2 (29{,}000 \text{ ksi})(533 \text{ in.}^4)}{\left[(20.0 \text{ ft})(12 \text{ in./ft})\right]^2}$$
$$= 2{,}650 \text{ kips}$$

From AISC *Specification* Commentary Equation C-A-7-5, for the column at line C:

LRFD	ASD
$K_x = \sqrt{\dfrac{P_{story}}{R_M P_r}\left(\dfrac{\pi^2 EI}{L^2}\right)\left(\dfrac{\Delta_H}{HL}\right)}$ $\geq \sqrt{\left(\dfrac{\pi^2 EI}{L^2}\right)\left(\dfrac{\Delta_H}{1.7H_{col}L}\right)}$ $= \sqrt{\left[\dfrac{288\text{ kips}}{(0.925)(72.5\text{ kips})}\right](2{,}650\text{ kips}) \times \left[\dfrac{0.145\text{ in.}}{(0.576\text{ kip})(20.0\text{ ft})(12\text{ in./ft})}\right]}$ $\geq \sqrt{(2{,}650\text{ kips}) \times \left[\dfrac{0.145\text{ in.}}{1.7(6.21\text{ kips})(20.0\text{ ft})(12\text{ in./ft})}\right]}$ $= 3.45 \geq 0.389$ Use K_x = 3.45	$K_x = \sqrt{\dfrac{1.6P_{story}}{R_M (1.6)P_r}\left(\dfrac{\pi^2 EI}{L^2}\right)\left(\dfrac{\Delta_H}{HL}\right)}$ $\geq \sqrt{\left(\dfrac{\pi^2 EI}{L^2}\right)\left(\dfrac{\Delta_H}{1.7(1.6)H_{col}L}\right)}$ $= \sqrt{\left[\dfrac{1.6(192\text{ kips})}{0.925(1.6)(48.4\text{ kips})}\right](2{,}650\text{ kips}) \times \left[\dfrac{0.155\text{ in.}}{(0.614\text{ kip})(20.0\text{ ft})(12\text{ in./ft})}\right]}$ $\geq \sqrt{(2{,}650\text{ kips}) \times \left[\dfrac{0.155\text{ in.}}{1.7(1.6)(4.14\text{ kips})(20.0\text{ ft})(12\text{ in./ft})}\right]}$ $= 3.46 \geq 0.390$ Use K_x = 3.46

Note that the column loads are multiplied by 1.6 for ASD in Equation C-A-7-5.

With K_x = 3.45 and K_y = 1.00, the column available strengths can be verified for the given member sizes for the second-order forces (calculations not shown), using the following effective lengths:

$$
\begin{aligned}
L_{cx} &= K_x L_x \\
&= 3.45(20.0\text{ ft}) \\
&= 69.0\text{ ft}
\end{aligned}
$$

$$
\begin{aligned}
L_{cy} &= K_y L_y \\
&= 1.00(20.0\text{ ft}) \\
&= 20.0\text{ ft}
\end{aligned}
$$

EXAMPLE C.1C DESIGN OF A MOMENT FRAME BY THE FIRST-ORDER METHOD

Given:

Repeat Example C.1A using the first-order analysis method.

Determine the required strengths and effective length factors for the columns in the moment frame shown in Figure C.1C-1 for the maximum gravity load combination, using LRFD and ASD. Use the first-order analysis method as given in AISC *Specification* Appendix 7, Section 7.3.

Columns are unbraced between the footings and roof in the *x*- and *y*-axes and have pinned bases.

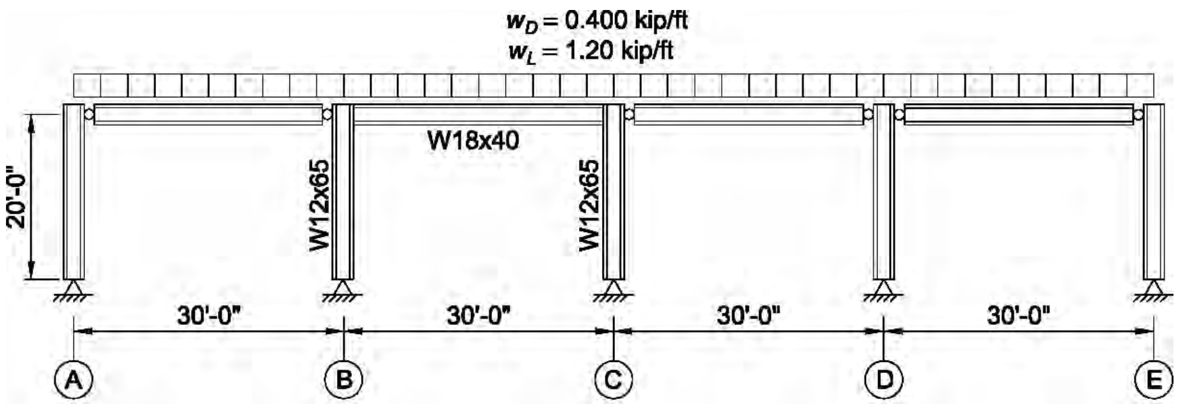


Fig. C.1C-1. Example C.1C moment frame elevation.

Solution:

From AISC *Manual* Table 1-1, the W12×65 has A = 19.1 in.^{2}

The beams from grid lines A to B and C to E and the columns at A, D and E are pinned at both ends and do not contribute to the lateral stability of the frame. There are no P-Δ effects to consider in these members and they may be designed using L_c=L.

The moment frame between grid lines B and C is the source of lateral stability and will be designed using the provisions of AISC *Specification* Appendix 7, Section 7.3. Although the columns at grid lines A, D and E do not contribute to lateral stability, the forces required to stabilize them must be considered in the moment-frame analysis. These members need not be included in the analysis model, except that the forces in the “leaning” columns must be included in the calculation of notional loads.

Check the limitations for the use of the first-order analysis method given in AISC *Specification* Appendix 7, Section 7.3.1:

(a) The structure supports gravity loads primarily through nominally vertical columns, walls or frames.
(b) The ratio of maximum second-order drift to the maximum first-order drift (both determined for LRFD load combinations or 1.6 times ASD load combinations, with stiffnesses not adjusted as specified in AISC *Specification* Section C2.3) in all stories will be assumed to be equal to or less than 1.5, subject to verification.
(c) The required axial compressive strength of all members whose flexural stiffnesses are considered to contribute to the lateral stability of the structure will be assumed to be no more than 50% of the cross-section strength, subject to verification.

Per AISC *Specification* Appendix 7, Section 7.3.2, the required strengths are determined from a first-order analysis using notional loads determined in the following, along with a B_1 multiplier to account for second-order effects, as determined from Appendix 8.

Loads

From Chapter 2 of ASCE/SEI 7, the maximum gravity load combinations are:

LRFD	ASD
$w_u = 1.2D + 1.6L$ $= 1.2(0.400 \text{ kip/ft}) + 1.6(1.20 \text{ kip/ft})$ $= 2.40 \text{ kip/ft}$	$w_u = D + L$ $= 0.400 \text{ kip/ft} + 1.20 \text{ kip/ft}$ $= 1.60 \text{ kip/ft}$

Concentrated gravity loads to be considered on the columns at B and C contributed by adjacent beams are:

LRFD	ASD
$P_u = \dfrac{w_u l}{2}$ $= \dfrac{(2.40 \text{ kip/ft})(30.0 \text{ ft})}{2}$ $= 36.0 \text{ kips}$	$P_a = \dfrac{w_a l}{2}$ $= \dfrac{(1.60 \text{ kip/ft})(30.0 \text{ ft})}{2}$ $= 24.0 \text{ kips}$

Using AISC *Specification* Appendix 7, Section 7.3.2, frame out-of-plumbness is accounted for by the application of an additional lateral load.

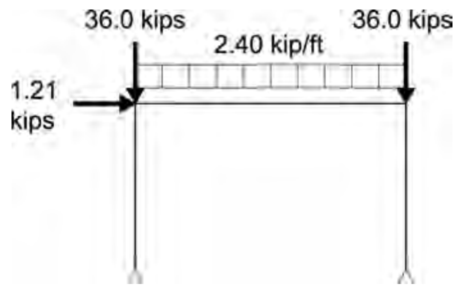
From AISC *Specification* Appendix Equation A-7-2, the additional lateral load is determined as follows:

LRFD	ASD
$\alpha = 1.0$ $Y_i = (120 \text{ ft})(2.40 \text{ kip/ft})$ $= 288 \text{ kips}$ $\Delta = 0$ in. (no drift for this load combination) $L = (20.0 \text{ ft})(12 \text{ in./ft})$ $= 240 \text{ in.}$ $N_i = 2.1\alpha(\Delta/L)Y_i \geq 0.0042Y_i$ (*Spec*. Eq. A-7-2) $= 2.1(1.0)\left(\dfrac{0 \text{ in.}}{240 \text{ in.}}\right)(288 \text{ kips})$ $\geq 0.0042(288 \text{ kips})$ $= 0 \text{ kip} < 1.21 \text{ kips}$ Use $N_i = 1.21$ kips	$\alpha = 1.6$ $Y_i = (120 \text{ ft})(1.60 \text{ kip/ft})$ $= 192 \text{ kips}$ $\Delta = 0$ in. (no drift for this load combination) $L = (20.0 \text{ ft})(12 \text{ in./ft})$ $= 240 \text{ in.}$ $N_i = 2.1\alpha(\Delta/L)Y_i \geq 0.0042Y_i$ (*Spec*. Eq. A-7-2) $= 2.1(1.6)\left(\dfrac{0 \text{ in.}}{240 \text{ in.}}\right)(192 \text{ kips})$ $\geq 0.0042(192 \text{ kips})$ $= 0 \text{ kip} < 0.806 \text{ kip}$ Use $N_i = 0.806$ kip

Summary of applied frame loads

The applied loads are shown in Figure C.1C-2.

LRFD



ASD

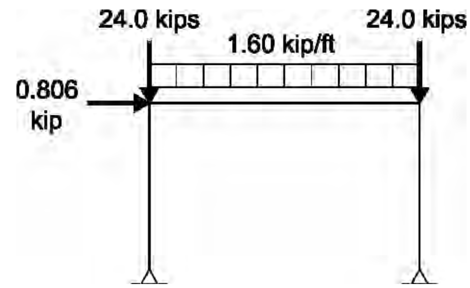


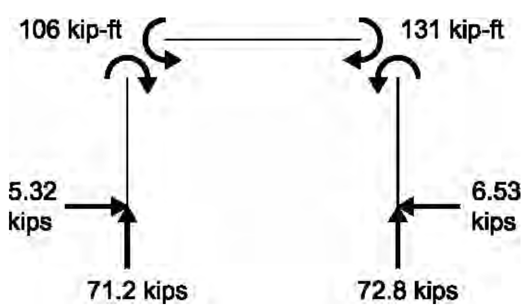
Fig. C.1C-2. Applied loads on the analysis model.

Conduct the analysis using the full nominal stiffnesses, as indicated in AISC *Specification* Commentary Appendix 7, Section 7.3.

Using analysis software, the first-order results shown in Figure C.1C-3 are obtained:

LRFD

$\Delta_{1st} = 0.304 \text{ in.}$



ASD

$\Delta_{1st} = 0.203 \text{ in.}$

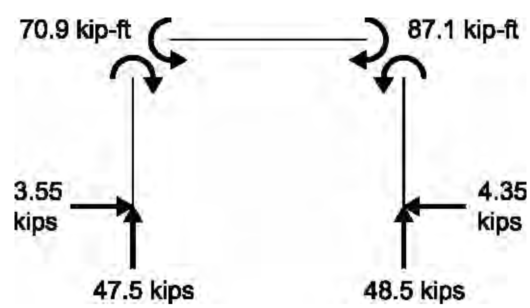


Fig. C.1C-3. Results of first-order analysis.

Check the assumption that the ratio of the second-order drift to the first-order drift does not exceed 1.5. B_2 can be used to check this limit. Calculate B_2 per Appendix 8, Section 8.2.2 using the results of the first-order analysis.

LRFD	ASD
$P_{mf} = 2(36.0 \text{ kips}) + (30.0 \text{ ft})(2.40 \text{ kip/ft})$ $= 144 \text{ kips}$ $P_{story} = 144 \text{ kips} + 4(36.0 \text{ kips})$ $= 288 \text{ kips}$	$P_{mf} = 2(24.0 \text{ kips}) + (30.0 \text{ ft})(1.60 \text{ kip/ft})$ $= 96.0 \text{ kips}$ $P_{story} = 96.0 \text{ kips} + 4(24.0 \text{ kips})$ $= 192 \text{ kips}$

LRFD	ASD
$R_M = 1-0.15\left(P_{mf}/P_{story}\right)$ (*Spec*. Eq. A-8-8) $= 1-0.15\left(144\text{ kips}/288\text{ kips}\right)$ $= 0.925$	$R_M = 1-0.15\left(P_{mf}/P_{story}\right)$ (*Spec*. Eq. A-8-8) $= 1-0.15\left(96.0\text{ kips}/192\text{ kips}\right)$ $= 0.925$
$\Delta_H = 0.304$ in.	$\Delta_H = 0.203$ in.
$H = 6.53\text{ kips} - 5.32\text{ kips}$ $= 1.21\text{ kips}$	$H = 4.35\text{ kips} - 3.55\text{ kips}$ $= 0.800\text{ kip}$
$L = (20\text{ ft})(12\text{ in./ft})$ $= 240\text{ in.}$	$L = (20\text{ ft})(12\text{ in./ft})$ $= 240\text{ in.}$
$P_{e\,story} = R_M \dfrac{HL}{\Delta_H}$ (*Spec*. Eq. A-8-7) $= 0.925\dfrac{(1.21\text{ kips})(240\text{ in.})}{0.304\text{ in.}}$ $= 884\text{ kips}$	$P_{e\,story} = R_M \dfrac{HL}{\Delta_H}$ (*Spec*. Eq. A-8-7) $= 0.925\dfrac{(0.800\text{ kip})(240\text{ in.})}{0.203\text{ in.}}$ $= 875\text{ kips}$
$\alpha = 1.0$	$\alpha = 1.6$
$B_2 = \dfrac{1}{1-\dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1$ (*Spec*. Eq. A-8-6) $= \dfrac{1}{1-\dfrac{1.0(288\text{ kips})}{884\text{ kips}}} \geq 1$ $= 1.48 > 1$	$B_2 = \dfrac{1}{1-\dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1$ (*Spec*. Eq. A-8-6) $= \dfrac{1}{1-\dfrac{1.6(192\text{ kips})}{875\text{ kips}}} \geq 1$ $= 1.54 > 1$

When a structure with a live-to-dead load ratio of 3 is analyzed by a first-order analysis the required strength for LRFD will always be 1.5 times the required strength for ASD. However, when a second-order analysis is used this ratio is not maintained. This is due to the use of the amplification factor, α, which is set equal to 1.6 for ASD, in order to capture the worst case second-order effects for any live-to-dead load ratio. Thus, in this example the limitation for applying the first-order analysis method, that the ratio of the maximum second-order drift to maximum first-order drift is not greater than 1.5, is verified for LRFD but is not verified for ASD. Therefore, for this example the first-order method is invalid for ASD and will proceed with LRFD only.

Check the assumption that $\alpha P_r \leq 0.5P_{ns}$ and, therefore, the first-order analysis method is permitted.

Because the W12×65 column does not contain elements that are slender for compression,

$$P_{ns} = F_y A_g$$

$$\begin{aligned} 0.5P_{ns} &= 0.5F_y A_g \\ &= 0.5(50\text{ ksi})\left(19.1\text{ in.}^2\right) \\ &= 478\text{ kips} \end{aligned}$$

$$\alpha P_r = 1.0(72.8 \text{ kips})$$
$$= 72.8 \text{ kips} < 478 \text{ kips} \quad \textbf{o.k.} \text{ (LRFD only)}$$

The assumption that the first-order analysis method can be used is verified for LRFD.

Although the second-order sway multiplier is 1.48, the change in bending moment is small because the only sway moments are those produced by the small notional loads. For load combinations with significant gravity and lateral loadings, the increase in bending moments is larger.

The column strengths can be verified after using the B_1 amplification given in Appendix 8, Section 8.2.1 to account for second-order effects (calculations not shown here). In the direction of sway, the effective length factor is taken equal to 1.00, and the column effective lengths are as follows:

$$L_{cx} = 20.0 \text{ ft}$$
$$L_{cy} = 20.0 \text{ ft}$$

Chapter D
Design of Members for Tension

D1. SLENDERNESS LIMITATIONS

AISC *Specification* Section D1 does not establish a slenderness limit for tension members, but recommends limiting L/r to a maximum of 300. This is not an absolute requirement. Rods and hangers are specifically excluded from this recommendation.

D2. TENSILE STRENGTH

Both tensile yielding strength and tensile rupture strength must be considered for the design of tension members. It is not unusual for tensile rupture strength to govern the design of a tension member, particularly for small members with holes or heavier sections with multiple rows of holes.

For preliminary design, tables are provided in Part 5 of the AISC *Manual* for W-shapes, L-shapes, WT-shapes, rectangular HSS, square HSS, round HSS, Pipe, and 2L-shapes. The calculations in these tables for available tensile rupture strength assume an effective area, A_e, of $0.75A_g$. The gross area, A_g, is the total cross-sectional area of the member. If the actual effective area is greater than $0.75A_g$, the tabulated values will be conservative and calculations can be performed to obtain higher available strengths. If the actual effective area is less than $0.75A_g$, the tabulated values will be unconservative and calculations are necessary to determine the available strength.

D3. EFFECTIVE NET AREA

In computing net area, A_n, AISC *Specification* Section B4.3b requires that an extra 1⁄16 in. be added to the bolt hole diameter. A computation of the effective area for a chain of holes is presented in Example D.9.

Unless all elements of the cross section are connected, $A_e = A_n U$, where U is a reduction factor to account for shear lag. The appropriate values of U can be obtained from AISC *Specification* Table D3.1.

D4. BUILT-UP MEMBERS

The limitations for connections of built-up members are discussed in Section D4 of the AISC *Specification*.

D5. PIN-CONNECTED MEMBERS

An example of a pin-connected member is given in Example D.7.

D6. EYEBARS

An example of an eyebar is given in Example D.8. The strength of an eyebar meeting the dimensional requirements of AISC *Specification* Section D6 is governed by tensile yielding of the body.

EXAMPLE D.1 W-SHAPE TENSION MEMBER

Given:

Select an ASTM A992 W-shape with 8 in. nominal depth to carry a dead load of 30 kips and a live load of 90 kips in tension. The member is 25.0 ft long. Verify the member strength by both LRFD and ASD with the bolted end connection as shown in Figure D.1-1. Verify that the member satisfies the recommended slenderness limit. Assume that connection limit states do not govern.

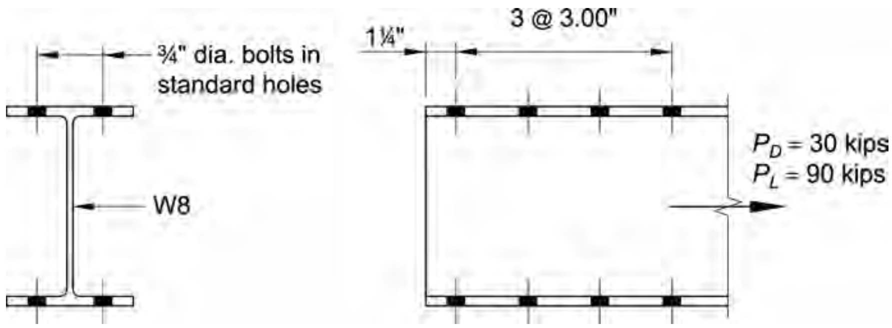


Fig D.1-1. Connection geometry for Example D.1.

Solution:

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(30 \text{ kips}) + 1.6(90 \text{ kips})$ $= 180 \text{ kips}$	$P_a = 30 \text{ kips} + 90 \text{ kips}$ $= 120 \text{ kips}$

From AISC *Manual* Table 5-1, try a W8×21.

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W8×21
A_g = 6.16 in.^{2}
b_f = 5.27 in.
t_f = 0.400 in.
d = 8.28 in.
r_y = 1.26 in.

The WT-shape corresponding to a W8×21 is a WT4×10.5. From AISC *Manual* Table 1-8, the geometric properties are as follows:

WT4×10.5
$\bar{y}$ = 0.831 in.

Tensile Yielding

From AISC *Manual* Table 5-1, the available tensile yielding strength of a W8×21 is:

LRFD	ASD
$\phi_t P_n = 277 \text{ kips} > 180 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = 184 \text{ kips} > 120 \text{ kips}$ **o.k.**

Tensile Rupture

Verify the table assumption that $A_e/A_g \geq 0.75$ for this connection.

From the description of the element in AISC *Specification* Table D3.1, Case 7, calculate the shear lag factor, *U*, as the larger of the values from AISC *Specification* Section D3, Table D3.1 Case 2 and Case 7.

From AISC *Specification* Section D3, for open cross sections, *U* need not be less than the ratio of the gross area of the connected element(s) to the member gross area.

$$U = \frac{2b_f t_f}{A_g}$$

$$= \frac{2(5.27 \text{ in.})(0.400 \text{ in.})}{6.16 \text{ in.}^2}$$

$$= 0.684$$

Case 2: Determine *U* based on two WT-shapes per AISC *Specification* Commentary Figure C-D3.1, with $\bar{x} = \bar{y} = 0.831$ in. and where *l* is the length of connection.

$$U = 1 - \frac{\bar{x}}{l}$$

$$= 1 - \frac{0.831 \text{ in.}}{9.00 \text{ in.}}$$

$$= 0.908$$

Case 7:

$$b_f = 5.27 \text{ in.}$$

$$\frac{2}{3}d = \frac{2}{3}(8.28 \text{ in.})$$

$$= 5.52 \text{ in.}$$

Because the flange is connected with three or more fasteners per line in the direction of loading and $b_f < \frac{2}{3}d$:

$$U = 0.85$$

Therefore, use the larger $U = 0.908$.

Calculate A_n using AISC *Specification* Section B4.3b.

$$A_n = A_g - 4(d_h + 1/16 \text{ in.})t_f$$
$$= 6.16 \text{ in.}^2 - 4(13/16 \text{ in.} + 1/16 \text{ in.})(0.400 \text{ in.})$$
$$= 4.76 \text{ in.}^2$$

Calculate A_e using AISC *Specification* Section D3.

$$A_e = A_n U \qquad \text{(Spec. Eq. D3-1)}$$
$$= (4.76 \text{ in.}^2)(0.908)$$
$$= 4.32 \text{ in.}^2$$

$$\frac{A_e}{A_g} = \frac{4.32 \text{ in.}^2}{6.16 \text{ in.}^2}$$
$$= 0.701 < 0.75$$

Because A_e/A_g < 0.75, the tensile rupture strength from AISC *Manual* Table 5-1 is not valid. The available tensile rupture strength is determined using AISC *Specification* Section D2 as follows:

$$P_n = F_u A_e \qquad \text{(Spec. Eq. D2-2)}$$
$$= (65 \text{ ksi})(4.32 \text{ in.}^2)$$
$$= 281 \text{ kips}$$

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(281 \text{ kips})$ $= 211 \text{ kips} > 180 \text{ kips}$ **o.k.**	$\Omega_t = 2.00$ $\frac{P_n}{\Omega_t} = \frac{281 \text{ kips}}{2.00}$ $= 141 \text{ kips} > 120 \text{ kips}$ **o.k.**

Note that the W8×21 available tensile strength is governed by the tensile rupture limit state at the end connection versus the tensile yielding limit state.

See Chapter J for illustrations of connection limit state checks.

Check Recommended Slenderness Limit

$$\frac{L}{r} = \frac{(25.0 \text{ ft})(12 \text{ in./ft})}{1.26 \text{ in.}}$$
$$= 238 < 300 \text{ from AISC } \textit{Specification} \text{ Section D1} \quad \textbf{o.k.}$$

EXAMPLE D.2 SINGLE-ANGLE TENSION MEMBER

Given:

Verify the tensile strength of an ASTM A36 L4×4×½ with one line of four ¾-in.-diameter bolts in standard holes, as shown in Figure D.2-1. The member carries a dead load of 20 kips and a live load of 60 kips in tension. Additionally, calculate at what length this tension member would cease to satisfy the recommended slenderness limit. Assume that connection limit states do not govern.

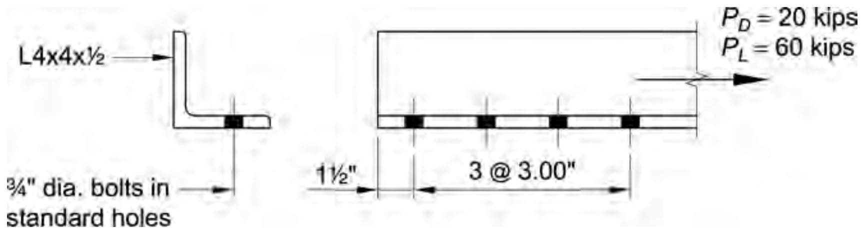


Fig. D.2-1. Connection geometry for Example D.2.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-7, the geometric properties are as follows:

L4×4×½
A_g = 3.75 in.^{2}
r_z = 0.776 in.
$\bar{x}$ = 1.18 in.

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(20 \text{ kips}) + 1.6(60 \text{ kips})$ $= 120 \text{ kips}$	$P_a = 20 \text{ kips} + 60 \text{ kips}$ $= 80.0 \text{ kips}$

Tensile Yielding

$$P_n = F_y A_g \quad \text{(Spec. Eq. D2-1)}$$
$$= (36 \text{ ksi})(3.75 \text{ in.}^2)$$
$$= 135 \text{ kips}$$

From AISC *Specification* Section D2, the available tensile yielding strength is:

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n = 0.90(135 \text{ kips})$ $= 122 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\Omega_t = 1.67$ $\dfrac{P_n}{\Omega_t} = \dfrac{135 \text{ kips}}{1.67}$ $= 80.8 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

Tensile Rupture

From the description of the element in AISC *Specification* Table D3.1 Case 8, calculate the shear lag factor, U, as the larger of the values from AISC *Specification* Section D3, Table D3.1 Case 2 and Case 8.

From AISC *Specification* Section D3, for open cross sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area. Half of the member is connected, therefore, the minimum value of U is:

$$U = 0.500$$

Case 2, where l is the length of connection and $\bar{y} = \bar{x}$:

$$\begin{aligned} U &= 1 - \frac{\bar{x}}{l} \\ &= 1 - \frac{1.18 \text{ in.}}{9.00 \text{ in.}} \\ &= 0.869 \end{aligned}$$

Case 8, with four or more fasteners per line in the direction of loading:

$$U = 0.80$$

Therefore, use the larger $U = 0.869$.

Calculate A_n using AISC *Specification* Section B4.3b.

$$\begin{aligned} A_n &= A_g - (d_h + \tfrac{1}{16} \text{ in.})t \\ &= 3.75 \text{ in.} - (\tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{1}{2} \text{ in.}) \\ &= 3.31 \text{ in.}^2 \end{aligned}$$

Calculate A_e using AISC *Specification* Section D3.

$$\begin{aligned} A_e &= A_n U \\ &= (3.31 \text{ in.}^2)(0.869) \\ &= 2.88 \text{ in.}^2 \end{aligned}$$ (*Spec*. Eq. D3-1)

$$\begin{aligned} P_n &= F_u A_e \\ &= (58 \text{ ksi})(2.88 \text{ in.}^2) \\ &= 167 \text{ kips} \end{aligned}$$ (*Spec*. Eq. D2-2)

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(167 \text{ kips})$ $= 125 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{167 \text{ kips}}{2.00}$ $= 83.5 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

The L4×4×½ available tensile strength is governed by the tensile yielding limit state.

LRFD	ASD
$\phi_t P_n = 122 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = 80.8 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

Recommended L_{max}

Using AISC *Specification* Section D1:

$$
\begin{aligned}
L_{max} &= 300 r_z \\
&= 300\left(\frac{0.776 \text{ in.}}{12 \text{ in./ft}}\right) \\
&= 19.4 \text{ ft}
\end{aligned}
$$

Note: The *L/r* limit is a recommendation, not a requirement.

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.3 WT-SHAPE TENSION MEMBER

Given:

An ASTM A992 WT6×20 member has a length of 30 ft and carries a dead load of 40 kips and a live load of 120 kips in tension. As shown in Figure D3-1, the end connection is fillet welded on each side for 16 in. Verify the member tensile strength by both LRFD and ASD. Assume that the gusset plate and the weld are satisfactory.

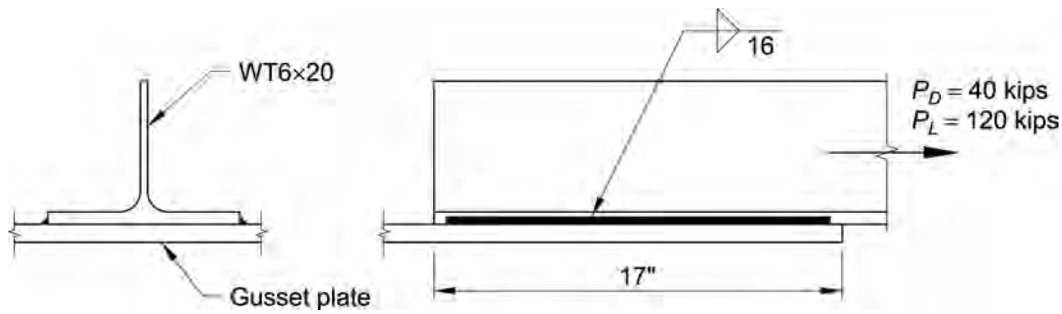


Fig. D.3-1. Connection geometry for Example D.3.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-8, the geometric properties are as follows:

WT6×20
A_g = 5.84 in.^{2}
b_f = 8.01 in.
t_f = 0.515 in.
r_x = 1.57 in.
$\bar{y}$ = 1.09 in.

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(40 \text{ kips}) + 1.6(120 \text{ kips})$ $= 240 \text{ kips}$	$P_a = 40 \text{ kips} + 120 \text{ kips}$ $= 160 \text{ kips}$

Tensile Yielding

Check tensile yielding limit state using AISC *Manual* Table 5-3.

LRFD	ASD
$\phi_t P_n = 263 \text{ kips} > 240 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = 175 \text{ kips} > 160 \text{ kips}$ **o.k.**

Tensile Rupture

Check tensile rupture limit state using AISC *Manual* Table 5-3.

LRFD	ASD
$\phi_t P_n = 214 \text{ kips} < 240 \text{ kips}$ **n.g.**	$\dfrac{P_n}{\Omega_t} = 142 \text{ kips} < 160 \text{ kips}$ **n.g.**

The tabulated available rupture strengths don't work and may be conservative for this case; therefore, calculate the exact solution.

Calculate U as the larger of the values from AISC *Specification* Section D3 and Table D3.1 Case 4.

From AISC *Specification* Section D3, for open cross sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area.

$$\begin{aligned} U &= \frac{b_f t_f}{A_g} \\ &= \frac{(8.01 \text{ in.})(0.515 \text{ in.})}{5.84 \text{ in.}^2} \\ &= 0.706 \end{aligned}$$

Case 4, where l is the length of the connection and $\bar{x} = \bar{y}$:

$$\begin{aligned} U &= \frac{3l^2}{3l^2 + w^2}\left(1 - \frac{\bar{x}}{l}\right) \\ &= \left[\frac{3(16.0 \text{ in.})^2}{3(16.0 \text{ in.})^2 + (8.01 \text{ in.})^2}\right]\left(1 - \frac{1.09 \text{ in.}}{16.0 \text{ in.}}\right) \\ &= 0.860 \end{aligned}$$

Therefore, use $U = 0.860$.

Calculate A_n using AISC *Specification* Section B4.3. Because there are no reductions due to bolt holes or notches:

$$\begin{aligned} A_n &= A_g \\ &= 5.84 \text{ in.}^2 \end{aligned}$$

Calculate A_e using AISC *Specification* Section D3.

$$\begin{aligned} A_e &= A_n U \\ &= (5.84 \text{ in.}^2)(0.860) \\ &= 5.02 \text{ in.}^2 \end{aligned}$$ (*Spec*. Eq. D3-1)

Calculate P_n.

$$\begin{aligned} P_n &= F_u A_e \\ &= (65 \text{ ksi})(5.02 \text{ in.}^2) \\ &= 326 \text{ kips} \end{aligned}$$ (*Spec*. Eq. D2-2)

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(326 \text{ kips})$ $= 245 \text{ kips} > 240 \text{ kips}$ **o.k.**	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{326 \text{ kips}}{2.00}$ $= 163 \text{ kips} > 160 \text{ kips}$ **o.k.**

Alternately, the available tensile rupture strengths can be determined by modifying the tabulated values. The available tensile rupture strengths published in the tension member selection tables are based on the assumption that $A_e = 0.75A_g$. The actual available strengths can be determined by adjusting the values from AISC *Manual* Table 5-3 as follows:

LRFD	ASD
$\phi_t P_n = (214 \text{ kips})\left(\dfrac{A_e}{0.75A_g}\right)$ $= (214 \text{ kips})\left[\dfrac{5.02 \text{ in.}^2}{0.75(5.84 \text{ in.}^2)}\right]$ $= 245 \text{ kips} > 240 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = (142 \text{ kips})\left(\dfrac{A_e}{0.75A_g}\right)$ $= (142 \text{ kips})\left[\dfrac{5.02 \text{ in.}^2}{0.75(5.84 \text{ in.}^2)}\right]$ $= 163 \text{ kips} > 160 \text{ kips}$ **o.k.**

Recommended Slenderness Limit

$$\frac{L}{r_x} = \frac{(30.0 \text{ ft})(12 \text{ in./ft})}{1.57 \text{ in.}}$$

$$= 229 < 300 \text{ from AISC } \textit{Specification} \text{ Section D1} \quad \textbf{o.k.}$$

Note: The L/r_x limit is a recommendation, not a requirement.

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.4 RECTANGULAR HSS TENSION MEMBER

Given:

Verify the tensile strength of an ASTM A500 Grade C HSS6×4×⅜ with a length of 30 ft. The member is carrying a dead load of 40 kips and a live load of 110 kips in tension. As shown in Figure D.4-1, the end connection is a fillet welded ½-in.-thick single concentric gusset plate with a weld length of 16 in. Assume that the gusset plate and weld are satisfactory.

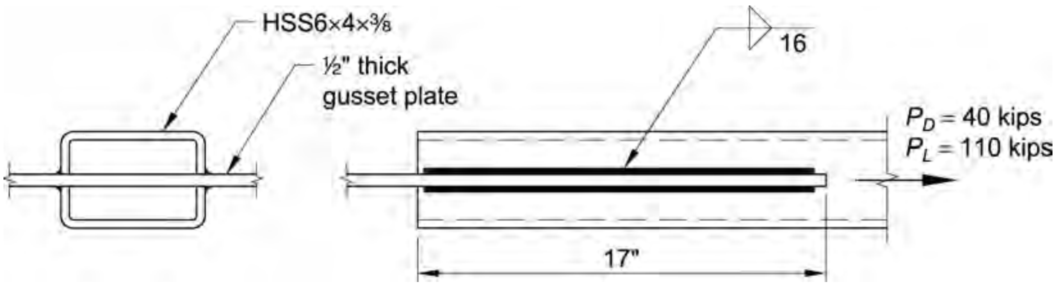


Fig. D.4-1. Connection geometry for Example D.4.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade C, rectangular HSS
F_y = 50 ksi
F_u = 62 ksi

From AISC *Manual* Table 1-11, the geometric properties are as follows:

HSS6×4×⅜
A_g = 6.18 in.^{2}
r_y = 1.55 in.
t = 0.349 in.

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(40 \text{ kips}) + 1.6(110 \text{ kips})$ $= 224 \text{ kips}$	$P_a = 40 \text{ kips} + 110 \text{ kips}$ $= 150 \text{ kips}$

Tensile Yielding

Check tensile yielding limit state using AISC *Manual* Table 5-4.

LRFD	ASD
$\phi_t P_n = 278 \text{ kips} > 224 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = 185 \text{ kips} > 150 \text{ kips}$ **o.k.**

Tensile Rupture

Check tensile rupture limit state using AISC *Manual* Table 5-4.

LRFD	ASD
$\phi_t P_n = 216 \text{ kips} < 224 \text{ kips}$ **n.g.**	$\dfrac{P_n}{\Omega_t} = 144 \text{ kips} < 150 \text{ kips}$ **n.g.**

The tabulated available rupture strengths may be conservative in this case; therefore, calculate the exact solution.

Calculate U from AISC *Specification* Section D3 and Table D3.1 Case 6.

$$\begin{aligned}\bar{x} &= \frac{B^2 + 2BH}{4(B+H)} \\ &= \frac{(4.00 \text{ in.})^2 + 2(4.00 \text{ in.})(6.00 \text{ in.})}{4(4.00 \text{ in.} + 6.00 \text{ in.})} \\ &= 1.60 \text{ in.}\end{aligned}$$

$$\begin{aligned}U &= 1 - \frac{\bar{x}}{l} \\ &= 1 - \frac{1.60 \text{ in.}}{16.0 \text{ in.}} \\ &= 0.900\end{aligned}$$

Allowing for a $\frac{1}{16}$-in. gap in fit-up between the HSS and the gusset plate:

$$\begin{aligned}A_n &= A_g - 2\left(t_p + \tfrac{1}{16} \text{ in.}\right)t \\ &= 6.18 \text{ in.}^2 - 2\left(\tfrac{1}{2} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)(0.349 \text{ in.}) \\ &= 5.79 \text{ in.}^2\end{aligned}$$

Calculate A_e using AISC *Specification* Section D3.

$$\begin{aligned}A_e &= A_n U \\ &= \left(5.79 \text{ in.}^2\right)(0.900) \\ &= 5.21 \text{ in.}^2\end{aligned}$$ (*Spec*. Eq. D3-1)

Calculate P_n.

$$\begin{aligned}P_n &= F_u A_e \\ &= (62 \text{ ksi})\left(5.21 \text{ in.}^2\right) \\ &= 323 \text{ kips}\end{aligned}$$ (*Spec*. Eq. D2-2)

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(323 \text{ kips})$ $= 242 \text{ kips} > 224 \text{ kips}$ **o.k.**	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{323 \text{ kips}}{2.00}$ $= 162 \text{ kips} > 150 \text{ kips}$ **o.k.**

The HSS available tensile strength is governed by the tensile rupture limit state.

Recommended Slenderness Limit

$$\frac{L}{r} = \frac{(30.0 \text{ ft})(12 \text{ in./ft})}{1.55 \text{ in.}}$$
$$= 232 < 300 \text{ from AISC } \textit{Specification} \text{ Section D1} \quad \textbf{o.k.}$$

Note: The L/r limit is a recommendation, not a requirement.

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.5 ROUND HSS TENSION MEMBER

Given:

Verify the tensile strength of an ASTM A500 Grade C HSS6.000×0.500 with a length of 30 ft. The member carries a dead load of 40 kips and a live load of 120 kips in tension. As shown in Figure D.5-1, the end connection is a fillet welded ½-in.-thick single concentric gusset plate with a weld length of 16 in. Assume that the gusset plate and weld are satisfactory.

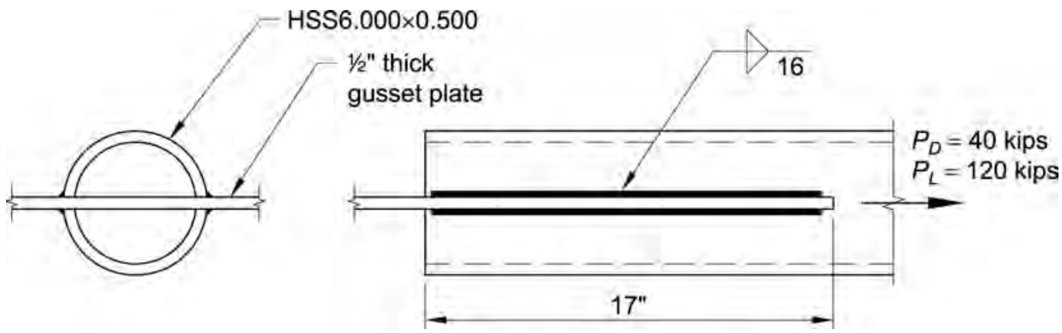


Fig. D.5-1. Connection geometry for Example D.5.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade C, round HSS
$F_y = 46$ ksi
$F_u = 62$ ksi

From AISC *Manual* Table 1-13, the geometric properties are as follows:

HSS6.000×0.500
$A_g = 8.09$ in.^{2}
$r = 1.96$ in.
$t = 0.465$ in.

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(40 \text{ kips}) + 1.6(120 \text{ kips})$ $= 240 \text{ kips}$	$P_a = 40 \text{ kips} + 120 \text{ kips}$ $= 160 \text{ kips}$

Tensile Yielding

Check tensile yielding limit state using AISC *Manual* Table 5-6.

LRFD	ASD
$\phi_t P_n = 335 \text{ kips} > 240 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = 223 \text{ kips} > 160 \text{ kips}$ **o.k.**

Tensile Rupture

Check tensile rupture limit state using AISC *Manual* Table 5-6.

LRFD	ASD
$\phi_t P_n = 282 \text{ kips} > 240 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = 188 \text{ kips} > 160 \text{ kips}$ **o.k.**

Check that $A_e/A_g \geq 0.75$ as assumed in table.

Determine U from AISC *Specification* Table D3.1 Case 5.

$$l = 16.0 \text{ in.}$$
$$D = 6.00 \text{ in.}$$
$$\frac{l}{D} = \frac{16.0 \text{ in.}}{6.00 \text{ in.}}$$
$$= 2.67 > 1.3, \text{ therefore } U = 1.0$$

Allowing for a 1⁄16-in. gap in fit-up between the HSS and the gusset plate,

$$A_n = A_g - 2\left(t_p + 1/16 \text{ in.}\right)t$$
$$= 8.09 \text{ in.}^2 - 2\left(1/2 \text{ in.} + 1/16 \text{ in.}\right)\left(0.465 \text{ in.}\right)$$
$$= 7.57 \text{ in.}^2$$

Calculate A_e using AISC *Specification* Section D3.

$$A_e = A_n U \qquad (\textit{Spec}. \text{ Eq. D3-1})$$
$$= \left(7.57 \text{ in.}^2\right)\left(1.0\right)$$
$$= 7.57 \text{ in.}^2$$

$$\frac{A_e}{A_g} = \frac{7.57 \text{ in.}^2}{8.09 \text{ in.}^2}$$
$$= 0.936 > 0.75 \quad \textbf{o.k.}$$

Because AISC *Manual* Table 5-6 provides an overly conservative estimate of the available tensile rupture strength for this example, calculate P_n using AISC *Specification* Section D2.

$$P_n = F_u A_e \qquad (\text{Spec. Eq. D2-2})$$
$$= \left(62 \text{ ksi}\right)\left(7.57 \text{ in.}^2\right)$$
$$= 469 \text{ kips}$$

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(469 \text{ kips})$ $= 352 \text{ kips} > 240 \text{ kips}$ **o.k.**	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{469 \text{ kips}}{2.00}$ $= 235 \text{ kips} > 160 \text{ kips}$ **o.k.**

The HSS available strength is governed by the tensile yielding limit state.

Recommended Slenderness Limit

$$\frac{L}{r} = \frac{(30.0 \text{ ft})(12 \text{ in./ft})}{1.96 \text{ in.}}$$
$$= 184 < 300 \text{ from AISC } \textit{Specification} \text{ Section D1} \quad \textbf{o.k.}$$

Note: The L/r limit is a recommendation, not a requirement.

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.6 DOUBLE-ANGLE TENSION MEMBER

Given:

An ASTM A36 2L4×4×½ (⅜-in. separation) has one line of eight ¾-in.-diameter bolts in standard holes and is 25 ft in length as shown in Figure D.6-1. The double angle is carrying a dead load of 40 kips and a live load of 120 kips in tension. Verify the member tensile strength. Assume that the gusset plate and bolts are satisfactory.

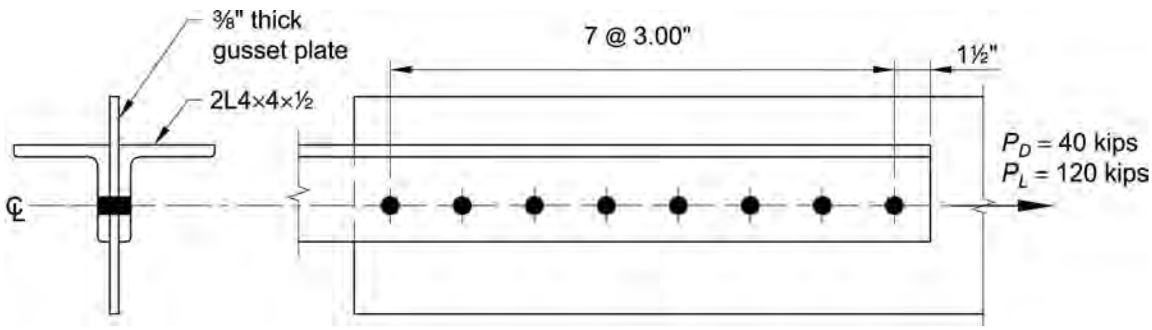


Fig. D.6-1. Connection geometry for Example D.6.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-7 and 1-15, the geometric properties are as follows:

L4×4×½
$\bar{x}$ = 1.18 in.

2L4×4×½ (s = ⅜ in.)
A_g = 7.50 in.^{2}
r_y = 1.83 in.
r_x = 1.21 in.

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(40\text{ kips}) + 1.6(120\text{ kips})$ $= 240\text{ kips}$	$P_a = 40\text{ kips} + 120\text{ kips}$ $= 160\text{ kips}$

Tensile Yielding

Check tensile yielding limit state using AISC *Manual* Table 5-8.

LRFD	ASD
$\phi_t P_n = 243\text{ kips} > 240\text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = 162\text{ kips} > 160\text{ kips}$ **o.k.**

Tensile Rupture

Determine the available tensile rupture strength using AISC *Specification* Section D2. Calculate U as the larger of the values from AISC *Specification* Section D3, Table D3.1 Case 2 and Case 8.

From AISC *Specification* Section D3, for open cross sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area. Half of the member is connected, therefore, the minimum U value is:

$$U = 0.500$$

From Case 2, where l is the length of connection:

$$\begin{aligned} U &= 1 - \frac{\bar{x}}{l} \\ &= 1 - \frac{1.18 \text{ in.}}{21.0 \text{ in.}} \\ &= 0.944 \end{aligned}$$

From Case 8, with four or more fasteners per line in the direction of loading:

$$U = 0.80$$

Therefore, use $U = 0.944$.

Calculate A_n using AISC *Specification* Section B4.3.

$$\begin{aligned} A_n &= A_g - 2\left(d_h + \tfrac{1}{16} \text{ in.}\right)t \\ &= 7.50 \text{ in.}^2 - 2\left(\tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)\left(\tfrac{1}{2} \text{ in.}\right) \\ &= 6.63 \text{ in.}^2 \end{aligned}$$

Calculate A_e using AISC *Specification* Section D3.

$$\begin{aligned} A_e &= A_n U \\ &= \left(6.63 \text{ in.}^2\right)(0.944) \\ &= 6.26 \text{ in.}^2 \end{aligned} \qquad \text{(}Spec\text{. Eq. D3-1)}$$

Calculate P_n.

$$\begin{aligned} P_n &= F_u A_e \\ &= (58 \text{ ksi})\left(6.26 \text{ in.}^2\right) \\ &= 363 \text{ kips} \end{aligned} \qquad \text{(}Spec\text{. Eq. D2-2)}$$

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(363 \text{ kips})$ $= 272 \text{ kips}$	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{363 \text{ kips}}{2.00}$ $= 182 \text{ kips}$

Note that AISC *Manual* Table 5-8 could also be conservatively used since $A_e \geq 0.75A_g$.

The double-angle available tensile strength is governed by the tensile yielding limit state.

LRFD	ASD
$243 \text{ kips} > 240 \text{ kips}$ **o.k.**	$162 \text{ kips} > 160 \text{ kips}$ **o.k.**

Recommended Slenderness Limit

$$\frac{L}{r_x} = \frac{(25.0 \text{ ft})(12 \text{ in./ft})}{1.21 \text{ in.}}$$
$$= 248 < 300 \text{ from AISC } \textit{Specification} \text{ Section D1} \quad \textbf{o.k.}$$

Note: From AISC *Specification* Section D4, the longitudinal spacing of connectors between components of built-up members should preferably limit the slenderness ratio in any component between the connectors to a maximum of 300.

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.7 PIN-CONNECTED TENSION MEMBER

Given:

An ASTM A36 pin-connected tension member with the dimensions shown in Figure D.7-1 carries a dead load of 4 kips and a live load of 12 kips in tension. The diameter of the pin is 1 in., in a 1⁄32-in. oversized hole. Assume that the pin itself is adequate. Verify the member tensile strength.

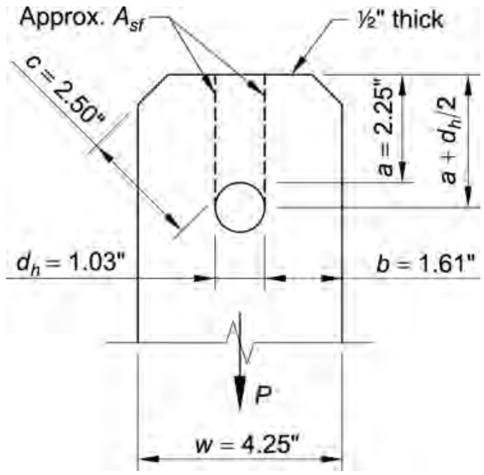


Fig. D.7-1. Connection geometry for Example D.7.

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

The geometric properties of the plate are as follows:

$$a = 2.25 \text{ in.}$$
$$b = 1.61 \text{ in.}$$
$$c = 2.50 \text{ in.}$$
$$d = 1.00 \text{ in.}$$
$$d_h = 1.03 \text{ in.}$$
$$t = \tfrac{1}{2} \text{ in.}$$
$$w = 4.25 \text{ in.}$$

The requirements given in AISC *Specification* Sections D5.2(a) and D5.2(b) are satisfied by the given geometry. Requirements given in AISC *Specification* Sections D5.2(c) and D5.2(d) are checked as follows:

$$\begin{aligned} b_e &= 2t + 0.63 \le b \\ &= 2\left(\tfrac{1}{2}\text{ in.}\right) + 0.63 \le 1.61 \text{ in.} \\ &= 1.63 \text{ in.} > 1.61 \text{ in.} \end{aligned}$$

Therefore, use b_e = 1.61 in.

$$a \geq 1.33b_e$$
$$2.25 \text{ in.} > 1.33(1.61 \text{ in.})$$
$$2.25 \text{ in.} > 2.14 \text{ in.} \quad \textbf{o.k.}$$

$$w \geq 2b_e + d$$
$$4.25 \text{ in.} > 2(1.61 \text{ in.}) + 1.00 \text{ in.}$$
$$4.25 \text{ in.} > 4.22 \text{ in.} \quad \textbf{o.k.}$$

$$c \geq a$$
$$2.50 \text{ in.} > 2.25 \text{ in.} \quad \textbf{o.k.}$$

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(4 \text{ kips}) + 1.6(12 \text{ kips})$ $= 24.0 \text{ kips}$	$P_a = 4 \text{ kips} + 12 \text{ kips}$ $= 16.0 \text{ kips}$

From AISC *Specification* Section D5.1, the available tensile strength is the lower value determined according to the limit states of tensile rupture, shear rupture, bearing and yielding.

Tensile Rupture

Calculate the available tensile rupture strength on the effective net area.

$$P_n = F_u(2tb_e) \quad \text{(}\textit{Spec}\text{. Eq. D5-1)}$$
$$= (58 \text{ ksi})(2)(\tfrac{1}{2} \text{ in.})(1.61 \text{ in.})$$
$$= 93.4 \text{ kips}$$

From AISC *Specification* Section D5.1, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(93.4 \text{ kips})$ $= 70.1 \text{ kips}$	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{93.4 \text{ kips}}{2.00}$ $= 46.7 \text{ kips}$

Shear Rupture

From AISC *Specification* Section D5.1, the area on the shear failure path is:

$$A_{sf} = 2t\left(a + \frac{d}{2}\right)$$
$$= 2(\tfrac{1}{2} \text{ in.})\left[2.25 \text{ in.} + \left(\frac{1.00 \text{ in.}}{2}\right)\right]$$
$$= 2.75 \text{ in.}^2$$

$$P_n = 0.6F_u A_{sf}$$ (*Spec*. Eq. D5-2)

$$= 0.6(58 \text{ ksi})(2.75 \text{ in.}^2)$$

$$= 95.7 \text{ kips}$$

From AISC *Specification* Section D5.1, the available shear rupture strength is:

LRFD	ASD
$\phi_{sf} = 0.75$ $\phi_{sf} P_n = 0.75(95.7 \text{ kips})$ $= 71.8 \text{ kips}$	$\Omega_{sf} = 2.00$ $\dfrac{P_n}{\Omega_{sf}} = \dfrac{95.7 \text{ kips}}{2.00}$ $= 47.9 \text{ kips}$

Bearing

Determine the available bearing strength using AISC *Specification* Section J7.

$$A_{pb} = td$$

$$= (\tfrac{1}{2} \text{ in.})(1.00 \text{ in.})$$

$$= 0.500 \text{ in.}^2$$

$$R_n = 1.8F_y A_{pb}$$ (*Spec*. Eq. J7-1)

$$= 1.8(36 \text{ ksi})(0.500 \text{ in.}^2)$$

$$= 32.4 \text{ kips}$$

From AISC *Specification* Section J7, the available bearing strength is:

LRFD	ASD
$\phi = 0.75$ $\phi P_n = 0.75(32.4 \text{ kips})$ $= 24.3 \text{ kips}$	$\Omega = 2.00$ $\dfrac{P_n}{\Omega} = \dfrac{32.4 \text{ kips}}{2.00}$ $= 16.2 \text{ kips}$

Tensile Yielding

Determine the available tensile yielding strength using AISC *Specification* Section D2(a).

$$A_g = wt$$

$$= (4.25 \text{ in.})(\tfrac{1}{2} \text{ in.})$$

$$= 2.13 \text{ in.}^2$$

$$P_n = F_y A_g$$ (*Spec*. Eq. D2-1)

$$= (36 \text{ ksi})(2.13 \text{ in.}^2)$$

$$= 76.7 \text{ kips}$$

From AISC *Specification* Section D2, the available tensile yielding strength is:

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n = 0.90\left(76.7 \text{ kips}\right)$ $= 69.0 \text{ kips}$	$\Omega_t = 1.67$ $\dfrac{P_n}{\Omega_t} = \dfrac{76.7 \text{ kips}}{1.67}$ $= 45.9 \text{ kips}$

The available tensile strength is governed by the bearing strength limit state.

LRFD	ASD
$\phi P_n = 24.3 \text{ kips} > 24.0 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega} = 16.2 \text{ kips} > 16.0 \text{ kips}$ **o.k.**

EXAMPLE D.8 EYEBAR TENSION MEMBER

Given:

A ⅝-in.-thick, ASTM A36 eyebar member as shown in Figure D.8, carries a dead load of 25 kips and a live load of 15 kips in tension. The pin diameter, d, is 3 in. Verify the member tensile strength.

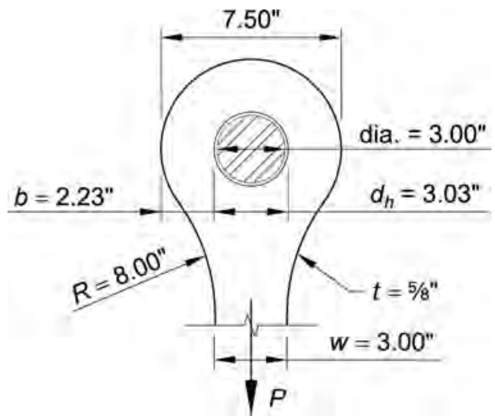


Fig. D.8-1. Connection geometry for Example D.8.

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

Plate
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

The geometric properties of the eyebar are as follows:

$R = 8.00$ in.
$b = 2.23$ in.
$d = 3.00$ in.
$d_h = 3.03$ in.
$d_{head} = 7.50$ in.
$t = ⅝$ in.
$w = 3.00$ in.

Check the dimensional requirement using AISC *Specification* Section D6.1.

$$w \le 8t$$
$$3.00 \text{ in.} < 8(⅝ \text{ in.})$$
$$3.00 \text{ in.} < 5.00 \text{ in.} \quad \textbf{o.k.}$$

Check the dimensional requirements using AISC *Specification* Section D6.2.

$$t \ge ½ \text{ in.}$$
$$⅝ \text{ in.} > ½ \text{ in.} \quad \textbf{o.k.}$$

$$d \geq \frac{7}{8}w$$

$$3.00 \text{ in.} > \frac{7}{8}(3.00 \text{ in.})$$

$3.00 \text{ in.} > 2.63 \text{ in.}$ **o.k.**

$$d_h \leq d + \tfrac{1}{32} \text{ in.}$$

$$3.03 \text{ in.} = 3.00 \text{ in.} + \tfrac{1}{32} \text{ in.}$$

$3.03 \text{ in.} = 3.03 \text{ in.}$ **o.k.**

$$R \geq d_{head}$$

$8.00 \text{ in.} > 7.50 \text{ in.}$ **o.k.**

$$\frac{2}{3}w < b \leq \frac{3}{4}w$$

$$\frac{2}{3}(3.00 \text{ in.}) < 2.23 \text{ in.} < \frac{3}{4}(3.00 \text{ in.})$$

$2.00 \text{ in.} < 2.23 \text{ in.} < 2.25 \text{ in.}$ **o.k.**

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(25 \text{ kips}) + 1.6(15 \text{ kips})$ $= 54.0 \text{ kips}$	$P_a = 25 \text{ kips} + 15 \text{ kips}$ $= 40.0 \text{ kips}$

Tensile Yielding

Determine the available tensile yielding strength using AISC *Specification* Section D2 at the eyebar body (at *w*).

$$\begin{aligned} A_g &= wt \\ &= (3.00 \text{ in.})(\tfrac{5}{8} \text{ in.}) \\ &= 1.88 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} P_n &= F_y A_g \\ &= (36 \text{ ksi})(1.88 \text{ in.}^2) \\ &= 67.7 \text{ kips} \end{aligned}$$ (*Spec*. Eq. D2-1)

The available tensile yielding strength is:

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n = 0.90(67.7 \text{ kips})$ $= 60.9 \text{ kips} > 54.0 \text{ kips}$ **o.k.**	$\Omega_t = 1.67$ $\frac{P_n}{\Omega_t} = \frac{67.7 \text{ kips}}{1.67}$ $= 40.5 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

The eyebar tension member available strength is governed by the tensile yielding limit state.

Note: The eyebar detailing limitations ensure that the tensile yielding limit state at the eyebar body will control the strength of the eyebar itself. The pin should also be checked for shear yielding, and, if the material strength is less than that of the eyebar, the bearing limit state should also be checked.

EXAMPLE D.9 PLATE WITH STAGGERED BOLTS

Given:

Compute A_n and A_e for a 14-in.-wide and ½-in.-thick plate subject to tensile loading with staggered holes as shown in Figure D.9-1.

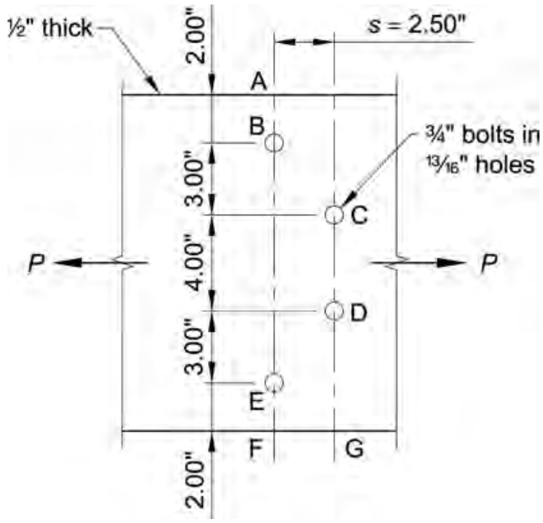


Fig. D.9-1. Connection geometry for Example D.9.

Solution:

Calculate the net hole diameter using AISC *Specification* Section B4.3b.

$$
\begin{aligned}
d_{net} &= d_h + \tfrac{1}{16} \text{ in.} \\
&= \tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.} \\
&= 0.875 \text{ in.}
\end{aligned}
$$

Compute the net width for all possible paths across the plate. Because of symmetry, many of the net widths are identical and need not be calculated.

$$w = 14.0 \text{ in.} - \Sigma d_{net} + \Sigma \frac{s^2}{4g} \quad \text{from AISC } \textit{Specification} \text{ Section B4.3b.}$$

Line A-B-E-F:

$$
\begin{aligned}
w &= 14.0 \text{ in.} - 2(0.875 \text{ in.}) \\
&= 12.3 \text{ in.}
\end{aligned}
$$

Line A-B-C-D-E-F:

$$
\begin{aligned}
w &= 14.0 \text{ in.} - 4(0.875 \text{ in.}) + \frac{(2.50 \text{ in.})^2}{4(3.00 \text{ in.})} + \frac{(2.50 \text{ in.})^2}{4(3.00 \text{ in.})} \\
&= 11.5 \text{ in.}
\end{aligned}
$$

Line A-B-C-D-G:

$$w = 14.0\text{ in.} - 3(0.875\text{ in.}) + \frac{(2.50\text{ in.})^2}{4(3.00\text{ in.})}$$
$$= 11.9\text{ in.}$$

Line A-B-D-E-F:

$$w = 14.0\text{ in.} - 3(0.875\text{ in.}) + \frac{(2.50\text{ in.})^2}{4(7.00\text{ in.})} + \frac{(2.50\text{ in.})^2}{4(3.00\text{ in.})}$$
$$= 12.1\text{ in.}$$

Line A-B-C-D-E-F controls the width, w, therefore:

$$A_n = wt$$
$$= (11.5\text{ in.})(\tfrac{1}{2}\text{ in.})$$
$$= 5.75\text{ in.}^2$$

Calculate U.

From AISC *Specification* Table D3.1 Case 1, because tension load is transmitted to all elements by the fasteners,

$$U = 1.0$$

$$A_e = A_n U \qquad \text{(}Spec\text{. Eq. D3-1)}$$
$$= (5.75\text{ in.}^2)(1.0)$$
$$= 5.75\text{ in.}^2$$

Chapter E
Design of Members for Compression

This chapter covers the design of compression members, the most common of which are columns. The AISC *Manual* includes design tables for the following compression member types in their most commonly available grades:

- W-shapes and HP-shapes
- Rectangular, square and round HSS
- Pipes
- WT-shapes
- Double angles
- Single angles

LRFD and ASD information is presented side-by-side for quick selection, design or verification. All of the tables account for the reduced strength of sections with slender elements.

The design and selection method for both LRFD and ASD is similar to that of previous editions of the AISC *Specification*, and will provide similar designs. In this AISC *Specification*, LRFD and ASD will provide identical designs when the live load is approximately three times the dead load.

The design of built-up shapes with slender elements can be tedious and time consuming, and it is recommended that standard rolled shapes be used whenever possible.

E1. GENERAL PROVISIONS

The design compressive strength, $\phi_c P_n$, and the allowable compressive strength, P_n/Ω_c, are determined as follows:

P_n = nominal compressive strength is the lowest value obtained based on the applicable limit states of flexural buckling, torsional buckling, and flexural-torsional buckling, kips

$\phi_c = 0.90$ (LRFD) $\qquad \Omega_c = 1.67$ (ASD)

Because the critical stress, F_{cr}, is used extensively in calculations for compression members, it has been tabulated in AISC *Manual* Table 4-14 for all of the common steel yield strengths.

E2. EFFECTIVE LENGTH

In the AISC *Specification*, there is no limit on slenderness, L_c/r. Per the User Note in AISC *Specification* Section E2, it is recommended that L_c/r not exceed 200, as a practical limit based on professional judgment and construction economics.

Although there is no restriction on the unbraced length of columns, the tables of the AISC *Manual* are stopped at common or practical lengths for ordinary usage. For example, a double L3×3×¼, with a ⅜-in. separation has an r_y of 1.38 in. At a L_c/r of 200, this strut would be 23 ft long. This is thought to be a reasonable limit based on fabrication and handling requirements.

Throughout the AISC *Manual*, shapes that contain slender elements for compression when supplied in their most common material grade are footnoted with the letter "c." For example, see a W14×22[c].

E3. FLEXURAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

Nonslender-element compression members, including nonslender built-up I-shaped columns and nonslender HSS columns, are governed by these provisions. The general design curve for critical stress versus L_c/r is shown in Figure E-1.

The term L_c is used throughout this chapter to describe the length between points that are braced against lateral and/or rotational displacement.

E4. TORSIONAL AND FLEXURAL-TORSIONAL BUCKLING OF SINGLE ANGLES AND MEMBERS WITHOUT SLENDER ELEMENTS

This section is most commonly applicable to double angles and WT sections, which are singly symmetric shapes subject to torsional and flexural-torsional buckling. The available strengths in axial compression of these shapes are tabulated in AISC *Manual* Part 4 and examples on the use of these tables have been included in this chapter for the shapes.

E5. SINGLE-ANGLE COMPRESSION MEMBERS

The available strength of single-angle compression members is tabulated in AISC *Manual* Part 4.

E6. BUILT-UP MEMBERS

The available strengths in axial compression for built-up double angles with intermediate connectors are tabulated in AISC *Manual* Part 4. There are no tables for other built-up shapes in the AISC *Manual*, due to the number of possible geometries.

E7. MEMBERS WITH SLENDER ELEMENTS

The design of these members is similar to members without slender elements except that a reduced effective area is used in lieu of the gross cross-sectional area.

The tables of AISC *Manual* Part 4 incorporate the appropriate reductions in available strength to account for slender elements.

Design examples have been included in this Chapter for built-up I-shaped members with slender webs and slender flanges**.** Examples have also been included for a double angle, WT and an HSS with slender elements.

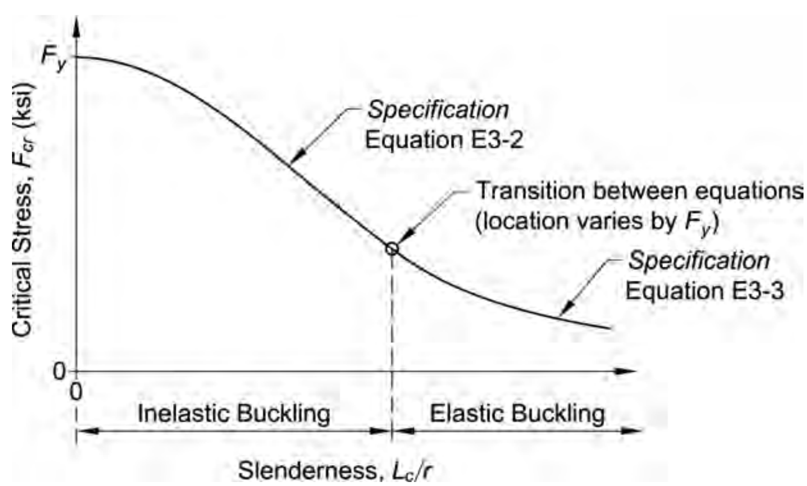


Fig. E-1. Standard column curve.

Table E-1
Limiting Values of L_c/r and F_e

F_y, ksi	Limiting L_c/r	F_e, ksi
36	134	15.9
50	113	22.4
65	99.5	28.9
70	95.9	31.1

EXAMPLE E.1A W-SHAPE COLUMN DESIGN WITH PINNED ENDS

Given:

Select a W-shape column to carry the loading as shown in Figure E.1A. The column is pinned top and bottom in both axes. Limit the column size to a nominal 14-in. shape. A column is selected for both ASTM A992 and ASTM A913 Grade 65 material.

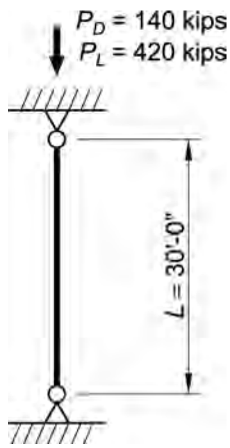


Fig. E.1A. Column loading and bracing.

Solution:

Note that ASTM A913 Grade 70 might also be used in this design. The requirement for higher preheat when welding and the need to use 90-ksi filler metals for complete-joint-penetration (CJP) welds to other 70-ksi pieces offset the advantage of the lighter column and should be considered in the selection of which grade to use.

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi

ASTM A913 Grade 65
F_y = 65 ksi

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(140\text{ kips}) + 1.6(420\text{ kips})$ $= 840\text{ kips}$	$P_a = 140\text{ kips} + 420\text{ kips}$ $= 560\text{ kips}$

Column Selection—ASTM A992

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K_x = K_y = 1.0. The effective length is:

$$L_c = K_x L_x$$
$$= K_y L_y$$
$$= 1.0(30 \text{ ft})$$
$$= 30.0 \text{ ft}$$

Because the unbraced length is the same in both the *x-x* and *y-y* directions and r_x exceeds r_y for all W-shapes, *y-y* axis bucking will govern.

Enter AISC *Manual* Table 4-1a with an effective length, L_c, of 30 ft, and proceed across the table until reaching the least weight shape with an available strength that equals or exceeds the required strength. Select a W14×132.

From AISC *Manual* Table 4-1a, the available strength for a *y-y* axis effective length of 30 ft is:

LRFD	ASD
$\phi_c P_n = 893 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 594 \text{ kips} > 560 \text{ kips}$ **o.k.**

Column Selection–ASTM A913 Grade 65

Enter AISC *Manual* Table 4-1b with an effective length, L_c, of 30 ft, and proceed across the table until reaching the least weight shape with an available strength that equals or exceeds the required strength. Select a W14×120.

From AISC *Manual* Table 4-1b, the available strength for a *y-y* axis effective length of 30 ft is:

LRFD	ASD
$\phi_c P_n = 856 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 569 \text{ kips} > 560 \text{ kips}$ **o.k.**

EXAMPLE E.1B W-SHAPE COLUMN DESIGN WITH INTERMEDIATE BRACING

Given:

Verify a W14×90 is adequate to carry the loading as shown in Figure E.1B. The column is pinned top and bottom in both axes and braced at the midpoint about the *y-y* axis and torsionally. The column is verified for both ASTM A992 and ASTM A913 Grade 65 material.

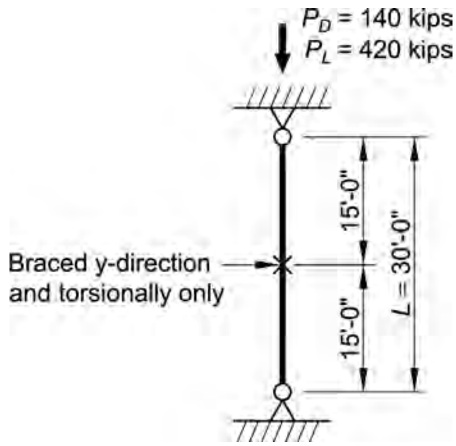


Fig. E.1B. Column loading and bracing.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi

ASTM A913 Grade 65
F_y = 65 ksi

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(140\text{ kips}) + 1.6(420\text{ kips})$ $= 840\text{ kips}$	$P_a = 140\text{ kips} + 420\text{ kips}$ $= 560\text{ kips}$

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, $K_x = K_y$ = 1.0. The effective length about the *y*-y axis is:

$$L_{cy} = K_y L_y$$
$$= 1.0(15\text{ ft})$$
$$= 15.0\text{ ft}$$

The values tabulated in AISC *Manual* Tables 4-1a, 4-1b and 4-1c are provided for buckling in the *y-y* direction. To determine the buckling strength in the *x-x* axis, an equivalent effective length for the *y-y* axis is determined using the r_x/r_y ratio provided at the the bottom of these tables. For a W14×90, r_x/r_y = 1.66, and the equivalent *y-y* axis effective length for *x-x* axis buckling is computed as:

$$L_{cx} = K_x L_x$$
$$= 1.0(30 \text{ ft})$$
$$= 30.0 \text{ ft}$$

$$L_{cy\,eq} = \frac{L_{cx}}{r_x / r_y} \quad \text{(\textit{Manual} Eq. 4-1)}$$
$$= \frac{30.0 \text{ ft}}{1.66}$$
$$= 18.1 \text{ ft}$$

Because 18.1 ft > 15.0 ft, the available compressive strength is governed by the *x-x* axis flexural buckling limit state.

Available Compressive Strength—ASTM A992

The available strength of a W14×90 is determined using AISC *Manual* Table 4-1a, conservatively using an unbraced length of L_c = 19.0 ft.

LRFD	ASD
$\phi_c P_n = 903 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\frac{P_n}{\Omega_c} = 601 \text{ kips} > 560 \text{ kips}$ **o.k.**

Available Compressive Strength—ASTM 913 Grade 65

The available strength of a W14×90 is determined using AISC *Manual* Table 4-1b, conservatively using an unbraced length of L_c = 19.0 ft.

LRFD	ASD
$\phi_c P_n = 1{,}080 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\frac{P_n}{\Omega_c} = 719 \text{ kips} > 560 \text{ kips}$ **o.k.**

The available strengths of the columns described in Examples E.1A and E.1B are easily selected directly from the AISC *Manual* Tables. The available strengths can also be determined as shown in the following Examples E.1C and E.1D.

EXAMPLE E.1C W-SHAPE AVAILABLE STRENGTH CALCULATION

Given:

Calculate the available strength of the column sizes selected in Example E.1A with unbraced lengths of 30 ft in both axes. The material properties and loads are as given in Example E.1A.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi

ASTM A913 Grade 65
F_y = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W14×120
A_g = 35.3 in.^{2}
r_x = 6.24 in.
r_y = 3.74 in.

W14×132
A_g = 38.8 in.^{2}
r_x = 6.28 in.
r_y = 3.76 in.

Column Compressive Strength—ASTM A992

Slenderness Check

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, $K_x = K_y$ = 1.0. The effective length about the *y*-y axis is:

$$\begin{aligned} L_{cy} &= K_y L_y \\ &= 1.0(30 \text{ ft}) \\ &= 30.0 \text{ ft} \end{aligned}$$

Because the unbraced length for the W14×132 column is the same for both axes, the *y-y* axis will govern.

$$\begin{aligned} \frac{L_{cy}}{r_y} &= \frac{(30.0 \text{ ft})(12 \text{ in./ft})}{3.76 \text{ in.}} \\ &= 95.7 \end{aligned}$$

Critical Stress

For F_y = 50 ksi, the available critical stresses, $\phi_c F_{cr}$ and F_{cr}/Ω_c for L_c/r = 95.7 are interpolated from AISC *Manual* Table 4-14 as follows. The available critical stress can also be determined as shown in Example E.1D.

LRFD	ASD
$\phi_c F_{cr} = 23.0 \text{ ksi}$	$\dfrac{F_{cr}}{\Omega_c} = 15.4 \text{ ksi}$

From AISC *Specification* Equation E3-1, the available compressive strength of the W14×132 column is:

LRFD	ASD
$\phi_c P_n = (\phi_c F_{cr}) A_g$ $= (23.0 \text{ ksi})(38.8 \text{ in.}^2)$ $= 892 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = \left(\dfrac{F_{cr}}{\Omega_c}\right) A_g$ $= (15.4 \text{ ksi})(38.8 \text{ in.}^2)$ $= 598 \text{ kips} > 560 \text{ kips}$ **o.k.**

Column Compressive Strength—ASTM A913 Grade 65

Slenderness Check

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, $K_x = K_y = 1.0$. The effective length about the *y*-y axis is:

$$\begin{aligned} L_{cy} &= K_y L_y \\ &= 1.0(30 \text{ ft}) \\ &= 30.0 \text{ ft} \end{aligned}$$

Because the unbraced length for the W14×120 column is the same for both axes, the *y-y* axis will govern.

$$\begin{aligned} \frac{L_{cy}}{r_y} &= \frac{(30.0 \text{ ft})(12 \text{ in./ft})}{3.74 \text{ in.}} \\ &= 96.3 \end{aligned}$$

Critical Stress

For $F_y = 65$ ksi, the available critical stresses, $\phi_c F_{cr}$ and F_{cr}/Ω_c for $L_c/r = 96.3$ are interpolated from AISC *Manual* Table 4-14 as follows. The available critical stress can also be determined as shown in Example E.1D.

LRFD	ASD
$\phi_c F_{cr} = 24.3 \text{ ksi}$	$\dfrac{F_{cr}}{\Omega_c} = 16.1 \text{ ksi}$

From AISC *Specification* Equation E3-1, the available compressive strength of the W14×120 column is:

LRFD	ASD
$\phi_c P_n = (\phi_c F_{cr}) A_g$ $= (24.3 \text{ ksi})(35.3 \text{ in.}^2)$ $= 858 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = \left(\dfrac{F_{cr}}{\Omega_c}\right) A_g$ $= (16.1 \text{ ksi})(35.3 \text{ in.}^2)$ $= 568 \text{ kips} > 560 \text{ kips}$ **o.k.**

Note that the calculated values are approximately equal to the tabulated values.

EXAMPLE E.1D W-SHAPE AVAILABLE STRENGTH CALCULATION

Given:

Calculate the available strength of a W14×90 with a *x-x* axis unbraced length of 30 ft and *y-y* axis and torsional unbraced lengths of 15 ft. The material properties and loads are as given in Example E.1A.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi

ASTM A913 Grade 65
F_y = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W14×90
A_g = 26.5 in.^{2}
r_x = 6.14 in.
r_y = 3.70 in.

$$\frac{b_f}{2t_f} = 10.2$$

$$\frac{h}{t_w} = 25.9$$

Slenderness Check

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, $K_x = K_y = 1.0$.

$$\begin{aligned} L_{cx} &= K_x L_x \\ &= 1.0(30 \text{ ft}) \\ &= 30.0 \text{ ft} \end{aligned}$$

$$\begin{aligned} \frac{L_{cx}}{r_x} &= \frac{(30.0 \text{ ft})(12 \text{ in./ft})}{6.14 \text{ in.}} \\ &= 58.6 \quad \textbf{governs} \end{aligned}$$

$$\begin{aligned} L_{cy} &= K_y L_y \\ &= 1.0(15 \text{ ft}) \\ &= 15.0 \text{ ft} \end{aligned}$$

$$\begin{aligned} \frac{L_{cy}}{r_y} &= \frac{(15.0 \text{ ft})(12 \text{ in./ft})}{3.70 \text{ in.}} \\ &= 48.6 \end{aligned}$$

Column Compressive Strength—ASTM A992

Width-to-Thickness Ratio

The width-to-thickness ratio of the flanges of the W14×90 is:

$$\frac{b_f}{2t_f} = 10.2$$

From AISC *Specification* Table B4.1a, Case 1, the limiting width-to-thickness ratio of the flanges is:

$$0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 13.5 > 10.2;\ \text{therefore, the flanges are nonslender}$$

The width-to-thickness ratio of the web of the W14×90 is:

$$\frac{h}{t_w} = 25.9$$

From AISC *Specification* Table B4.1a, Case 5, the limiting width-to-thickness ratio of the web is:

$$1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 35.9 > 25.9;\ \text{therefore, the web is nonslender}$$

Because the web and flanges are nonslender, the limit state of local buckling does not apply.

Critical Stresses

The available critical stresses may be interpolated from AISC *Manual* Table 4-14 or calculated directly as follows.

Calculate the elastic critical buckling stress, F_e, according to AISC *Specification* Section E3. As noted in AISC *Specification* Commentary Section E4, torsional buckling of symmetric shapes is a failure mode usually not considered in the design of hot-rolled columns. This failure mode generally does not govern unless the section is manufactured from relatively thin plates or a torsional unbraced length significantly larger than the *y-y* axis flexural unbraced length is present.

$$F_e = \frac{\pi^2 E}{\left(\dfrac{L_c}{r}\right)^2} \qquad \text{(}\textit{Spec}\text{. Eq. E3-4)}$$
$$= \frac{\pi^2 (29{,}000\text{ ksi})}{(58.6)^2}$$
$$= 83.3\text{ ksi}$$

Calculate the flexural buckling stress, F_{cr}.

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 113$$

Because $\dfrac{L_c}{r} = 58.6 < 113,$

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \qquad (Spec.\ \text{Eq. E3-2})$$

$$= \left(0.658^{\frac{50\text{ ksi}}{83.3\text{ ksi}}}\right)(50\text{ ksi})$$

$$= 38.9\text{ ksi}$$

Nominal Compressive Strength

$$P_n = F_{cr}A_g \qquad (Spec.\ \text{Eq. E3-1})$$

$$= (38.9\text{ ksi})(26.5\text{ in.}^2)$$

$$= 1{,}030\text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(1{,}030\text{ kips})$ $= 927\text{ kips} > 840\text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\dfrac{P_n}{\Omega_c} = \dfrac{1{,}030\text{ kips}}{1.67}$ $= 617\text{ kips} > 560\text{ kips}$ **o.k.**

Column Compressive Strength—ASTM A913 Grade 65

Width-to-Thickness Ratio

The width-to-thickness ratio of the flanges of the W14×90 is:

$$\frac{b_f}{2t_f} = 10.2$$

From AISC *Specification* Table B4.1a, Case 1, the limiting width-to-thickness ratio of the flanges is:

$$0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29{,}000\text{ ksi}}{65\text{ ksi}}}$$

$$= 11.8 > 10.2;\ \text{therefore, the flanges are nonslender}$$

The width-to-thickness ratio of the web of the W14×90 is:

$$\frac{h}{t_w} = 25.9$$

From AISC *Specification* Table B4.1a, Case 5, the limiting width-to-thickness ratio of the web is:

$$1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29{,}000 \text{ ksi}}{65 \text{ ksi}}}$$
$$= 31.5 > 25.9; \text{ therefore, the web is nonslender}$$

Because the web and flanges are nonslender, the limit state of local buckling does not apply.

Critical Stress

$F_e = 83.3$ ksi (calculated previously)

Calculate the flexural buckling stress, F_{cr}.

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000 \text{ ksi}}{65 \text{ ksi}}}$$
$$= 99.5$$

Because $\frac{L_c}{r} = 58.6 < 99.5$,

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \qquad \text{(\textit{Spec}. Eq. E3-2)}$$
$$= \left(0.658^{\frac{65 \text{ ksi}}{83.3 \text{ ksi}}}\right)(65 \text{ ksi})$$
$$= 46.9 \text{ ksi}$$

Nominal Compressive Strength

$$P_n = F_{cr}A_g \qquad \text{(\textit{Spec}. Eq. E3-1)}$$
$$= (46.9 \text{ ksi})(26.5 \text{ in.}^2)$$
$$= 1{,}240 \text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$	$\Omega_c = 1.67$
$\phi_c P_n = 0.90(1{,}240 \text{ kips})$ $= 1{,}120 \text{ kips} > 840 \text{ kips}$ **o.k.**	$\frac{P_n}{\Omega_c} = \frac{1{,}240 \text{ kips}}{1.67}$ $= 743 \text{ kips} > 560 \text{ kips}$ **o.k.**

EXAMPLE E.2 BUILT-UP COLUMN WITH A SLENDER WEB

Given:

Verify that a built-up, ASTM A572 Grade 50 column with PL1 in. × 8 in. flanges and a PL¼ in. × 15 in. web, as shown in Figure E2-1, is sufficient to carry a dead load of 70 kips and live load of 210 kips in axial compression. The column's unbraced length is 15 ft and the ends are pinned in both axes.

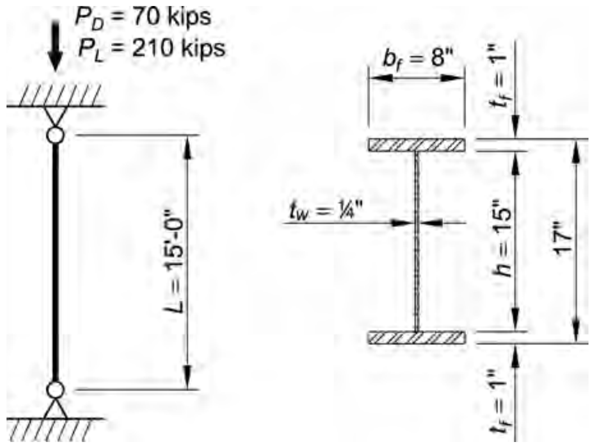


Fig. E.2-1. Column geometry for Example E.2.

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

Built-Up Column
ASTM A572 Grade 50
F_y = 50 ksi
F_u = 65 ksi

The geometric properties are as follows:

Built-Up Column
d = 17.0 in.
b_f = 8.00 in.
t_f = 1.00 in.
h = 15.0 in.
t_w = ¼ in.

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(70\text{ kips}) + 1.6(210\text{ kips})$ $= 420\text{ kips}$	$P_a = 70\text{ kips} + 210\text{ kips}$ $= 280\text{ kips}$

Built-Up Section Properties (ignoring fillet welds)

$$A_g = 2b_f t_f + ht_w$$
$$= 2(8.00\text{ in.})(1.00\text{ in.}) + (15.0\text{ in.})(\tfrac{1}{4}\text{ in.})$$
$$= 19.8\text{ in.}^2$$

$$I_y = \sum \frac{bh^3}{12}$$
$$= 2\left[\frac{(1.00\text{ in.})(8.00\text{ in.})^3}{12}\right] + \frac{(15.0\text{ in.})(\tfrac{1}{4}\text{ in.})^3}{12}$$
$$= 85.4\text{ in.}^4$$

$$r_y = \sqrt{\frac{I_y}{A}}$$
$$= \sqrt{\frac{85.4\text{ in.}^4}{19.8\text{ in.}^2}}$$
$$= 2.08\text{ in.}$$

$$I_x = \sum Ad^2 + \sum \frac{bh^3}{12}$$
$$= 2\left[(8.00\text{ in.}^2)(8.00\text{ in.})^2\right] + \frac{(\tfrac{1}{4}\text{ in.})(15.0\text{ in.})^3}{12} + 2\left[\frac{(8.00\text{ in.})(1.00\text{ in.})^3}{12}\right]$$
$$= 1{,}100\text{ in.}^4$$

Elastic Flexural Buckling Stress

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K_y = 1.0.

Because the unbraced length is the same for both axes, the *y-y* axis will govern by inspection. With $L_{cy} = K_y L_y$ = 1.0(15 ft) = 15.0 ft:

$$\frac{L_{cy}}{r_y} = \frac{(15.0\text{ ft})(12\text{ in./ft})}{2.08\text{ in.}}$$
$$= 86.5$$

$$F_e = \frac{\pi^2 E}{\left(\frac{L_{cy}}{r_y}\right)^2} \quad \text{(from } Spec. \text{ Eq. E3-4)}$$
$$= \frac{\pi^2 (29{,}000\text{ ksi})}{(86.5)^2}$$
$$= 38.3\text{ ksi}$$

Elastic Critical Torsional Buckling Stress

Note: Torsional buckling generally will not govern for doubly symmetric members if $L_{cy} \geq L_{cz}$; however, the check is included here to illustrate the calculation.

From the User Note in AISC *Specification* Section E4:

$$C_w = \frac{I_y h_o^2}{4}$$
$$= \frac{(85.4 \text{ in.}^4)(16.0 \text{ in.})^2}{4}$$
$$= 5{,}470 \text{ in.}^6$$

From AISC Design Guide 9, Equation 3.4:

$$J = \sum \frac{bt^3}{3}$$
$$= 2\left[\frac{(8.00 \text{ in.})(1.00 \text{ in.})^3}{3}\right] + \frac{(15.0 \text{ in.})(\text{¼ in.})^3}{3}$$
$$= 5.41 \text{ in.}^4$$

$$F_e = \left(\frac{\pi^2 E C_w}{L_{cz}^2} + GJ\right)\frac{1}{I_x + I_y} \qquad (\textit{Spec}.\text{ Eq. E4-2})$$
$$= \left\{\frac{\pi^2 (29{,}000 \text{ ksi})(5{,}470 \text{ in.}^6)}{[1.0(15 \text{ ft})(12 \text{ in./ft})]^2} + (11{,}200 \text{ ksi})(5.41 \text{ in.}^4)\right\}\left(\frac{1}{1{,}100 \text{ in.}^4 + 85.4 \text{ in.}^4}\right)$$
$$= 91.9 \text{ ksi} > 38.3 \text{ ksi}$$

Therefore, the flexural buckling limit state controls.

Use F_e = 38.3 ksi.

Flexural Buckling Stress

$$\frac{F_y}{F_e} = \frac{50 \text{ ksi}}{38.3 \text{ ksi}}$$
$$= 1.31$$

Because $\frac{F_y}{F_e} < 2.25$,

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \qquad (\textit{Spec}.\text{ Eq. E3-2})$$
$$= \left(0.658^{1.31}\right)(50 \text{ ksi})$$
$$= 28.9 \text{ ksi}$$

Slenderness

Check for slender flanges using AISC *Specification* Table B4.1a.

Calculate k_c using AISC *Specification* Table B4.1a, note [a].

$$k_c = \frac{4}{\sqrt{h/t_w}}$$
$$= \frac{4}{\sqrt{\dfrac{15.0\text{ in.}}{\frac{1}{4}\text{ in.}}}}$$
$$= 0.516,\ \text{which is between 0.35 and 0.76.}$$

For the flanges:

$$\lambda = \frac{b}{t}$$
$$= \frac{4.00\text{ in.}}{1.00\text{ in.}}$$
$$= 4.00$$

Determine the flange limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 2:

$$\lambda_r = 0.64\sqrt{\frac{k_c E}{F_y}}$$
$$= 0.64\sqrt{\frac{0.516(29{,}000\text{ ksi})}{50\text{ ksi}}}$$
$$= 11.1$$

Because $\lambda < \lambda_r$, the flanges are not slender and there is no reduction in effective area due to local buckling of the flanges.

Check for a slender web, and then determine the effective area for compression, A_e, using AISC *Specification* Section E7.1.

$$\lambda = \frac{h}{t_w}$$
$$= \frac{15.0\text{ in.}}{\frac{1}{4}\text{ in.}}$$
$$= 60.0$$

Determine the slender web limit from AISC *Specification* Table B4.1a, Case 5:

$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}}$$
$$= 1.49\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 35.9$$

Because $\lambda > \lambda_r$, the web is slender.

Determine the slenderness limit from AISC *Specification* Section E7.1 for a fully effective element:

$$\lambda_r \sqrt{\frac{F_y}{F_{cr}}} = 35.9\sqrt{\frac{50 \text{ ksi}}{28.9 \text{ ksi}}}$$
$$= 47.2$$

Because $\lambda > \lambda_r \sqrt{\frac{F_y}{F_{cr}}}$, the effective width is determined from AISC *Specification* Equation E7-3. Determine the effective width imperfection adjustment factors from AISC *Specification* Table E7.1, Case (a):

$$c_1 = 0.18$$
$$c_2 = 1.31$$

The elastic local buckling stress is:

$$F_{el} = \left(c_2 \frac{\lambda_r}{\lambda}\right)^2 F_y \qquad (\textit{Spec}. \text{ Eq. E7-5})$$
$$= \left[1.31\left(\frac{35.9}{60.0}\right)\right]^2 (50 \text{ ksi})$$
$$= 30.7 \text{ ksi}$$

Determine the effective width of the web and the resulting effective area:

$$h_e = h\left(1 - c_1\sqrt{\frac{F_{el}}{F_{cr}}}\right)\sqrt{\frac{F_{el}}{F_{cr}}} \qquad (\text{from } \textit{Spec}. \text{ Eq. E7-3})$$
$$= (15.0 \text{ in.})\left(1 - 0.18\sqrt{\frac{30.7 \text{ ksi}}{28.9 \text{ ksi}}}\right)\sqrt{\frac{30.7 \text{ ksi}}{28.9 \text{ ksi}}}$$
$$= 12.6 \text{ in.}$$

$$A_e = A_g - (h - h_e)t_w$$
$$= 19.8 \text{ in.}^2 - (15.0 \text{ in.} - 12.6 \text{ in.})(\tfrac{1}{4} \text{ in.})$$
$$= 19.2 \text{ in.}^2$$

Available Compressive Strength

$$P_n = F_{cr} A_e \qquad (\textit{Spec}. \text{ Eq. E7-1})$$
$$= (28.9 \text{ ksi})(19.2 \text{ in.}^2)$$
$$= 555 \text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(555 \text{ kips})$ $= 500 \text{ kips} > 420 \text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\frac{P_n}{\Omega_c} = \frac{555 \text{ kips}}{1.67}$ $= 332 \text{ kips} > 280 \text{ kips}$ **o.k.**

EXAMPLE E.3 BUILT-UP COLUMN WITH SLENDER FLANGES

Given:

Determine if a built-up, ASTM A572 Grade 50 column with PL⅜ in. × 10½ in. flanges and a PL¼ in. × 7¼ in. web, as shown in Figure E.3-1, has sufficient available strength to carry a dead load of 40 kips and a live load of 120 kips in axial compression. The column's unbraced length is 15 ft and the ends are pinned in both axes.

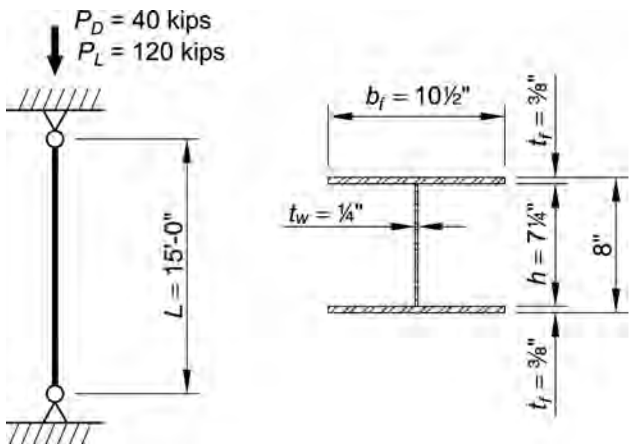


Fig. E.3-1. Column geometry for Example E.3.

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

Built-Up Column
ASTM A572 Grade 50
F_y = 50 ksi
F_u = 65 ksi

The geometric properties are as follows:

Built-Up Column
d = 8.00 in.
b_f = 10½ in.
t_f = ⅜ in.
h = 7¼ in.
t_w = ¼ in.

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(40 \text{ kips}) + 1.6(120 \text{ kips})$ $= 240 \text{ kips}$	$P_a = 40 \text{ kips} + 120 \text{ kips}$ $= 160 \text{ kips}$

Built-Up Section Properties (ignoring fillet welds)

$$A_g = 2(10\tfrac{1}{2} \text{ in.})(\tfrac{3}{8} \text{ in.}) + (7\tfrac{1}{4} \text{ in.})(\tfrac{1}{4} \text{ in.})$$
$$= 9.69 \text{ in.}^2$$

Because the unbraced length is the same for both axes, the weak axis will govern.

$$I_y = \sum \frac{bh^3}{12}$$
$$= 2\left[\frac{(\tfrac{3}{8}\text{ in.})(10\tfrac{1}{2}\text{ in.})^3}{12}\right] + \frac{(7\tfrac{1}{4}\text{ in.})(\tfrac{1}{4}\text{ in.})^3}{12}$$
$$= 72.4\text{ in.}^4$$

$$r_y = \sqrt{\frac{I_y}{A_g}}$$
$$= \sqrt{\frac{72.4\text{ in.}^4}{9.69\text{ in.}^2}}$$
$$= 2.73\text{ in.}$$

$$I_x = \sum Ad^2 + \sum \frac{bh^3}{12}$$
$$= 2\left[(10\tfrac{1}{2}\text{ in.})(\tfrac{3}{8}\text{ in.})(3.81\text{ in.})^2\right] + \frac{(\tfrac{1}{4}\text{ in.})(7\tfrac{1}{4}\text{ in.})^3}{12} + 2\left[\frac{(10\tfrac{1}{2}\text{ in.})(\tfrac{3}{8}\text{ in.})^3}{12}\right]$$
$$= 122\text{ in.}^4$$

Web Slenderness

Determine the limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 5:

$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}}$$
$$= 1.49\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 35.9$$

$$\lambda = \frac{h}{t_w}$$
$$= \frac{7\tfrac{1}{4}\text{ in.}}{\tfrac{1}{4}\text{ in.}}$$
$$= 29.0$$

Because $\lambda < \lambda_r$, the web is not slender.

Note that the fillet welds are ignored in the calculation of h for built up sections.

Flange Slenderness

Calculate k_c using AISC *Specification* Table B4.1a, note [a]:

$$k_c = \frac{4}{\sqrt{h/t_w}} = \frac{4}{\sqrt{\frac{7\tfrac{1}{4}\text{ in.}}{\tfrac{1}{4}\text{ in.}}}} = 0.743, \text{ which is between 0.35 and 0.76}$$

Determine the limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 2:

$$\lambda_r = 0.64\sqrt{\frac{k_c E}{F_y}} = 0.64\sqrt{\frac{0.743(29{,}000\text{ ksi})}{50\text{ ksi}}} = 13.3$$

$$\lambda = \frac{b}{t} = \frac{5.25\text{ in.}}{\tfrac{3}{8}\text{ in.}} = 14.0$$

Because $\lambda > \lambda_r$, the flanges are slender.

For compression members with slender elements, AISC *Specification* Section E7 applies. The nominal compressive strength, P_n, is determined based on the limit states of flexural, torsional and flexural-torsional buckling. Depending on the slenderness of the column, AISC *Specification* Equation E3-2 or E3-3 applies. F_e is used in both equations and is calculated as the lesser of AISC *Specification* Equations E3-4 and E4-2.

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

Because the unbraced length is the same for both axes, the weak axis will govern. With $L_{cy} = K_y L_y$ = 1.0(15 ft) = 15.0 ft:

$$\frac{L_{cy}}{r_y} = \frac{(15.0\text{ ft})(12\text{ in./ft})}{2.73\text{ in.}} = 65.9$$

Elastic Critical Stress, F_e, for Flexural Buckling

$$F_e = \frac{\pi^2 E}{\left(\frac{L_{cy}}{r_y}\right)^2} = \frac{\pi^2(29{,}000\text{ ksi})}{(65.9)^2} = 65.9\text{ ksi}$$ (from *Spec*. Eq. E3-4)

Elastic Critical Stress, F_e, for Torsional Buckling

Note: This limit state is not likely to govern, but the check is included here for completeness.

From the User Note in AISC *Specification* Section E4:

$$C_w = \frac{I_y h_o^2}{4}$$
$$= \frac{\left(72.4\text{ in.}^4\right)\left(7.63\text{ in.}\right)^2}{4}$$
$$= 1{,}050\text{ in.}^6$$

From AISC Design Guide 9, Equation 3.4:

$$J = \sum\frac{bt^3}{3}$$
$$= \frac{2\left(10\tfrac{1}{2}\text{ in.}\right)\left(\tfrac{3}{8}\text{ in.}\right)^3 + \left(7\tfrac{1}{4}\text{ in.}\right)\left(\tfrac{1}{4}\text{ in.}\right)^3}{3}$$
$$= 0.407\text{ in.}^4$$

With $L_{cz} = K_z L_z = 1.0(15\text{ ft}) = 15\text{ ft}$:

$$F_e = \left(\frac{\pi^2 E C_w}{L_{cz}^2} + GJ\right)\frac{1}{I_x + I_y} \qquad (\textit{Spec}.\text{ Eq. E4-2})$$
$$= \left\{\frac{\pi^2\left(29{,}000\text{ ksi}\right)\left(1{,}050\text{ in.}^6\right)}{\left[\left(15\text{ ft}\right)\left(12\text{ in./ft}\right)\right]^2} + \left(11{,}200\text{ ksi}\right)\left(0.407\text{ in.}^4\right)\right\}\left(\frac{1}{122\text{ in.}^4 + 72.4\text{ in.}^4}\right)$$
$$= 71.2\text{ ksi} > 65.9\text{ ksi}$$

Therefore, use $F_e = 65.9$ ksi.

Flexural Buckling Stress

$$\frac{F_y}{F_e} = \frac{50\text{ ksi}}{65.9\text{ ksi}}$$
$$= 0.759$$

Because $\frac{F_y}{F_e} < 2.25$:

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \qquad (\textit{Spec}.\text{ Eq. E3-2})$$
$$= \left(0.658^{0.759}\right)\left(50\text{ ksi}\right)$$
$$= 36.4\text{ ksi}$$

Effective Area, A_e

The effective area, A_e, is the summation of the effective areas of the cross section based on the reduced effective widths, b_e or h_e. Since the web is nonslender, there is no reduction in the effective area due to web local buckling and $h_e = h$.

Determine the slender web limit from AISC *Specification* Section E7.1.

$$\lambda_r \sqrt{\frac{F_y}{F_{cr}}} = 13.3\sqrt{\frac{50 \text{ ksi}}{36.4 \text{ ksi}}}$$
$$= 15.6$$

Because $\lambda < \lambda_r \sqrt{\frac{F_y}{F_{cr}}}$ for all elements,

$$b_e = b \qquad \text{(Spec. Eq. E7-2)}$$

Therefore, $A_e = A_g$.

Available Compressive Strength

$$P_n = F_{cr} A_e$$
$$= (36.4 \text{ ksi})(9.69 \text{ in.}^2) \qquad \text{(Spec. Eq. E7-1)}$$
$$= 353 \text{ kips}$$

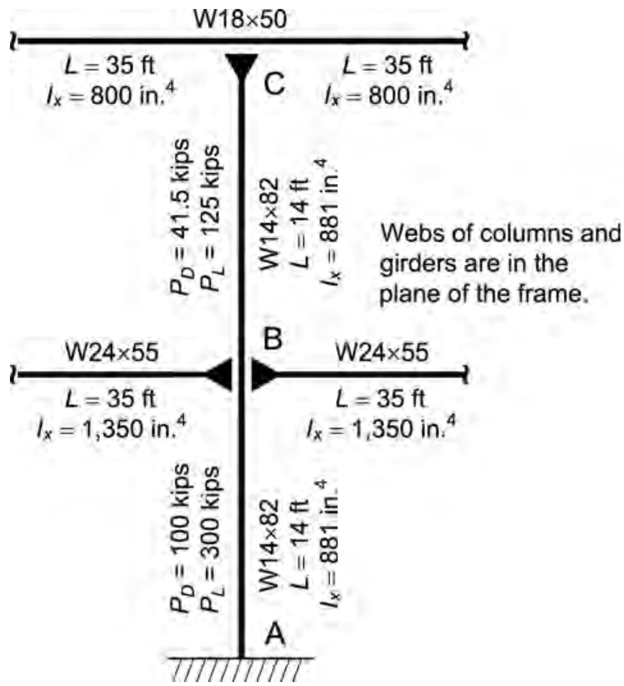
From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(353 \text{ kips})$ $= 318 \text{ kips} > 240 \text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\frac{P_n}{\Omega_c} = \frac{353 \text{ kips}}{1.67}$ $= 211 \text{ kips} > 160 \text{ kips}$ **o.k.**

Note: Built-up sections are generally more expensive than standard rolled shapes; therefore, a standard compact shape, such as a W8×35 might be a better choice even if the weight is somewhat higher. This selection could be taken directly from AISC *Manual* Table 4-1a.

EXAMPLE E.4A W-SHAPE COMPRESSION MEMBER (MOMENT FRAME)

This example is primarily intended to illustrate the use of the alignment chart for sidesway uninhibited columns in conjunction with the effective length method.



Given:

The member sizes shown for the moment frame illustrated here (sidesway uninhibited in the plane of the frame) have been determined to be adequate for lateral loads. The material for both the column and the girders is ASTM A992. The loads shown at each level are the accumulated dead loads and live loads at that story. The column is fixed at the base about the *x-x* axis of the column.

Determine if the column is adequate to support the gravity loads shown. Assume the column is continuously supported in the transverse direction (the *y-y* axis of the column).

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
I_x = 800 in.^{4}

W24×55
I_x = 1,350 in.^{4}

W14×82
A_g = 24.0 in.^{2}
I_x = 881 in.^{4}

Column B-C

From ASCE/SEI 7, Chapter 2, the required compressive strength for the column between the roof and floor is:

LRFD	ASD
$P_u = 1.2(41.5\text{ kips}) + 1.6(125\text{ kips})$ $= 250\text{ kips}$	$P_a = 41.5\text{ kips} + 125\text{ kips}$ $= 167\text{ kips}$

Effective Length Factor

Using the effective length method, the effective length factor is determined using AISC *Specification* Commentary Appendix 7, Section 7.2. As discussed there, column inelasticity should be addressed by incorporating the stiffness reduction parameter, τ_b. Determine G_{top} and G_{bottom} accounting for column inelasticity by replacing $E_{col}I_{col}$ with $\tau_b(E_{col}I_{col})$. Calculate the stiffness reduction parameter, τ_b, for the column B-C using AISC *Manual* Table 4-13.

LRFD	ASD
$\frac{P_u}{A_g}=\frac{250\text{ kips}}{24.0\text{ in.}^2}$ $=10.4\text{ ksi}$ $\tau_b=1.00$	$\frac{P_a}{A_g}=\frac{167\text{ kips}}{24.0\text{ in.}^2}$ $=6.96\text{ ksi}$ $\tau_b=1.00$

Therefore, no reduction in stiffness for inelastic buckling will be required.

Determine G_{top} and G_{bottom}.

$$G_{top}=\tau_b\left[\frac{\sum(EI/L)_{col}}{\sum(EI/L)_g}\right] \qquad \text{(from } Spec. \text{ Comm. Eq. C-A-7-3)}$$

$$=1.00\left\{\frac{\left[\frac{(29{,}000\text{ ksi})(881\text{ in.}^4)}{14.0\text{ ft}}\right]}{2\left[\frac{(29{,}000\text{ ksi})(800\text{ in.}^4)}{35.0\text{ ft}}\right]}\right\}$$

$$=1.38$$

$$G_{bottom}=\tau_b\left[\frac{\sum(EI/L)_{col}}{\sum(EI/L)_g}\right] \qquad \text{(from } Spec. \text{ Comm. Eq. C-A-7-3)}$$

$$=1.00\left\{\frac{2\left[\frac{(29{,}000\text{ ksi})(881\text{ in.}^4)}{14.0\text{ ft}}\right]}{2\left[\frac{(29{,}000\text{ ksi})(1{,}350\text{ in.}^4)}{35.0\text{ ft}}\right]}\right\}$$

$$=1.63$$

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, K is slightly less than 1.5; therefore use K = 1.5. Because the column available strength tables are based on the L_c about the y-y axis, the equivalent effective column length of the upper segment for use in the table is:

$$L_{cx}=(KL)_x$$
$$=1.5(14\text{ ft})$$
$$=21.0\text{ ft}$$

From AISC *Manual* Table 4-1a, for a W14×82:

$$\frac{r_x}{r_y} = 2.44$$

$$L_c = \frac{L_{cx}}{\left(\dfrac{r_x}{r_y}\right)} = \frac{21.0 \text{ ft}}{2.44} = 8.61 \text{ ft}$$

Take the available strength of the W14×82 from AISC *Manual* Table 4-1a.

At L_c = 9 ft, the available strength in axial compression is:

LRFD	ASD
$\phi_c P_n = 940 \text{ kips} > 250 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 626 \text{ kips} > 167 \text{ kips}$ **o.k.**

Column A-B

From Chapter 2 of ASCE/SEI 7, the required compressive strength for the column between the floor and the foundation is:

LRFD	ASD
$P_u = 1.2(100 \text{ kips}) + 1.6(300 \text{ kips})$ $= 600 \text{ kips}$	$P_a = 100 \text{ kips} + 300 \text{ kips}$ $= 400 \text{ kips}$

Effective Length Factor

Determine the stiffness reduction parameter, τ_b, for column A-B using AISC *Manual* Table 4-13.

LRFD	ASD
$\dfrac{P_u}{A_g} = \dfrac{600 \text{ kips}}{24.0 \text{ in.}^2} = 25.0 \text{ ksi}$ $\tau_b = 1.00$	$\dfrac{P_a}{A_g} = \dfrac{400 \text{ kips}}{24.0 \text{ in.}^2} = 16.7 \text{ ksi}$ $\tau_b = 0.994$

Use τ_b = 0.994.

$$G_{top} = \tau_b \left[\frac{\Sigma(EI/L)_{col}}{\Sigma(EI/L)_g} \right] \qquad \text{(from } \textit{Spec.} \text{ Comm. Eq. C-A-7-3)}$$

$$= 0.994 \left\{ \frac{2\left[\dfrac{(29{,}000 \text{ ksi})(881 \text{ in.}^4)}{14.0 \text{ ft}} \right]}{2\left[\dfrac{(29{,}000 \text{ ksi})(1{,}350 \text{ in.}^4)}{35.0 \text{ ft}} \right]} \right\}$$

$$= 1.62$$

$G_{bottom} = 1.0$ (fixed), from AISC *Specification* Commentary Appendix 7, Section 7.2

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, K is approximately 1.4. Because the column available strength tables are based on L_c about the y-y axis, the effective column length of the lower segment for use in the table is:

$$L_{cx} = (KL)_x$$
$$= 1.4(14 \text{ ft})$$
$$= 19.6 \text{ ft}$$

$$L_c = \frac{L_{cx}}{\left(\dfrac{r_x}{r_y}\right)}$$
$$= \frac{19.6 \text{ ft}}{2.44}$$
$$= 8.03 \text{ ft}$$

Take the available strength of the W14×82 from AISC *Manual* Table 4-1a.

At $L_c = 9$ ft, (conservative) the available strength in axial compression is:

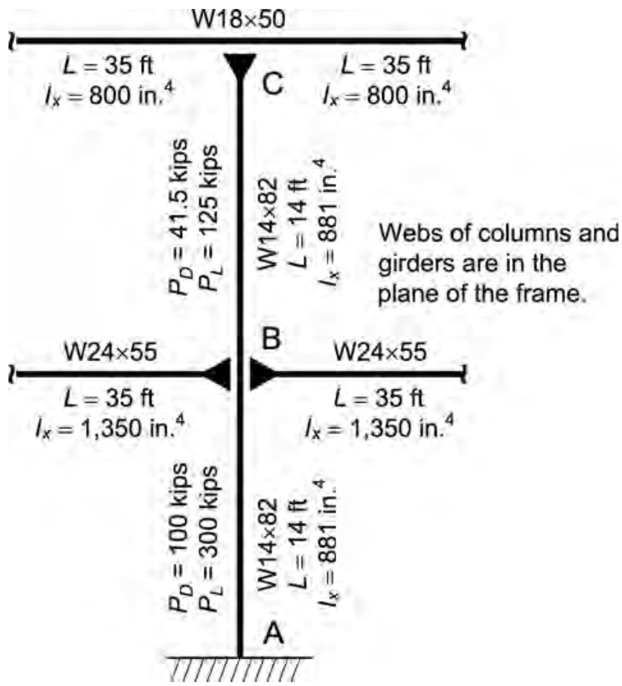
LRFD	ASD
$\phi_c P_n = 940 \text{ kips} > 600 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 626 \text{ kips} > 400 \text{ kips}$ **o.k.**

A more accurate strength could be determined by interpolation from AISC *Manual* Table 4-1a.

EXAMPLE E.4B W-SHAPE COMPRESSION MEMBER (MOMENT FRAME)

Given:

Using the effective length method, determine the available strength of the column shown subject to the same gravity loads shown in Example E.4A with the column pinned at the base about the *x-x* axis. All other assumptions remain the same.



Solution:

As determined in Example E.4A, for the column segment B-C between the roof and the floor, the column strength is adequate.

As determined in Example E.4A, for the column segment A-B between the floor and the foundation,

$$G_{top} = 1.62$$

At the base,

$G_{bottom} = 10$ (pinned) from AISC *Specification* Commentary Appendix 7, Section 7.2

Note: this is the only change in the analysis.

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, K is approximately equal to 2.0. Because the column available strength tables are based on the effective length, L_c, about the *y-y* axis, the effective column length of the segment A-B for use in the table is:

$$\begin{aligned} L_{cx} &= (KL)_x \\ &= 2.0(14 \text{ ft}) \\ &= 28.0 \text{ ft} \end{aligned}$$

From AISC *Manual* Table 4-1a, for a W14×82:

$$\frac{r_x}{r_y} = 2.44$$

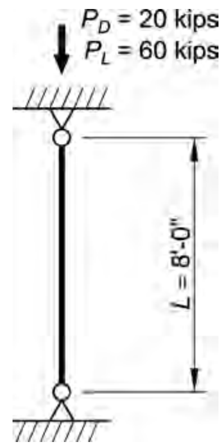
$$L_c = \frac{L_{cx}}{\left(\frac{r_x}{r_y}\right)}$$
$$= \frac{28.0 \text{ ft}}{2.44}$$
$$= 11.5 \text{ ft}$$

Interpolate the available strength of the W14×82 from AISC *Manual* Table 4-1a.

LRFD	ASD
$\phi_c P_n = 861 \text{ kips} > 600 \text{ kips}$ **o.k.**	$\frac{P_n}{\Omega_c} = 573 \text{ kips} > 400 \text{ kips}$ **o.k.**

EXAMPLE E.5 DOUBLE-ANGLE COMPRESSION MEMBER WITHOUT SLENDER ELEMENTS

Given:



Verify the strength of a 2L4×3½×⅜ LLBB (¾-in. separation) strut, ASTM A36, with a length of 8 ft and pinned ends carrying an axial dead load of 20 kips and live load of 60 kips. Also, calculate the required number of pretensioned bolted or welded intermediate connectors required. The solution will be provided using:

(1) AISC *Manual* Tables
(2) Calculations using AISC *Specification* provisions

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-7 and 1-15, the geometric properties are as follows:

L4×3½×⅜
r_z = 0.719 in.

2L4×3½×⅜ LLBB
r_x = 1.25 in.
r_y = 1.55 in. for ⅜-in. separation
r_y = 1.69 in. for ¾-in. separation

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(20 \text{ kips}) + 1.6(60 \text{ kips})$ $= 120 \text{ kips}$	$P_a = 20 \text{ kips} + 60 \text{ kips}$ $= 80.0 \text{ kips}$

(1) AISC *Manual* Table Solution

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0. Therefore, $L_{cx} = L_{cy} = KL$ = 1.0(8 ft) = 8.00 ft. The available strength in axial compression is taken from the upper (X-X Axis) portion of AISC *Manual* Table 4-9:

LRFD	ASD
$\phi_c P_n = 127 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 84.7 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

For buckling about the y-y axis, the values are tabulated for a separation of ⅜ in.

To adjust to a spacing of ¾ in., L_{cy} is multiplied by the ratio of the r_y for a ⅜-in. separation to the r_y for a ¾-in. separation, where $L_{cy} = K_y L_y$ = 1.0(8 ft) = 8.00 ft . Thus:

$$L_{cy} = (8.00 \text{ ft})\left(\frac{1.55 \text{ in.}}{1.69 \text{ in.}}\right)$$
$$= 7.34 \text{ ft}$$

The calculation of the equivalent L_{cy} in the preceding text is a simplified approximation of AISC *Specification* Section E6.1. To ensure a conservative adjustment for a ¾-in. separation, take L_{cy} = 8 ft. The available strength in axial compression is taken from the lower (Y-Y Axis) portion of AISC *Manual* Table 4-9 as:

LRFD	ASD
$\phi_c P_n = 132 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 87.9 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

Therefore, *x-x* axis flexural buckling governs.

Intermediate Connectors

From AISC *Manual* Table 4-9, at least two welded or pretensioned bolted intermediate connectors are required. This can be verified as follows:

$$\begin{aligned} a &= \text{distance between connectors} \\ &= \frac{(8.00 \text{ ft})(12 \text{ in./ft})}{3 \text{ spaces}} \\ &= 32.0 \text{ in.} \end{aligned}$$

From AISC *Specification* Section E6.2, the effective slenderness ratio of the individual components of the built-up member based upon the distance between intermediate connectors, a, must not exceed three-fourths of the governing slenderness ratio of the built-up member.

Therefore, $\dfrac{a}{r_i} \le \dfrac{3}{4}\left(\dfrac{L_c}{r}\right)_{max}$.

Solving for a gives:

$$a \le \frac{3r_i\left(\dfrac{L_c}{r}\right)_{max}}{4}$$

$$\begin{aligned} \frac{L_{cx}}{r_x} &= \frac{(8.00 \text{ ft})(12 \text{ in./ft})}{1.25 \text{ in.}} \\ &= 76.8 \quad \textbf{controls} \end{aligned}$$

$$\begin{aligned} \frac{L_{cy}}{r_y} &= \frac{(8.00 \text{ ft})(12 \text{ in./ft})}{1.69 \text{ in.}} \\ &= 56.8 \end{aligned}$$

$$\begin{aligned} a &= \frac{3r_z\left(\dfrac{L_c}{r}\right)_{max}}{4} \\ &= \frac{3(0.719 \text{ in.})(76.8)}{4} \\ &= 41.4 \text{ in.} \end{aligned}$$

Therefore, two welded or pretensioned bolted connectors are adequate since 32.0 in. < 41.4 in.

Note that one connector would not be adequate as 48.0 in. > 41.4 in. Available strength can also be determined by hand calculations, as demonstrated in the following.

(2) Calculations Using AISC *Specification* Provisions

From AISC *Manual* Tables 1-7 and 1-15, the geometric properties are as follows:

L4×3½×⅜
$J = 0.132 \text{ in.}^4$

2L4×3½×⅜ LLBB (¾ in. separation)
$A_g = 5.36 \text{ in.}^2$
$r_y = 1.69 \text{ in.}$
$\bar{r}_o = 2.33 \text{ in.}$
$H = 0.813$

Slenderness Check

$$\lambda = \frac{b}{t} = \frac{4.00 \text{ in.}}{3/8 \text{ in.}} = 10.7$$

Determine the limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 3:

$$\lambda_r = 0.45\sqrt{\frac{E}{F_y}} = 0.45\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}} = 12.8$$

$\lambda < \lambda_r$; therefore, there are no slender elements.

For double-angle compression members without slender elements, AISC *Specification* Sections E3, E4 and E6 apply.

The nominal compressive strength, P_n, is determined based on the limit states of flexural, torsional and flexural-torsional buckling.

Flexural Buckling about the x-x Axis

$$\frac{L_{cx}}{r_x} = \frac{(8.00 \text{ ft})(12 \text{ in./ft})}{1.25 \text{ in.}} = 76.8$$

$$F_{ex} = \frac{\pi^2 E}{\left(\frac{L_{cx}}{r_x}\right)^2} \qquad \text{(Spec. Eq. E4-5)}$$

$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{(76.8)^2}$$

$$= 48.5 \text{ ksi}$$

Flexural Buckling about the y-y Axis

$$\frac{L_{cy}}{r_y} = \frac{(8.00 \text{ ft})(12 \text{ in./ft})}{1.69 \text{ in.}}$$

$$= 56.8$$

Using AISC *Specification* Section E6, compute the modified L_c/r for built up members with pretensioned bolted or welded connectors. Assume two connectors are required.

$$a = \frac{(8.00 \text{ ft})(12 \text{ in./ft})}{3}$$

$$= 32.0 \text{ in.}$$

$$r_i = r_z \text{ (single angle)}$$

$$= 0.719 \text{ in.}$$

$$\frac{a}{r_i} = \frac{32.0 \text{ in.}}{0.719 \text{ in.}}$$

$$= 44.5 > 40$$

Therefore:

$$\left(\frac{L_c}{r}\right)_m = \sqrt{\left(\frac{L_c}{r}\right)_o^2 + \left(\frac{K_i a}{r_i}\right)^2} \qquad \text{(Spec. Eq. E6-2b)}$$

where K_i = 0.50 for angles back-to-back

$$\left(\frac{L_c}{r}\right)_m = \sqrt{(56.8)^2 + \left[\frac{0.50(32.0 \text{ in.})}{0.719 \text{ in.}}\right]^2}$$

$$= 61.0$$

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{L_{cy}}{r_y}\right)^2} \qquad \text{(Spec. Eq. E4-6)}$$

$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{(61.0)^2}$$

$$= 76.9 \text{ ksi}$$

Torsional and Flexural-Torsional Buckling

For nonslender double-angle compression members, AISC *Specification* Equation E4-3 applies. Per the User Note for AISC *Specification* Section E4, the term with C_w is omitted when computing F_{ez} and x_o is taken as zero. The flexural buckling term about the *y-y* axis, F_{ey}, was computed in the preceding section.

$$F_{ez} = \left(\frac{\pi^2 EC_w}{L_{cz}^2} + GJ\right)\frac{1}{A_g \bar{r}_o^2} \qquad (Spec. \text{ Eq. E4-7})$$

$$= \left[0 + (11{,}200 \text{ ksi})(0.132 \text{ in.}^4)(2 \text{ angles})\right]\frac{1}{(5.36 \text{ in.}^2)(2.33 \text{ in.})^2}$$

$$= 102 \text{ ksi}$$

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right] \qquad (Spec. \text{ Eq. E4-3})$$

$$= \left[\frac{76.9 \text{ ksi} + 102 \text{ ksi}}{2(0.813)}\right]\left[1 - \sqrt{1 - \frac{4(76.9 \text{ ksi})(102 \text{ ksi})(0.813)}{(76.9 \text{ ksi} + 102 \text{ ksi})^2}}\right]$$

$$= 60.5 \text{ ksi}$$

Critical Buckling Stress

The critical buckling stress for the member could be controlled by flexural buckling about either the *x-x* axis or *y-y* axis, F_{ex} or F_{ey}, respectively. Note that AISC *Specification* Equations E4-5 and E4-6 reflect the same buckling modes as calculated in AISC *Specification* Equation E3-4. Or, the critical buckling stress for the member could be controlled by torsional or flexural-torsional buckling calculated per AISC *Specification* Equation E4-3. In this example, F_e calculated in accordance with AISC *Specification* Equation E4-5 (or Equation E3-4) is less than that calculated in accordance with AISC *Specification* Equation E4-3 or E4-6, and controls. Therefore:

$$F_e = 48.5 \text{ ksi}$$

$$\frac{F_y}{F_e} = \frac{36 \text{ ksi}}{48.5 \text{ ksi}}$$

$$= 0.742$$

Per the AISC *Specification* User Note for Section E3, the two inequalities for calculating limits of applicability of Sections E3(a) and E3(b) provide the same result for flexural buckling only. When the elastic buckling stress, F_e, is controlled by torsional or flexural-torsional buckling, the L_c/r limits would not be applicable unless an equivalent L_c/r ratio is first calculated by substituting the governing F_e into AISC *Specification* Equation E3-4 and solving for L_c/r. The F_y/F_e limits may be used regardless of which buckling mode governs.

Because $\frac{F_y}{F_e} < 2.25$:

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \qquad (Spec. \text{ Eq. E3-2})$$

$$= \left(0.658^{0.742}\right)(36 \text{ ksi})$$

$$= 26.4 \text{ ksi}$$

Available Compressive Strength

$$P_n = F_{cr} A_g$$ (*Spec*. Eq. E3-1, Eq. E4-1)

$$= (26.4 \text{ ksi})(5.36 \text{ in.}^2)$$

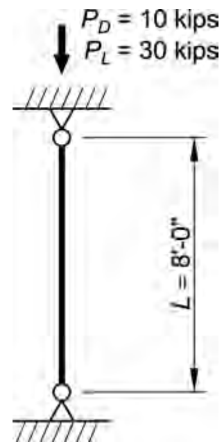
$$= 142 \text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(142 \text{ kips})$ $= 128 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\dfrac{P_n}{\Omega_c} = \dfrac{142 \text{ kips}}{1.67}$ $= 85.0 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

EXAMPLE E.6 DOUBLE-ANGLE COMPRESSION MEMBER WITH SLENDER ELEMENTS

Given:



Determine if a 2L5×3×¼ LLBB (¾-in. separation) strut, ASTM A36, with a length of 8 ft and pinned ends has sufficient available strength to support a dead load of 10 kips and live load of 30 kips in axial compression. Also, calculate the required number of pretensioned bolted or welded intermediate connectors. The solution will be provided using:

(1) AISC *Manual* Tables
(2) Calculations using AISC *Specification* provisions

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-7 and 1-15, the geometric properties are as follows:

L5×3×¼
r_z = 0.652 in.

2L5×3×¼ LLBB
r_x = 1.62 in.
r_y = 1.19 in. for ⅜-in. separation
r_y = 1.33 in. for ¾-in. separation

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(10\text{ kips}) + 1.6(30\text{ kips})$ $= 60.0\text{ kips}$	$P_a = 10\text{ kips} + 30\text{ kips}$ $= 40.0\text{ kips}$

(1) AISC *Manual* Table Solution

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0. Therefore, $L_{cx} = L_{cy} = KL$ = 1.0(8 ft) = 8.00 ft. The available strength in axial compression is taken from the upper (X-X Axis) portion of AISC *Manual* Table 4-9:

LRFD	ASD
$\phi_c P_{nx} = 91.2\text{ kips} > 60.0\text{ kips}$ **o.k.**	$\dfrac{P_{nx}}{\Omega_c} = 60.7\text{ kips} > 40.0\text{ kips}$ **o.k.**

For buckling about the y-y axis, the tabulated values are based on a separation of ⅜ in. To adjust for a spacing of ¾ in., L_{cy} is multiplied by the ratio of r_y for a ⅜-in. separation to r_y for a ¾-in. separation.

$$L_{cy} = (8.00\text{ ft})\left(\frac{1.19\text{ in.}}{1.33\text{ in.}}\right)$$
$$= 7.16\text{ ft}$$

This calculation of the equivalent L_{cy} does not completely take into account the effect of AISC *Specification* Section E6.1 and is slightly unconservative.

From the lower portion of AISC *Manual* Table 4-9, interpolate for a value at L_{cy} = 7.16 ft.

The available strength in compression is:

LRFD	ASD
$\phi_c P_{ny} = 68.3 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\dfrac{P_{ny}}{\Omega_c} = 45.4 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

These strengths are approximate due to the linear interpolation from the table and the approximate value of the equivalent L_{cy} noted in the preceding text. These can be compared to the more accurate values calculated in detail as follows.

Intermediate Connectors

From AISC *Manual* Table 4-9, it is determined that at least two welded or pretensioned bolted intermediate connectors are required. This can be confirmed by calculation, as follows:

$$\begin{aligned} a &= \text{distance between connectors} \\ &= \frac{(8.00 \text{ ft})(12 \text{ in./ft})}{3 \text{ spaces}} \\ &= 32.0 \text{ in.} \end{aligned}$$

From AISC *Specification* Section E6.2, the effective slenderness ratio of the individual components of the built-up member based upon the distance between intermediate connectors, a, must not exceed three-fourths of the governing slenderness ratio of the built-up member.

Therefore, $\dfrac{a}{r_i} \leq \dfrac{3}{4}\left(\dfrac{L_c}{r}\right)_{max}$.

Solving for a gives:

$$a \leq \frac{3r_i\left(\dfrac{L_c}{r}\right)_{max}}{4}$$

$$\begin{aligned} r_i &= r_z \\ &= 0.652 \text{ in.} \end{aligned}$$

$$\begin{aligned} \frac{L_{cx}}{r_x} &= \frac{(8.00 \text{ ft})(12 \text{ in./ft})}{1.62 \text{ in.}} \\ &= 59.3 \end{aligned}$$

$$\begin{aligned} \frac{L_{cy}}{r_y} &= \frac{(8.00 \text{ ft})(12 \text{ in./ft})}{1.33 \text{ in.}} \\ &= 72.2 \quad \textbf{controls} \end{aligned}$$

$$a = \frac{3r_z\left(\frac{L_c}{r}\right)_{max}}{4}$$
$$= \frac{3(0.652\text{ in.})(72.2)}{4}$$
$$= 35.3\text{ in.}$$

Therefore, two welded or pretensioned bolted connectors are adequate since 32.0 in. < 35.3 in.

Available strength can also be determined by hand calculations, as determined in the following.

(2) Calculations Using AISC *Specification* Provisions

From AISC *Manual* Tables 1-7 and 1-15, the geometric properties are as follows.

L5×3×¼
$J = 0.0438\text{ in.}^4$
$r_z = 0.652\text{ in.}$

2L5×3×¼ LLBB
$A_g = 3.88\text{ in.}^2$
$r_x = 1.62\text{ in.}$
$r_y = 1.33$ in. for ¾-in. separation
$\bar{r}_o = 2.59\text{ in.}$
$H = 0.657$

Slenderness Check

For the 5-in. leg:

$$\lambda = \frac{b}{t}$$
$$= \frac{5.00\text{ in.}}{1/4\text{ in.}}$$
$$= 20.0$$

For the 3-in. leg:

$$\lambda = \frac{b}{t}$$
$$= \frac{3.00\text{ in.}}{1/4\text{ in.}}$$
$$= 12.0$$

Calculate the limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 3:

$$\lambda_r = 0.45\sqrt{\frac{E}{F_y}}$$
$$= 0.45\sqrt{\frac{29{,}000\text{ ksi}}{36\text{ ksi}}}$$
$$= 12.8$$

For the longer leg, $\lambda > \lambda_r$, and therefore it is classified as a slender element. For the shorter leg, $\lambda < \lambda_r$, and therefore it is classified as a nonslender element.

For a double-angle compression member with slender elements, AISC *Specification* Section E7 applies. The nominal compressive strength, P_n, is determined based on the limit states of flexural, torsional and flexural-torsional buckling. A_e will be determined by AISC *Specification* Section E7.1.

Elastic Buckling Stress about the x-x Axis

With $L_{cx} = K_xL_x = 1.0(8 \text{ ft}) = 8.00 \text{ ft}$:

$$\frac{L_{cx}}{r_x} = \frac{(8.00 \text{ ft})(12 \text{ in./ft})}{1.62 \text{ in.}}$$
$$= 59.3$$

$$F_{ex} = \frac{\pi^2 E}{\left(\frac{L_{cx}}{r_x}\right)^2} \quad \text{(Spec. Eq. 3-4 or E4-5)}$$
$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{(59.3)^2}$$
$$= 81.4$$

Elastic Buckling Stress about the y-y Axis

With $L_{cy} = K_yL_y = 1.0(8 \text{ ft}) = 8.00 \text{ ft}$:

$$\frac{L_{cy}}{r_y} = \frac{(8.00 \text{ ft})(12 \text{ in./ft})}{1.33 \text{ in.}}$$
$$= 72.2$$

Using AISC *Specification* Section E6, compute the modified L_{cy}/r_y for built-up members with pretensioned bolted or welded connectors. Assuming two connectors are required:

$$a = \frac{(8.00 \text{ ft})(12 \text{ in./ft})}{3}$$
$$= 32.0 \text{ in.}$$

$$r_i = r_z \text{ (single angle)}$$
$$= 0.652 \text{ in.}$$

$$\frac{a}{r_i} = \frac{32.0 \text{ in.}}{0.652 \text{ in.}}$$
$$= 49.1 > 40$$

Therefore:

$$\left(\frac{L_c}{r}\right)_m = \sqrt{\left(\frac{L_c}{r}\right)_o^2 + \left(\frac{K_i a}{r_i}\right)^2} \quad \text{(Spec. Eq. E6-2b)}$$

where K_i = 0.50 for angles back-to-back

$$\left(\frac{L_c}{r}\right)_m = \sqrt{(72.2)^2 + \left[\frac{0.50(32.0\text{ in.})}{0.652\text{ in.}}\right]^2}$$
$$= 76.3$$

$$F_{ey} = \frac{\pi^2 E}{\left(\dfrac{L_{cy}}{r_y}\right)^2} \qquad \text{(}Spec\text{. Eq. E3-4 or E4-6)}$$
$$= \frac{\pi^2(29{,}000\text{ ksi})}{(76.3)^2}$$
$$= 49.2\text{ ksi}$$

Torsional and Flexural-Torsional Elastic Buckling Stress

Per the User Note in AISC *Specification* Section E4, the term with C_w is omitted when computing F_{ez}, and x_o is taken as zero. The flexural buckling term about the *y-y* axis, F_{ey}, was computed in the preceding section.

$$F_{ez} = \left(\frac{\pi^2 EC_w}{L_{cz}^2} + GJ\right)\frac{1}{A_g \bar{r}_o^2} \qquad \text{(}Spec\text{. Eq. E4-7)}$$
$$= \left[0 + (11{,}200\text{ ksi})(0.0438\text{ in.}^4)(2\text{ angles})\right]\frac{1}{(3.88\text{ in.}^2)(2.59\text{ in.})^2}$$
$$= 37.7\text{ ksi}$$

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right] \qquad \text{(}Spec\text{. Eq. E4-3)}$$
$$= \left[\frac{49.2\text{ ksi} + 37.7\text{ ksi}}{2(0.657)}\right]\left[1 - \sqrt{1 - \frac{4(49.2\text{ ksi})(37.7\text{ ksi})(0.657)}{(49.2\text{ ksi} + 37.7\text{ ksi})^2}}\right]$$
$$= 26.8\text{ ksi} \quad \textbf{controls}$$

Critical Buckling Stress

The critical buckling stress for the member could be controlled by flexural buckling about either the *x-x* axis or *y-y* axis, F_{ex} or F_{ey}, respectively. Note that AISC *Specification* Equations E4-5 and E4-6 reflect the same buckling modes as calculated in AISC *Specification* Equation E3-4. Or, the critical buckling stress for the member could be controlled by torsional or flexural-torsional buckling calculated per AISC *Specification* Equation E4-3. In this example, F_e calculated in accordance with AISC *Specification* Equation E4-3 is less than that calculated in accordance with AISC *Specification* Equation E4-5 or E4-6, and controls. Therefore:

$$F_e = 26.8\text{ ksi}$$

$$\frac{F_y}{F_e} = \frac{36\text{ ksi}}{26.8\text{ ksi}}$$
$$= 1.34$$

Per the AISC *Specification* User Note for Section E3, the two inequalities for calculating limits of applicability of Sections E3(a) and E3(b) provide the same result for flexural buckling only. When the elastic buckling stress, F_e, is controlled by torsional or flexural-torsional buckling, the L_c/r limits would not be applicable unless an equivalent L_c/r ratio is first calculated by substituting the governing F_e into AISC *Specification* Equation E3-4 and solving for L_c/r. The F_y/F_e limits may be used regardless of which buckling mode governs.

Because $\dfrac{F_y}{F_e} < 2.25$:

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right) F_y \qquad (\textit{Spec}.\ \text{Eq. E3-2})$$

$$= \left(0.658^{1.34}\right)(36\text{ ksi})$$

$$= 20.5\text{ ksi}$$

Effective Area

Determine the limits of applicability for local buckling in accordance with AISC *Specification* Section E7.1. The shorter leg was shown previously to be nonslender and therefore no reduction in effective area due to local buckling of the shorter leg is required. The longer leg was shown previously to be slender and therefore the limits of AISC *Specification* Section E7.1 need to be evaluated.

$$\lambda = 20.0$$

$$\lambda_r \sqrt{\frac{F_y}{F_{cr}}} = 12.8\sqrt{\frac{36\text{ ksi}}{20.5\text{ ksi}}}$$

$$= 17.0$$

Because $\lambda > \lambda_r \sqrt{\dfrac{F_y}{F_{cr}}}$, determine the effective width imperfection adjustment factors per AISC *Specification* Table E7.1, Case (c).

$$c_1 = 0.22$$
$$c_2 = 1.49$$

Determine the elastic local buckling stress from AISC *Specification* Section E7.1.

$$F_{el} = \left(c_2 \frac{\lambda_r}{\lambda}\right)^2 F_y \qquad (\textit{Spec}.\ \text{Eq. E7-5})$$

$$= \left[1.49\left(\frac{12.8}{20.0}\right)\right]^2 (36\text{ ksi})$$

$$= 32.7\text{ ksi}$$

Determine the effective width of the angle leg and the resulting effective area.

$$b_e = b\left(1 - c_1\sqrt{\frac{F_{el}}{F_{cr}}}\right)\sqrt{\frac{F_{el}}{F_{cr}}}$$ (*Spec*. Eq. E7-3)

$$= (5.00 \text{ in.})\left(1 - 0.22\sqrt{\frac{32.7 \text{ ksi}}{20.5 \text{ ksi}}}\right)\sqrt{\frac{32.7 \text{ ksi}}{20.5 \text{ ksi}}}$$

$$= 4.56 \text{ in.}$$

$$A_e = A_g - t\sum(b - b_e)$$

$$= \left(3.88 \text{ in.}^2\right) - (\tfrac{1}{4} \text{ in.})(5.00 \text{ in.} - 4.56 \text{ in.})(2 \text{ angles})$$

$$= 3.66 \text{ in.}^2$$

Available Compressive Strength

$$P_n = F_{cr}A_e$$ (*Spec*. Eq. E7-1)

$$= (20.5 \text{ ksi})\left(3.66 \text{ in.}^2\right)$$

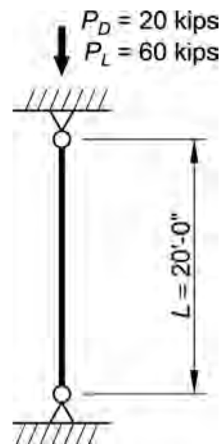
$$= 75.0 \text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$	$\Omega_c = 1.67$
$\phi_c P_n = 0.90(75.0 \text{ kips})$ $= 67.5 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = \dfrac{75.0 \text{ kips}}{1.67}$ $= 44.9 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

EXAMPLE E.7 WT COMPRESSION MEMBER WITHOUT SLENDER ELEMENTS

Given:



Select an ASTM A992 nonslender WT-shape compression member with a length of 20 ft to support a dead load of 20 kips and live load of 60 kips in axial compression. The ends are pinned. The solution will be provided using:
(1) AISC *Manual* Tables
(2) Calculations using AISC *Specification* provisions

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(20 \text{ kips}) + 1.6(60 \text{ kips})$ $= 120 \text{ kips}$	$P_a = 20 \text{ kips} + 60 \text{ kips}$ $= 80.0 \text{ kips}$

(1) AISC *Manual* Table Solution

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0. Therefore, $L_{cx} = L_{cy} = KL$ = 1.0(20 ft) = 20.0 ft.

Select the lightest nonslender member from AISC *Manual* Table 4-7 with sufficient available strength about both the *x-x* axis (upper portion of the table) and the *y-y* axis (lower portion of the table) to support the required strength.

Try a WT7×34.

The available strength in compression is:

LRFD	ASD
$\phi_c P_{nx} = 128 \text{ kips} > 120 \text{ kips}$ **o.k.** **controls**	$\dfrac{P_{nx}}{\Omega_c} = 85.5 \text{ kips} > 80.0 \text{ kips}$ **o.k.** **controls**
$\phi_c P_{ny} = 222 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\dfrac{P_{ny}}{\Omega_c} = 147 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

Available strength can also be determined by hand calculations, as demonstrated in the following.

(2) Calculation Using AISC *Specification* Provisions

From AISC *Manual* Table 1-8, the geometric properties are as follows.

WT7×34
A_g = 10.0 in.^{2}
r_x = 1.81 in.
r_y = 2.46 in.
J = 1.50 in.^{4}

$\bar{y} = 1.29$ in.
$I_x = 32.6$ in.^{4}
$I_y = 60.7$ in.^{4}
$d = 7.02$ in.
$t_w = 0.415$ in.
$b_f = 10.0$ in.
$t_f = 0.720$ in.

Stem Slenderness Check

$$\lambda = \frac{d}{t_w}$$
$$= \frac{7.02 \text{ in.}}{0.415 \text{ in.}}$$
$$= 16.9$$

Determine the stem limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 4:

$$\lambda_r = 0.75\sqrt{\frac{E}{F_y}}$$
$$= 0.75\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 18.1$$

$\lambda < \lambda_r$; therefore, the stem is not slender

Flange Slenderness Check

$$\lambda = \frac{b_f}{2t_f}$$
$$= \frac{10.0 \text{ in.}}{2(0.720 \text{ in.})}$$
$$= 6.94$$

Determine the flange limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 1:

$$\lambda_r = 0.56\sqrt{\frac{E}{F_y}}$$
$$= 0.56\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 13.5$$

$\lambda < \lambda_r$; therefore, the flange is not slender

There are no slender elements.

For compression members without slender elements, AISC *Specification* Sections E3 and E4 apply. The nominal compressive strength, P_n, is determined based on the limit states of flexural, torsional and flexural-torsional buckling.

Elastic Flexural Buckling Stress about the x-x Axis

$$\frac{L_{cx}}{r_x} = \frac{(20.0\text{ ft})(12\text{ in./ft})}{1.81\text{ in.}}$$
$$= 133$$

$$F_{ex} = \frac{\pi^2 E}{\left(\frac{L_{cx}}{r_x}\right)^2}$$ (*Spec*. Eq. E3-4 or E4-5)

$$= \frac{\pi^2(29{,}000\text{ ksi})}{(133)^2}$$
$$= 16.2\text{ ksi} \quad \textbf{controls}$$

Elastic Flexural Buckling Stress about the y-y Axis

$$\frac{L_{cy}}{r_y} = \frac{(20.0\text{ ft})(12\text{ in./ft})}{2.46\text{ in.}}$$
$$= 97.6$$

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{L_{cy}}{r_y}\right)^2}$$ (*Spec*. Eq. E3-4 or E4-6)

$$= \frac{\pi^2(29{,}000\text{ ksi})}{(97.6)^2}$$
$$= 30.0\text{ ksi}$$

Torsional and Flexural-Torsional Elastic Buckling Stress

Because the WT7×34 section does not have any slender elements, AISC *Specification* Section E4 will be applicable for torsional and flexural-torsional buckling. F_e will be calculated using AISC *Specification* Equation E4-3. Per the User Note for AISC *Specification* Section E4, the term with C_w is omitted when computing F_{ez}, and x_o is taken as zero. The flexural buckling term about the *y-y* axis, F_{ey}, was computed in the preceding section.

$$x_o = 0$$

$$y_o = \bar{y} - \frac{t_f}{2}$$
$$= 1.29\text{ in.} - \frac{0.720\text{ in.}}{2}$$
$$= 0.930\text{ in.}$$

$$\bar{r}_o^2 = x_o^2 + y_o^2 + \frac{I_x + I_y}{A_g} \qquad (Spec.\ \text{Eq. E4-9})$$

$$= 0 + (0.930\text{ in.})^2 + \frac{32.6\text{ in.}^4 + 60.7\text{ in.}^4}{10.0\text{ in.}^2}$$

$$= 10.2\text{ in.}^2$$

$$F_{ez} = \left(\frac{\pi^2 EC_w}{L_{cz}^2} + GJ\right)\frac{1}{A_g \bar{r}_o^2} \qquad (Spec.\ \text{Eq. E4-7})$$

$$= \left[0 + (11{,}200\text{ ksi})(1.50\text{ in.}^4)\right]\frac{1}{(10.0\text{ in.}^2)(10.2\text{ in.}^2)}$$

$$= 165\text{ ksi}$$

$$H = 1 - \frac{x_o^2 + y_o^2}{\bar{r}_o^2} \qquad (Spec.\ \text{Eq. E4-8})$$

$$= 1 - \frac{0 + (0.930\text{ in.})^2}{10.2\text{ in.}^2}$$

$$= 0.915$$

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right] \qquad (Spec.\ \text{Eq. E4-3})$$

$$= \left[\frac{30.0\text{ ksi} + 165\text{ ksi}}{2(0.915)}\right]\left[1 - \sqrt{1 - \frac{4(30.0\text{ ksi})(165\text{ ksi})(0.915)}{(30.0\text{ ksi} + 165\text{ ksi})^2}}\right]$$

$$= 29.5\text{ ksi}$$

Critical Buckling Stress

The critical buckling stress for the member could be controlled by flexural buckling about either the *x-x* axis or *y-y* axis, F_{ex} or F_{ey}, respectively. Note that AISC *Specification* Equations E4-5 and E4-6 reflect the same buckling modes as calculated in AISC *Specification* Equation E3-4. Or, the critical buckling stress for the member could be controlled by torsional or flexural-torsional buckling calculated per AISC *Specification* Equation E4-3. In this example, F_e calculated in accordance with AISC *Specification* Equation E4-5 is less than that calculated in accordance with AISC *Specification* Equation E4-3 or E4-6 and controls. Therefore:

$$F_e = 16.2\text{ ksi}$$

$$\frac{F_y}{F_e} = \frac{50\text{ ksi}}{16.2\text{ ksi}}$$

$$= 3.09$$

Per the AISC *Specification* User Note for Section E3, the two inequalities for calculating limits of applicability of Sections E3(a) and E3(b) provide the same result for flexural buckling only. When the elastic buckling stress, F_e, is controlled by torsional or flexural-torsional buckling, the L_c/r limits would not be applicable unless an equivalent L_c/r ratio is first calculated by substituting the governing F_e into AISC *Specification* Equation E3-4 and solving for L_c/r. The F_y/F_e limits may be used regardless of which buckling mode governs.

Because $\frac{F_y}{F_e} > 2.25$:

$$F_{cr} = 0.877F_e \quad \text{(Spec. Eq. E3-3)}$$
$$= 0.877(16.2 \text{ ksi})$$
$$= 14.2 \text{ ksi}$$

Available Compressive Strength

$$P_n = F_{cr}A_g \quad \text{(Spec. Eq. E3-1)}$$
$$= (14.2 \text{ ksi})(10.0 \text{ in.}^2)$$
$$= 142 \text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

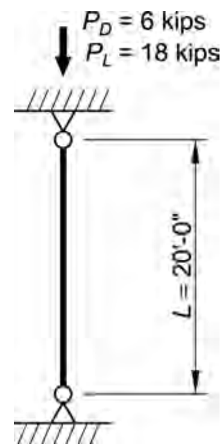
LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(142 \text{ kips})$ $= 128 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\frac{P_n}{\Omega_c} = \frac{142 \text{ kips}}{1.67}$ $= 85.0 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

EXAMPLE E.8 WT COMPRESSION MEMBER WITH SLENDER ELEMENTS

Given:

Select an ASTM A992 WT-shape compression member with a length of 20 ft to support a dead load of 6 kips and live load of 18 kips in axial compression. The ends are pinned. The solution will be provided using:

(1) AISC *Manual* Tables
(2) Calculations using AISC *Specification* provisions



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From ASCE/SEI 7, Chapter 2 , the required compressive strength is:

LRFD	ASD
$P_u = 1.2(6\text{ kips}) + 1.6(18\text{ kips})$ $= 36.0\text{ kips}$	$P_a = 6\text{ kips} + 18\text{ kips}$ $= 24.0\text{ kips}$

(1) AISC *Manual* Table Solution

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0. Therefore, $L_{cx} = L_{cy} = KL = 1.0(20\text{ ft}) = 20.0$ ft.

Select the lightest member from AISC *Manual* Table 4-7 with sufficient available strength about the both the x-x axis (upper portion of the table) and the y-y axis (lower portion of the table) to support the required strength.

Try a WT7×15.

The available strength in axial compression from AISC *Manual* Table 4-7 is:

LRFD	ASD
$\phi_c P_{nx} = 74.3\text{ kips} > 36.0\text{ kips}$ **o.k.** $\phi_c P_{ny} = 36.6\text{ kips} > 36.0\text{ kips}$ **o.k. controls**	$\dfrac{P_{nx}}{\Omega_c} = 49.4\text{ kips} > 24.0\text{ kips}$ **o.k.** $\dfrac{P_{ny}}{\Omega_c} = 24.4\text{ kips} > 24.0\text{ kips}$ **o.k. controls**

Available strength can also be determined by hand calculations, as demonstrated in the following.

(2) Calculation Using AISC *Specification* Provisions

From AISC *Manual* Table 1-8, the geometric properties are as follows:

WT7×15
$A_g = 4.42\text{ in.}^2$
$r_x = 2.07$ in.
$r_y = 1.49$ in.

$J = 0.190 \text{ in.}^4$
$\bar{y} = 1.58 \text{ in.}$
$I_x = 19.0 \text{ in.}^4$
$I_y = 9.79 \text{ in.}^4$
$d = 6.92 \text{ in.}$
$t_w = 0.270 \text{ in.}$
$b_f = 6.73 \text{ in.}$
$t_f = 0.385 \text{ in.}$

Stem Slenderness Check

$$\lambda = \frac{d}{t_w} = \frac{6.92 \text{ in.}}{0.270 \text{ in.}} = 25.6$$

Determine stem limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 4:

$$\lambda_r = 0.75\sqrt{\frac{E}{F_y}} = 0.75\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}} = 18.1$$

$\lambda > \lambda_r$; therefore, the stem is slender

Flange Slenderness Check

$$\lambda = \frac{b_f}{2t_f} = \frac{6.73 \text{ in.}}{2(0.385 \text{ in.})} = 8.74$$

Determine flange limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 1:

$$\lambda_r = 0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}} = 13.5$$

$\lambda < \lambda_r$; therefore, the flange is not slender

Because this WT7×15 has a slender web, AISC *Specification* Section E7 is applicable. The nominal compressive strength, P_n, is determined based on the limit states of flexural, torsional and flexural-torsional buckling.

Elastic Flexural Buckling Stress about the x-x Axis

$$\frac{L_{cx}}{r_x} = \frac{(20.0\text{ ft})(12\text{ in./ft})}{2.07\text{ in.}}$$
$$= 116$$

$$F_{ex} = \frac{\pi^2 E}{\left(\frac{L_{cx}}{r_x}\right)^2}$$ (*Spec*. Eq. E3-4 or E4-5)

$$= \frac{\pi^2 (29{,}000\text{ ksi})}{(116)^2}$$
$$= 21.3$$

Elastic Flexural Buckling Stress about the y-y Axis

$$\frac{L_{cy}}{r_y} = \frac{(20.0\text{ ft})(12\text{ in./ft})}{1.49\text{ in.}}$$
$$= 161$$

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{L_{cy}}{r_y}\right)^2}$$ (*Spec*. Eq. E3-4 or E4-6)

$$= \frac{\pi^2 (29{,}000\text{ ksi})}{(161)^2}$$
$$= 11.0\text{ ksi}$$

Torsional and Flexural-Torsional Elastic Buckling Stress

F_e will be calculated using AISC *Specification* Equation E4-3. Per the User Note for AISC *Specification* Section E4, the term with C_w is omitted when computing F_{ez}, and x_o is taken as zero. The flexural buckling term about the *y-y* axis, F_{ey}, was computed in the preceding section.

$$x_o = 0$$

$$y_o = \bar{y} - \frac{t_f}{2}$$
$$= 1.58\text{ in.} - \frac{0.385\text{ in.}}{2}$$
$$= 1.39\text{ in.}$$

$$\bar{r}_o^{\,2} = x_o^{\,2} + y_o^{\,2} + \frac{I_x + I_y}{A_g}$$ (*Spec*. Eq. E4-9)

$$= 0 + (1.39\text{ in.})^2 + \frac{19.0\text{ in.}^4 + 9.79\text{ in.}^4}{4.42\text{ in.}^2}$$
$$= 8.45\text{ in.}^2$$

$$F_{ez} = \left(\frac{\pi^2 EC_w}{L_{cz}^2} + GJ\right)\frac{1}{A_g \bar{r}_o^2} \qquad \text{(}Spec\text{. Eq. E4-7)}$$

$$= \left[0 + (11{,}200 \text{ ksi})(0.190 \text{ in.}^4)\right]\frac{1}{(4.42 \text{ in.}^2)(8.45 \text{ in.}^2)}$$

$$= 57.0 \text{ ksi}$$

$$H = 1 - \frac{x_o^2 + y_o^2}{\bar{r}_o^2} \qquad \text{(}Spec\text{. Eq. E4-8)}$$

$$= 1 - \frac{0 + (1.39 \text{ in.})^2}{8.45 \text{ in.}^2}$$

$$= 0.771$$

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right] \qquad \text{(}Spec\text{. Eq. E4-3)}$$

$$= \left[\frac{11.0 \text{ ksi} + 57.0 \text{ ksi}}{2(0.771)}\right]\left[1 - \sqrt{1 - \frac{4(11.0 \text{ ksi})(57.0 \text{ ksi})(0.771)}{(11.0 \text{ ksi} + 57.0 \text{ ksi})^2}}\right]$$

$$= 10.5 \text{ ksi} \quad \textbf{controls}$$

Critical Buckling Stress

The critical buckling stress for the member could be controlled by flexural buckling about either the x-x axis or y-y axis, F_{ex} or F_{ey}, respectively. Note that AISC *Specification* Equations E4-5 and E4-6 reflect the same buckling modes as calculated in AISC *Specification* Equation E3-4. Or, the critical buckling stress for the member could be controlled by torsional or flexural-torsional buckling calculated per AISC *Specification* Equation E4-3. In this example, F_e calculated in accordance with AISC *Specification* Equation E4-3 is less than that calculated in accordance with AISC *Specification* Equation E4-5 or E4-6 and controls. Therefore:

$$F_e = 10.5 \text{ ksi}$$

$$\frac{F_y}{F_e} = \frac{50 \text{ ksi}}{10.5 \text{ ksi}}$$

$$= 4.76$$

Per the AISC *Specification* User Note for Section E3, the two inequalities for calculating limits of applicability of Sections E3(a) and E3(b) provide the same result for flexural buckling only. When the elastic buckling stress, F_e, is controlled by torsional or flexural-torsional buckling, the L_c/r limits would not be applicable unless an equivalent L_c/r ratio is first calculated by substituting the governing F_e into AISC *Specification* Equation E3-4 and solving for L_c/r. The F_y/F_e limits may be used regardless of which buckling mode governs.

Because $\dfrac{F_y}{F_e} > 2.25$:

$$F_{cr} = 0.877F_e \qquad \text{(}Spec\text{. Eq. E3-3)}$$

$$= 0.877(10.5 \text{ ksi})$$

$$= 9.21 \text{ ksi}$$

Effective Area

Because this section was found to have a slender element, the limits of AISC *Specification* Section E7.1 must be evaluated to determine if there is a reduction in effective area due to local buckling. Since the flange was found to not be slender, no reduction in effective area due to local buckling in the flange is required. Only a reduction in effective area due to local buckling in the stem may be required.

$$\lambda = 25.6$$

$$\lambda_r \sqrt{\frac{F_y}{F_{cr}}} = 18.1\sqrt{\frac{50 \text{ ksi}}{9.21 \text{ ksi}}}$$
$$= 42.2$$

Because $\lambda < \lambda_r \sqrt{\frac{F_y}{F_{cr}}}$,

$$b_e = b \qquad \text{(Spec. Eq. E7-2)}$$

There is no reduction in effective area due to local buckling of the stem at the critical stress level and $A_e = A_g$.

Available Compressive Strength

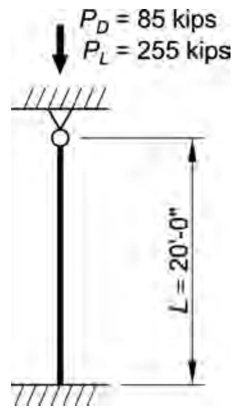
$$P_n = F_{cr} A_e \qquad \text{(Spec. Eq. E7-1)}$$
$$= (9.21 \text{ ksi})(4.42 \text{ in.}^2)$$
$$= 40.7 \text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(40.7 \text{ kips})$ $= 36.6 \text{ kips} > 36.0 \text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\frac{P_n}{\Omega_c} = \frac{40.7 \text{ kips}}{1.67}$ $= 24.4 \text{ kips} > 24.0 \text{ kips}$ **o.k.**

EXAMPLE E.9 RECTANGULAR HSS COMPRESSION MEMBER WITHOUT SLENDER ELEMENTS

Given:



Select an ASTM A500 Grade C rectangular HSS compression member, with a length of 20 ft, to support a dead load of 85 kips and live load of 255 kips in axial compression. The base is fixed and the top is pinned. The solution will be provided using:

(1) AISC *Manual* Tables
(2) Calculations using AISC *Specification* provisions

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade C, rectangular HSS
F_y = 50 ksi
F_u = 62 ksi

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(85 \text{ kips}) + 1.6(255 \text{ kips})$ $= 510 \text{ kips}$	$P_a = 85 \text{ kips} + 255 \text{ kips}$ $= 340 \text{ kips}$

(1) AISC *Manual* Table Solution

From AISC *Specification* Commentary Table C-A-7.1, for a fixed-pinned condition, $K_x = K_y = 0.80$.

$$\begin{aligned} L_c &= K_x L_x \\ &= K_y L_y \\ &= 0.80(20 \text{ ft}) \\ &= 16.0 \text{ ft} \end{aligned}$$

Enter AISC *Manual* Table 4-3 for rectangular sections.

Try a HSS12×10×3⁄8.

From AISC *Manual* Table 4-3, the available strength in axial compression is:

LRFD	ASD
$\phi_c P_n = 556 \text{ kips} > 510 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 370 \text{ kips} > 340 \text{ kips}$ **o.k.**

Available strength can also be determined by hand calculations, as demonstrated in the following.

(2) Calculation Using AISC *Specification* Provisions

From AISC *Manual* Table 1-11, the geometric properties are as follows:

HSS12×10×3⁄8

A_g = 14.6 in.^{2}
t = 0.349 in.
r_x = 4.61 in.
r_y = 4.01 in.
b/t = 25.7
h/t = 31.4

Slenderness Check

Determine the wall limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 6:

$$\begin{aligned}\lambda_r &= 1.40\sqrt{\frac{E}{F_y}}\\ &= 1.40\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}\\ &= 33.7\end{aligned}$$

For the narrow side:

$$\lambda = b/t = 25.7$$

For the wide side:

$$\lambda = h/t = 31.4$$

$\lambda < \lambda_r$; therefore, the section does not contain slender elements.

Elastic Buckling Stress

Because $r_y < r_x$ and $L_{cx} = L_{cy}$, r_y will govern the available strength.

Determine the applicable equation:

$$\begin{aligned}\frac{L_{cy}}{r_y} &= \frac{(16.0 \text{ ft})(12 \text{ in./ft})}{4.01 \text{ in.}}\\ &= 47.9\end{aligned}$$

$$\begin{aligned}4.71\sqrt{\frac{E}{F_y}} &= 4.71\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}\\ &= 113 \geq 47.9\end{aligned}$$

Therefore, use AISC *Specification* Equation E3-2.

$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} \qquad \text{(Spec. Eq. E3-4)}$$

$$\begin{aligned}&= \frac{\pi^2(29{,}000 \text{ ksi})}{(47.9)^2}\\ &= 125 \text{ ksi}\end{aligned}$$

Critical Buckling Stress

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right) F_y \qquad \text{(}Spec\text{. Eq. E3-2)}$$

$$= \left(0.658^{\frac{50 \text{ ksi}}{125 \text{ ksi}}}\right)(50 \text{ ksi})$$

$$= 42.3 \text{ ksi}$$

Available Compressive Strength

$$P_n = F_{cr} A_g \qquad \text{(}Spec\text{. Eq. E3-1)}$$

$$= (42.3 \text{ ksi})(14.6 \text{ in.}^2)$$

$$= 618 \text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(618 \text{ kips})$ $= 556 \text{ kips} > 510 \text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\frac{P_n}{\Omega_c} = \frac{618 \text{ kips}}{1.67}$ $= 370 \text{ kips} > 340 \text{ kips}$ **o.k.**

EXAMPLE E.10 RECTANGULAR HSS COMPRESSION MEMBER WITH SLENDER ELEMENTS

Given:

Using the AISC *Specification* provisions, calculate the available strength of a HSS12×8×3⁄16 compression member with an effective length of L_c = 24 ft with respect to both axes. Use ASTM A500 Grade C.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade C, rectangular HSS
F_y = 50 ksi
F_u = 62 ksi

From AISC *Manual* Table 1-11 the geometric properties of an HSS12×8×3⁄16 are as follows:

$A = 6.76 \text{ in.}^2$
$t = 0.174 \text{ in.}$
$r_x = 4.56 \text{ in.}$
$r_y = 3.35 \text{ in.}$
$\dfrac{b}{t} = 43.0$
$\dfrac{h}{t} = 66.0$

Slenderness Check

Calculate the limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 6 for walls of rectangular HSS.

$$\lambda_r = 1.40\sqrt{\frac{E}{F_y}}$$
$$= 1.40\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 33.7$$

Determine the width-to-thickness ratios of the HSS walls.

For the narrow side:

$$\lambda = \frac{b}{t}$$
$$= 43.0 > \lambda_r = 33.7$$

For the wide side:

$$\lambda = \frac{h}{t}$$
$$= 66.0 > \lambda_r = 33.7$$

All walls of the HSS12×8×3⁄16 are slender elements and the provisions of AISC *Specification* Section E7 apply.

Critical Stress, F_{cr}

From AISC *Specification* Section E7, the critical stress, F_{cr}, is calculated using the gross section properties and following the provisions of AISC *Specification* Section E3. The effective slenderness ratio about the *y*-axis will control. From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0. Therefore, $L_{cy} = K_y L_y$ = 1.0(24 ft) = 24.0 ft.

$$\left(\frac{L_c}{r}\right)_{max} = \frac{L_{cy}}{r_y}$$
$$= \frac{(24.0 \text{ ft})(12 \text{ in./ft})}{3.35 \text{ in.}}$$
$$= 86.0$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 113 \geq 86.0$$

Therefore, use AISC *Specification* Equation E3-2.

$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} \qquad \text{(\textit{Spec}. Eq. E3-4)}$$
$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{(86.0)^2}$$
$$= 38.7 \text{ ksi}$$

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right) F_y \qquad \text{(\textit{Spec}. Eq. E3-2)}$$
$$= \left[0.658^{\left(\frac{50 \text{ ksi}}{38.7 \text{ ksi}}\right)}\right](50 \text{ ksi})$$
$$= 29.1 \text{ ksi}$$

Effective Area, A_e

Compute the effective wall widths, h_e and b_e, in accordance with AISC *Specification* Section E7.1. Compare λ for each wall with the following limit to determine if a local buckling reduction applies.

$$\lambda_r \sqrt{\frac{F_y}{F_{cr}}} = 33.7\sqrt{\frac{50 \text{ ksi}}{29.1 \text{ ksi}}}$$
$$= 44.2$$

For the narrow walls:

$$\lambda = \frac{b}{t}$$
$$= 43.0 < 44.2$$

Therefore, the narrow wall width does not need to be reduced ($b_e = b$) per AISC *Specification* Equation E7-2.

For the wide walls:

$$\lambda = \frac{h}{t}$$
$$= 66.0 > 44.2$$

Therefore, use AISC *Specification* Equation E7-3, with $h = \left(\frac{h}{t}\right)t = (66.0)(0.174 \text{ in.}) = 11.5 \text{ in.}$

The effective width imperfection adjustment factors, c_1 and c_2, are selected from AISC *Specification* Table E7.1, Case (b):

$$c_1 = 0.20$$
$$c_2 = 1.38$$

$$F_{el} = \left(c_2 \frac{\lambda_r}{\lambda}\right)^2 F_y \qquad \text{(Spec. Eq. E7-5)}$$
$$= \left[1.38\left(\frac{33.7}{66.0}\right)\right]^2 (50 \text{ ksi})$$
$$= 24.8 \text{ ksi}$$

$$h_e = h\left(1 - c_1\sqrt{\frac{F_{el}}{F_{cr}}}\right)\sqrt{\frac{F_{el}}{F_{cr}}} \qquad \text{(Spec. Eq. E7-3)}$$
$$= (11.5 \text{ in.})\left(1 - 0.20\sqrt{\frac{24.8 \text{ ksi}}{29.1 \text{ ksi}}}\right)\sqrt{\frac{24.8 \text{ ksi}}{29.1 \text{ ksi}}}$$
$$= 8.66 \text{ in.}$$

The effective area, A_e, is determined using the effective width h_e = 8.66 in. and the design wall thickness t = 0.174 in. As shown in Figure E.10-1, $h - h_e$ is the width of the wall segments that must be reduced from the gross area, A, to compute the effective area, A_e. Note that a similar deduction would be required for the narrow walls if $b_e < b$.

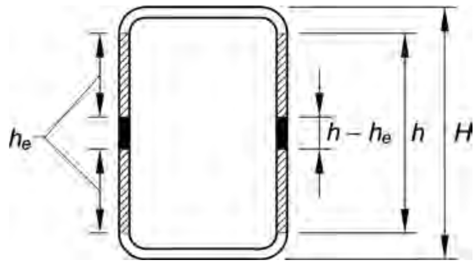


Fig. E.10-1. HSS Effective Area.

$$A_e = A - 2(h - h_e)t$$
$$= 6.76 \text{ in.}^2 - 2(11.5 \text{ in.} - 8.66 \text{ in.})(0.174 \text{ in.})$$
$$= 5.77 \text{ in.}^2$$

Available Compressive Strength

The effective area is used to compute nominal compressive strength:

$$P_n = F_{cr} A_e \qquad \text{(Spec. Eq. E7-1)}$$
$$= (29.1 \text{ ksi})(5.77 \text{ in.}^2)$$
$$= 168 \text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(168 \text{ kips})$ $= 151 \text{ kips}$	$\Omega_c = 1.67$ $\dfrac{P_n}{\Omega_c} = \dfrac{168 \text{ kips}}{1.67}$ $= 101 \text{ kips}$

Discussion

The width-to-thickness criterion, $\lambda_r = 1.40\sqrt{\dfrac{E}{F_y}}$ for HSS in Table B4.1a is based on the assumption that the element will be stressed to F_y. If the critical flexural buckling stress is less than F_y, which it always is for compression members of reasonable length, wall local buckling may or may not occur before member flexural buckling occurs. For the case where the flexural buckling stress is low enough, wall local buckling will not occur. This is the case addressed in AISC *Specification* Section E7.1(a). For members where the flexural buckling stress is high enough, wall local buckling will occur. This is the case addressed in AISC *Specification* Section E7.1(b).

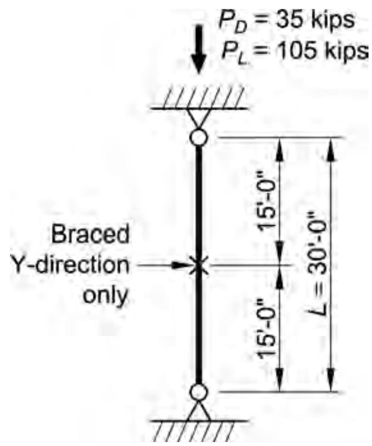
The HSS12×8×3⁄16 in this example is slender according to Table B4.1a. For effective length L_c = 24.0 ft, the flexural buckling critical stress was F_{cr} = 29.1 ksi. By Section E7.1, at F_{cr} = 29.1 ksi, the wide wall effective width must be determined but the narrow wall is fully effective. Thus, the axial strength is reduced because of local buckling of the wide wall. Table E.10 repeats the example analysis for two other column effective lengths and compares those results to the results for L_c = 24 ft calculated previously. For L_c = 18.0 ft, the flexural buckling critical stress, F_{cr} = 36.9 ksi, is high enough that both the wide and narrow walls must have their effective width determined according to Equation E7-3. For L_c = 40.0 ft the flexural buckling critical stress, F_{cr} = 12.2 ksi, is low enough that there will be no local buckling of either wall and the actual widths will be used according to Equation E7-2.

Table E.10. Analysis of HSS12×8×3⁄16 Column at Different Effective Lengths			
Effective length, L_c (ft)	18.0	24.0	40.0
Check Table B4.1 criterion (same as for L_c = 24.0 ft).			
λ_r	33.7	33.7	33.7
λ (narrow wall) = 43.0 > λ_r	Yes	Yes	Yes
λ (wide wall) = 66.0 > λ_r	Yes	Yes	Yes
F_{cr} (ksi)	36.9	29.1	12.2
Check AISC *Specification* Section E7.1 criteria.			
Narrow wall:			
$\lambda_r \sqrt{\frac{F_y}{F_{cr}}}$	$39.2 \le \lambda = 43.0$	$44.2 > \lambda = 43.0$	$68.2 > \lambda = 43.0$
Local buckling reduction per AISC *Specification* Section E7.1?	Yes	No	No
F_{el} (ksi)	58.5	–	–
b_e (in.)	7.05	–	–
Wide wall:			
$\lambda_r \sqrt{\frac{F_y}{F_{cr}}}$	$39.2 \le \lambda = 66.0$	$44.2 \le \lambda = 66.0$	$68.2 > \lambda = 66.0$
Local buckling reduction per AISC *Specification* Section E7.1?	Yes	Yes	No
F_{el} (ksi)	24.8	24.8	–
h_e (in.)	7.88	8.66	–
Effective area, A_e (in.^{2})	5.35	5.77	6.76
Compressive strength			
P_n (kips)	197	168	82.5
LRFD, $\phi_c P_n$ (kips)	177	151	74.2
ASD, P_n/Ω_c (kips)	118	101	49.4

EXAMPLE E.11 PIPE COMPRESSION MEMBER

Given:

Select an ASTM A53 Grade B Pipe compression member with a length of 30 ft to support a dead load of 35 kips and live load of 105 kips in axial compression. The column is pin-connected at the ends in both axes and braced at the midpoint in the *y-y* direction. The solution will be provided using:
(1) AISC *Manual* Tables
(2) Calculations using AISC *Specification* provisions



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A53 Grade B
$F_y = 35$ ksi
$F_u = 60$ ksi

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(35\text{ kips}) + 1.6(105\text{ kips})$ $= 210\text{ kips}$	$P_a = 35\text{ kips} + 105\text{ kips}$ $= 140\text{ kips}$

(1) AISC *Manual* Table Solution

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0. Therefore, $L_{cx} = K_xL_x$ = 1.0(30 ft) = 30.0 ft and $L_{cy} = K_yL_y$ = 1.0(15 ft) = 15.0 ft. Buckling about the *x-x* axis controls.

Enter AISC *Manual* Table 4-6 with L_c = 30.0 ft and select the lightest section with sufficient available strength to support the required strength.

Try a 10-in. Standard Pipe.

From AISC *Manual* Table 4-6, the available strength in axial compression is:

LRFD	ASD
$\phi_c P_n = 222\text{ kips} > 210\text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 148\text{ kips} > 140\text{ kips}$ **o.k.**

Available strength can also be determined by hand calculations, as demonstrated in the following.

(2) Calculation Using AISC *Specification* Provisions

From AISC *Manual* Table 1-14, the geometric properties are as follows:

Pipe 10 Std.
$A_g = 11.5\text{ in.}^2$
$r = 3.68$ in.

$$\lambda = \frac{D}{t} = 31.6$$

No Pipes shown in AISC *Manual* Table 4-6 are slender at 35 ksi, so no local buckling check is required; however, some round HSS are slender at higher steel strengths. The following calculations illustrate the required check.

Limiting Width-to-Thickness Ratio

Determine the wall limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 9:

$$\begin{aligned}\lambda_r &= 0.11\frac{E}{F_y}\\ &= 0.11\left(\frac{29{,}000\text{ ksi}}{35\text{ ksi}}\right)\\ &= 91.1\end{aligned}$$

$\lambda < \lambda_r$; therefore, the pipe is not slender

Critical Stress, F_{cr}

$$\begin{aligned}\frac{L_c}{r} &= \frac{(30.0\text{ ft})(12\text{ in./ft})}{3.68\text{ in.}}\\ &= 97.8\end{aligned}$$

$$\begin{aligned}4.71\sqrt{\frac{E}{F_y}} &= 4.71\sqrt{\frac{29{,}000\text{ ksi}}{35\text{ ksi}}}\\ &= 136 > 97.8,\text{ therefore, use AISC } \textit{Specification} \text{ Equation E3-2}\end{aligned}$$

$$\begin{aligned}F_e &= \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2}\\ &= \frac{\pi^2(29{,}000\text{ ksi})}{(97.8)^2}\\ &= 29.9\text{ ksi}\end{aligned}$$ (*Spec*. Eq. E3-4)

$$\begin{aligned}F_{cr} &= \left(0.658^{\frac{F_y}{F_e}}\right)F_y\\ &= \left[0.658^{\left(\frac{35\text{ ksi}}{29.9\text{ ksi}}\right)}\right](35\text{ ksi})\\ &= 21.4\text{ ksi}\end{aligned}$$ (*Spec*. Eq. E3-2)

Available Compressive Strength

$$\begin{aligned}P_n &= F_{cr}A_g\\ &= (21.4\text{ ksi})(11.5\text{ in.}^2)\\ &= 246\text{ kips}\end{aligned}$$ (*Spec*. Eq. E3-1)

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(246 \text{ kips})$ $= 221 \text{ kips} > 210 \text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\dfrac{P_n}{\Omega_c} = \dfrac{246 \text{ kips}}{1.67}$ $= 147 \text{ kips} > 140 \text{ kips}$ **o.k.**

Note that the design procedure would be similar for a round HSS column.

EXAMPLE E.12 BUILT-UP I-SHAPED MEMBER WITH DIFFERENT FLANGE SIZES

Given:

Compute the available strength of a built-up compression member with a length of 14 ft, as shown in Figure E.12-1. The ends are pinned. The outside flange is PL¾ in. × 5 in., the inside flange is PL¾ in. × 8 in., and the web is PL⅜ in. × 10½ in. The material is ASTM A572 Grade 50.

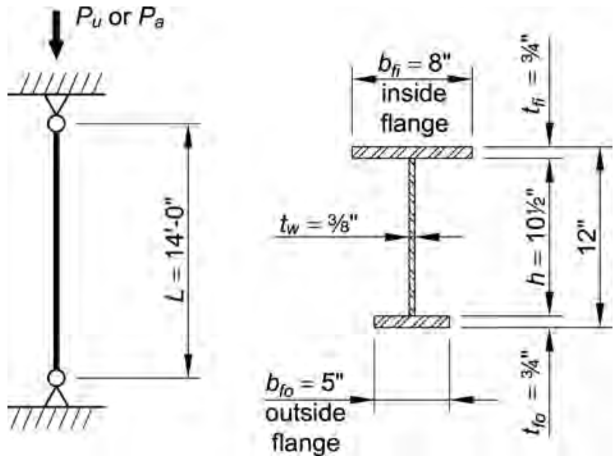


Fig. E.12-1. Column geometry for Example E.12.

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

ASTM A572 Grade 50
$F_y = 50$ ksi
$F_u = 65$ ksi

There are no tables for special built-up shapes; therefore, the available strength is calculated as follows.

Slenderness Check

Check outside flange slenderness.

From AISC *Specification* Table B4.1a note [a], calculate k_c.

$$
\begin{aligned}
k_c &= \frac{4}{\sqrt{h/t_w}} \\
&= \frac{4}{\sqrt{\dfrac{10\tfrac{1}{2}\text{ in.}}{\tfrac{3}{8}\text{ in.}}}} \\
&= 0.756,\ \ 0.35 \le k_c \le 0.76 \quad \textbf{o.k.}
\end{aligned}
$$

For the outside flange, the slenderness ratio is:

$$\lambda = \frac{b}{t}$$
$$= \frac{2.50 \text{ in.}}{\frac{3}{4} \text{ in.}}$$
$$= 3.33$$

Determine the limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a, Case 2:

$$\lambda_r = 0.64\sqrt{\frac{k_c E}{F_y}}$$
$$= 0.64\sqrt{\frac{0.756(29{,}000 \text{ ksi})}{50 \text{ ksi}}}$$
$$= 13.4$$

$\lambda \leq \lambda_r$; therefore, the outside flange is not slender

Check inside flange slenderness.

$$\lambda = \frac{b}{t}$$
$$= \frac{4.00 \text{ in.}}{\frac{3}{4} \text{ in.}}$$
$$= 5.33$$

$\lambda \leq \lambda_r$; therefore, the inside flange is not slender

Check web slenderness.

$$\lambda = \frac{h}{t}$$
$$= \frac{10\frac{1}{2} \text{ in.}}{\frac{3}{8} \text{ in.}}$$
$$= 28.0$$

Determine the limiting slenderness ratio, λ_r, for the web from AISC *Specification* Table B4.1a, Case 5:

$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}}$$
$$= 1.49\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 35.9$$

$\lambda \leq \lambda_r$; therefore, the web is not slender

Section Properties (ignoring welds)

$$
\begin{aligned}
A_g &= b_{fi}t_{fi} + ht_w + b_{fo}t_{fo} \\
&= (8.00 \text{ in.})(\tfrac{3}{4} \text{ in.}) + (10\tfrac{1}{2} \text{ in.})(\tfrac{3}{8} \text{ in.}) + (5.00 \text{ in.})(\tfrac{3}{4} \text{ in.}) \\
&= 13.7 \text{ in.}^2
\end{aligned}
$$

$$
\begin{aligned}
\bar{y} &= \frac{\Sigma A_i y_i}{\Sigma A_i} \\
&= \frac{(6.00 \text{ in.}^2)(11.6 \text{ in.}) + (3.94 \text{ in.}^2)(6.00 \text{ in.}) + (3.75 \text{ in.}^2)(0.375 \text{ in.})}{6.00 \text{ in.}^2 + 3.94 \text{ in.}^2 + 3.75 \text{ in.}^2} \\
&= 6.91 \text{ in.}
\end{aligned}
$$

Note that the center of gravity about the x-axis is measured from the bottom of the outside flange.

$$
\begin{aligned}
I_x &= \sum\left(\frac{bh^3}{12} + Ad^2\right) \\
&= \left[\frac{(8.00 \text{ in.})(\tfrac{3}{4} \text{ in.})^3}{12} + (8.00 \text{ in.})(\tfrac{3}{4} \text{ in.})(4.72 \text{ in.})^2\right] + \left[\frac{(\tfrac{3}{8} \text{ in.})(10\tfrac{1}{2} \text{ in.})^3}{12} + (\tfrac{3}{8} \text{ in.})(10\tfrac{1}{2} \text{ in.})(0.910 \text{ in.})^2\right] \\
&\quad + \left[\frac{(5.00 \text{ in.})(\tfrac{3}{4} \text{ in.})^3}{12} + (5.00 \text{ in.})(\tfrac{3}{4} \text{ in.})(6.54 \text{ in.})^2\right] \\
&= 334 \text{ in.}^4
\end{aligned}
$$

$$
\begin{aligned}
r_x &= \sqrt{\frac{I_x}{A}} \\
&= \sqrt{\frac{334 \text{ in.}^4}{13.7 \text{ in.}^2}} \\
&= 4.94 \text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
I_y &= \sum\frac{bh^3}{12} \\
&= \frac{(\tfrac{3}{4} \text{ in.})(8.00 \text{ in.})^3}{12} + \frac{(10\tfrac{1}{2} \text{ in.})(\tfrac{3}{8} \text{ in.})^3}{12} + \frac{(\tfrac{3}{4} \text{ in.})(5.00 \text{ in.})^3}{12} \\
&= 39.9 \text{ in.}^4
\end{aligned}
$$

$$
\begin{aligned}
r_y &= \sqrt{\frac{I_y}{A}} \\
&= \sqrt{\frac{39.9 \text{ in.}^4}{13.7 \text{ in.}^2}} \\
&= 1.71 \text{ in.}
\end{aligned}
$$

Elastic Buckling Stress about the x-x Axis

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0. Therefore, $L_{cx} = L_{cy} = L_{cz} = KL$ = 1.0(14 ft) = 14.0 ft.

The effective slenderness ratio about the x-axis is:

$$\frac{L_{cx}}{r_x} = \frac{(14.0 \text{ ft})(12 \text{ in./ft})}{4.94 \text{ in.}}$$
$$= 34.0$$

$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} \quad \text{(Spec. Eq. E3-4)}$$
$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{(34.0)^2}$$
$$= 248 \text{ ksi} \quad \textbf{does not control}$$

Flexural-Torsional Elastic Buckling Stress

Calculate the torsional constant, J, using AISC Design Guide 9, Equation 3.4:

$$J = \sum \frac{bt^3}{3}$$
$$= \frac{(8.00 \text{ in.})(\tfrac{3}{4} \text{ in.})^3}{3} + \frac{(10\tfrac{1}{2} \text{ in.})(\tfrac{3}{8} \text{ in.})^3}{3} + \frac{(5.00 \text{ in.})(\tfrac{3}{4} \text{ in.})^3}{3}$$
$$= 2.01 \text{ in.}^4$$

Distance between flange centroids:

$$h_o = d - \frac{t_{fi}}{2} - \frac{t_{fo}}{2}$$
$$= 12.0 \text{ in.} - \frac{\tfrac{3}{4} \text{ in.}}{2} - \frac{\tfrac{3}{4} \text{ in.}}{2}$$
$$= 11.3 \text{ in.}$$

Warping constant:

$$C_w = \frac{t_f h_o^2}{12}\left(\frac{b_{fi}^3 b_{fo}^3}{b_{fi}^3 + b_{fo}^3}\right)$$
$$= \frac{(\tfrac{3}{4} \text{ in.})(11.3 \text{ in.})^2}{12}\left[\frac{(8.00 \text{ in.})^3 (5.00 \text{ in.})^3}{(8.00 \text{ in.})^3 + (5.00 \text{ in.})^3}\right]$$
$$= 802 \text{ in.}^6$$

Due to symmetry, both the centroid and the shear center lie on the y-axis. Therefore, $x_o = 0$. The distance from the center of the outside flange to the shear center is:

$$e = h_o\left(\frac{b_{fi}^3}{b_{fi}^3 + b_{fo}^3}\right)$$
$$= (11.3 \text{ in.})\left[\frac{(8.00 \text{ in.})^3}{(8.00 \text{ in.})^3 + (5.00 \text{ in.})^3}\right]$$
$$= 9.08 \text{ in.}$$

Add one-half the flange thickness to determine the shear center location measured from the bottom of the outside flange.

$$e+\frac{t_f}{2}=9.08\text{ in.}+\frac{\tfrac{3}{4}\text{ in.}}{2}$$
$$=9.46\text{ in.}$$

$$y_o=\left(e+\frac{t_f}{2}\right)-\bar{y}$$
$$=9.46\text{ in.}-6.91\text{ in.}$$
$$=2.55\text{ in.}$$

$$\bar{r}_o^{\,2}=x_o^2+y_o^2+\frac{I_x+I_y}{A_g}$$ (*Spec*. Eq. E4-9)
$$=(0)^2+(2.55\text{ in.})^2+\frac{334\text{ in.}^4+39.9\text{ in.}^4}{13.7\text{ in.}^2}$$
$$=33.8\text{ in.}^2$$

$$H=1-\frac{x_o^2+y_o^2}{\bar{r}_o^{\,2}}$$ (*Spec*. Eq. E4-8)
$$=1-\frac{(0)^2+(2.55\text{ in.})^2}{33.8\text{ in.}^2}$$
$$=0.808$$

The effective slenderness ratio about the y-axis is:

$$\frac{L_{cy}}{r_y}=\frac{(14.0\text{ ft})(12\text{ in./ft})}{1.71\text{ in.}}$$
$$=98.2$$

$$F_{ey}=\frac{\pi^2E}{\left(\dfrac{L_{cy}}{r_y}\right)^2}$$ (*Spec*. Eq. E4-6)
$$=\frac{\pi^2(29{,}000\text{ ksi})}{(98.2)^2}$$
$$=29.7\text{ ksi}$$

$$F_{ez}=\left(\frac{\pi^2EC_w}{L_{cz}^{\,2}}+GJ\right)\frac{1}{A_g\bar{r}_o^{\,2}}$$ (*Spec*. Eq. E4-7)
$$=\left\{\frac{\pi^2(29{,}000\text{ ksi})(802\text{ in.}^6)}{\left[(14.0\text{ ft})(12\text{ in./ft})\right]^2}+(11{,}200\text{ ksi})(2.01\text{ in.}^4)\right\}\left[\frac{1}{(13.7\text{ in.}^2)(33.8\text{ in.}^2)}\right]$$
$$=66.2\text{ ksi}$$

$$F_e = \left(\frac{F_{ey}+F_{ez}}{2H}\right)\left[1-\sqrt{1-\frac{4F_{ey}F_{ez}H}{\left(F_{ey}+F_{ez}\right)^2}}\right]$$ (*Spec*. Eq. E4-3)

$$= \left[\frac{29.7\text{ ksi}+66.2\text{ ksi}}{2(0.808)}\right]\left[1-\sqrt{1-\frac{4(29.7\text{ ksi})(66.2\text{ ksi})(0.808)}{(29.7\text{ ksi}+66.2\text{ ksi})^2}}\right]$$

$$= 26.4\text{ ksi} \quad \textbf{controls}$$

Torsional and flexural-torsional buckling governs.

$$\frac{F_y}{F_e} = \frac{50\text{ ksi}}{26.4\text{ ksi}}$$
$$= 1.89$$

Because $\frac{F_y}{F_e} < 2.25$:

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y$$ (*Spec*. Eq. E3-2)

$$= \left(0.658^{1.89}\right)(50\text{ ksi})$$
$$= 22.7\text{ ksi}$$

Available Compressive Strength

$$P_n = F_{cr}A_g$$ (*Spec*. Eq. E3-1)

$$= (22.7\text{ ksi})\left(13.7\text{ in.}^2\right)$$
$$= 311\text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(311\text{ kips})$ $= 280\text{ kips}$	$\Omega_c = 1.67$ $\frac{P_n}{\Omega_c} = \frac{311\text{ kips}}{1.67}$ $= 186\text{ kips}$

EXAMPLE E.13 DOUBLE-WT COMPRESSION MEMBER

Given:

Determine the available compressive strength for an ASTM A992 double-WT9×20 compression member, as shown in Figure E.13-1. Assume that ½-in.-thick connectors are welded in position at the ends and at equal intervals, "*a*", along the length. Use the minimum number of intermediate connectors needed to force the two WT-shapes to act as a single built-up compression member.

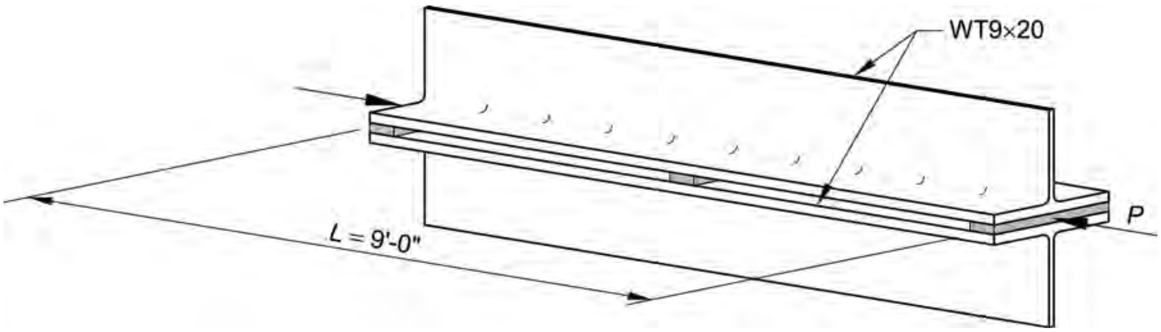


Fig. E.13-1. Double-WT compression member in Example E.13.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Tee
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-8 the geometric properties for a single WT9×20 are as follows:

A = 5.88 in.^{2}
d = 8.95 in.
t_w = 0.315 in.
d/t_w = 28.4
I_x = 44.8 in.^{4}
I_y = 9.55 in.^{4}
r_x = 2.76 in.
r_y = 1.27 in.
$\bar{y}$ = 2.29 in.
J = 0.404 in.^{4}
C_w = 0.788 in.^{6}

From mechanics of materials, the combined section properties for two WT9×20's, flange-to-flange, spaced ½-in. apart, are as follows:

$$A = \Sigma A_{single\ tee}$$
$$= 2\left(5.88 \text{ in.}^2\right)$$
$$= 11.8 \text{ in.}^2$$

$$I_x = \Sigma\left(I_x + A\bar{y}^2\right)$$
$$= 2\left[44.8 \text{ in.}^4 + \left(5.88 \text{ in.}^2\right)\left(2.29 \text{ in.} + \tfrac{1}{4} \text{ in.}\right)^2\right]$$
$$= 165 \text{ in.}^4$$

$$r_x = \sqrt{\frac{I_x}{A}}$$
$$= \sqrt{\frac{165 \text{ in.}^4}{11.8 \text{ in.}^2}}$$
$$= 3.74 \text{ in.}$$

$$I_y = \Sigma I_{y\ single\ tee}$$
$$= 2\left(9.55 \text{ in.}^4\right)$$
$$= 19.1 \text{ in.}^4$$

$$r_y = \sqrt{\frac{I_y}{A}}$$
$$= \sqrt{\frac{19.1 \text{ in.}^4}{11.8 \text{ in.}^2}}$$
$$= 1.27 \text{ in.}$$

$$J = \Sigma J_{single\ tee}$$
$$= 2\left(0.404 \text{ in.}^4\right)$$
$$= 0.808 \text{ in.}^4$$

For the double-WT (cruciform) shape shown in Figure E.13-2 it is reasonable to take $C_w = 0$ and ignore any warping contribution to column strength.

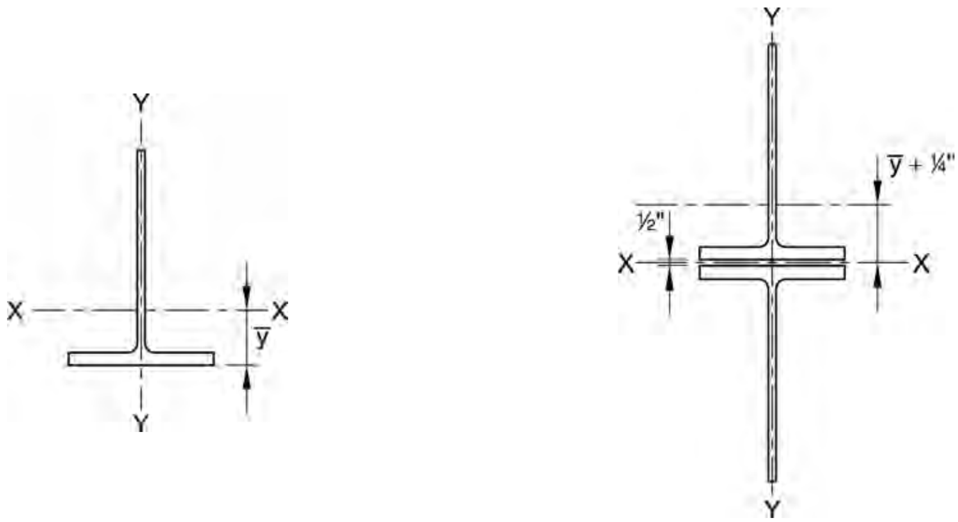


Fig. E.13-2. Double-WT shape cross section.

The y-axis of the combined section is the same as the y-axis of the single section. When buckling occurs about the y-axis, there is no relative slip between the two WTs. For buckling about the x-axis of the combined section, the WTs will slip relative to each other unless restrained by welded or slip-critical end connections.

Intermediate Connectors Dimensional Requirements

Determine the minimum number of intermediate connectors required.

From AISC *Specification* Section E6.2, the maximum slenderness ratio of each tee should not exceed three-fourths times the maximum slenderness ratio of the double-WT built-up section. For a WT9×20, the minimum radius of gyration is:

$$\begin{aligned} r_i &= r_y \\ &= 1.27 \text{ in.} \end{aligned}$$

Use K = 1.0 for both the single tee and the double tee; therefore, $L_{cy} = K_yL_y$ = 1.0(9 ft) = 9.00 ft:

$$\left(\frac{a}{r_i}\right)_{single\ tee} \leq \frac{3}{4}\left(\frac{L_{cy}}{r_{min}}\right)_{double\ tee}$$

$$\begin{aligned} a &\leq \frac{3(r_y)_{single\ tee}}{4(r_y)_{double\ tee}}(L_{cy})_{double\ tee} \\ &= \frac{3}{4}\left(\frac{1.27 \text{ in.}}{1.27 \text{ in.}}\right)\left[(9.00 \text{ ft})(12 \text{ in./ft})\right] \\ &= 81.0 \text{ in.} \end{aligned}$$

Thus, one intermediate connector at mid-length [a = (4.5 ft)(12 in./ft) = 54.0 in.] satisfies AISC *Specification* Section E6.2 as shown in Figure E.13-3.

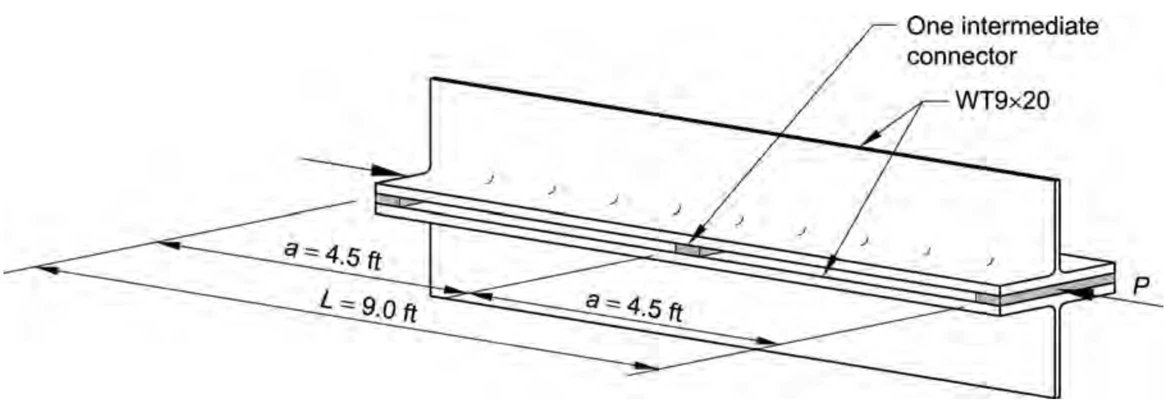


Figure E.13-3. Minimum connectors required for double-WT compression member.

Flexural Buckling and Torsional Buckling Strength

For the WT9×20, the stem is slender because $d/t_w = 28.4 > 0.75\sqrt{29{,}000 \text{ ksi}/50 \text{ ksi}} = 18.1$ (from AISC *Specification* Table B4.1a, Case 4). Therefore, the member is a slender element member and the provisions of Section E7 are followed. Determine the elastic buckling stress for flexural buckling about the *y*- and *x*-axes, and torsional buckling. Then, determine the effective area considering local buckling, the critical buckling stress, and the nominal strength.

Elastic Buckling Stress about the y-y Axis

$$\frac{L_{cy}}{r_y} = \frac{(9.00 \text{ ft})(12 \text{ in./ft})}{1.27 \text{ in.}}$$
$$= 85.0$$

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{L_{cy}}{r_y}\right)^2} \qquad \text{(Spec. Eq. E4-6)}$$
$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{(85.0)^2}$$
$$= 39.6 \text{ ksi} \quad \textbf{controls}$$

Elastic Buckling Stress about the x-x Axis

Flexural buckling about the *x*-axis is determined using the modified slenderness ratio to account for shear deformation of the intermediate connectors.

Note that the provisions of AISC *Specification* Section E6.1, which require that L_c/r be replaced with $(L_c/r)_m$, apply if "the buckling mode involves relative deformations that produce shear forces in the connectors between individual shapes…". Relative slip between the two sections occurs for buckling about the *x*-axis so the provisions of the section apply only to buckling about the *x*-axis.

The connectors are welded at the ends and the intermediate point. The modified slenderness is calculated using the spacing between intermediate connectors:

$$a = (4.5 \text{ ft})(12.0 \text{ in./ft})$$
$$= 54.0 \text{ in.}$$

$$r_i = r_y$$
$$= 1.27 \text{ in.}$$

$$\frac{a}{r_i} = \frac{54.0 \text{ in.}}{1.27 \text{ in.}}$$
$$= 42.5$$

Because $a/r_i > 40$, use AISC *Specification* Equation E6-2b.

$$\left(\frac{L_c}{r}\right)_m = \sqrt{\left(\frac{L_c}{r}\right)_o^2 + \left(\frac{K_i a}{r_i}\right)^2} \qquad \text{(Spec. Eq. E6-2b)}$$

where

$$\left(\frac{L_c}{r}\right)_o = \frac{L_{cx}}{r_x}$$
$$= \frac{(9.00\text{ ft})(12\text{ in./ft})}{3.74\text{ in.}}$$
$$= 28.9$$

$$\frac{K_i a}{r_i} = \frac{0.86(4.50\text{ ft})(12\text{ in./ft})}{1.27\text{ in.}}$$
$$= 36.6$$

Thus,

$$\left(\frac{L_c}{r}\right)_m = \sqrt{(28.9)^2 + (36.6)^2}$$
$$= 46.6$$

$$F_{ex} = \frac{\pi^2 E}{\left(\frac{L_{cx}}{r_x}\right)^2} \qquad \text{(\textit{Spec}. Eq. E4-5)}$$
$$= \frac{\pi^2(29{,}000\text{ ksi})}{(46.6)^2}$$
$$= 132\text{ ksi}$$

Torsional Buckling Elastic Stress

$$F_e = \left(\frac{\pi^2 E C_w}{L_{cz}^2} + GJ\right)\frac{1}{I_x + I_y} \qquad \text{(\textit{Spec}. Eq. E4-2)}$$

The cruciform section made up of two back-to-back WT's has virtually no warping resistance, thus the warping contribution is ignored and *Specification* Equation E4-2 becomes:

$$F_e = \frac{GJ}{I_x + I_y}$$
$$= \frac{(11{,}200\text{ ksi})(0.808\text{ in.}^4)}{165\text{ in.}^4 + 19.1\text{ in.}^4}$$
$$= 49.2\text{ ksi}$$

Critical Stress

Use the smallest elastic buckling stress, F_e, from the limit states considered above to determine F_{cr} by AISC *Specification* Equation E3-2 or Equation E3-3, as follows:

$$F_e = 39.6\text{ ksi}$$

$$\frac{F_y}{F_e} = \frac{50 \text{ ksi}}{39.6 \text{ ksi}}$$
$$= 1.26$$

Because $\frac{F_y}{F_e} < 2.25$,

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \qquad \text{(\textit{Spec}. Eq. E3-2)}$$
$$= \left(0.658^{1.26}\right)(50 \text{ ksi})$$
$$= 29.5 \text{ ksi}$$

Effective Area

Since the stem was previously shown to be slender, calculate the limits of AISC *Specification* Section E7.1 to determine if the stem is fully effective or if there is a reduction in effective area due to local buckling of the stem.

$$\lambda = 28.4$$

$$\lambda_r = 0.75\sqrt{\frac{E}{F_y}}$$
$$= 0.75\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 18.1$$

$$\lambda_r\sqrt{\frac{F_y}{F_{cr}}} = 18.1\sqrt{\frac{50 \text{ ksi}}{29.5 \text{ ksi}}}$$
$$= 23.6$$

Because $\lambda > \lambda_r\sqrt{F_y/F_{cr}}$, the stem will not be fully effective and there will be a reduction in effective area due to local buckling of the stem. The effective width imperfection adjustment factors can be determined from AISC *Specification* Table E7.1, Case (c), as follows.

$$c_1 = 0.22$$
$$c_2 = 1.49$$

Determine the elastic local buckling stress from AISC *Specification* Section E7.1.

$$F_{el} = \left(c_2\frac{\lambda_r}{\lambda}\right)^2 F_y \qquad \text{(\textit{Spec}. Eq. E7-5)}$$
$$= \left[1.49\left(\frac{18.1}{28.4}\right)\right]^2 (50 \text{ ksi})$$
$$= 45.1 \text{ ksi}$$

Determine the effective width of the tee stem and the resulting effective area, where $b = d = 8.95$ in.

$$b_e = b\left(1 - c_1\sqrt{\frac{F_{el}}{F_{cr}}}\right)\sqrt{\frac{F_{el}}{F_{cr}}}$$ (*Spec*. Eq. E7-3)

$$= (8.95 \text{ in.})\left(1 - 0.22\sqrt{\frac{45.1 \text{ ksi}}{29.5 \text{ ksi}}}\right)\sqrt{\frac{45.1 \text{ ksi}}{29.5 \text{ ksi}}}$$

$$= 8.06 \text{ in.}$$

$$A_e = \sum A - \sum\left[t_w\left(b - b_e\right)\right]$$

$$= (2)\left(5.88 \text{ in.}^2\right) - (2)(0.315 \text{ in.})(8.95 \text{ in.} - 8.06 \text{ in.})$$

$$= 11.2 \text{ in.}^2$$

Available Compressive Strength

$$P_n = F_{cr} A_e$$ (*Spec*. Eq. E7-1)

$$= (29.5 \text{ ksi})\left(11.2 \text{ in.}^2\right)$$

$$= 330 \text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(330 \text{ kips})$ $= 297 \text{ kips}$	$\Omega_c = 1.67$ $\dfrac{P_n}{\Omega_c} = \dfrac{330 \text{ kips}}{1.67}$ $= 198 \text{ kips}$

EXAMPLE E.14 ECCENTRICALLY LOADED SINGLE-ANGLE COMPRESSION MEMBER (LONG LEG ATTACHED)

Given:

Determine the available strength of an eccentrically loaded ASTM A36 L8×4×½ single angle compression member, as shown in Figure E.14-1, with an effective length of 5 ft. The long leg of the angle is the attached leg, and the eccentric load is applied at 0.75t as shown. Use the provisions of the AISC *Specification* and compare the results to the available strength found in AISC *Manual* Table 4-12.

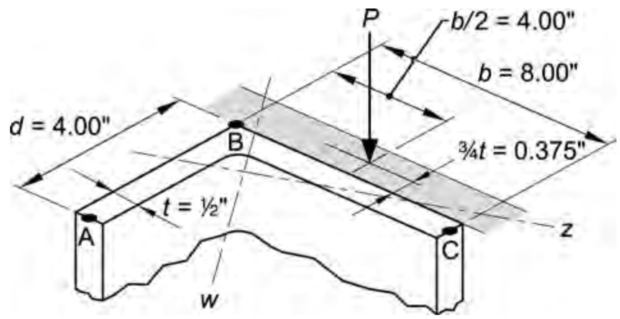


Fig. E.14-1. Eccentrically loaded single-angle compression member in Example E.14.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-7:

L8×4×½
$\bar{x} = 0.854$ in.
$\bar{y} = 2.84$ in.
$A = 5.80$ in.^{2}
$I_x = 38.6$ in.^{4}
$I_y = 6.75$ in.^{4}
$I_z = 4.32$ in.^{4}
$r_z = 0.863$ in.
$\tan \alpha = 0.266$

From AISC Shapes Database V15.0:

$I_w = 41.0$ in.^{4}
$S_{wA} = 12.4$ in.^{3}
$S_{wB} = 16.3$ in.^{3}
$S_{wC} = 7.98$ in.^{3}
$S_{zA} = 1.82$ in.^{3}
$S_{zB} = 2.77$ in.^{3}
$S_{wC} = 5.81$ in.^{3}

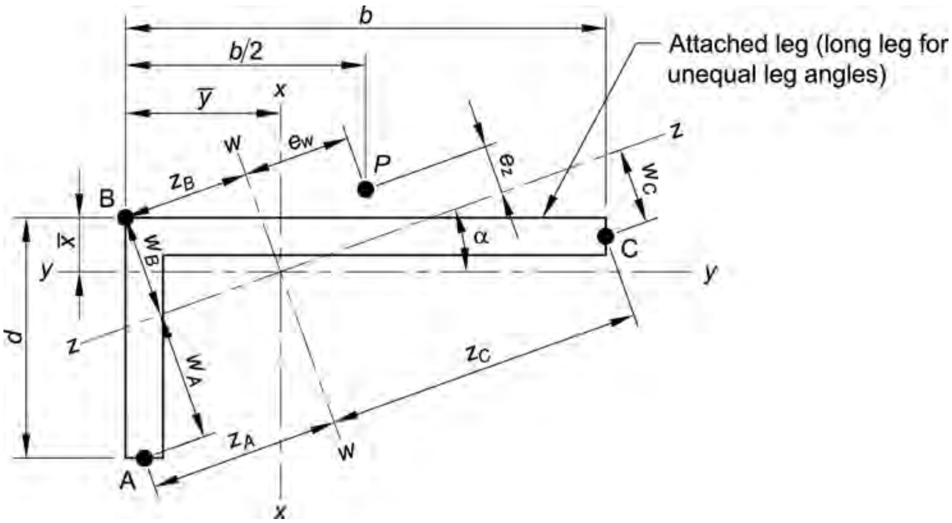


Fig. E.14-2. Geometry about principal axes.

The load is applied at the location shown in Figure E.14-2. Determine the eccentricities about the major (w-w axis) and minor (z-z axis) principal axes for the load, P. From AISC *Manual* Table 1-7, the angle of the principal axes is found to be $\alpha = \tan^{-1}(0.266) = 14.9°$.

Using the geometry shown in Figures E.14-2 and E.14-3:

$$
\begin{aligned}
e_w &= \left[\left(\bar{x}+0.75t\right)-\left(0.5b-\bar{y}\right)\tan\alpha\right]\sin\alpha+\left(\frac{0.5b-\bar{y}}{\cos\alpha}\right)\\
&=\left\{\left[0.854\text{ in.}+0.75\left(\tfrac{1}{2}\text{ in.}\right)\right]-\left[0.5\left(8.00\text{ in.}\right)-2.84\text{ in.}\right]\left(0.266\right)\right\}\left(\sin 14.9°\right)+\left[\frac{0.5\left(8.00\text{ in.}\right)-2.84\text{ in.}}{\left(\cos 14.9°\right)}\right]\\
&=1.44\text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
e_z &= \left(\bar{x}+0.75t\right)\cos\alpha-\left(0.5b-\bar{y}\right)\sin\alpha\\
&=\left[0.854\text{ in.}+0.75\left(\tfrac{1}{2}\text{ in.}\right)\right]\left(\cos 14.9°\right)-\left[0.5\left(8.00\text{ in.}\right)-2.84\text{ in.}\right]\left(\sin 14.9°\right)\\
&=0.889\text{ in.}
\end{aligned}
$$

Because of these eccentricities, the moment resultant has components about both principal axes; therefore, the combined stress provisions of AISC *Specification* Section H2 must be followed.

$$\left|\frac{f_{ra}}{F_{ca}}+\frac{f_{rbw}}{F_{cbw}}+\frac{f_{rbz}}{F_{cbz}}\right|\leq 1.0 \qquad \text{(\textit{Spec.} Eq. H2-1)}$$

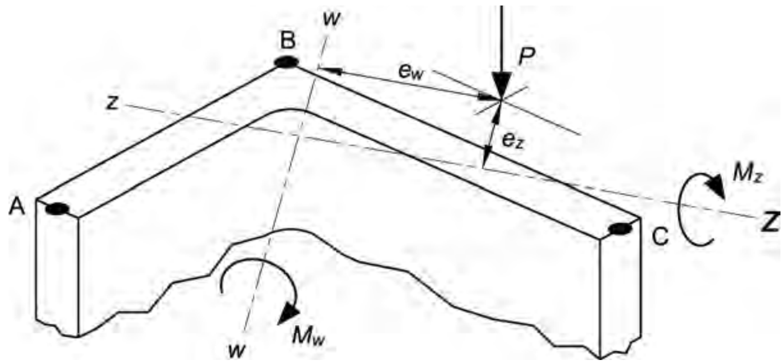


Fig. E.14-3. Applied moments and eccentric axial load.

Due to the load and the given eccentricities, moments about the *w-w* and *z-z* axes will have different effects on points A, B and C. The axial force will produce a compressive stress and the moments, where positive moments are in the direction shown in Figure E.14-3, will produce stresses with a sign indicated by the sense given in the following. In this example, compressive stresses will be taken as positive and tensile stresses will be taken as negative.

Point	Caused by M_w	Caused by M_z
A	tension	tension
B	tension	compression
C	compression	tension

Available Compressive Strength

Check the slenderness of the longest leg for uniform compression.

$$\begin{aligned}\lambda &= \frac{b}{t} \\ &= \frac{8.00\text{ in.}}{½\text{ in.}} \\ &= 16.0\end{aligned}$$

Check the slenderness of the shorter leg for uniform compression.

$$\begin{aligned}\lambda &= \frac{d}{t} \\ &= \frac{4.00\text{ in.}}{½\text{ in.}} \\ &= 8.00\end{aligned}$$

From *AISC Specification* Table B4.1a, Case 3, the limiting width-to-thickness ratio is:

$$\begin{aligned}\lambda_r &= 0.45\sqrt{\frac{E}{F_y}} \\ &= 0.45\sqrt{\frac{29{,}000\text{ ksi}}{36\text{ ksi}}} \\ &= 12.8\end{aligned}$$

Because b/t = 16.0 > 12.8, the longer leg is classified as a slender element for compression. Because d/t = 8.00 < 12.8, the shorter leg is classified as a nonslender element for compression.

Determine if torsional and flexural-torsional buckling is applicable, using the provisions of AISC *Specification* Section E4.

$$\lambda = 16.0$$

$$0.71\sqrt{\frac{E}{F_y}} = 0.71\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$
$$= 20.2$$

Because $\lambda < 0.71\sqrt{E/F_y}$, torsional and flexural-torsional buckling is not applicable.

Determine the critical stress, F_{cr} , with L_c = (5.00 ft)(12 in./ft) = 60.0 in. for buckling about the z-z axis.

$$\frac{L_{cz}}{r_z} = \frac{60.0 \text{ in.}}{0.863 \text{ in.}}$$
$$= 69.5$$

$$F_e = \frac{\pi^2 E}{\left(\frac{L_{cz}}{r_z}\right)^2} \qquad (\textit{Spec}. \text{ Eq. E3-4})$$
$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{(69.5)^2}$$
$$= 59.3 \text{ ksi}$$

$$\frac{F_y}{F_e} = \frac{36 \text{ ksi}}{59.3 \text{ ksi}}$$
$$= 0.607$$

Because $\frac{F_y}{F_e} < 2.25$:

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \qquad (\textit{Spec}. \text{ Eq. E3-2})$$
$$= \left(0.658^{0.607}\right)(36 \text{ ksi})$$
$$= 27.9 \text{ ksi}$$

Because the longer leg was found to be slender, the limits of AISC *Specification* Section E7.1 must be evaluated to determine if the leg is fully effective for compression or if a reduction in effective area must be taken to account for local buckling in the longer leg.

$$\lambda = 16.0$$

$$\lambda_r \sqrt{\frac{F_y}{F_{cr}}} = 12.8\sqrt{\frac{36\text{ ksi}}{27.9\text{ ksi}}}$$
$$= 14.5$$

Because $\lambda > 14.5,$ there will be a reduction in effective area due to local buckling in the longer leg. Determine the effective width imperfection adjustment factors per AISC *Specification* Table E7.1 as follows.

$$c_1 = 0.22$$
$$c_2 = 1.49$$

Determine the elastic local buckling stress from AISC *Specification* Section E7.1.

$$F_{el} = \left(c_2 \frac{\lambda_r}{\lambda}\right)^2 F_y \qquad \text{(}\textit{Spec}\text{. Eq. E7-5)}$$
$$= \left[1.49\left(\frac{12.8}{16.0}\right)\right]^2 (36\text{ ksi})$$
$$= 51.2\text{ ksi}$$

Determine the effective width of the angle leg and the resulting effective area.

$$b_e = b\left(1 - c_1\sqrt{\frac{F_{el}}{F_{cr}}}\right)\sqrt{\frac{F_{el}}{F_{cr}}} \qquad \text{(}\textit{Spec}\text{. Eq. E7-3)}$$
$$= (8.00\text{ in.})\left(1 - 0.22\sqrt{\frac{51.2\text{ ksi}}{27.9\text{ ksi}}}\right)\sqrt{\frac{51.2\text{ ksi}}{27.9\text{ ksi}}}$$
$$= 7.61\text{ in.}$$

$$A_e = A_g - t\sum(b - b_e)$$
$$= 5.80\text{ in.}^2 - (\tfrac{1}{2}\text{ in.})(8.00\text{ in.} - 7.61\text{ in.})$$
$$= 5.61\text{ in.}^2$$

Available Compressive Strength

$$P_n = F_{cr}A_e \qquad \text{(}\textit{Spec}\text{. Eq. E7-1)}$$
$$= (27.9\text{ ksi})(5.61\text{ in.}^2)$$
$$= 157\text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(157\text{ kips})$ $= 141\text{ kips}$	$\Omega_c = 1.67$ $\dfrac{P_n}{\Omega_c} = \dfrac{157\text{ kips}}{1.67}$ $= 94.0\text{ kips}$

Determine the available flexural strengths, M_{cbw} and M_{cbz}, and the available flexural stresses at each point on the cross section.

Yielding

Consider the limit state of yielding for bending about the *w*-*w* and *z*-*z* axes at points A, B and C, according to AISC *Specification* Section F10.1.

w-*w* axis:

$$\begin{aligned} M_{ywA} &= F_y S_{wA} \\ &= (36 \text{ ksi})(12.4 \text{ in.}^3) \\ &= 446 \text{ kip-in.} \end{aligned}$$

$$\begin{aligned} M_{nwA} &= 1.5 M_{ywA} \\ &= 1.5(446 \text{ kip-in.}) \\ &= 669 \text{ kip-in.} \end{aligned}$$ (from *Spec*. Eq. F10-1)

$$\begin{aligned} M_{ywB} &= F_y S_{wB} \\ &= (36 \text{ ksi})(16.3 \text{ in.}^3) \\ &= 587 \text{ kip-in.} \end{aligned}$$

$$\begin{aligned} M_{nwB} &= 1.5 M_{ywB} \\ &= 1.5(587 \text{ kip-in.}) \\ &= 881 \text{ kip-in.} \end{aligned}$$ (from *Spec*. Eq. F10-1)

$$\begin{aligned} M_{ywC} &= F_y S_{wC} \\ &= (36 \text{ ksi})(7.98 \text{ in.}^3) \\ &= 287 \text{ kip-in.} \end{aligned}$$

$$\begin{aligned} M_{nwC} &= 1.5 M_{ywC} \\ &= 1.5(287 \text{ kip-in.}) \\ &= 431 \text{ kip-in.} \end{aligned}$$ (from *Spec*. Eq. F10-1)

z-*z* axis:

$$\begin{aligned} M_{yzA} &= F_y S_{zA} \\ &= (36 \text{ ksi})(1.82 \text{ in.}^3) \\ &= 65.5 \text{ kip-in.} \end{aligned}$$

$$\begin{aligned} M_{nzA} &= 1.5 M_{yzA} \\ &= 1.5(65.5 \text{ kip-in.}) \\ &= 98.3 \text{ kip-in.} \end{aligned}$$ (from *Spec*. Eq. F10-1)

$$\begin{aligned} M_{yzB} &= F_y S_{zB} \\ &= (36 \text{ ksi})(2.77 \text{ in.}^3) \\ &= 99.7 \text{ kip-in.} \end{aligned}$$

$$\begin{aligned} M_{nzB} &= 1.5 M_{yzB} \\ &= 1.5(99.7 \text{ kip-in.}) \\ &= 150 \text{ kip-in.} \end{aligned}$$ (from *Spec.* Eq. F10-1)

$$\begin{aligned} M_{yzC} &= F_y S_{zC} \\ &= (36 \text{ ksi})(5.81 \text{ in.}^3) \\ &= 209 \text{ kip-in.} \end{aligned}$$

$$\begin{aligned} M_{nzC} &= 1.5 M_{yzC} \\ &= 1.5(209 \text{ kip-in.}) \\ &= 314 \text{ kip-in.} \end{aligned}$$ (from *Spec.* Eq. F10-1)

Select the least M_n for each axis.

For the limit state of yielding about the *w*-*w* axis:

$$M_{nw} = 431 \text{ kip-in. at point C}$$

For the limit state of yielding about the *z*-*z* axis:

$$M_{nz} = 98.3 \text{ kip-in. at point A}$$

Lateral-Torsional Buckling

From AISC *Specification* Section F10.2, the limit state of lateral-torsional buckling of a single angle without continuous restraint along its length is a function of the elastic lateral-torsional buckling moment about the major principal axis. For bending about the major principal axis for a single angle:

$$M_{cr} = \frac{9EAr_z tC_b}{8L_b}\left[\sqrt{1+\left(4.4\frac{\beta_w r_z}{L_b t}\right)^2}+4.4\frac{\beta_w r_z}{L_b t}\right]$$ (*Spec.* Eq. F10-4)

From AISC *Specification* Section F1, for uniform moment along the member length, C_b = 1.0. From AISC *Specification* Commentary Table C-F10.1, an L8×4×½ has β_w = 5.48 in. From AISC *Specification* Commentary Figure C-F10.4b, with the tip of the long leg (point C) in compression for bending about the *w*-axis, β_w is taken as negative. Thus:

$$\begin{aligned} M_{cr} &= \frac{9(29{,}000 \text{ ksi})(5.80 \text{ in.}^2)(0.863 \text{ in.})(\tfrac{1}{2} \text{ in.})(1.0)}{8(60.0 \text{ in.})} \\ &\quad \times\left\{\sqrt{1+\left[4.4\frac{(-5.48 \text{ in.})(0.863 \text{ in.})}{(60.0 \text{ in.})(\tfrac{1}{2} \text{ in.})}\right]^2}+4.4\frac{(-5.48 \text{ in.})(0.863 \text{ in.})}{(60.0 \text{ in.})(\tfrac{1}{2} \text{ in.})}\right\} \\ &= 712 \text{ kip-in.} \end{aligned}$$

$$\frac{M_{ywC}}{M_{cr}} = \frac{287 \text{ kip-in.}}{712 \text{ kip-in.}}$$
$$= 0.403$$

Because $M_{ywC}/M_{cr} < 1.0$, determine M_n as follows:

$$M_{nwC} = \left(1.92 - 1.17\sqrt{\frac{M_{ywC}}{M_{cr}}}\right)M_{ywC} \leq 1.5M_{ywC} \quad \text{(from } \textit{Spec.} \text{ Eq. F10-2)}$$
$$= \left(1.92 - 1.17\sqrt{0.403}\right)(287 \text{ kip-in.}) < 1.5(287 \text{ kip-in.})$$
$$= 338 \text{ kip-in.} < 431 \text{ kip-in.}$$
$$= 338 \text{ kip-in.}$$

Leg Local Buckling

From AISC *Specification* Section F10.3, the limit state of leg local buckling applies when the toe of the leg is in compression. As discussed previously and indicated in Table E.14-1, the only case in which a toe is in compression is point C for bending about the *w*-*w* axis. Thus, determine the slenderness of the long leg as a compression element subject to flexure. From AISC *Specification* Table B4.1b, Case 12:

$$\lambda_p = 0.54\sqrt{\frac{E}{F_y}}$$
$$= 0.54\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$
$$= 15.3$$

$$\lambda_r = 0.91\sqrt{\frac{E}{F_y}}$$
$$= 0.91\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$
$$= 25.8$$

$$\lambda = \frac{b}{t}$$
$$= \frac{8.0 \text{ in.}}{\frac{1}{2} \text{ in.}}$$
$$= 16.0$$

Because $\lambda_p < \lambda < \lambda_r$, the angle is noncompact for flexure for this loading. From AISC *Specification* Equation F10-6:

$$M_{nwC} = F_y S_{wC}\left(2.43 - 1.72\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}}\right) \quad \text{(from } \textit{Spec.} \text{ Eq. F10-6)}$$
$$= (36 \text{ ksi})\left(7.98 \text{ in.}^3\right)\left[2.43 - 1.72(16.0)\sqrt{\frac{36 \text{ ksi}}{29{,}000 \text{ ksi}}}\right]$$
$$= 420 \text{ kip-in.}$$

Table E.14-1 provides a summary of nominal flexural strength at each point. T indicates the point is in tension and C indicates it is in compression.

Table E.14-1						
Point	Yielding		Lateral-Torsional Buckling		Leg Local Buckling	
	M_{nw}, kip-in.	M_{nz}, kip-in.	M_{nw}, kip-in.	M_{nz}, kip-in.	M_{nw}, kip-in.	M_{nz}, kip-in.
A	669 T	98.3 T	–	–	–	–
B	881 T	150 C	–	–	–	–
C	431 C	314 T	338 C	–	420 C	–
Note: (–) indicates that the limit state is not applicable to this point.						

Available Flexural Strength

Select the controlling nominal flexural strength for the *w-w* and *z-z* axes.

For the *w-w* axis:

$$M_{nw} = 338 \text{ kip-in.}$$

For the *z-z* axis:

$$M_{nz} = 98.3 \text{ kip-in.}$$

From AISC *Specification* Section F1, determine the available flexural strength for each axis, *w-w* and *z-z*, as follows:

LRFD	ASD
$\phi_b = 0.90$	$\Omega_b = 1.67$
$M_{cbw} = \phi_b M_{nw}$ $= 0.90(338 \text{ kip-in.})$ $= 304 \text{ kip-in.}$	$M_{cbw} = \dfrac{M_{nw}}{\Omega_b}$ $= \dfrac{338 \text{ kip-in.}}{1.67}$ $= 202 \text{ kip-in.}$
$M_{cbz} = \phi_b M_{nz}$ $= 0.90(98.3 \text{ kip-in.})$ $= 88.5 \text{ kip-in.}$	$M_{cbz} = \dfrac{M_{nz}}{\Omega_b}$ $= \dfrac{98.3 \text{ kip-in.}}{1.67}$ $= 58.9 \text{ kip-in.}$

Required Flexural Strength

The load on the column is applied at eccentricities about the *w-w* and *z-z* axes resulting in the following moments:

$$M_w = P_r e_w$$
$$= P_r(1.44 \text{ in.})$$

and

$$M_z = P_r e_z$$
$$= P_r (0.889 \text{ in.})$$

The combination of axial load and moment will produce second-order effects in the column which must be accounted for.

Using AISC *Specification* Appendix 8.2, an approximate second-order analysis can be performed. The required second-order flexural strengths will be $B_{1w} M_w$ and $B_{1z} M_z$, respectively, where

$$B_1 = \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}} \geq 1.0 \qquad \text{(\textit{Spec}. Eq. A-8-3)}$$

and

$\alpha = 1.0$ (LRFD)

$\alpha = 1.6$ (ASD)

C_m = 1.0 for a column with uniform moment along its length

For each axis, parameters P_{e1w} and P_{e1z}, as used in the moment magnification terms, B_{1w} and B_{1z}, are:

$$P_{e1w} = \frac{\pi^2 EI_w}{(L_{c1})^2} \qquad \text{(from \textit{Spec}. Eq. A-8-5)}$$
$$= \frac{\pi^2 (29{,}000 \text{ ksi})(41.0 \text{ in.}^4)}{(60.0 \text{ in.})^2}$$
$$= 3{,}260 \text{ kips}$$

$$P_{e1z} = \frac{\pi^2 EI_z}{(L_{c1})^2} \qquad \text{(from \textit{Spec}. Eq. A-8-5)}$$
$$= \frac{\pi^2 (29{,}000 \text{ ksi})(4.32 \text{ in.}^4)}{(60.0 \text{ in.})^2}$$
$$= 343 \text{ kips}$$

and

$$B_{1w} = \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1w}}} \qquad \text{(\textit{Spec}. Eq. A-8-3)}$$
$$= \frac{1.0}{1 - \dfrac{\alpha P_r}{3{,}260 \text{ kips}}}$$

$$B_{1z} = \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1z}}} \qquad \text{(\textit{Spec}. Eq. A-8-3)}$$
$$= \frac{1.0}{1 - \dfrac{\alpha P_r}{343 \text{ kips}}}$$

Thus, the required second-order flexural strengths are:

$$M_{rw} = P_r\left(1.44\text{ in.}\right)\left(\frac{1.0}{1-\dfrac{\alpha P_r}{3{,}260\text{ kips}}}\right)$$

$$M_{rz} = P_r\left(0.889\text{ in.}\right)\left(\frac{1.0}{1-\dfrac{\alpha P_r}{343\text{ kips}}}\right)$$

Interaction of Axial and Flexural Strength

Evaluate the interaction of axial and flexural stresses according to the provisions of AISC *Specification* Section H2.

The interaction equation is given as:

$$\left|\frac{f_{ra}}{F_{ca}} + \frac{f_{rbw}}{F_{cbw}} + \frac{f_{rbz}}{F_{cbz}}\right| \le 1.0 \qquad \text{(\textit{Spec.} Eq. H2-1)}$$

where the stresses are to be considered at each point on the cross section with the appropriate sign representing the sense of the stress. Because the required stress and available stress at any point are both functions of the same section property, *A* or *S*, it is possible to convert Equation H2-1 from a stress based equation to a force based equation where the section properties will cancel.

Substituting the available strengths and the expressions for the required second-order flexural strengths into AISC *Specification* Equation H2-1 yields:

LRFD	ASD
$\left\lvert\dfrac{P_u}{141\text{ kips}}+\dfrac{P_u\left(1.44\text{ in.}\right)}{304\text{ kip-in.}}\left(\dfrac{1.0}{1-\dfrac{1.0P_u}{3{,}260\text{ kips}}}\right)+\left[\dfrac{P_u\left(0.889\text{ in.}\right)}{88.5\text{ kip-in.}}\right]\left(\dfrac{1}{1-\dfrac{1.0P_u}{343\text{ kips}}}\right)\right\rvert \le 1.0$	$\left\lvert\dfrac{P_a}{94.0\text{ kips}}+\dfrac{P_a\left(1.44\text{ in.}\right)}{202\text{ kip-in.}}\left(\dfrac{1.0}{1-\dfrac{1.6P_a}{3{,}260\text{ kips}}}\right)+\left[\dfrac{P_a\left(0.889\text{ in.}\right)}{58.9\text{ kip-in.}}\right]\left(\dfrac{1}{1-\dfrac{1.6P_a}{343\text{ kips}}}\right)\right\rvert \le 1.0$

These interaction equations must now be applied at each critical point on the section, points A, B and C using the appropriate sign for the sense of the resulting stress, with compression taken as positive.

For point A, the *w* term is negative and the *z* term is negative. Thus:

LRFD	ASD
$\left\lvert\dfrac{P_u}{141\text{ kips}}-\dfrac{P_u\left(1.44\text{ in.}\right)}{304\text{ kip-in.}}\left(\dfrac{1.0}{1-\dfrac{1.0P_u}{3{,}260\text{ kips}}}\right)-\left[\dfrac{P_u\left(0.889\text{ in.}\right)}{88.5\text{ kip-in.}}\right]\left(\dfrac{1}{1-\dfrac{1.0P_u}{343\text{ kips}}}\right)\right\rvert \le 1.0$ By iteration, $P_u = 88.4$ kips.	$\left\lvert\dfrac{P_a}{94.0\text{ kips}}-\dfrac{P_a\left(1.44\text{ in.}\right)}{202\text{ kip-in.}}\left(\dfrac{1.0}{1-\dfrac{1.6P_a}{3{,}260\text{ kips}}}\right)-\left[\dfrac{P_a\left(0.889\text{ in.}\right)}{58.9\text{ kip-in.}}\right]\left(\dfrac{1}{1-\dfrac{1.6P_a}{343\text{ kips}}}\right)\right\rvert \le 1.0$ By iteration, $P_a = 57.7$ kips.

For point B, the *w* term is negative and the *z* term is positive. Thus:

LRFD	ASD
$\left\| \dfrac{P_u}{141\text{ kips}} - \dfrac{P_u(1.44\text{ in.})}{304\text{ kip-in.}}\left(\dfrac{1.0}{1-\dfrac{1.0P_u}{3{,}260\text{ kips}}}\right) + \left[\dfrac{P_u(0.889\text{ in.})}{88.5\text{ kip-in.}}\right]\left(\dfrac{1}{1-\dfrac{1.0P_u}{343\text{ kips}}}\right) \right\| \le 1.0$ By iteration, P_u = 67.7 kips.	$\left\| \dfrac{P_a}{94.0\text{ kips}} - \dfrac{P_a(1.44\text{ in.})}{202\text{ kip-in.}}\left(\dfrac{1.0}{1-\dfrac{1.6P_a}{3{,}260\text{ kips}}}\right) + \left[\dfrac{P_a(0.889\text{ in.})}{58.9\text{ kip-in.}}\right]\left(\dfrac{1}{1-\dfrac{1.6P_a}{343\text{ kips}}}\right) \right\| \le 1.0$ By iteration, P_a = 44.6 kips.

For point C, the *w* term is positive and the *z* term is negative. Thus:

LRFD	ASD
$\left\| \dfrac{P_u}{141\text{ kips}} + \dfrac{P_u(1.44\text{ in.})}{304\text{ kip-in.}}\left(\dfrac{1.0}{1-\dfrac{1.0P_u}{3{,}260\text{ kips}}}\right) - \left[\dfrac{P_u(0.889\text{ in.})}{88.5\text{ kip-in.}}\right]\left(\dfrac{1}{1-\dfrac{1.0P_u}{343\text{ kips}}}\right) \right\| \le 1.0$ By iteration, P_u = 156 kips.	$\left\| \dfrac{P_a}{94.0\text{ kips}} + \dfrac{P_a(1.44\text{ in.})}{202\text{ kip-in.}}\left(\dfrac{1.0}{1-\dfrac{1.6P_a}{3{,}260\text{ kips}}}\right) - \left[\dfrac{P_a(0.889\text{ in.})}{58.9\text{ kip-in.}}\right]\left(\dfrac{1}{1-\dfrac{1.6P_a}{343\text{ kips}}}\right) \right\| \le 1.0$ By iteration, P_a = 99.5 kips.

Governing Available Strength

LRFD	ASD
From the above iterations, P_u = 67.7 kips From AISC *Manual* Table 4-12, $\phi P_n = 67.7$ kips	From the above iterations, P_a = 44.6 kips From AISC *Manual* Table 4-12, $\dfrac{P_n}{\Omega} = 44.6$ kips

Thus, the calculations demonstrate how the values for this member in AISC *Manual* Table 4-12 can be confirmed.

Chapter F
Design of Members for Flexure

INTRODUCTION

This *Specification* chapter contains provisions for calculating the flexural strength of members subject to simple bending about one principal axis. Included are specific provisions for I-shaped members, channels, HSS, box sections, tees, double angles, single angles, rectangular bars, rounds and unsymmetrical shapes. Also included is a section with proportioning requirements for beams and girders.

There are selection tables in the AISC *Manual* for standard beams in the commonly available yield strengths. The section property tables for most cross sections provide information that can be used to conveniently identify noncompact and slender element sections. LRFD and ASD information is presented side-by-side.

Most of the formulas from this chapter are illustrated by the following examples. The design and selection techniques illustrated in the examples for both LRFD and ASD will result in similar designs.

F1. GENERAL PROVISIONS

Selection and evaluation of all members is based on deflection requirements and strength, which is determined as the design flexural strength, $\phi_b M_n$, or the allowable flexural strength, M_n/Ω_b,

where
M_n = the lowest nominal flexural strength based on the limit states of yielding, lateral torsional-buckling, and local buckling, where applicable
ϕ_b = 0.90 (LRFD)
Ω_b = 1.67 (ASD)

This design approach is followed in all examples.

The term L_b is used throughout this chapter to describe the length between points which are either braced against lateral displacement of the compression flange or braced against twist of the cross section. Requirements for bracing systems and the required strength and stiffness at brace points are given in AISC *Specification* Appendix 6.

The use of C_b is illustrated in several of the following examples. AISC *Manual* Table 3-1 provides tabulated C_b values for some common situations.

F2. DOUBLY SYMMETRIC COMPACT I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MAJOR AXIS

AISC *Specification* Section F2 applies to the design of compact beams and channels. As indicated in the User Note in Section F2 of the AISC *Specification*, the vast majority of rolled I-shaped beams and channels fall into this category. The curve presented as a solid line in Figure F-1 is a generic plot of the nominal flexural strength, M_n, as a function of the unbraced length, L_b. The horizontal segment of the curve at the far left, between L_b = 0 ft and L_p, is the range where the strength is limited by flexural yielding. In this region, the nominal strength is taken as the full plastic moment strength of the section as given by AISC *Specification* Equation F2-1. In the range of the curve at the far right, starting at L_r, the strength is limited by elastic buckling. The strength in this region is given by AISC *Specification* Equation F2-3. Between these regions, within the linear region of the curve between $M_n = M_p$ at L_p on the left, and $M_n = 0.7M_y = 0.7F_yS_x$ at L_r on the right, the strength is limited by inelastic buckling. The strength in this region is provided in AISC *Specification* Equation F2-2.

The curve plotted as a heavy solid line represents the case where C_b = 1.0, while the heavy dashed line represents the case where C_b exceeds 1.0. The nominal strengths calculated in both AISC *Specification* Equations F2-2 and F2-3 are linearly proportional to C_b, but are limited to M_p as shown in the figure.

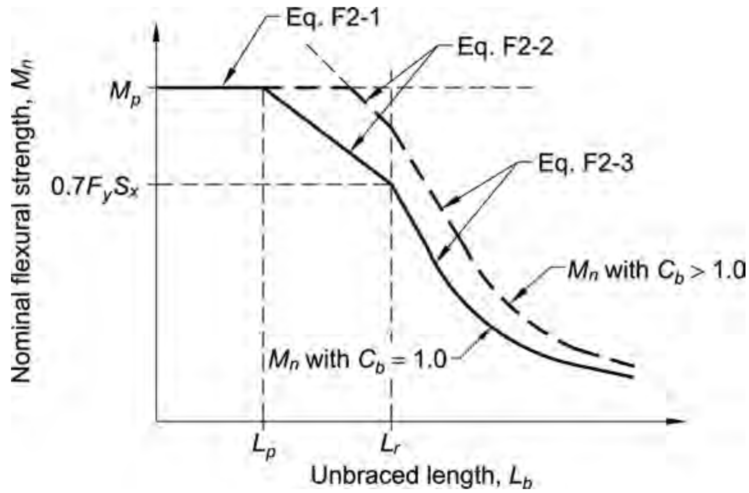


Fig. F-1. Nominal flexural strength versus unbraced length.

$$M_n = M_p = F_y Z_x \qquad (Spec.\ \text{Eq. F2-1})$$

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad (Spec.\ \text{Eq. F2-2})$$

$$M_n = F_{cr}S_x \le M_p \qquad (Spec.\ \text{Eq. F2-3})$$

where

$$F_{cr} = \frac{C_b\pi^2 E}{\left(\dfrac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_x h_o}\left(\frac{L_b}{r_{ts}}\right)^2} \qquad (Spec.\ \text{Eq. F2-4})$$

The provisions of this section are illustrated in Example F.1 (W-shape beam) and Example F.2 (channel).

Inelastic design provisions are given in AISC *Specification* Appendix 1. L_{pd}, the maximum unbraced length for prismatic member segments containing plastic hinges is less than L_p.

F3. DOUBLY SYMMETRIC I-SHAPED MEMBERS WITH COMPACT WEBS AND NONCOMPACT OR SLENDER FLANGES BENT ABOUT THEIR MAJOR AXIS

The strength of shapes designed according to this section is limited by local buckling of the compression flange. Only a few standard wide-flange shapes have noncompact flanges. For these sections, the strength reduction for F_y = 50 ksi steel varies. The approximate percentages of M_p about the strong axis that can be developed by noncompact members when braced such that $L_b \le L_p$ are shown as follows:

W21×48 = 99%	W14×99 = 99%	W14×90 = 97%	W12×65 = 98%
W10×12 = 99%	W8×31 = 99%	W8×10 = 99%	W6×15 = 94%
W6×8.5 = 97%			

The strength curve for the flange local buckling limit state, shown in Figure F-2, is similar in nature to that of the lateral-torsional buckling curve. The horizontal axis parameter is $\lambda = b_f/2t_f$. The flat portion of the curve to the left of λ_{pf} is the plastic yielding strength, M_p. The curved portion to the right of λ_{rf} is the strength limited by elastic buckling of the flange. The linear transition between these two regions is the strength limited by inelastic flange buckling.

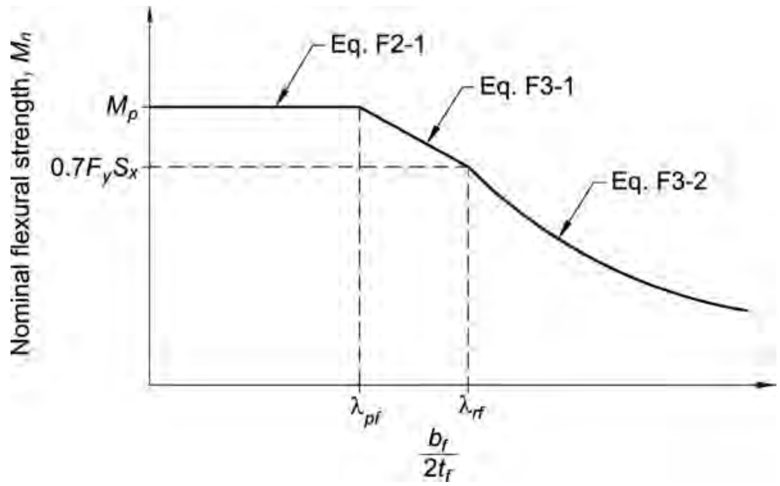


Fig. F-2. Flange local buckling strength.

$$M_n = M_p = F_y Z_x \qquad (Spec.\text{ Eq. F2-1})$$

$$M_n = M_p - \left(M_p - 0.7F_y S_x\right)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right) \qquad (Spec.\text{ Eq. F3-1})$$

$$M_n = \frac{0.9Ek_c S_x}{\lambda^2} \qquad (Spec.\text{ Eq. F3-2})$$

where

$k_c = \frac{4}{\sqrt{h/t_w}}$ and shall not be taken less than 0.35 nor greater than 0.76 for calculation purposes.

The strength reductions due to flange local buckling of the few standard rolled shapes with noncompact flanges are incorporated into the design tables in Part 3 and Part 6 of the AISC *Manual*.

There are no standard I-shaped members with slender flanges. The noncompact flange provisions of this section are illustrated in Example F.3.

F4. OTHER I-SHAPED MEMBERS WITH COMPACT OR NONCOMPACT WEBS BENT ABOUT THEIR MAJOR AXIS

This section of the AISC *Specification* applies to doubly symmetric I-shaped members with noncompact webs and singly symmetric I-shaped members (those having different flanges) with compact or noncompact webs.

F5. DOUBLY SYMMETRIC AND SINGLY SYMMETRIC I-SHAPED MEMBERS WITH SLENDER WEBS BENT ABOUT THEIR MAJOR AXIS

This section applies to doubly symmetric and singly symmetric I-shaped members with slender webs, formerly designated as "plate girders".

F6. I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MINOR AXIS

I-shaped members and channels bent about their minor axis are not subject to lateral-torsional buckling. Rolled or built-up shapes with noncompact or slender flanges, as determined by AISC *Specification* Table B4.1b, must be checked for strength based on the limit state of flange local buckling using Equations F6-2 or F6-3 as applicable.

The vast majority of W, M, C and MC shapes have compact flanges, and can therefore develop the full plastic moment, M_p, about the minor axis. The provisions of this section are illustrated in Example F.5.

F7. SQUARE AND RECTANGULAR HSS AND BOX SECTIONS

Square and rectangular HSS need to be checked for the limit states of yielding, and flange and web local buckling. Lateral-torsional buckling is also possible for rectangular HSS or box sections bent about the strong axis; however, as indicated in the User Note in AISC *Specification* Section F7, deflection will usually control the design before there is a significant reduction in flexural strength due to lateral-torsional buckling.

The design and section property tables in the AISC *Manual* were calculated using a design wall thickness of 93% of the nominal wall thickness (see AISC *Specification* Section B4.2). Strength reductions due to local buckling have been accounted for in the AISC *Manual* design tables. The selection of a square HSS with compact flanges is illustrated in Example F.6. The provisions for a rectangular HSS with noncompact flanges is illustrated in Example F.7. The provisions for a square HSS with slender flanges are illustrated in Example F.8. Available flexural strengths of rectangular and square HSS are listed in Tables 3-12 and 3-13, respectively. If HSS members are specified using ASTM A1065 or ASTM A1085 material, the design wall thickness may be taken equal to the nominal wall thickness.

F8. ROUND HSS

The definition of HSS encompasses both tube and pipe products. The lateral-torsional buckling limit state does not apply, but round HSS are subject to strength reductions from local buckling. Available strengths of round HSS and Pipes are listed in AISC *Manual* Tables 3-14 and 3-15, respectively. The tabulated properties and available flexural strengths of these shapes in the AISC *Manual* are calculated using a design wall thickness of 93% of the nominal wall thickness. The design of a Pipe is illustrated in Example F.9. If round HSS members are specified using ASTM A1085 material, the design wall thickness may be taken equal to the nominal wall thickness.

F9. TEES AND DOUBLE ANGLES LOADED IN THE PLANE OF SYMMETRY

The AISC *Specification* provides a check for flange local buckling, which applies only when a noncompact or slender flange is in compression due to flexure. This limit state will seldom govern. A check for local buckling of the tee stem in flexural compression was added in the 2010 edition of the *Specification*. The provisions were expanded to include local buckling of double-angle web legs in flexural compression in the 2016 edition. Attention should be given to end conditions of tees to avoid inadvertent fixed end moments that induce compression in the web unless this limit state is checked. The design of a WT-shape in bending is illustrated in Example F.10.

F10. SINGLE ANGLES

Section F10 of the AISC *Specification* permits the flexural design of single angles using either the principal axes or geometric axes (*x*- and *y*-axes). When designing single angles without continuous bracing using the geometric axis design provisions, M_y must be multiplied by 0.80 for use in Equations F10-1, F10-2 and F10-3. The design of a single angle in bending is illustrated in Example F.11.

F11. RECTANGULAR BARS AND ROUNDS

The AISC *Manual* does not include design tables for these shapes. The local buckling limit state does not apply to any bars. With the exception of rectangular bars bent about the strong axis, solid square, rectangular and round bars are not subject to lateral-torsional buckling and are governed by the yielding limit state only. Rectangular bars bent about the strong axis are subject to lateral-torsional buckling and are checked for this limit state with Equations F11-2 and F11-3, as applicable.

These provisions can be used to check plates and webs of tees in connections. A design example of a rectangular bar in bending is illustrated in Example F.12. A design example of a round bar in bending is illustrated in Example F.13.

F12. UNSYMMETRICAL SHAPES

Due to the wide range of possible unsymmetrical cross sections, specific lateral-torsional and local buckling provisions are not provided in this *Specification* section. A general template is provided, but appropriate literature investigation and engineering judgment are required for the application of this section. A design example of a Z-shaped section in bending is illustrated in Example F.14.

F13. PROPORTIONS OF BEAMS AND GIRDERS

This section of the *Specification* includes a limit state check for tensile rupture due to holes in the tension flange of beams, proportioning limits for I-shaped members, detail requirements for cover plates and connection requirements for built-up beams connected side-to-side. Also included are unbraced length requirements for beams designed using the moment redistribution provisions of AISC *Specification* Section B3.3.

EXAMPLE F.1-1A W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, CONTINUOUSLY BRACED

Given:

Select a W-shape beam for span and uniform dead and live loads as shown in Figure F.1-1A. Limit the member to a maximum nominal depth of 18 in. Limit the live load deflection to *L*/360. The beam is simply supported and continuously braced. The beam is ASTM A992 material.

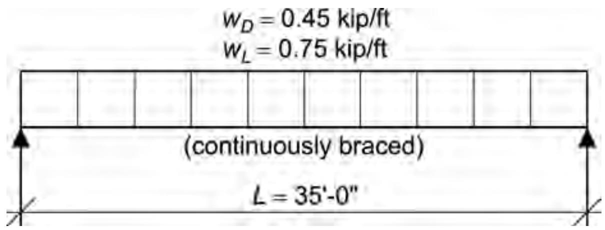


Fig. F.1-1A. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.45\text{ kip/ft}) + 1.6(0.75\text{ kip/ft})$ $= 1.74\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(1.74\text{ kip/ft})(35\text{ ft})^2}{8}$ $= 266\text{ kip-ft}$	$w_a = 0.45\text{ kip/ft} + 0.75\text{ kip/ft}$ $= 1.20\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(1.20\text{ kip/ft})(35\text{ ft})^2}{8}$ $= 184\text{ kip-ft}$

Required Moment of Inertia for Live-Load Deflection Criterion of L/360

$$\Delta_{max} = \frac{L}{360}$$
$$= \frac{(35\text{ ft})(12\text{ in./ft})}{360}$$
$$= 1.17\text{ in.}$$

$$I_{x(reqd)} = \frac{5w_L L^4}{384E\Delta_{max}}$$ (from AISC *Manual* Table 3-23, Case 1)

$$= \frac{5\left(0.75 \text{ kip/ft}\right)\left(35 \text{ ft}\right)^4\left(12 \text{ in./ft}\right)^3}{384\left(29{,}000 \text{ ksi}\right)\left(1.17 \text{ in.}\right)}$$

$$= 746 \text{ in.}^4$$

Beam Selection

Select a W18×50 from AISC *Manual* Table 3-3.

$I_x = 800 \text{ in.}^4 > 746 \text{ in.}^4$ **o.k.**

Per the User Note in AISC *Specification* Section F2, the section is compact. Because the beam is continuously braced and compact, only the yielding limit state applies.

From AISC *Manual* Table 3-2, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = \phi_b M_{px}$ $= 379 \text{ kip-ft} > 266 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = \dfrac{M_{px}}{\Omega_b}$ $= 252 \text{ kip-ft} > 184 \text{ kip-ft}$ **o.k.**

EXAMPLE F.1-1B W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, CONTINUOUSLY BRACED

Given:

Verify the available flexural strength of the ASTM A992 W18×50 beam selected in Example F.1-1A by directly applying the requirements of the AISC *Specification*.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
Z_x = 101 in.^{3}

The required flexural strength from Example F.1-1A is:

LRFD	ASD
$M_u = 266$ kip-ft	$M_a = 184$ kip-ft

Nominal Flexural Strength

Per the User Note in AISC *Specification* Section F2, the section is compact. Because the beam is continuously braced and compact, only the yielding limit state applies.

$$M_n = M_p = F_y Z_x \qquad \text{(}\textit{Spec}\text{. Eq. F2-1)}$$
$$= (50 \text{ ksi})(101 \text{ in.}^3)$$
$$= 5{,}050 \text{ kip-in. or } 421 \text{ kip-ft}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(421 \text{ kip-ft})$ $= 379 \text{ kip-ft} > 266 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{421 \text{ kip-ft}}{1.67}$ $= 252 \text{ kip-ft} > 184 \text{ kip-ft}$ **o.k.**

EXAMPLE F.1-2A W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, BRACED AT THIRD POINTS

Given:

Use the AISC *Manual* tables to verify the available flexural strength of the W18×50 beam size selected in Example F.1-1A for span and uniform dead and live loads as shown in Figure F.1-2A. The beam is simply supported and braced at the ends and third points. The beam is ASTM A992 material.

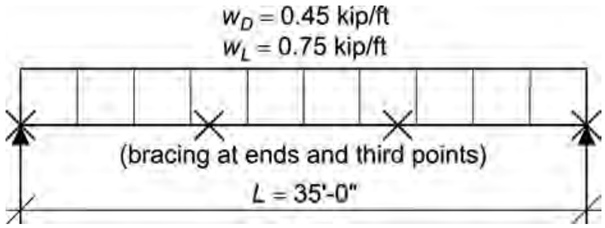


Fig. F.1-2A. Beam loading and bracing diagram.

Solution:

The required flexural strength at midspan from Example F.1-1A is:

LRFD	ASD
$M_u = 266$ kip-ft	$M_a = 184$ kip-ft

Unbraced Length

$$L_b = \frac{35 \text{ ft}}{3} = 11.7 \text{ ft}$$

By inspection, the middle segment will govern. From AISC *Manual* Table 3-1, for a uniformly loaded beam braced at the ends and third points, C_b = 1.01 in the middle segment. Conservatively neglect this small adjustment in this case.

Available Flexural Strength

Enter AISC *Manual* Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 11.7 ft. Obtain the available strength from the appropriate vertical scale to the left.

From AISC *Manual* Table 3-10, the available flexural strength is:

LRFD	ASD
$\phi_b M_n \approx 302$ kip-ft > 266 kip-ft **o.k.**	$\dfrac{M_n}{\Omega_b} \approx 201$ kip-ft > 184 kip-ft **o.k.**

EXAMPLE F.1-2B W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, BRACED AT THIRD POINTS

Given:

Verify the available flexural strength of the W18×50 beam selected in Example F.1-1A with the beam braced at the ends and third points by directly applying the requirements of the AISC *Specification*. The beam is ASTM A992 material.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
r_y = 1.65 in.
S_x = 88.9 in.^{3}
J = 1.24 in.^{4}
r_{ts} = 1.98 in.
h_o = 17.4 in.

The required flexural strength from Example F.1-1A is:

LRFD	ASD
$M_u = 266$ kip-ft	$M_a = 184$ kip-ft

Nominal Flexural Strength

Calculate C_b. For the lateral-torsional buckling limit state, the nonuniform moment modification factor can be calculated using AISC *Specification* Equation F1-1. For the center segment of the beam, the required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: M_{max} = 1.00, M_A = 0.972, M_B = 1.00, and M_C = 0.972.

$$C_b = \frac{12.5M_{max}}{2.5M_{max}+3M_A+4M_B+3M_C}$$ (*Spec*. Eq. F1-1)

$$= \frac{12.5(1.00)}{2.5(1.00)+3(0.972)+4(1.00)+3(0.972)}$$

$$= 1.01$$

For the end-span beam segments, the required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: M_{max} = 0.889, M_A = 0.306, M_B = 0.556, and M_C = 0.750.

$$C_b = \frac{12.5M_{max}}{2.5M_{max}+3M_A+4M_B+3M_C}$$ (*Spec*. Eq. F1-1)

$$= \frac{12.5(0.889)}{2.5(0.889)+3(0.306)+4(0.556)+3(0.750)}$$

$$= 1.46$$

Thus, the center span, with the higher required strength and lower C_b, will govern.

The limiting laterally unbraced length for the limit state of yielding is:

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} \qquad (Spec.\ \text{Eq. F2-5})$$

$$= 1.76(1.65\text{ in.})\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$

$$= 69.9\text{ in. or }5.83\text{ ft}$$

The limiting unbraced length for the limit state of inelastic lateral-torsional buckling, with c = 1 from AISC *Specification* Equation F2-8a for doubly symmetric I-shaped members, is:

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_x h_o} + \sqrt{\left(\frac{Jc}{S_x h_o}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}} \qquad (Spec.\ \text{Eq. F2-6})$$

$$= 1.95(1.98\text{ in.})\left[\frac{29{,}000\text{ ksi}}{0.7(50\text{ ksi})}\right]\sqrt{\frac{(1.24\text{ in.}^4)(1.0)}{(88.9\text{ in.}^3)(17.4\text{ in.})} + \sqrt{\left[\frac{(1.24\text{ in.}^4)(1.0)}{(88.9\text{ in.}^3)(17.4\text{ in.})}\right]^2 + 6.76\left[\frac{0.7(50\text{ ksi})}{29{,}000\text{ ksi}}\right]^2}}$$

$$= 203\text{ in. or }16.9\text{ ft}$$

For a compact beam with an unbraced length of $L_p < L_b \le L_r$, the lesser of either the flexural yielding limit state or the inelastic lateral-torsional buckling limit state controls the nominal strength.

M_p = 5,050 kip-in. (from Example F.1-1B)

$$M_n = C_b\left[M_p - (M_p - 0.7F_y S_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad (Spec.\ \text{Eq. F2-2})$$

$$= 1.01\left\{5{,}050\text{ kip-in.} - \left[5{,}050\text{ kip-in.} - 0.7(50\text{ ksi})(88.9\text{ in.}^3)\right]\left(\frac{11.7\text{ ft} - 5.83\text{ ft}}{16.9\text{ ft} - 5.83\text{ ft}}\right)\right\} \le 5{,}050\text{ kip-in.}$$

$$= 4{,}060\text{ kip-in.} < 5{,}050\text{ kip-in.}$$

$$= 4{,}060\text{ kip-in. or }339\text{ kip-ft}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(339\text{ kip-ft})$ $= 305\text{ kip-ft} > 266\text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{339\text{ kip-ft}}{1.67}$ $= 203\text{ kip-ft} > 184\text{ kip-ft}$ **o.k.**

EXAMPLE F.1-3A W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR AXIS BENDING, BRACED AT MIDSPAN

Given:

Use the AISC *Manual* tables to verify the available flexural strength of the W18×50 beam size selected in Example F.1-1A for span and uniform dead and live loads as shown in Figure F.1-3A. The beam is simply supported and braced at the ends and midpoint. The beam is ASTM A992 material.

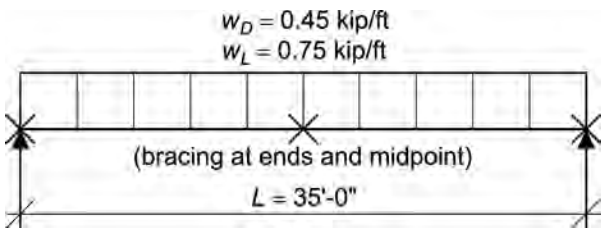


Fig. F.1-3A. Beam loading and bracing diagram.

Solution:

The required flexural strength at midspan from Example F.1-1A is:

LRFD	ASD
$M_u = 266$ kip-ft	$M_a = 184$ kip-ft

Unbraced Length

$$L_b = \frac{35 \text{ ft}}{2} = 17.5 \text{ ft}$$

From AISC *Manual* Table 3-1, for a uniformly loaded beam braced at the ends and at the center point, C_b = 1.30. There are several ways to make adjustments to AISC *Manual* Table 3-10 to account for C_b greater than 1.0.

Procedure A

Available moments from the sloped and curved portions of the plots from AISC *Manual* Table 3-10 may be multiplied by C_b, but may not exceed the value of the horizontal portion (ϕM_p for LRFD, M_p/Ω for ASD).

Obtain the available strength of a W18×50 with an unbraced length of 17.5 ft from AISC *Manual* Table 3-10.

Enter AISC *Manual* Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 17.5 ft. Obtain the available strength from the appropriate vertical scale to the left.

LRFD	ASD
$\phi_b M_n \approx 222$ kip-ft From AISC *Manual* Table 3-2: $\phi_b M_p = 379$ kip-ft (upper limit on $C_b \phi_b M_n$)	$\dfrac{M_n}{\Omega_b} \approx 148$ kip-ft From AISC *Manual* Table 3-2: $\dfrac{M_p}{\Omega_b} = 252$ kip-ft (upper limit on $C_b \dfrac{M_n}{\Omega_b}$)

LRFD	ASD
Adjust for C_b. $1.30(222 \text{ kip-ft}) = 289 \text{ kip-ft}$ Check limit. $289 \text{ kip-ft} < \phi_b M_p = 379 \text{ kip-ft}$ **o.k.** Check available versus required strength. $289 \text{ kip-ft} > 266 \text{ kip-ft}$ **o.k.**	Adjust for C_b. $1.30(148 \text{ kip-ft}) = 192 \text{ kip-ft}$ Check limit. $192 \text{ kip-ft} < \dfrac{M_p}{\Omega_b} = 252 \text{ kip-ft}$ **o.k.** Check available versus required strength. $192 \text{ kip-ft} > 184 \text{ kip-ft}$ **o.k.**

Procedure B

For preliminary selection, the required strength can be divided by C_b and directly compared to the strengths in AISC *Manual* Table 3-10. Members selected in this way must be checked to ensure that the required strength does not exceed the available plastic moment strength of the section.

Calculate the adjusted required strength.

LRFD	ASD
$M'_u = \dfrac{266 \text{ kip-ft}}{1.30}$ $= 205 \text{ kip-ft}$	$M'_a = \dfrac{184 \text{ kip-ft}}{1.30}$ $= 142 \text{ kip-ft}$

Obtain the available strength for a W18×50 with an unbraced length of 17.5 ft from AISC *Manual* Table 3-10.

LRFD	ASD
$\phi_b M_n \approx 222 \text{ kip-ft} > 205 \text{ kip-ft}$ **o.k.** $\phi_b M_p = 379 \text{ kip-ft} > 266 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} \approx 148 \text{ kip-ft} > 142 \text{ kip-ft}$ **o.k.** $\dfrac{M_p}{\Omega_b} = 252 \text{ kip-ft} > 184 \text{ kip-ft}$ **o.k.**

EXAMPLE F.1-3B W-SHAPE FLEXURAL MEMBER DESIGN IN MAJOR-AXIS BENDING, BRACED AT MIDSPAN

Given:

Verify the available flexural strength of the W18×50 beam selected in Example F.1-1A with the beam braced at the ends and center point by directly applying the requirements of the AISC *Specification*. The beam is ASTM A992 material.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
r_{ts} = 1.98 in.
S_x = 88.9 in.^{3}
J = 1.24 in.^{4}
h_o = 17.4 in.

The required flexural strength from Example F.1-1A is:

LRFD	ASD
$M_u = 266$ kip-ft	$M_a = 184$ kip-ft

Nominal Flexural Strength

Calculate C_b. The required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: M_{max} = 1.00, M_A = 0.438, M_B = 0.750, and M_C = 0.938.

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \qquad (Spec.\ \text{Eq. F1-1})$$

$$= \frac{12.5(1.00)}{2.5(1.00) + 3(0.438) + 4(0.750) + 3(0.938)}$$

$$= 1.30$$

From AISC *Manual* Table 3-2:

L_p = 5.83 ft
L_r = 16.9 ft

From Example F.1-3A:

L_b = 17.5 ft

For a compact beam with an unbraced length $L_b > L_r$, the limit state of elastic lateral-torsional buckling applies.

Calculate F_{cr}, where c = 1.0 for doubly symmetric I-shapes.

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2} \qquad \text{(\textit{Spec}. Eq. F2-4)}$$

$$= \frac{1.30\pi^2 (29{,}000 \text{ ksi})}{\left[\frac{(17.5 \text{ ft})(12 \text{ in./ft})}{1.98 \text{ in.}}\right]^2} \sqrt{1 + 0.078 \frac{(1.24 \text{ in.}^4)(1.0)}{(88.9 \text{ in.}^3)(17.4 \text{ in.})} \left[\frac{(17.5 \text{ ft})(12 \text{ in./ft})}{1.98 \text{ in.}}\right]^2}$$

$$= 43.2 \text{ ksi}$$

$$M_p = 5{,}050 \text{ kip-in. (from Example F.1-1B)}$$

$$M_n = F_{cr} S_x \le M_p \qquad \text{(\textit{Spec}. Eq. F2-3)}$$

$$= (43.2 \text{ ksi})(88.9 \text{ in.}^3) \le 5{,}050 \text{ kip-in.}$$

$$= 3{,}840 \text{ kip-in.} < 5{,}050 \text{ kip-in.}$$

$$= 3{,}840 \text{ kip-in. or } 320 \text{ kip-ft}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(320 \text{ kip-ft})$ $= 288 \text{ kip-ft} > 266 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{320 \text{ kip-ft}}{1.67}$ $= 192 \text{ kip-ft} > 184 \text{ kip-ft}$ **o.k.**

EXAMPLE F.2-1A COMPACT CHANNEL FLEXURAL MEMBER, CONTINUOUSLY BRACED

Given:

Using the AISC *Manual* tables, select a channel to serve as a roof edge beam for span and uniform dead and live loads as shown in Figure F.2-1A. The beam is simply supported and continuously braced. Limit the live load deflection to L/360. The channel is ASTM A36 material.

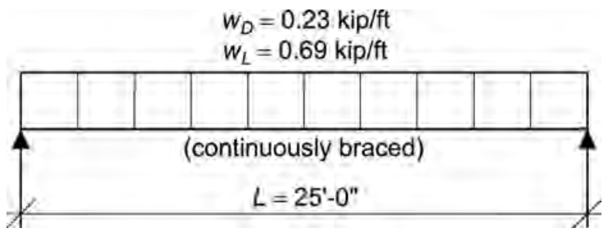


Fig. F.2-1A. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.23\text{ kip/ft}) + 1.6(0.69\text{ kip/ft})$ $= 1.38\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(1.38\text{ kip/ft})(25\text{ ft})^2}{8}$ $= 108\text{ kip-ft}$	$w_a = 0.23\text{ kip/ft} + 0.69\text{ kip/ft}$ $= 0.920\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(0.920\text{ kip/ft})(25\text{ ft})^2}{8}$ $= 71.9\text{ kip-ft}$

Beam Selection

Per the User Note in AISC *Specification* Section F2, all ASTM A36 channels are compact. Because the beam is compact and continuously braced, the yielding limit state governs and $M_n = M_p$. Try C15×33.9 from AISC *Manual* Table 3-8.

LRFD	ASD
$\phi_b M_n = \phi_b M_p$ $= 137\text{ kip-ft} > 108\text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = \dfrac{M_p}{\Omega_b}$ $= 91.3\text{ kip-ft} > 71.9\text{ kip-ft}$ **o.k.**

Live Load Deflection

Limit the live load deflection at the center of the beam to $L/360$.

$$\Delta_{max} = \frac{L}{360} = \frac{(25\text{ ft})(12\text{ in./ft})}{360} = 0.833\text{ in.}$$

For C15×33.9, I_x = 315 in.^{4} from AISC *Manual* Table 1-5.

The maximum calculated deflection is:

$$\Delta_{max} = \frac{5w_L L^4}{384EI}$$ (from AISC *Manual* Table 3-23, Case 1)

$$= \frac{5(0.69\text{ kip/ft})(25\text{ ft})^4(12\text{ in./ft})^3}{384(29{,}000\text{ ksi})(315\text{ in.}^4)}$$

$$= 0.664\text{ in.} < 0.833\text{ in.} \quad \textbf{o.k.}$$

EXAMPLE F.2-1B COMPACT CHANNEL FLEXURAL MEMBER, CONTINUOUSLY BRACED

Given:

Verify the available flexural strength of the C15×33.9 beam selected in Example F.2-1A by directly applying the requirements of the AISC *Specification*. The channel is ASTM A36 material.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-5, the geometric properties are as follows:

C15×33.9
Z_x = 50.8 in.^{3}

The required flexural strength from Example F.2-1A is:

LRFD	ASD
$M_u = 108$ kip-ft	$M_a = 71.9$ kip-ft

Nominal Flexural Strength

Per the User Note in AISC *Specification* Section F2, all ASTM A36 C- and MC-shapes are compact.

A channel that is continuously braced and compact is governed by the yielding limit state.

$$M_n = M_p = F_y Z_x \quad \text{(Spec. Eq. F2-1)}$$
$$= (36 \text{ ksi})(50.8 \text{ in.}^3)$$
$$= 1{,}830 \text{ kip-in. or } 152 \text{ kip-ft}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(152 \text{ kip-ft})$ $= 137 \text{ kip-ft} > 108 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{152 \text{ kip-ft}}{1.67}$ $= 91.0 \text{ kip-ft} > 71.9 \text{ kip-ft}$ **o.k.**

EXAMPLE F.2-2A COMPACT CHANNEL FLEXURAL MEMBER WITH BRACING AT ENDS AND FIFTH POINTS

Given:

Use the AISC *Manual* tables to verify the available flexural strength of the C15×33.9 beam selected in Example F.2-1A for span and uniform dead and live loads as shown in Figure F.2-2A. The beam is simply supported and braced at the ends and fifth points. The channel is ASTM A36 material.

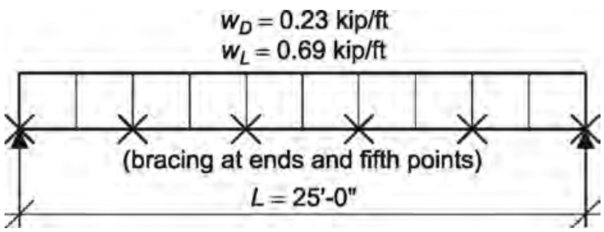


Fig. F.2-2A. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

The center segment will govern by inspection.

The required flexural strength at midspan from Example F.2-1A is:

LRFD	ASD
$M_u = 108$ kip-ft	$M_a = 71.9$ kip-ft

From AISC *Manual* Table 3-1, with an almost uniform moment across the center segment, C_b = 1.00; therefore, no adjustment is required.

Unbraced Length

$$L_b = \frac{25\text{ ft}}{5}$$
$$= 5.00\text{ ft}$$

Obtain the strength of the C15×33.9 with an unbraced length of 5.00 ft from AISC *Manual* Table 3-11.

Enter AISC *Manual* Table 3-11 and find the intersection of the curve for the C15×33.9 with an unbraced length of 5.00 ft. Obtain the available strength from the appropriate vertical scale to the left.

LRFD	ASD
$\phi_b M_n \approx 130$ kip-ft $>$ 108 kip-ft **o.k.**	$\dfrac{M_n}{\Omega_b} \approx 87.0$ kip-ft $>$ 71.9 kip-ft **o.k.**

EXAMPLE F.2-2B COMPACT CHANNEL FLEXURAL MEMBER WITH BRACING AT ENDS AND FIFTH POINTS

Given:

Verify the results from Example F.2-2A by directly applying the requirements of the AISC *Specification*.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-5, the geometric properties are as follows:

C15×33.9
S_x = 42.0 in.^{3}

The required flexural strength from Example F.2-1A is:

LRFD	ASD
$M_u = 108$ kip-ft	$M_a = 71.9$ kip-ft

Available Flexural Strength

Per the User Note in AISC *Specification* Section F2, all ASTM A36 C- and MC-shapes are compact.

From AISC *Manual* Table 3-1, for the center segment of a uniformly loaded beam braced at the ends and the fifth points:

C_b = 1.00

From AISC *Manual* Table 3-8, for a C15×33.9:

L_p = 3.75 ft
L_r = 14.5 ft

From Example F2.2A:

L_b = 5.00 ft

For a compact channel with $L_p < L_b \le L_r$, the lesser of the flexural yielding limit state or the inelastic lateral-torsional buckling limit state controls the available flexural strength.

The nominal flexural strength based on the flexural yielding limit state, from Example F.2-1B, is:

$$M_n = M_p$$
$$= 1{,}830 \text{ kip-in.}$$

The nominal flexural strength based on the lateral-torsional buckling limit state is:

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(\textit{Spec}. Eq. F2-2)}$$

$$= 1.00\left\{1{,}830 \text{ kip-in.} - \left[1{,}830 \text{ kip-in.} - 0.7(36 \text{ ksi})\left(42.0 \text{ in.}^3\right)\right]\left(\frac{5.00 \text{ ft} - 3.75 \text{ ft}}{14.5 \text{ ft} - 3.75 \text{ ft}}\right)\right\} \le 1{,}830 \text{ kip-in.}$$

$$= 1{,}740 \text{ kip-in.} < 1{,}830 \text{ kip-in.}$$

$$= 1{,}740 \text{ kip-in. or } 145 \text{ kip-ft}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(145 \text{ kip-ft})$ $= 131 \text{ kip-ft} > 108 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{145 \text{ kip-ft}}{1.67}$ $= 86.8 \text{ kip-ft} > 71.9 \text{ kip-ft}$ **o.k.**

EXAMPLE F.3A W-SHAPE FLEXURAL MEMBER WITH NONCOMPACT FLANGES IN MAJOR AXIS BENDING

Given:

Using the AISC *Manual* tables, select a W-shape beam for span, uniform dead load, and concentrated live loads as shown in Figure F.3A. The beam is simply supported and continuously braced. Also calculate the deflection. The beam is ASTM A992 material.

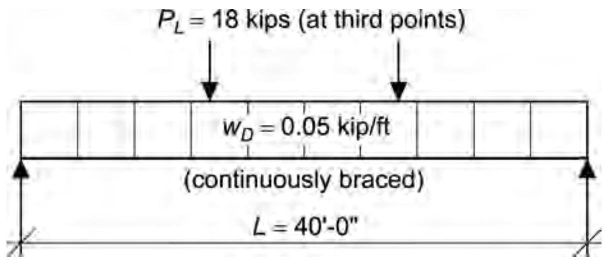


Fig. F.3A. Beam loading and bracing diagram.

Note: A beam with noncompact flanges will be selected to demonstrate that the tabulated values of the AISC *Manual* account for flange compactness.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength at midspan is:

LRFD	ASD
$w_u = 1.2(0.05\text{ kip/ft}) = 0.0600\text{ kip/ft}$	$w_a = 0.05\text{ kip/ft}$
$P_u = 1.6(18\text{ kips}) = 28.8\text{ kips}$	$P_a = 18\text{ kips}$
From AISC *Manual* Table 3-23, Cases 1 and 9:	From AISC *Manual* Table 3-23, Cases 1 and 9:
$M_u = \dfrac{w_u L^2}{8} + P_u a = \dfrac{(0.0600\text{ kip/ft})(40\text{ ft})^2}{8} + (28.8\text{ kips})\left(\dfrac{40\text{ ft}}{3}\right) = 396\text{ kip-ft}$	$M_a = \dfrac{w_a L^2}{8} + P_a a = \dfrac{(0.05\text{ kip/ft})(40\text{ ft})^2}{8} + (18\text{ kips})\left(\dfrac{40\text{ ft}}{3}\right) = 250\text{ kip-ft}$

Beam Selection

For a continuously braced W-shape, the available flexural strength equals the available plastic flexural strength.

Select the lightest section providing the required strength from the bold entries in AISC *Manual* Table 3-2.

Try a W21×48.

This beam has a noncompact compression flange at F_y = 50 ksi as indicated by footnote "f" in AISC *Manual* Table 3-2. This shape is also footnoted in AISC *Manual* Table 1-1.

From AISC *Manual* Table 3-2, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = \phi_b M_{px}$ $= 398 \text{ kip-ft} > 396 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = \dfrac{M_{px}}{\Omega_b}$ $= 265 \text{ kip-ft} > 250 \text{ kip-ft}$ **o.k.**

Note: The value M_{px} in AISC *Manual* Table 3-2 includes the strength reductions due to the shape being noncompact.

Deflection

From AISC *Manual* Table 1-1:

$I_x = 959 \text{ in.}^4$

The maximum deflection occurs at the center of the beam.

$$\Delta_{max} = \frac{5w_D L^4}{384EI} + \frac{23P_L L^3}{648EI} \quad \text{(AISC } \textit{Manual} \text{ Table 3-23, Cases 1 and 9)}$$

$$= \frac{5(0.05 \text{ kip/ft})(40 \text{ ft})^4 (12 \text{ in./ft})^3}{384(29{,}000 \text{ ksi})(959 \text{ in.}^4)} + \frac{23(18 \text{ kips})(40 \text{ ft})^3 (12 \text{ in./ft})^3}{648(29{,}000 \text{ ksi})(959 \text{ in.}^4)}$$

$$= 2.64 \text{ in.}$$

This deflection can be compared with the appropriate deflection limit for the application. Deflection will often be more critical than strength in beam design.

EXAMPLE F.3B W-SHAPE FLEXURAL MEMBER WITH NONCOMPACT FLANGES IN MAJOR AXIS BENDING

Given:

Verify the results from Example F.3A by directly applying the requirements of the AISC *Specification*.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W21×48
$S_x = 93.0 \text{ in.}^3$
$Z_x = 107 \text{ in.}^3$
$\dfrac{b_f}{2t_f} = 9.47$

The required flexural strength from Example F.3A is:

LRFD	ASD
$M_u = 396$ kip-ft	$M_a = 250$ kip-ft

Flange Slenderness

$$\lambda = \frac{b_f}{2t_f} = 9.47$$

The limiting width-to-thickness ratios for the compression flange are:

$$\lambda_{pf} = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}} = 9.15$$ (*Spec*. Table B4.1b, Case 10)

$$\lambda_{rf} = 1.0\sqrt{\frac{E}{F_y}} = 1.0\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}} = 24.1$$ (*Spec*. Table B4.1b, Case 10)

$\lambda_{pf} < \lambda < \lambda_{rf}$, therefore, the compression flange is noncompact. This could also be determined from the footnote "f" in AISC *Manual* Table 1-1.

Nominal Flexural Strength

Because the beam is continuously braced, and therefore not subject to lateral-torsional buckling, the available strength is based on the limit state of compression flange local buckling. From AISC *Specification* Section F3.2:

$$M_p = F_y Z_x \quad (Spec.\ \text{Eq. F2-1})$$
$$= (50 \text{ ksi})(107 \text{ in.}^3)$$
$$= 5{,}350 \text{ kip-in. or } 446 \text{ kip-ft}$$

$$M_n = \left[M_p - (M_p - 0.7F_y S_x)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)\right] \quad (Spec.\ \text{Eq. F3-1})$$
$$= \left\{5{,}350 \text{ kip-in.} - \left[5{,}350 \text{ kip-in.} - 0.7(50 \text{ ksi})(93.0 \text{ in.}^3)\right]\left(\frac{9.47 - 9.15}{24.1 - 9.15}\right)\right\}$$
$$= 5{,}310 \text{ kip-in. or } 442 \text{ kip-ft}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(442 \text{ kip-ft})$ $= 398 \text{ kip-ft} > 396 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{442 \text{ kip-ft}}{1.67}$ $= 265 \text{ kip-ft} > 250 \text{ kip-ft}$ **o.k.**

Note that these available strengths are identical to the tabulated values in AISC *Manual* Table 3-2, as shown in Example F.3A, which account for the noncompact flange.

EXAMPLE F.4 W-SHAPE FLEXURAL MEMBER, SELECTION BY MOMENT OF INERTIA FOR MAJOR AXIS BENDING

Given:

Using the AISC *Manual* tables, select a W-shape using the moment of inertia required to limit the live load deflection to 1.00 in. for span and uniform dead and live loads as shown in Figure F.4. The beam is simply supported and continuously braced. The beam is ASTM A992 material.

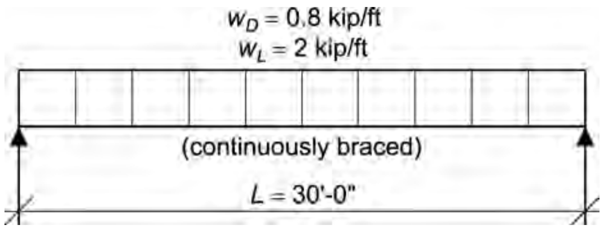


Fig. F.4. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.8\text{ kip/ft}) + 1.6(2\text{ kip/ft})$ $= 4.16\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(4.16\text{ kip/ft})(30\text{ ft})^2}{8}$ $= 468\text{ kip-ft}$	$w_a = 0.8\text{ kip/ft} + 2\text{ kip/ft}$ $= 2.80\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(2.80\text{ kip/ft})(30\text{ ft})^2}{8}$ $= 315\text{ kip-ft}$

Minimum Required Moment of Inertia

The maximum live load deflection, Δ_{max}, occurs at midspan and is calculated as:

$$\Delta_{max} = \frac{5 w_L L^4}{384 EI}$$ (AISC *Manual* Table 3-23, Case 1)

Rearranging and substituting Δ_{max} = 1.00 in.,

$$I_{min} = \frac{5w_L L^4}{384E\Delta_{max}}$$
$$= \frac{5(2\text{ kip/ft})(30\text{ ft})^4(12\text{ in./ft})^3}{384(29,000\text{ ksi})(1.00\text{ in.})}$$
$$= 1,260\text{ in.}^4$$

Beam Selection

Select the lightest section with the required moment of inertia from the bold entries in AISC *Manual* Table 3-3.

Try a W24×55.

I_x = 1,350 in.^{4} > 1,260 in.^{4} **o.k.**

Because the W24×55 is continuously braced and compact, its strength is governed by the yielding limit state and AISC *Specification* Section F2.1.

From AISC *Manual* Table 3-2, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = \phi_b M_{px}$ $= 503\text{ kip-ft} > 468\text{ kip-ft}$ **o.k.**	$\frac{M_n}{\Omega_b} = \frac{M_{px}}{\Omega_b}$ $= 334\text{ kip-ft} > 315\text{ kip-ft}$ **o.k.**

EXAMPLE F.5 I-SHAPED FLEXURAL MEMBER IN MINOR AXIS BENDING

Given:

Using the AISC *Manual* tables, select a W-shape beam loaded on its minor axis for span and uniform dead and live loads as shown in Figure F.5. Limit the live load deflection to L/240. The beam is simply supported and braced only at the ends. The beam is ASTM A992 material.

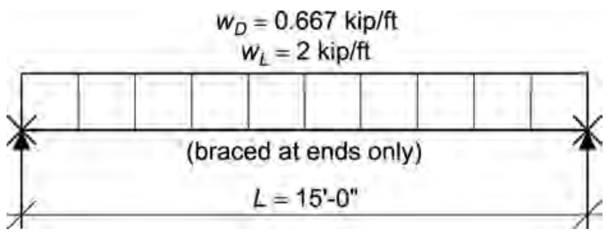


Fig. F.5. Beam loading and bracing diagram.

Note: Although not a common design case, this example is being used to illustrate AISC *Specification* Section F6 (I-shaped members and channels bent about their minor axis).

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.667\text{ kip/ft}) + 1.6(2\text{ kip/ft})$ $= 4.00\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(4.00\text{ kip/ft})(15\text{ ft})^2}{8}$ $= 113\text{ kip-ft}$	$w_a = 0.667\text{ kip/ft} + 2\text{ kip/ft}$ $= 2.67\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(2.67\text{ kip/ft})(15\text{ ft})^2}{8}$ $= 75.1\text{ kip-ft}$

Minimum Required Moment of Inertia

The maximum live load deflection permitted is:

$$\Delta_{max} = \frac{L}{240}$$
$$= \frac{(15\text{ ft})(12\text{ in./ft})}{240}$$
$$= 0.750\text{ in.}$$

$$I_{y,reqd} = \frac{5w_L L^4}{384E\Delta_{max}}$$ (modified AISC *Manual* Table 3-23, Case 1)

$$= \frac{5(2 \text{ kip/ft})(15 \text{ ft})^4 (12 \text{ in./ft})^3}{384(29{,}000 \text{ ksi})(0.750 \text{ in.})}$$

$$= 105 \text{ in.}^4$$

Beam Selection

Select the lightest section from the bold entries in AISC *Manual* Table 3-5.

Try a W12×58.

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W12×58
$S_y = 21.4 \text{ in.}^3$
$Z_y = 32.5 \text{ in.}^3$
$I_y = 107 \text{ in.}^4 > 105 \text{ in.}^4$ **o.k. (**for deflection requirement)

Nominal Flexural Strength

AISC *Specification* Section F6 applies. Because the W12×58 has compact flanges per the User Note in this Section, the yielding limit state governs the design.

$$M_n = M_p = F_y Z_y \le 1.6 F_y S_y$$ (*Spec*. Eq. F6-1)

$$= (50 \text{ ksi})(32.5 \text{ in.}^3) \le 1.6(50 \text{ ksi})(21.4 \text{ in.}^3)$$

$$= 1{,}630 \text{ kip-in.} < 1{,}710 \text{ kip-in.}$$

$$= 1{,}630 \text{ kip-in or } 136 \text{ kip-ft}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(136 \text{ kip-ft})$ $= 122 \text{ kip-ft} > 113 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{136 \text{ kip-ft}}{1.67}$ $= 81.4 \text{ kip-ft} > 75.1 \text{ kip-ft}$ **o.k.**

EXAMPLE F.6 SQUARE HSS FLEXURAL MEMBER WITH COMPACT FLANGES

Given:

Using the AISC *Manual* tables, select a square HSS beam for span and uniform dead and live loads as shown in Figure F.6. Limit the live load deflection to L/240. The beam is simply supported and continuously braced. The HSS is ASTM A500 Grade C material.

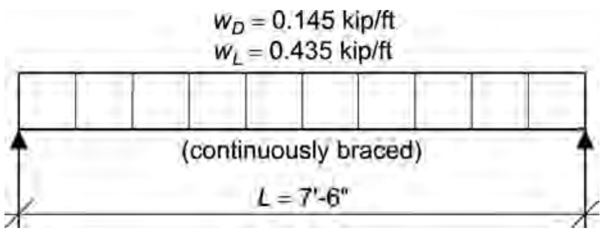


Fig. F.6. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade C, rectangular HSS
F_y = 50 ksi
F_u = 62 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.145 \text{ kip/ft}) + 1.6(0.435 \text{ kip/ft})$ $= 0.870 \text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(0.870 \text{ kip/ft})(7.5 \text{ ft})^2}{8}$ $= 6.12 \text{ kip-ft}$	$w_a = 0.145 \text{ kip/ft} + 0.435 \text{ kip/ft}$ $= 0.580 \text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(0.580 \text{ kip/ft})(7.5 \text{ ft})^2}{8}$ $= 4.08 \text{ kip-ft}$

Minimum Required Moment of Inertia

The maximum live load deflection permitted is:

$$\Delta_{max} = \frac{L}{240}$$
$$= \frac{(7.5 \text{ ft})(12 \text{ in./ft})}{240}$$
$$= 0.375 \text{ in.}$$

Determine the minimum required moment of inertia as follows.

$$I_{req} = \frac{5w_L L^4}{384E\Delta_{max}}$$ (from AISC *Manual* Table 3-23, Case 1)

$$= \frac{5(0.435 \text{ kip/ft})(7.5 \text{ ft})^4 (12 \text{ in./ft})^3}{384(29,000 \text{ ksi})(0.375 \text{ in.})}$$

$$= 2.85 \text{ in.}^4$$

Beam Selection

Select an HSS with a minimum I_x of 2.85 in.^{4}, using AISC *Manual* Table 1-12, and having adequate available strength, using AISC *Manual* Table 3-13.

Try an HSS3½×3½×⅛.

From AISC *Manual* Table 1-12,

$I_x = 2.90 \text{ in.}^4 > 2.85 \text{ in.}^4$ **o.k.**

From AISC *Manual* Table 3-13, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = 7.21$ kip-ft > 6.12 kip-ft **o.k.**	$\frac{M_n}{\Omega_b} = 4.79$ kip-ft > 4.08 kip-ft **o.k.**

EXAMPLE F.7A RECTANGULAR HSS FLEXURAL MEMBER WITH NONCOMPACT FLANGES

Given:

Using the AISC *Manual* tables, select a rectangular HSS beam for span and uniform dead and live loads as shown in Figure F.7A. Limit the live load deflection to L/240. The beam is simply supported and braced at the end points only. A noncompact member was selected here to illustrate the relative ease of selecting noncompact shapes from the AISC *Manual*, as compared to designing a similar shape by applying the AISC *Specification* requirements directly, as shown in Example F.7B. The HSS is ASTM A500 Grade C material.

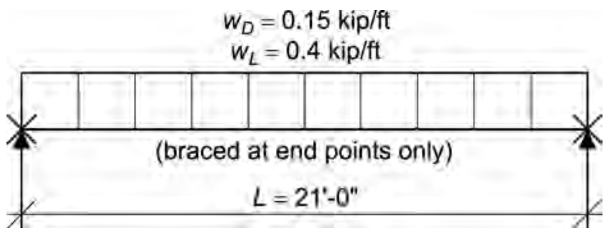


Fig. F.7A. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade C, rectangular HSS
F_y = 50 ksi
F_u = 62 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.15 \text{ kip/ft}) + 1.6(0.4 \text{ kip/ft})$ $= 0.820 \text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(0.820 \text{ kip/ft})(21 \text{ ft})^2}{8}$ $= 45.2 \text{ kip-ft}$	$w_a = 0.15 \text{ kip/ft} + 0.4 \text{ kip/ft}$ $= 0.550 \text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(0.550 \text{ kip/ft})(21 \text{ ft})^2}{8}$ $= 30.3 \text{ kip-ft}$

Minimum Required Moment of Inertia

The maximum live load deflection permitted is:

$$\Delta_{max} = \frac{L}{240}$$
$$= \frac{(21 \text{ ft})(12 \text{ in./ft})}{240}$$
$$= 1.05 \text{ in.}$$

Determine the minimum required moment of inertia as follows:

$$I_{min} = \frac{5w_L L^4}{384E\Delta_{max}}$$ (from AISC *Manual* Table 3-23, Case 1)

$$= \frac{5(0.4 \text{ kip/ft})(21 \text{ ft})^4 (12 \text{ in./ft})^3}{384(29{,}000 \text{ ksi})(1.05 \text{ in.})}$$

$$= 57.5 \text{ in.}^4$$

Beam Selection

Select a rectangular HSS with a minimum I_x of 57.5 in.^{4}, using AISC *Manual* Table 1-11, and having adequate available strength, using AISC *Manual* Table 3-12.

Try an HSS10×6×3⁄16 oriented in the strong direction. This rectangular HSS section was purposely selected for illustration purposes because it has a noncompact flange. See AISC *Manual* Table 1-12A for compactness criteria.

$I_x = 74.6 \text{ in.}^4 > 57.5 \text{ in.}^4$ **o.k.**

From AISC *Manual* Table 3-12, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = 59.7 \text{ kip-ft} > 45.2 \text{ kip-ft}$ **o.k.**	$\frac{M_n}{\Omega_b} = 39.7 \text{ kip-ft} > 30.3 \text{ kip-ft}$ **o.k.**

Note: Because AISC *Manual* Table 3-12 does not account for lateral-torsional buckling, it needs to be checked using AISC *Specification* Section F7.4.

As discussed in the User Note to AISC *Specification* Section F7.4, lateral-torsional buckling will not occur in square sections or sections bending about their minor axis. In HSS sizes, deflection will often occur before there is a significant reduction in flexural strength due to lateral-torsional buckling. See Example F.7B for the calculation accounting for lateral-torsional buckling for the HSS10×6×3⁄16.

EXAMPLE F.7B RECTANGULAR HSS FLEXURAL MEMBER WITH NONCOMPACT FLANGES

Given:

In Example F.7A the required information was easily determined by consulting the tables of the AISC *Manual*. The purpose of the following calculation is to demonstrate the use of the AISC *Specification* to calculate the flexural strength of an HSS member with a noncompact compression flange. The HSS is ASTM A500 Grade C material.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade C, rectangular HSS
$F_y = 50$ ksi
$F_u = 62$ ksi

From AISC *Manual* Table 1-11, the geometric properties are as follows:

HSS10×6×3⁄16
$A_g = 5.37 \text{ in.}^2$
$Z_x = 18.0 \text{ in.}^3$
$S_x = 14.9 \text{ in.}^3$
$r_y = 2.52$ in.
$J = 73.8 \text{ in.}^4$
$b/t = 31.5$
$h/t = 54.5$

Flange Compactness

$$\lambda = \frac{b}{t_f}$$
$$= \frac{b}{t}$$
$$= 31.5$$

From AISC *Specification* Table B4.1b, Case 17, the limiting width-to-thickness ratios for the flange are:

$$\lambda_p = 1.12\sqrt{\frac{E}{F_y}}$$
$$= 1.12\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 27.0$$

$$\lambda_r = 1.40\sqrt{\frac{E}{F_y}}$$
$$= 1.40\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 33.7$$

$\lambda_p < \lambda < \lambda_r$; therefore, the flange is noncompact and AISC *Specification* Equation F7-2 applies.

Web Compactness

$$\lambda = \frac{h}{t}$$
$$= 54.5$$

From AISC *Specification* Table B4.1b, Case 19, the limiting width-to-thickness ratio for the web is:

$$\lambda_p = 2.42\sqrt{\frac{E}{F_y}}$$
$$= 2.42\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 58.3$$

$\lambda < \lambda_p$; therefore, the web is compact and the limit state of web local buckling does not apply.

Nominal Flexural Strength

Flange Local Buckling

From AISC *Specification* Section F7.2(b), the limit state of flange local buckling applies for HSS with noncompact flanges and compact webs.

$$M_p = F_y Z_x \quad \text{(from } Spec. \text{ Eq. F7-1)}$$
$$= (50 \text{ ksi})(18.0 \text{ in.}^3)$$
$$= 900 \text{ kip-in.}$$

$$M_n = M_p - (M_p - F_y S)\left(3.57\frac{b}{t_f}\sqrt{\frac{F_y}{E}} - 4.0\right) \le M_p \quad (Spec. \text{ Eq. F7-2})$$

$$= 900 \text{ kip-in.} - \left[900 \text{ kip-in.} - (50 \text{ ksi})(14.9 \text{ in.}^3)\right]\left[3.57(31.5)\sqrt{\frac{50 \text{ ksi}}{29{,}000 \text{ ksi}}} - 4.0\right] \le 900 \text{ kip-in.}$$
$$= 796 \text{ kip-in.} < 900 \text{ kip-in.}$$
$$= 796 \text{ kip-in. or } 66.4 \text{ kip-ft}$$

Yielding and Lateral-Torsional Buckling

Determine the limiting laterally unbraced lengths for the limit state of yielding and the limit state of inelastic lateral-torsional buckling using AISC *Specification* Section F7.4.

$$L_b = (21 \text{ ft})(12 \text{ in./ft})$$
$$= 252 \text{ in.}$$

$$L_p = 0.13 E r_y \frac{\sqrt{J A_g}}{M_p} \quad (Spec. \text{ Eq. F7-12})$$

$$= 0.13(29{,}000 \text{ ksi})(2.52 \text{ in.})\frac{\sqrt{(73.8 \text{ in.}^4)(5.37 \text{ in.}^2)}}{900 \text{ kip-in.}}$$
$$= 210 \text{ in.}$$

$$L_r = 2Er_y \frac{\sqrt{JA_g}}{0.7F_yS_x} \qquad (Spec.\ \text{Eq. F7-13})$$

$$= 2(29{,}000 \text{ ksi})(2.52 \text{ in.}) \frac{\sqrt{(73.8 \text{ in.}^4)(5.37 \text{ in.}^2)}}{0.7(50 \text{ ksi})(14.9 \text{ in.}^3)}$$

$$= 5{,}580 \text{ in.}$$

For the lateral-torsional buckling limit state, the lateral-torsional buckling modification factor can be calculated using AISC *Specification* Equation F1-1. For the beam, the required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: M_{max} = 1.00, M_A = 0.750, M_B = 1.00, and M_C = 0.750.

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \qquad (Spec.\ \text{Eq. F1-1})$$

$$= \frac{12.5(1.00)}{2.5(1.00) + 3(0.750) + 4(1.00) + 3(0.750)}$$

$$= 1.14$$

Since $L_p < L_b < L_r$, the nominal moment strength considering lateral-torsional buckling is given by:

$$M_n = C_b \left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right) \right] \le M_p \qquad (Spec.\ \text{Eq. F7-10})$$

$$= 1.14 \left\{ 900 \text{ kip-in.} - \left[900 \text{ kip-in.} - 0.7(50 \text{ ksi})(14.9 \text{ in.}^3) \right] \left(\frac{252 \text{ in.} - 210 \text{ in.}}{5{,}580 \text{ in.} - 210 \text{ in.}} \right) \right\} \le 900 \text{ kip-in.}$$

$$= 1{,}020 \text{ kip-in.} > 900 \text{ kip-in.}$$

$$= 900 \text{ kip-in. or } 75.0 \text{ kip-ft}$$

Available Flexural Strength

The nominal strength is controlled by flange local buckling and therefore:

$$M_n = 66.4 \text{ kip-ft}$$

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(66.4 \text{ kip-ft})$ $= 59.8 \text{ kip-ft}$	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{66.4 \text{ kip-ft}}{1.67}$ $= 39.8 \text{ kip-ft}$

EXAMPLE F.8A SQUARE HSS FLEXURAL MEMBER WITH SLENDER FLANGES

Given:

Using AISC *Manual* tables, verify the strength of an HSS8×8×3⁄16 beam for span and uniform dead and live loads as shown in Figure F.8A. Limit the live load deflection to L/240. The beam is simply supported and continuously braced. The HSS is ASTM A500 Grade C material.

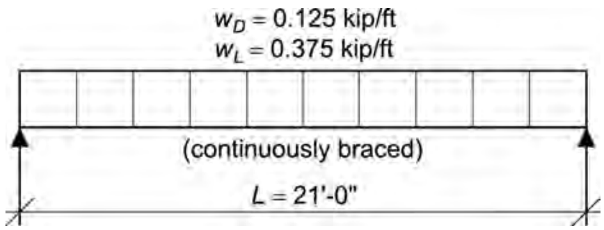


Fig. F.8A. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade C
F_y = 50 ksi
F_u = 62 ksi

From AISC *Manual* Table 1-12, the geometric properties are as follows:

HSS8×8×3⁄16
$I_x = I_y = 54.4$ in.^{4}

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.125\text{ kip/ft}) + 1.6(0.375\text{ kip/ft})$ $= 0.750\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(0.750\text{ kip/ft})(21.0\text{ ft})^2}{8}$ $= 41.3\text{ kip-ft}$	$w_a = 0.125\text{ kip/ft} + 0.375\text{ kip/ft}$ $= 0.500\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(0.500\text{ kip/ft})(21.0\text{ ft})^2}{8}$ $= 27.6\text{ kip-ft}$

From AISC *Manual* Table 3-13, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = 46.3\text{ kip-ft} > 41.3\text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = 30.8\text{ kip-ft} > 27.6\text{ kip-ft}$ **o.k.**

Note that the strengths given in AISC *Manual* Table 3-13 incorporate the effects of noncompact and slender elements.

Deflection

The maximum live load deflection permitted is:

$$\Delta_{max} = \frac{L}{240}$$
$$= \frac{(21.0 \text{ ft})(12 \text{ in./ft})}{240}$$
$$= 1.05 \text{ in.}$$

The calculated deflection is:

$$\Delta = \frac{5w_L L^4}{384EI}$$ (modified AISC *Manual* Table 3-23 Case 1)

$$= \frac{5(0.375 \text{ kip/ft})(21.0 \text{ ft})^4 (12 \text{ in./ft})^3}{384(29{,}000 \text{ ksi})(54.4 \text{ in.}^4)}$$
$$= 1.04 \text{ in.} < 1.05 \text{ in.}$$ **o.k.**

EXAMPLE F.8B SQUARE HSS FLEXURAL MEMBER WITH SLENDER FLANGES

Given:

In Example F.8A the available strengths were easily determined from the tables of the AISC *Manual*. The purpose of the following calculation is to demonstrate the use of the AISC *Specification* to calculate the flexural strength of the HSS beam given in Example F.8A. The HSS is ASTM A500 Grade C material.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade C, rectangular HSS
F_y = 50 ksi
F_u = 62 ksi

From AISC *Manual* Table 1-12, the geometric properties are as follows:

HSS8×8×3⁄16
I = 54.4 in.^{4}
Z = 15.7 in.^{3}
S = 13.6 in.^{3}
B = 8.00 in.
H = 8.00 in.
t = 0.174 in.
b/t = 43.0
h/t = 43.0

The required flexural strength from Example F.8A is:

LRFD	ASD
$M_u = 41.3$ kip-ft	$M_a = 27.6$ kip-ft

Flange Slenderness

The outside corner radii of HSS shapes are taken as 1.5t and the design thickness is used in accordance with AISC *Specification* Section B4.1b to check compactness.

Determine the limiting ratio for a slender HSS flange in flexure from AISC *Specification* Table B4.1b, Case 17.

$$\lambda_r = 1.40\sqrt{\frac{E}{F_y}}$$
$$= 1.40\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 33.7$$

$$\lambda = \frac{b}{t}$$
$$= \frac{b}{t_f}$$
$$= 43.0 > \lambda_r; \text{ therefore, the flange is slender}$$

Web Slenderness

Determine the limiting ratio for a compact web in flexure from AISC *Specification* Table B4.1b, Case 19.

$$\lambda_p = 2.42\sqrt{\frac{E}{F_y}}$$
$$= 2.42\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 58.3$$

$$\lambda = \frac{h}{t}$$
$$= 43.0 < \lambda_p;\ \text{therefore, the web is compact and the limit state of web local buckling does not apply}$$

Nominal Flexural Strength

Flange Local Buckling

For HSS sections with slender flanges and compact webs, AISC *Specification* Section F7.2(c) applies.

$$M_n = F_y S_e \qquad \text{(}\textit{Spec}\text{. Eq. F7-3)}$$

From AISC *Specification* Section B4.1b(d), the width of the compression flange is determined as follows:

$$b = 8.00\text{ in.} - 3(0.174\text{ in.})$$
$$= 7.48\text{ in.}$$

Where the effective section modulus, S_e, is determined using the effective width of the compression flange as follows:

$$b_e = 1.92t_f\sqrt{\frac{E}{F_y}}\left(1-\frac{0.38}{b/t_f}\sqrt{\frac{E}{F_y}}\right) \le b \qquad \text{(}\textit{Spec}\text{. Eq. F7-4)}$$
$$= 1.92(0.174\text{ in.})\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}\left[1-\left(\frac{0.38}{43.0}\right)\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}\right] \le 7.48\text{ in.}$$
$$= 6.33\text{ in.}$$

The ineffective width of the compression flange is:

$$b - b_e = 7.48\text{ in.} - 6.33\text{ in.}$$
$$= 1.15\text{ in.}$$

An exact calculation of the effective moment of inertia and section modulus could be performed taking into account the ineffective width of the compression flange and the resulting neutral axis shift. Alternatively, a simpler but slightly conservative calculation can be performed by removing the ineffective width symmetrically from both the top and bottom flanges.

$$I_{eff} \approx I_x - \left(\sum \frac{bt^3}{12} + \sum ad^2\right)$$

$$= 54.4 \text{ in.}^4 - 2\left[\frac{(1.15 \text{ in.})(0.174 \text{ in.})^3}{12} + (1.15 \text{ in.})(0.174 \text{ in.})\left(\frac{8.00 \text{ in.} - 0.174 \text{ in.}}{2}\right)^2\right]$$

$$= 48.3 \text{ in.}^4$$

The effective section modulus is calculated as follows:

$$S_e = \frac{I_{eff}}{\left(\frac{H}{2}\right)}$$

$$= \frac{48.3 \text{ in.}^4}{\left(\frac{8.00 \text{ in.}}{2}\right)}$$

$$= 12.1 \text{ in.}^3$$

$$M_n = F_y S_e$$ (*Spec*. Eq. F7-3)

$$= (50 \text{ ksi})(12.1 \text{ in.}^3)$$

$$= 605 \text{ kip-in. or } 50.4 \text{ kip-ft}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(50.4 \text{ kip-ft})$ $= 45.4 \text{ kip-ft} > 41.3 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{50.4 \text{ kip-ft}}{1.67}$ $= 30.2 \text{ kip-ft} > 27.6 \text{ kip-ft}$ **o.k.**

Note that the calculated available strengths are somewhat lower than those in AISC *Manual* Table 3-13 due to the use of the conservative calculation of the effective section modulus. Also, note that per the User Note in AISC *Specification* Section F7.4, lateral-torsional buckling is not applicable to square HSS.

EXAMPLE F.9A PIPE FLEXURAL MEMBER

Given:

Using AISC *Manual* tables, select a Pipe shape with an 8-in. nominal depth for span and uniform dead and live loads as shown in Figure F.9A. There is no deflection limit for this beam. The beam is simply supported and braced at end points only. The Pipe is ASTM A53 Grade B material.

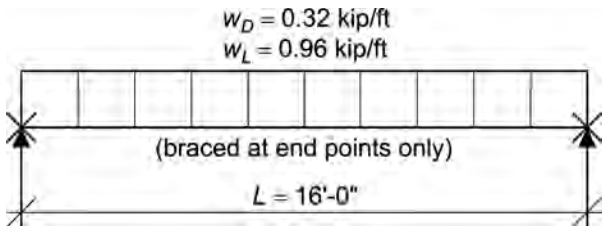


Fig. F.9A. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A53 Grade B
F_y = 35 ksi
F_u = 60 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.32\text{ kip/ft}) + 1.6(0.96\text{ kip/ft})$ $= 1.92\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(1.92\text{ kip/ft})(16\text{ ft})^2}{8}$ $= 61.4\text{ kip-ft}$	$w_a = 0.32\text{ kip/ft} + 0.96\text{ kip/ft}$ $= 1.28\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(1.28\text{ kip/ft})(16\text{ ft})^2}{8}$ $= 41.0\text{ kip-ft}$

Pipe Selection

Select a member from AISC *Manual* Table 3-15 having the required strength.

Select Pipe 8 x-Strong.

From AISC *Manual* Table 3-15, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = 81.4\text{ kip-ft} > 61.4\text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = 54.1\text{ kip-ft} > 41.0\text{ kip-ft}$ **o.k.**

EXAMPLE F.9B PIPE FLEXURAL MEMBER

Given:

The available strength in Example F.9A was easily determined using AISC *Manual* Table 3-15. The following example demonstrates the calculation of the available strength by directly applying the AISC *Specification*.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A53 Grade B
F_y = 35 ksi
F_u = 60 ksi

From AISC *Manual* Table 1-14, the geometric properties are as follows:

Pipe 8 x-Strong
Z = 31.0 in.^{3}
D/t = 18.5

The required flexural strength from Example F.9A is:

LRFD	ASD
$M_u = 61.4$ kip-ft	$M_a = 41.0$ kip-ft

Slenderness Check

Determine the limiting diameter-to-thickness ratio for a compact section from AISC *Specification* Table B4.1b Case 20.

$$\lambda_p = 0.07\frac{E}{F_y}$$
$$= 0.07\left(\frac{29{,}000 \text{ ksi}}{35 \text{ ksi}}\right)$$
$$= 58.0$$

$$\lambda = \frac{D}{t}$$
$$= 18.5 < \lambda_p; \text{ therefore, the section is compact and the limit state of flange local buckling does not apply}$$

$$\frac{0.45E}{F_y} = \frac{0.45(29{,}000 \text{ ksi})}{35 \text{ ksi}}$$
$$= 373 > 18.5; \text{ therefore, AISC } \textit{Specification} \text{ Section F8 applies}$$

Nominal Flexural Strength

Based on the limit state of yielding given in AISC *Specification* Section F8.1:

$$M_n = M_p = F_y Z$$ (*Spec*. Eq. F8-1)

$$= (35\text{ ksi})(31.0\text{ in.}^3)$$

$$= 1{,}090\text{ kip-in. or }90.4\text{ kip-ft}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(90.4\text{ kip-ft})$ $= 81.4\text{ kip-ft} > 61.4\text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{90.4\text{ kip-ft}}{1.67}$ $= 54.1\text{ kip-ft} > 41.0\text{ kip-ft}$ **o.k.**

EXAMPLE F.10 WT-SHAPE FLEXURAL MEMBER

Given:

Directly applying the requirements of the AISC *Specification*, select a WT beam with a 5-in. nominal depth for span and uniform dead and live loads as shown in Figure F.10. The toe of the stem of the WT is in tension. There is no deflection limit for this member. The beam is simply supported and continuously braced. The WT is ASTM A992 material.

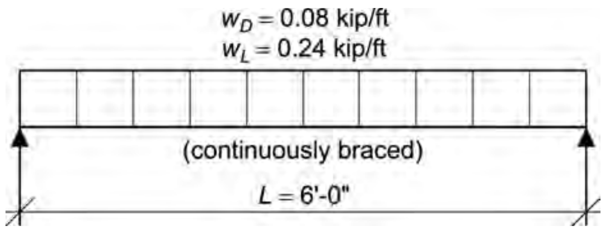


Fig. F.10. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.08 \text{ kip/ft}) + 1.6(0.24 \text{ kip/ft})$ $= 0.480 \text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(0.480 \text{ kip/ft})(6 \text{ ft})^2}{8}$ $= 2.16 \text{ kip-ft}$	$w_a = 0.08 \text{ kip/ft} + 0.24 \text{ kip/ft}$ $= 0.320 \text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(0.320 \text{ kip/ft})(6 \text{ ft})^2}{8}$ $= 1.44 \text{ kip-ft}$

Try a WT5×6.

From AISC *Manual* Table 1-8, the geometric properties are as follows:

WT5×6
d = 4.94 in.
I_x = 4.35 in.^{4}
Z_x = 2.20 in.^{3}
S_x = 1.22 in.^{3}
b_f = 3.96 in.
t_f = 0.210 in.
$\bar{y}$ = 1.36 in.

$b_f/2t_f = 9.43$

$$S_{xc} = \frac{I_x}{\bar{y}}$$
$$= \frac{4.35 \text{ in.}^4}{1.36 \text{ in.}}$$
$$= 3.20 \text{ in.}^3$$

Nominal Flexural Strength

Yielding

From AISC *Specification* Section F9.1, for the limit state of yielding:

$$M_n = M_p \qquad \text{(Spec. Eq. F9-1)}$$

$$M_y = F_y S_x \qquad \text{(Spec. Eq. F9-3)}$$
$$= (50 \text{ ksi})(1.22 \text{ in.}^3)$$
$$= 61.0 \text{ kip-in.}$$

$$M_p = F_y Z_x \le 1.6 M_y \text{ (for stems in tension)} \qquad \text{(Spec. Eq. F9-2)}$$
$$= (50 \text{ ksi})(2.20 \text{ in.}^3) \le 1.6(61.0 \text{ kip-in.})$$
$$= 110 \text{ kip-in.} > 97.6 \text{ kip-in.}$$
$$= 97.6 \text{ kip-in. or } 8.13 \text{ kip-ft}$$

Lateral-Torsional Buckling

From AISC *Specification* Section F9.2, because the WT is continuously braced, the limit state of lateral-torsional buckling does not apply.

Flange Local Buckling

The limit state of flange local buckling is checked using AISC *Specification* Section F9.3.

Flange Slenderness

$$\lambda = \frac{b_f}{2t_f}$$
$$= 9.43$$

From AISC *Specification* Table B4.1b, Case 10, the limiting width-to-thickness ratio for the flange is:

$$\lambda_{pf} = 0.38\sqrt{\frac{E}{F_y}}$$
$$= 0.38\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 9.15$$

$$\lambda_{rf} = 1.0\sqrt{\frac{E}{F_y}}$$
$$= 1.0\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 24.1$$

Because $\lambda_{pf} < \lambda < \lambda_{rf}$, the flange is noncompact and the limit state of flange local buckling will apply.

From AISC *Specification* Section F9.3, the nominal flexural strength of a tee with a noncompact flange is:

$$M_n = \left[M_p - \left(M_p - 0.7F_yS_{xc}\right)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)\right] \le 1.6M_y \qquad \text{(}Spec\text{. Eq. F9-14)}$$
$$= \left\{110\text{ kip-in.} - \left[110\text{ kip-in.} - 0.7\left(50\text{ ksi}\right)\left(3.20\text{ in.}^3\right)\right]\left(\frac{9.43 - 9.15}{24.1 - 9.15}\right)\right\} \le 97.6\text{ kip-in.}$$
$$= 110\text{ kip-in.} > 97.6\text{ kip-in.}$$
$$= 97.6\text{ kip-in.}$$

Flexural yielding controls:

$$M_n = 97.6\text{ kip-in. or } 8.13\text{ kip-ft}$$

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_bM_n = 0.90\left(8.13\text{ kip-ft}\right)$ $= 7.32\text{ kip-ft} > 2.16\text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{8.13\text{ kip-ft}}{1.67}$ $= 4.87\text{ kip-ft} > 1.44\text{ kip-ft}$ **o.k.**

EXAMPLE F.11A SINGLE-ANGLE FLEXURAL MEMBER WITH BRACING AT ENDS ONLY

Given:

Directly applying the requirements of the AISC *Specification*, select a single angle for span and uniform dead and live loads as shown in Figure F.11A. The vertical leg of the single angle is up and the toe is in compression. There are no horizontal loads. There is no deflection limit for this angle. The beam is simply supported and braced at the end points only. Assume bending about the geometric *x-x* axis and that there is no lateral-torsional restraint. The angle is ASTM A36 material.

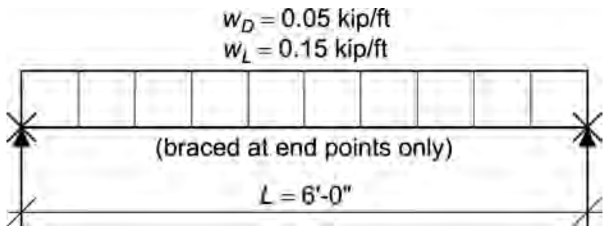


Fig. F.11A. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_{ux} = 1.2(0.05\text{ kip/ft}) + 1.6(0.15\text{ kip/ft})$ $= 0.300\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_{ux} = \dfrac{w_{ux}L^2}{8}$ $= \dfrac{(0.300\text{ kip/ft})(6\text{ ft})^2}{8}$ $= 1.35\text{ kip-ft}$	$w_{ax} = 0.05\text{ kip/ft} + 0.15\text{ kip/ft}$ $= 0.200\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_{ax} = \dfrac{w_{ax}L^2}{8}$ $= \dfrac{(0.200\text{ kip/ft})(6\text{ ft})^2}{8}$ $= 0.900\text{ kip-ft}$

Try a L4×4×¼.

From AISC *Manual* Table 1-7, the geometric properties are as follows:

L4×4×¼
S_x = 1.03 in.^{3}

Nominal Flexural Strength

Yielding

From AISC *Specification* Section F10.1, the nominal flexural strength due to the limit state of flexural yielding is:

$$\begin{aligned} M_n &= 1.5M_y \\ &= 1.5F_yS_x \\ &= 1.5(36\text{ ksi})(1.03\text{ in.}^3) \\ &= 55.6\text{ kip-in.} \end{aligned} \qquad (\textit{Spec}.\text{ Eq. F10-1})$$

Lateral-Torsional Buckling

From AISC *Specification* Section F10.2, for single angles bending about a geometric axis with no lateral-torsional restraint, M_y is taken as 0.80 times the yield moment calculated using the geometric section modulus.

$$\begin{aligned} M_y &= 0.80F_yS_x \\ &= 0.80(36\text{ ksi})(1.03\text{ in.}^3) \\ &= 29.7\text{ kip-in.} \end{aligned}$$

Determine M_{cr}.

For bending moment about one of the geometric axes of an equal-leg angle with no axial compression, with no lateral-torsional restraint, and with maximum compression at the toe, use AISC *Specification* Equation F10-5a.

C_b = 1.14 from AISC *Manual* Table 3-1

$$M_{cr} = \frac{0.58Eb^4tC_b}{L_b^2}\left(\sqrt{1+0.88\left(\frac{L_bt}{b^2}\right)^2}-1\right) \qquad (\textit{Spec}.\text{ Eq. F10-5a})$$

$$\begin{aligned} &= \frac{0.58(29{,}000\text{ ksi})(4.00\text{ in.})^4(\tfrac{1}{4}\text{ in.})(1.14)}{\left[(6\text{ ft})(12\text{ in./ft})\right]^2}\left\{\sqrt{1+0.88\left[\frac{(6\text{ ft})(12\text{ in./ft})(\tfrac{1}{4}\text{ in.})}{(4.00\text{ in.})^2}\right]^2}-1\right\} \\ &= 107\text{ kip-in.} \end{aligned}$$

$$\begin{aligned} \frac{M_y}{M_{cr}} &= \frac{29.7\text{ kip-in.}}{107\text{ kip-in.}}; \\ &= 0.278 < 1.0;\text{ therefore, AISC } \textit{Specification}\text{ Equation F10-2 is applicable} \end{aligned}$$

$$M_n = \left(1.92-1.17\sqrt{\frac{M_y}{M_{cr}}}\right)M_y \le 1.5M_y \qquad (\textit{Spec}.\text{ Eq. F10-2})$$

$$\begin{aligned} &= \left(1.92-1.17\sqrt{\frac{29.7\text{ kip-in.}}{107\text{ kip-in.}}}\right)(29.7\text{ kip-in.}) \le 1.5(29.7\text{ kip-in.}) \\ &= 38.7\text{ kip-in.} < 44.6\text{ kip-in.} \\ &= 38.7\text{ kip-in.} \end{aligned}$$

Leg Local Buckling

AISC *Specification* Section F10.3 applies when the toe of the leg is in compression.

Check slenderness of the leg in compression.

$$\lambda = \frac{b}{t}$$
$$= \frac{4.00 \text{ in.}}{\frac{1}{4} \text{ in.}}$$
$$= 16.0$$

Determine the limiting compact slenderness ratios from AISC *Specification* Table B4.1b, Case 12.

$$\lambda_p = 0.54\sqrt{\frac{E}{F_y}}$$
$$= 0.54\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$
$$= 15.3$$

Determine the limiting noncompact slenderness ratios from AISC *Specification* Table B4.1b, Case 12.

$$\lambda_r = 0.91\sqrt{\frac{E}{F_y}}$$
$$= 0.91\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$
$$= 25.8$$

$\lambda_p < \lambda < \lambda_r$, therefore, the leg is noncompact in flexure

$$S_c = 0.80S_x$$
$$= 0.80\left(1.03 \text{ in.}^3\right)$$
$$= 0.824 \text{ in.}^3$$

$$M_n = F_y S_c\left[2.43 - 1.72\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}}\right] \qquad \text{(\textit{Spec}. Eq. F10-6)}$$
$$= \left(36 \text{ ksi}\right)\left(0.824 \text{ in.}^3\right)\left[2.43 - 1.72\left(16.0\right)\sqrt{\frac{36 \text{ ksi}}{29{,}000 \text{ ksi}}}\right]$$
$$= 43.3 \text{ kip-in.}$$

The lateral-torsional buckling limit state controls.

M_n = 38.7 kip-in. or 3.23 kip-ft

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$	$\Omega_b = 1.67$
$\phi_b M_n = 0.90(3.23 \text{ kip-ft})$ $= 2.91 \text{ kip-ft} > 1.35 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = \dfrac{3.23 \text{ kip-ft}}{1.67}$ $= 1.93 \text{ kip-ft} > 0.900 \text{ kip-ft}$ **o.k.**

EXAMPLE F.11B SINGLE-ANGLE FLEXURAL MEMBER WITH BRACING AT ENDS AND MIDSPAN

Given:

Directly applying the requirements of the AISC *Specification*, select a single angle for span and uniform dead and live loads as shown in Figure F.11B. The vertical leg of the single angle is up and the toe is in compression. There are no horizontal loads. There is no deflection limit for this angle. The beam is simply supported and braced at the end points and midspan. Assume bending about the geometric *x*-*x* axis and that there is lateral-torsional restraint at the midspan and ends only. The angle is ASTM A36 material.

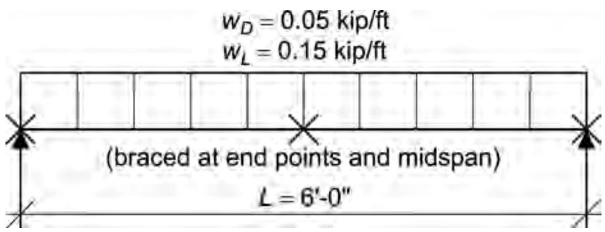


Fig. F.11B. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_{ux} = 1.2(0.05 \text{ kip/ft}) + 1.6(0.15 \text{ kip/ft})$ $= 0.300 \text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_{ux} = \dfrac{w_{ux}L^2}{8}$ $= \dfrac{(0.300 \text{ kip/ft})(6 \text{ ft})^2}{8}$ $= 1.35 \text{ kip-ft}$	$w_{ax} = 0.05 \text{ kip/ft} + 0.15 \text{ kip/ft}$ $= 0.200 \text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_{ax} = \dfrac{w_{ax}L^2}{8}$ $= \dfrac{(0.200 \text{ kip/ft})(6 \text{ ft})^2}{8}$ $= 0.900 \text{ kip-ft}$

Try a L4×4×¼.

From AISC *Manual* Table 1-7, the geometric properties are as follows:

L4×4×¼
S_x = 1.03 in.^{3}

Nominal Flexural Strength

Flexural Yielding

From AISC *Specification* Section F10.1, the nominal flexural strength due to the limit state of flexural yielding is:

$$\begin{aligned} M_n &= 1.5M_y \\ &= 1.5F_yS_x \\ &= 1.5(36\text{ ksi})(1.03\text{ in.}^3) \\ &= 55.6\text{ kip-in.} \end{aligned}$$ (*Spec*. Eq. F10-1)

Lateral-Torsional Buckling

From AISC *Specification* Section F10.2(b)(2)(ii), for single angles with lateral-torsional restraint at the point of maximum moment, M_y is taken as the yield moment calculated using the geometric section modulus.

$$\begin{aligned} M_y &= F_yS_x \\ &= (36\text{ ksi})(1.03\text{ in.}^3) \\ &= 37.1\text{ kip-in.} \end{aligned}$$

Determine M_{cr}.

For bending moment about one of the geometric axes of an equal-leg angle with no axial compression, with lateral-torsional restraint at the point of maximum moment only (at midspan in this case), and with maximum compression at the toe, M_{cr} shall be taken as 1.25 times M_{cr} computed using AISC *Specification* Equation F10-5a.

C_b = 1.30 from AISC *Manual* Table 3-1

$$M_{cr} = 1.25\left(\frac{0.58Eb^4tC_b}{L_b{}^2}\right)\left(\sqrt{1+0.88\left(\frac{L_bt}{b^2}\right)^2}-1\right)$$ (from *Spec*. Eq. F10-5a)

$$\begin{aligned} &= 1.25\left[\frac{0.58(29{,}000\text{ ksi})(4.00\text{ in.})^4(\tfrac{1}{4}\text{ in.})(1.30)}{\left[(3\text{ ft})(12\text{ in./ft})\right]^2}\right]\left\{\sqrt{1+0.88\left[\frac{(3\text{ ft})(12\text{ in./ft})(\tfrac{1}{4}\text{ in.})}{(4.00\text{ in.})^2}\right]^2}-1\right\} \\ &= 176\text{ kip-in.} \end{aligned}$$

$$\begin{aligned} \frac{M_y}{M_{cr}} &= \frac{37.1\text{ kip-in.}}{176\text{ kip-in.}} \\ &= 0.211 < 1.0;\ \text{therefore, AISC } \textit{Specification} \text{ Equation F10-2 is applicable} \end{aligned}$$

$$M_n = \left(1.92-1.17\sqrt{\frac{M_y}{M_{cr}}}\right)M_y \le 1.5M_y$$ (*Spec*. Eq. F10-2)

$$\begin{aligned} &= \left(1.92-1.17\sqrt{\frac{37.1\text{ kip-in.}}{176\text{ kip-in.}}}\right)(37.1\text{ kip-in.}) \le 1.5(37.1\text{ kip-in.}) \\ &= 51.3\text{ kip-in.} < 55.7\text{ kip-in.} \\ &= 51.3\text{ kip-in.} \end{aligned}$$

Leg Local Buckling

M_n = 43.3 kip-in. from Example F.11A.

The leg local buckling limit state controls.

M_n = 43.3 kip-in. or 3.61 kip-ft

Available Flexural Strength

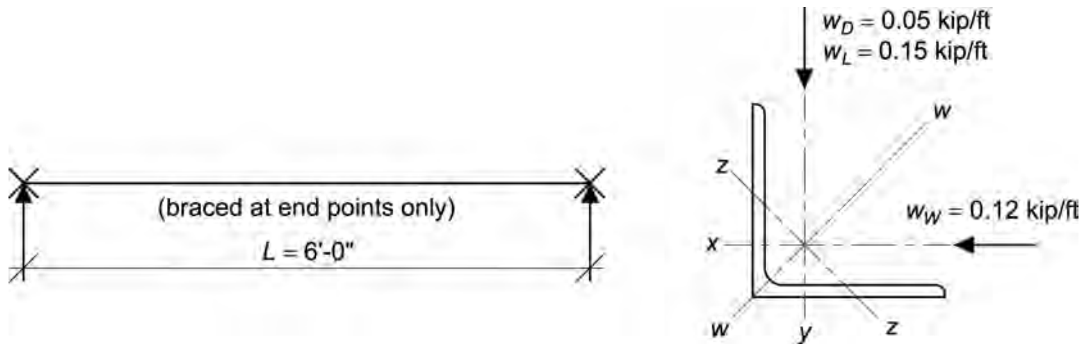
From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(3.61 \text{ kip-ft})$ $= 3.25 \text{ kip-ft} > 1.35 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{3.61 \text{ kip-ft}}{1.67}$ $= 2.16 \text{ kip-ft} > 0.900 \text{ kip-ft}$ **o.k.**

EXAMPLE F.11C SINGLE-ANGLE FLEXURAL MEMBER WITH VERTICAL AND HORIZONTAL LOADING

Given:

Directly applying the requirements of the AISC *Specification*, select a single angle for span and uniform vertical dead and live loads as shown in Figure F.11C-1. The horizontal load is a uniform wind load. There is no deflection limit for this angle. The angle is simply supported and braced at the end points only and there is no lateral-torsional restraint. Use load combination 4 from Section 2.3.1 of ASCE/SEI 7 for LRFD and load combination 6 from Section 2.4.1 of ASCE/SEI 7 for ASD. The angle is ASTM A36 material.



(a) Beam bracing diagram *(b) Beam loading*

Fig. F.11C-1. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_{ux} = 1.2(0.05\text{ kip/ft}) + 0.15\text{ kip/ft}$ $= 0.210\text{ kip/ft}$ $w_{uy} = 1.0(0.12\text{ kip/ft})$ $= 0.120\text{ kip/ft}$ $M_{ux} = \dfrac{w_{ux}L^2}{8}$ $= \dfrac{(0.210\text{ kip/ft})(6\text{ ft})^2}{8}$ $= 0.945\text{ kip-ft}$	$w_{ax} = 0.05\text{ kip/ft} + 0.75(0.15\text{ kip/ft})$ $= 0.163\text{ kip/ft}$ $w_{ay} = 0.75[0.6(0.12\text{ kip/ft})]$ $= 0.0540\text{ kip/ft}$ $M_{ax} = \dfrac{w_{ax}L^2}{8}$ $= \dfrac{(0.163\text{ kip/ft})(6\text{ ft})^2}{8}$ $= 0.734\text{ kip-ft}$

LRFD	ASD
$M_{uy} = \dfrac{w_{uy}L^2}{8}$ $= \dfrac{(0.120 \text{ kip/ft})(6 \text{ ft})^2}{8}$ $= 0.540 \text{ kip-ft}$	$M_{ay} = \dfrac{w_{ay}L^2}{8}$ $= \dfrac{(0.0540 \text{ kip/ft})(6 \text{ ft})^2}{8}$ $= 0.243 \text{ kip-ft}$

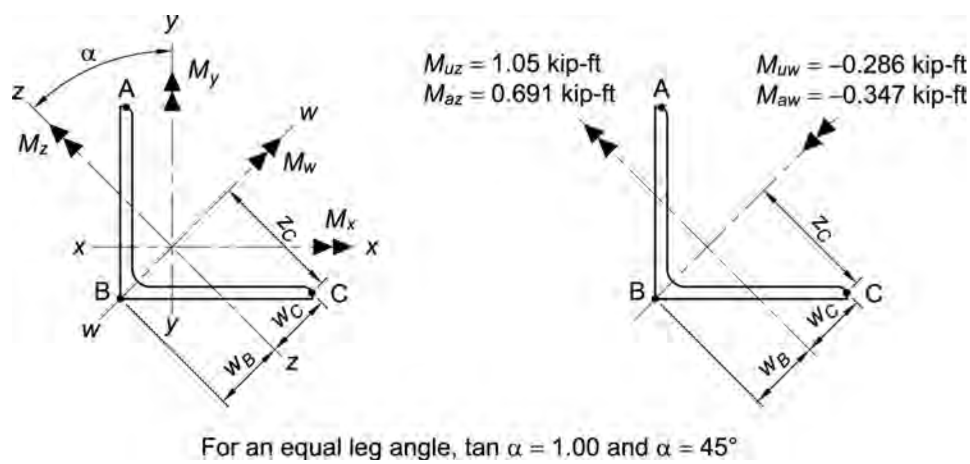
Try a L4×4×¼.

Sign convention for geometric axes moments are:

LRFD	ASD
$M_{ux} = -0.945 \text{ kip-ft}$ $M_{uy} = 0.540 \text{ kip-ft}$	$M_{ax} = -0.734 \text{ kip-ft}$ $M_{ay} = 0.243 \text{ kip-ft}$

As shown in Figure F.11C-2, the principal axes moments are:

LRFD	ASD
$M_{uw} = M_{ux}\cos\alpha + M_{uy}\sin\alpha$ $= (-0.945 \text{ kip-ft})(\cos 45°) + (0.540 \text{ kip-ft})(\sin 45°)$ $= -0.286 \text{ kip-ft}$ $M_{uz} = -M_{ux}\sin\alpha + M_{uy}\cos\alpha$ $= -(-0.945 \text{ kip-ft})(\sin 45°) + (0.540 \text{ kip-ft})(\cos 45°)$ $= 1.05 \text{ kip-ft}$	$M_{aw} = M_{ax}\cos\alpha + M_{ay}\sin\alpha$ $= (-0.734 \text{ kip-ft})(\cos 45°) + (0.243 \text{ kip-ft})(\sin 45°)$ $= -0.347 \text{ kip-ft}$ $M_{az} = -M_{ax}\sin\alpha + M_{ay}\cos\alpha$ $= -(-0.734 \text{ kip-ft})(\sin 45°) + (0.243 \text{ kip-ft})(\cos 45°)$ $= 0.691 \text{ kip-ft}$



(a) Positive geometric and principal axes *(b) Principal axis moments*

Fig. F.11C-2. Example F.11C single angle geometric and principal axes moments.

From AISC *Manual* Table 1-7, the geometric properties are as follows:

L4×4×¼
A = 1.93 in.^{2}
$S_x = S_y$ = 1.03 in.^{3}
$I_x = I_y$ = 3.00 in.^{4}
I_z = 1.19 in.^{4}
r_z = 0.783 in.

Additional principal axes properties from the AISC *Shapes Database* are as follows:

w_B = 1.53 in.
w_C = 1.39 in.
z_C = 2.74 in.
I_w = 4.82 in.^{4}
S_{zB} = 0.778 in.^{3}
S_{zC} = 0.856 in.^{3}
S_{wC} = 1.76 in.^{3}

Z-Axis Nominal Flexural Strength

Note that M_{uz} and M_{az} are positive; therefore, the toes of the angle are in compression.

Flexural Yielding

From AISC *Specification* Section F10.1, the nominal flexural strength due to the limit state of flexural yielding is:

$$\begin{aligned} M_{nz} &= 1.5M_y \\ &= 1.5F_y S_{zB} \\ &= 1.5(36 \text{ ksi})(0.778 \text{ in.}^3) \\ &= 42.0 \text{ kip-in.} \end{aligned}$$ (from *Spec*. Eq. F10-1)

Lateral-Torsional Buckling

From the User Note in AISC *Specification* Section F10, the limit state of lateral-torsional buckling does not apply for bending about the minor axis.

Leg Local Buckling

Check slenderness of outstanding leg in compression.

$$\begin{aligned} \lambda &= \frac{b}{t} \\ &= \frac{4.00 \text{ in.}}{\frac{1}{4} \text{ in.}} \\ &= 16.0 \end{aligned}$$

From AISC *Specification* Table B4.1b, Case 12, the limiting width-to-thickness ratios are:

$$\lambda_p = 0.54\sqrt{\frac{E}{F_y}}$$
$$= 0.54\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$
$$= 15.3$$

$$\lambda_r = 0.91\sqrt{\frac{E}{F_y}}$$
$$= 0.91\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$
$$= 25.8$$

Because $\lambda_p < \lambda < \lambda_r$, the leg is noncompact in flexure.

$$S_c = S_{zC} \text{ (to toe in compression)}$$
$$= 0.856 \text{ in.}^3$$

$$M_{nz} = F_y S_c \left[2.43 - 1.72\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}}\right] \quad \text{(}Spec\text{. Eq. F10-6)}$$
$$= (36 \text{ ksi})(0.856 \text{ in.}^3)\left[2.43 - 1.72(16.0)\sqrt{\frac{36 \text{ ksi}}{29{,}000 \text{ ksi}}}\right]$$
$$= 45.0 \text{ kip-in.}$$

The flexural yielding limit state controls.

$$M_{nz} = 42.0 \text{ kip-in. or } 3.50 \text{ kip-ft}$$

Z-Axis Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_{nz} = 0.90(3.50 \text{ kip-ft})$ $= 3.15 \text{ kip-ft}$	$\Omega_b = 1.67$ $\frac{M_{nz}}{\Omega_b} = \frac{3.50 \text{ kip-ft}}{1.67}$ $= 2.10 \text{ kip-ft}$

W-Axis Nominal Flexural Strength

Flexural Yielding

$$M_{nw} = 1.5M_y \quad \text{(from }Spec\text{. Eq. F10-1)}$$
$$= 1.5F_y S_{wC}$$
$$= 1.5(36 \text{ ksi})(1.76 \text{ in.}^3)$$
$$= 95.0 \text{ kip-in.}$$

Lateral-Torsional Buckling

Determine M_{cr}.

For bending about the major principal axis of an equal-leg angle without continuous lateral-torsional restraint, use AISC *Specification* Equation F10-4.

C_b = 1.14 from *Manual* Table 3-1

From AISC *Specification* Section F10.2(b)(1), $\beta_w = 0$ for equal leg angles.

$$M_{cr} = \frac{9EAr_z tC_b}{8L_b}\left[\sqrt{1+\left(4.4\frac{\beta_w r_z}{L_b t}\right)^2}+4.4\frac{\beta_w r_z}{L_b t}\right] \quad \text{(Spec. Eq. F10-4)}$$

$$= \frac{9(29{,}000\text{ ksi})(1.93\text{ in.}^2)(0.783\text{ in.})(\tfrac{1}{4}\text{ in.})(1.14)}{8(6\text{ ft})(12\text{ in./ft})}$$
$$\times\left\{\sqrt{1+\left[4.4\frac{0(0.783\text{ in.})}{(6\text{ ft})(12\text{ in./ft})(\tfrac{1}{4}\text{ in.})}\right]^2}+4.4\left[\frac{0(0.783\text{ in.})}{(6\text{ ft})(12\text{ in./ft})(\tfrac{1}{4}\text{ in.})}\right]\right\}$$
$$= 195\text{ kip-in.}$$

$$M_y = F_y S_{wC}$$
$$= (36\text{ ksi})(1.76\text{ in.}^3)$$
$$= 63.4\text{ kip-in.}$$

$$\frac{M_y}{M_{cr}} = \frac{63.4\text{ kip-in.}}{195\text{ kip-in.}}$$
$$= 0.325 < 1.0, \text{ therefore, AISC } \textit{Specification} \text{ Equation F10-2 is applicable}$$

$$M_{nw} = \left(1.92-1.17\sqrt{\frac{M_y}{M_{cr}}}\right)M_y \le 1.5M_y \quad \text{(Spec. Eq. F10-2)}$$
$$= \left(1.92-1.17\sqrt{\frac{63.4\text{ kip-in.}}{195\text{ kip-in.}}}\right)(63.4\text{ kip-in.}) \le 1.5(63.4\text{ kip-in.})$$
$$= 79.4\text{ kip-in.} < 95.1\text{ kip-in.}$$
$$= 79.4\text{ kip-in.}$$

Leg Local Buckling

From the preceding calculations, the leg is noncompact in flexure.

$$S_c = S_{wC} \text{ (to toe in compression)}$$
$$= 1.76\text{ in.}^3$$

$$M_{nw} = F_y S_c \left[2.43 - 1.72\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}} \right]$$ (*Spec*. Eq. F10-6)

$$= (36\text{ ksi})(1.76\text{ in.}^3)\left[2.43 - 1.72(16.0)\sqrt{\frac{36\text{ ksi}}{29{,}000\text{ ksi}}}\right]$$

$$= 92.5\text{ kip-in.}$$

The lateral-torsional buckling limit state controls.

$$M_{nw} = 79.4\text{ kip-in. or } 6.62\text{ kip-ft}$$

W-Axis Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_{nw} = 0.90(6.62\text{ kip-ft})$ $= 5.96\text{ kip-ft}$	$\Omega_b = 1.67$ $\dfrac{M_{nw}}{\Omega_b} = \dfrac{6.62\text{ kip-ft}}{1.67}$ $= 3.96\text{ kip-ft}$

Combined Loading

The moment resultant has components about both principal axes; therefore, the combined stress ratio must be checked using the provisions of AISC *Specification* Section H2.

$$\left| \frac{f_{ra}}{F_{ca}} + \frac{f_{rbw}}{F_{cbw}} + \frac{f_{rbz}}{F_{cbz}} \right| \le 1.0$$ (*Spec*. Eq. H2-1)

Note: Rather than convert moments into stresses, it is acceptable to simply use the moments in the interaction equation because the section properties that would be used to convert the moments to stresses are the same in the numerator and denominator of each term. It is also important for the designer to keep track of the signs of the stresses at each point so that the proper sign is applied when the terms are combined. The sign of the moments used to convert geometric axis moments to principal axis moments will indicate which points are in tension and which are in compression but those signs will not be used in the interaction equations directly.

Based on Figure F.11C-2, the required flexural strength and available flexural strength for this beam can be summarized as:

LRFD	ASD
$M_{uw} = 0.286\text{ kip-ft}$ $\phi_b M_{nw} = 5.96\text{ kip-ft}$ $M_{uz} = 1.05\text{ kip-ft}$ $\phi_b M_{nz} = 3.15\text{ kip-ft}$	$M_{aw} = 0.347\text{ kip-ft}$ $\dfrac{M_{nw}}{\Omega_b} = 3.96\text{ kip-ft}$ $M_{az} = 0.691\text{ kip-ft}$ $\dfrac{M_{nz}}{\Omega_b} = 2.10\text{ kip-ft}$

At point B:

M_w causes no stress at point B; therefore, the stress ratio is set to zero. M_z causes tension at point B; therefore it will be taken as negative.

LRFD	ASD
$\left\lvert 0 - \dfrac{1.05 \text{ kip-ft}}{3.15 \text{ kip-ft}} \right\rvert = 0.333 \le 1.0$ **o.k.**	$\left\lvert 0 - \dfrac{0.691 \text{ kip-ft}}{2.10 \text{ kip-ft}} \right\rvert = 0.329 \le 1.0$ **o.k.**

At point C:

M_w causes tension at point C; therefore, it will be taken as negative. M_z causes compression at point C; therefore, it will be taken as positive.

LRFD	ASD
$\left\lvert -\dfrac{0.286 \text{ kip-ft}}{5.96 \text{ kip-ft}} + \dfrac{1.05 \text{ kip-ft}}{3.15 \text{ kip-ft}} \right\rvert = 0.285 \le 1.0$ **o.k.**	$\left\lvert -\dfrac{0.347 \text{ kip-ft}}{3.96 \text{ kip-ft}} + \dfrac{0.691 \text{ kip-ft}}{2.10 \text{ kip-ft}} \right\rvert = 0.241 \le 1.0$ **o.k.**

At point A:

M_w and M_z cause compression at point A; therefore, both will be taken as positive.

LRFD	ASD
$\left\lvert \dfrac{0.286 \text{ kip-ft}}{5.96 \text{ kip-ft}} + \dfrac{1.05 \text{ kip-ft}}{3.15 \text{ kip-ft}} \right\rvert = 0.381 \le 1.0$ **o.k.**	$\left\lvert \dfrac{0.347 \text{ kip-ft}}{3.96 \text{ kip-ft}} + \dfrac{0.691 \text{ kip-ft}}{2.10 \text{ kip-ft}} \right\rvert = 0.417 \le 1.0$ **o.k.**

Thus, the interaction of stresses at each point is seen to be less than 1.0 and this member is adequate to carry the required load. Although all three points were checked, it was expected that point A would be the controlling point because compressive stresses add at this point.

EXAMPLE F.12 RECTANGULAR BAR IN MAJOR AXIS BENDING

Given:

Directly applying the requirements of the AISC *Specification*, select a rectangular bar for span and uniform vertical dead and live loads as shown in Figure F.12. The beam is simply supported and braced at the end points and midspan. Conservatively use C_b = 1.0. Limit the depth of the member to 5 in. The bar is ASTM A36 material.

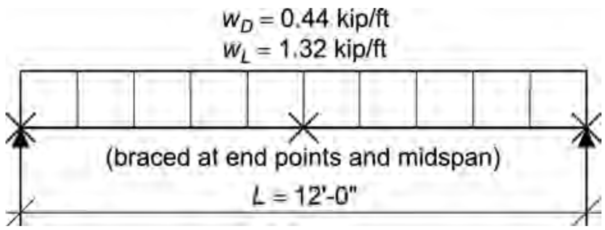


Fig. F.12. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.44\text{ kip/ft}) + 1.6(1.32\text{ kip/ft})$ $= 2.64\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(2.64\text{ kip/ft})(12\text{ ft})^2}{8}$ $= 47.5\text{ kip-ft}$	$w_a = 0.44\text{ kip/ft} + 1.32\text{ kip/ft}$ $= 1.76\text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(1.76\text{ kip/ft})(12\text{ ft})^2}{8}$ $= 31.7\text{ kip-ft}$

Try a BAR 5 in. × 3 in.

From AISC *Manual* Table 17-27, the geometric properties are as follows:

$$S_x = \frac{bd^2}{6}$$
$$= \frac{(3.00\text{ in.})(5.00\text{ in.})^2}{6}$$
$$= 12.5\text{ in.}^3$$

$$Z_x = \frac{bd^2}{4}$$
$$= \frac{(3.00\text{ in.})(5.00\text{ in.})^2}{4}$$
$$= 18.8\text{ in.}^3$$

Nominal Flexural Strength

Flexural Yielding

Check limit from AISC *Specification* Section F11.1.

$$\frac{L_b d}{t^2} = \frac{(6\text{ ft})(12\text{ in./ft})(5.00\text{ in.})}{(3.00\text{ in.})^2}$$
$$= 40.0$$

$$\frac{0.08E}{F_y} = \frac{0.08(29{,}000\text{ ksi})}{36\text{ ksi}}$$
$$= 64.4 > 40.0;\ \text{therefore, the yielding limit state applies}$$

$$M_n = M_p = F_y Z \le 1.6F_y S \qquad (Spec.\text{ Eq. F11-1})$$

$$1.6F_y S = 1.6F_y S_x$$
$$= 1.6(36\text{ ksi})(12.5\text{ in.}^3)$$
$$= 720\text{ kip-in.}$$

$$F_y Z = F_y Z_x$$
$$= (36\text{ ksi})(18.8\text{ in.}^3)$$
$$= 677\text{ kip-in.} < 720\text{ kip-in.}$$

Use M_n = 677 kip-in. or 56.4 kip-ft.

Lateral-Torsional Buckling

From AISC *Specification* Section F11.2(a), because $\frac{L_b d}{t^2} \le \frac{0.08E}{F_y}$, the lateral-torsional buckling limit state does not apply.

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(56.4\text{ kip-ft})$ $= 50.8\text{ kip-ft} > 47.5\text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{56.4\text{ kip-ft}}{1.67}$ $= 33.8\text{ kip-ft} > 31.7\text{ kip-ft}$ **o.k.**

EXAMPLE F.13 ROUND BAR IN BENDING

Given:

Select a round bar for span and concentrated dead and live loads, at midspan, as shown in Figure F.13. The beam is simply supported and braced at the end points only. Conservatively use C_b = 1.0. Limit the diameter of the member to 2 in. The weight of the bar is negligible. The bar is ASTM A36 material.

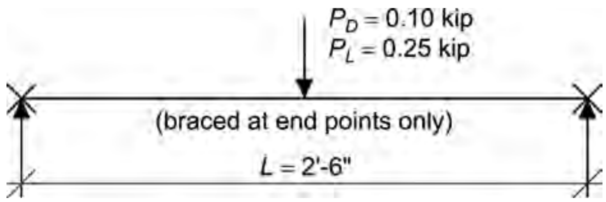


Fig. F.13. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7 the required flexural strength is:

LRFD	ASD
$P_u = 1.2(0.10\text{ kip}) + 1.6(0.25\text{ kip})$ $= 0.520\text{ kip}$ From AISC *Manual* Table 3-23, Case 7: $M_u = \dfrac{P_u L}{4}$ $= \dfrac{(0.520\text{ kip})(2.5\text{ ft})}{4}$ $= 0.325\text{ kip-ft}$	$P_a = 0.10\text{ kip} + 0.25\text{ kip}$ $= 0.350\text{ kip}$ From AISC *Manual* Table 3-23, Case 7: $M_a = \dfrac{P_a L}{4}$ $= \dfrac{(0.350\text{ kip})(2.5\text{ ft})}{4}$ $= 0.219\text{ kip-ft}$

Try a BAR 1-in.-diameter.

From AISC *Manual* Table 17-27, the geometric properties are as follows:

$$S = \frac{\pi d^3}{32}$$
$$= \frac{\pi(1.00\text{ in.})^3}{32}$$
$$= 0.0982\text{ in.}^3$$

$$Z = \frac{d^3}{6}$$
$$= \frac{(1.00 \text{ in.})^3}{6}$$
$$= 0.167 \text{ in.}^3$$

Nominal Flexural Strength

Flexural Yielding

From AISC *Specification* Section F11.1, the nominal flexural strength based on the limit state of flexural yielding is:

$$M_n = M_p = F_y Z \leq 1.6 F_y S_x \qquad \text{(Spec. Eq. F11-1)}$$

$$1.6 F_y S = 1.6(36 \text{ ksi})(0.0982 \text{ in.}^3)$$
$$= 5.66 \text{ kip-in.}$$

$$F_y Z = (36 \text{ ksi})(0.167 \text{ in.}^3)$$
$$= 6.01 \text{ kip-in.} > 5.66 \text{ kip-in, therefore, } M_n = 5.66 \text{ kip-in.}$$

From *AISC Specification* Section F11.2, the limit state lateral-torsional buckling need not be considered for rounds.

The flexural yielding limit state controls.

M_n = 5.66 kip-in. or 0.472 kip-ft

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(0.472 \text{ kip-ft})$ $= 0.425 \text{ kip-ft} > 0.325 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{0.472 \text{ kip-ft}}{1.67}$ $= 0.283 \text{ kip-ft} > 0.219 \text{ kip-ft}$ **o.k.**

EXAMPLE F.14 POINT-SYMMETRICAL Z-SHAPE IN MAJOR AXIS BENDING

Given:

Directly applying the requirements of the AISC *Specification*, determine the available flexural strength of a Z-shaped flexural member for the span and loading shown in Figure F.14-1. The beam is simply supported and braced at the third and end points. Assume C_b = 1.0. Assume the beam is loaded through the shear center. The geometry for the member is shown in Figure F.14-2. The member is ASTM A36 material.

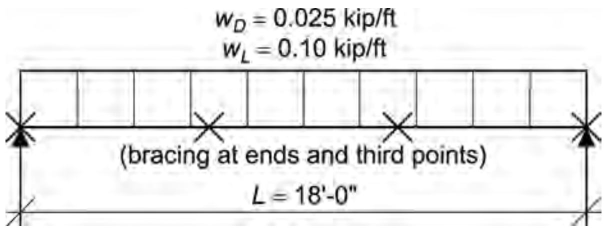


Fig. F.14-1. Beam loading and bracing diagram.

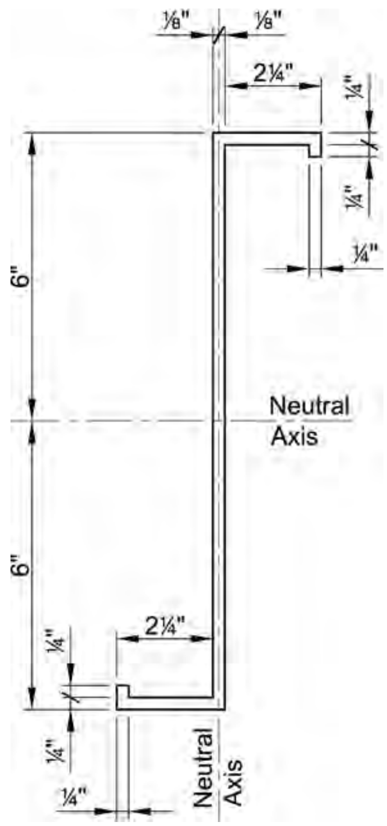


Fig. F.14-2. Beam geometry for Example F.14.

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

The geometric properties are as follows:

$$t_w = t_f$$
$$= \tfrac{1}{4} \text{ in.}$$

$$A = 2(2.50 \text{ in.})(\tfrac{1}{4} \text{ in.}) + 2(\tfrac{1}{4} \text{ in.})(\tfrac{1}{4} \text{ in.}) + (11.5 \text{ in.})(\tfrac{1}{4} \text{ in.})$$
$$= 4.25 \text{ in.}^2$$

$$I_x = 2\left[\frac{(\tfrac{1}{4} \text{ in.})(\tfrac{1}{4} \text{ in.})^3}{12} + (\tfrac{1}{4} \text{ in.})^2(5.63 \text{ in.})^2\right] + 2\left[\frac{(2.50 \text{ in.})(\tfrac{1}{4} \text{ in.})^3}{12} + (2.50 \text{ in.})(\tfrac{1}{4} \text{ in.})(5.88 \text{ in.})^2\right]$$
$$+ \frac{(\tfrac{1}{4} \text{ in.})(11.5 \text{ in.})^3}{12}$$
$$= 78.9 \text{ in.}^4$$

$$\bar{y} = 6.00 \text{ in.}$$

$$S_x = \frac{I_x}{\bar{y}}$$
$$= \frac{78.9 \text{ in.}^4}{6.00 \text{ in.}}$$
$$= 13.2 \text{ in.}^3$$

$$I_y = 2\left[\frac{(\tfrac{1}{4} \text{ in.})(\tfrac{1}{4} \text{ in.})^3}{12} + (\tfrac{1}{4} \text{ in.})^2(2.25 \text{ in.})^2\right] + 2\left[\frac{(\tfrac{1}{4} \text{ in.})(2.50 \text{ in.})^3}{12} + (2.50 \text{ in.})(\tfrac{1}{4} \text{ in.})(1.13 \text{ in.})^2\right]$$
$$+ \frac{(11.5 \text{ in.})(\tfrac{1}{4} \text{ in.})^3}{12}$$
$$= 2.90 \text{ in.}^4$$

$$r_y = \sqrt{\frac{I_y}{A}}$$
$$= \sqrt{\frac{2.90 \text{ in.}^4}{4.25 \text{ in.}^2}}$$
$$= 0.826 \text{ in.}$$

The effective radius of gyration, r_{ts}, may be conservatively approximated from the User Note in AISC *Specification* Section F2.2. A more exact method may be derived as discussed in AISC Design Guide 9, *Torsional Analysis of Structural Steel Members* (Seaburg and Carter, 1997), for a Z-shape that excludes lips.

From AISC *Specification* Section F2.2 User Note:

$$r_{ts} \approx \frac{b_f}{\sqrt{12\left(1+\dfrac{1}{6}\dfrac{ht_w}{b_f t_f}\right)}}$$

$$= \frac{2.50 \text{ in.}}{\sqrt{12\left\{1+\left(\dfrac{1}{6}\right)\left[\dfrac{(11.5 \text{ in.})(\frac{1}{4} \text{ in.})}{(2.50 \text{ in.})(\frac{1}{4} \text{ in.})}\right]\right\}}}$$

$$= 0.543 \text{ in.}$$

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.025 \text{ kip/ft}) + 1.6(0.10 \text{ kip/ft})$ $= 0.190 \text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(0.190 \text{ kip/ft})(18 \text{ ft})^2}{8}$ $= 7.70 \text{ kip-ft}$	$w_a = 0.025 \text{ kip/ft} + 0.10 \text{ kip/ft}$ $= 0.125 \text{ kip/ft}$ From AISC *Manual* Table 3-23, Case 1: $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(0.125 \text{ kip/ft})(18 \text{ ft})^2}{8}$ $= 5.06 \text{ kip-ft}$

Nominal Flexural Strength

Flexural Yielding

From AISC *Specification* Section F12.1, the nominal flexural strength based on the limit state of flexural yielding is,

$$F_n = F_y \qquad \text{(}\textit{Spec}\text{. Eq. F12-2)}$$
$$= 36 \text{ ksi}$$

$$M_n = F_n S_{min} \qquad \text{(}\textit{Spec}\text{. Eq. F12-1)}$$
$$= (36 \text{ ksi})(13.2 \text{ in.}^3)$$
$$= 475 \text{ kip-in.}$$

Local Buckling

There are no specific local buckling provisions for Z-shapes in the AISC *Specification*. Use provisions for rolled channels from AISC *Specification* Table B4.1b, Cases 10 and 15.

Flange Slenderness

Conservatively neglecting the end return,

$$\lambda = \frac{b}{t_f}$$
$$= \frac{2.50\text{ in.}}{\frac{1}{4}\text{ in.}}$$
$$= 10.0$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}}$$ (*Spec.* Table B4.1b, Case 10)
$$= 0.38\sqrt{\frac{29{,}000\text{ ksi}}{36\text{ ksi}}}$$
$$= 10.8$$

$\lambda < \lambda_p$; therefore, the flange is compact

Web Slenderness

$$\lambda = \frac{h}{t_w}$$
$$= \frac{11.5\text{ in.}}{\frac{1}{4}\text{ in.}}$$
$$= 46.0$$

$$\lambda_p = 3.76\sqrt{\frac{E}{F_y}}$$ (*Spec.* Table B4.1b, Case 15)
$$= 3.76\sqrt{\frac{29{,}000\text{ ksi}}{36\text{ ksi}}}$$
$$= 107$$

$\lambda < \lambda_p$; therefore, the web is compact

Therefore, the local buckling limit state does not apply.

Lateral-Torsional Buckling

Per the User Note in AISC *Specification* Section F12, take the critical lateral-torsional buckling stress as half that of the equivalent channel. This is a conservative approximation of the lateral-torsional buckling strength which accounts for the rotation between the geometric and principal axes of a Z-shaped cross section, and is adopted from the *North American Specification for the Design of Cold-Formed Steel Structural Members* (AISI, 2016).

Calculate limiting unbraced lengths.

For bracing at 6 ft on center,

$$L_b = (6\text{ ft})(12\text{ in./ft})$$
$$= 72.0\text{ in.}$$

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}}$$ (*Spec*. Eq. F2-5)

$$= 1.76(0.826 \text{ in.})\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$

$$= 41.3 \text{ in.} < 72.0 \text{ in.}$$

Per the User Note in AISC *Specification* Section F2, the square root term in AISC *Specification* Equation F2-4 can conservatively be taken equal to one. Therefore, Equation F2-6 can also be simplified. Substituting $0.7F_y$ for F_{cr} (where F_{cr} is half of the critical lateral-torsional buckling stress of the equivalent channel) in Equation F2-4 and solving for $L_b = L_r$, AISC *Specification* Equation F2-6 becomes:

$$L_r = \pi r_{ts}\sqrt{\frac{0.5E}{0.7F_y}}$$

$$= \pi(0.543 \text{ in.})\sqrt{\frac{0.5(29{,}000 \text{ ksi})}{0.7(36 \text{ ksi})}}$$

$$= 40.9 \text{ in.} < 72.0 \text{ in.}$$

Calculate one half of the critical lateral-torsional buckling stress of the equivalent channel.

$L_b > L_r$, therefore,

$$F_{cr} = (0.5)\frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\left(\frac{Jc}{S_x h_o}\right)\left(\frac{L_b}{r_{ts}}\right)^2}$$ (from *Spec*. Eq. F2-4)

Conservatively taking the square root term as 1.0,

$$F_{cr} = (0.5)\left[\frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2}\right](1.0)$$

$$= (0.5)\left[\frac{1.0\pi^2(29{,}000 \text{ ksi})}{\left(\frac{72.0 \text{ in.}}{0.543 \text{ in.}}\right)^2}\right](1.0)$$

$$= 8.14 \text{ ksi}$$

$$F_n = F_{cr} \le F_y$$ (*Spec*. Eq. F12-3)

$$= 8.14 \text{ ksi} \le 36 \text{ ksi}$$ **o.k.**

$$M_n = F_n S_{min}$$ (*Spec*. Eq. F12-1)

$$= (8.14 \text{ ksi})(13.2 \text{ in.}^3)$$

$$= 107 \text{ kip-in.}$$

The lateral-torsional buckling limit state controls.

M_n = 107 kip-in. or 8.92 kip-ft

Available Flexural Strength

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(8.92 \text{ kip-ft})$ $= 8.03 \text{ kip-ft} > 7.70 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{8.92 \text{ kip-ft}}{1.67}$ $= 5.34 \text{ kip-ft} > 5.06 \text{ kip-ft}$ **o.k.**

Because the beam is loaded through the shear center, consideration of a torsional moment is unnecessary. If the loading produced torsion, the torsional effects should be evaluated using AISC Design Guide 9, *Torsional Analysis of Structural Steel Members* (Seaburg and Carter, 1997).

EXAMPLE F.15 PLATE GIRDER FLEXURAL MEMBER

Given:

Verify the built-up plate girder for the span and loads as shown in Figure F.15-1 with a cross section as shown in Figure F.15-2. The beam has a concentrated dead and live load at midspan and a uniformly distributed self weight. The plate girder is simply supported and is laterally braced at quarter and end points. The deflection of the girder is limited to 1 in. The plate girder is ASTM A572 Grade 50 material. The flange-to-web welds will be designed for both continuous and intermittent fillet welds using 70-ksi electrodes.

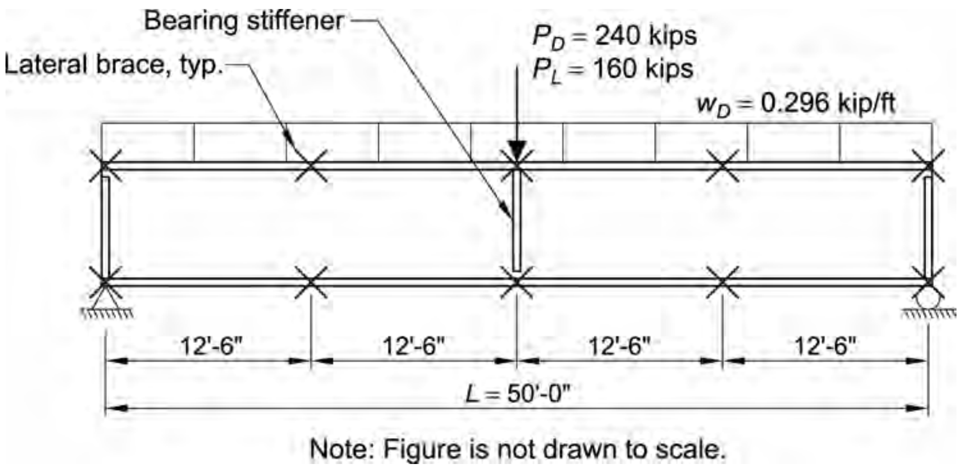


Fig. F.15-1. Beam loading and bracing diagram.

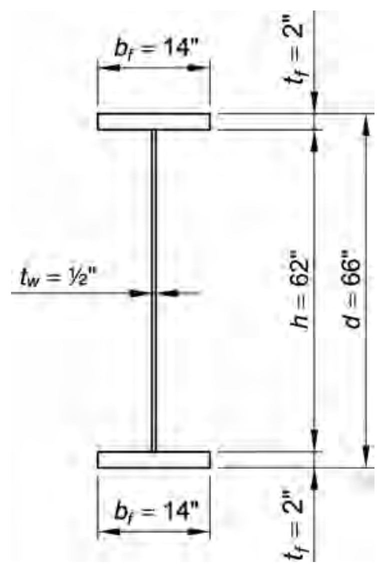


Fig. F.15-2. Plate girder geometry.

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

ASTM A572 Grade 50
F_y = 50 ksi
F_u = 65 ksi

From ASCE/SEI 7, Chapter 2, the required shear and flexural strengths are:

LRFD	ASD
$P_u = 1.2(240\text{ kips}) + 1.6(160\text{ kips})$ $= 544\text{ kips}$	$P_a = 240\text{ kips} + 160\text{ kips}$ $= 400\text{ kips}$
$w_u = 1.2(0.296\text{ kip/ft})$ $= 0.355\text{ kip/ft}$	$w_a = 0.296\text{ kip/ft}$
$V_u = \dfrac{P_u}{2} + \dfrac{w_u L}{2}$ $= \dfrac{544\text{ kips}}{2} + \dfrac{(0.355\text{ kip/ft})(50\text{ ft})}{2}$ $= 281\text{ kips}$	$V_a = \dfrac{P_a}{2} + \dfrac{w_a L}{2}$ $= \dfrac{400\text{ kips}}{2} + \dfrac{(0.296\text{ kip/ft})(50\text{ ft})}{2}$ $= 207\text{ kips}$
$M_u = \dfrac{P_u L}{4} + \dfrac{w_u L^2}{8}$ $= \dfrac{(544\text{ kips})(50\text{ ft})}{4} + \dfrac{(0.355\text{ kip/ft})(50\text{ ft})^2}{8}$ $= 6{,}910\text{ kip-ft}$	$M_a = \dfrac{P_a L}{4} + \dfrac{w_a L^2}{8}$ $= \dfrac{(400\text{ kips})(50\text{ ft})}{4} + \dfrac{(0.296\text{ kip/ft})(50\text{ ft})^2}{8}$ $= 5{,}090\text{ kip-ft}$

Proportioning Limits

The proportioning limits from AISC *Specification* Section F13.2 are evaluated as follows, where a is the clear distance between transverse stiffeners.

$$\frac{a}{h} = \frac{(25\text{ ft})(12\text{ in./ft})}{62\text{ in.}} = 4.84$$

Because $a/h > 1.5$, use AISC *Specification* Equation F13-4.

$$\left(\frac{h}{t_w}\right)_{max} = \frac{0.40E}{F_y} = \frac{0.40(29{,}000\text{ ksi})}{50\text{ ksi}} = 232 \qquad \text{(}Spec.\text{ Eq. F13-3)}$$

$$\frac{h}{t_w} = \frac{62\text{ in.}}{½\text{ in.}} = 124 < 232 \quad \textbf{o.k.}$$

From AISC *Specification* Section F13.2, the following limit applies to all built-up I-shaped members:

$$\frac{h_c t_w}{b_f t_f} = \frac{(62\text{ in.})(½\text{ in.})}{(14\text{ in.})(2\text{ in.})} < 10$$
$$= 1.11 < 10 \quad \textbf{o.k.}$$

Section Properties

$$I_x = \sum \frac{bh^3}{12} + \sum Ad^2$$
$$= \frac{(\tfrac{1}{2}\text{ in.})(62\text{ in.})^3}{12} + 2\left[\frac{(14\text{ in.})(2\text{ in.})^3}{12}\right] + 2\left[(2\text{ in.})(14\text{ in.})(32.0\text{ in.})^2\right]$$
$$= 67{,}300\text{ in.}^4$$

$$S_{xt} = S_{xc}$$
$$= \frac{I_x}{(d/2)}$$
$$= \frac{67{,}300\text{ in.}^4}{(66\text{ in.}/2)}$$
$$= 2{,}040\text{ in.}^3$$

$$Z_x = \sum A\bar{y}$$
$$= (2)(\tfrac{1}{2}\text{ in.})(31.0\text{ in.})(31.0\text{ in.}/2) + (2)(2\text{ in.})(14\text{ in.})(32.0\text{ in.})$$
$$= 2{,}270\text{ in.}^3$$

$$J = \sum \frac{bt^3}{3}$$
$$= 2\left[\frac{(14\text{ in.})(2\text{ in.})^3}{3}\right] + \frac{(62\text{ in.})(\tfrac{1}{2}\text{ in.})^3}{3}$$
$$= 77.3\text{ in.}^4$$

$$h_o = h + t_f$$
$$= 62\text{ in.} + 2\text{ in.}$$
$$= 64.0\text{ in.}$$

Deflection

The maximum deflection is:

$$\Delta = \frac{(P_D + P_L)L^3}{48EI} + \frac{5w_D L^4}{384EI}$$
$$= \frac{(240\text{ kips} + 160\text{ kips})(50\text{ ft})^3(12\text{ in./ft})^3}{48(29{,}000\text{ ksi})(67{,}300\text{ in.}^4)} + \frac{5(0.296\text{ kip/ft})(50\text{ ft})^4(12\text{ in./ft})^3}{384(29{,}000\text{ ksi})(67{,}300\text{ in.}^4)}$$
$$= 0.944\text{ in.} < 1.00\text{ in.} \quad \textbf{o.k.}$$

Web Slenderness

$$\lambda = \frac{h}{t_w}$$
$$= \frac{62 \text{ in.}}{½ \text{ in.}}$$
$$= 124$$

The limiting width-to-thickness ratios for the web are:

$$\lambda_{pw} = 3.76\sqrt{\frac{E}{F_y}} \text{ from AISC } \textit{Specification} \text{ Table B4.1b, Case 15}$$
$$= 3.76\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 90.6$$

$$\lambda_{rw} = 5.70\sqrt{\frac{E}{F_y}} \text{ from AISC } \textit{Specification} \text{ Table B4.1b, Case 15}$$
$$= 5.70\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 137$$

$\lambda_{pw} < \lambda < \lambda_{rw}$, therefore the web is noncompact and AISC *Specification* Section F4 applies.

Flange Slenderness

$$\lambda = \frac{b}{t}$$
$$= \frac{b_f}{2t_f}$$
$$= \frac{14 \text{ in.}}{2(2 \text{ in.})}$$
$$= 3.50$$

$$\lambda_{pf} = 0.38\sqrt{\frac{E}{F_y}} \text{ from AISC } \textit{Specification} \text{ Table B4.1b, Case 11}$$
$$= 0.38\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 9.15 > \lambda, \text{ therefore the flanges are compact}$$

Nominal Flexural Strength

Compression Flange Yielding

The web plastification factor is determined using AISC *Specification* Section F4.2(c)(6).

$$I_{yc} = \frac{t_f b_f^3}{12}$$
$$= \frac{(2 \text{ in.})(14 \text{ in.})^3}{12}$$
$$= 457 \text{ in.}^4$$

$$I_y = 2\left(\frac{t_f b_f^3}{12}\right) + \frac{h t_w^3}{12}$$
$$= 2\left[\frac{(2 \text{ in.})(14 \text{ in.})^3}{12}\right] + \frac{(62 \text{ in.})(\tfrac{1}{2} \text{ in.})^3}{12}$$
$$= 915 \text{ in.}^4$$

$$\frac{I_{yc}}{I_y} = \frac{457 \text{ in.}^4}{915 \text{ in.}^4}$$
$$= 0.499$$

Because $I_{yc}/I_y > 0.23$, AISC *Specification* Section F4.2(c)(6)(i) applies.

$$M_p = F_y Z_x \leq 1.6 F_y S_x$$
$$= (50 \text{ ksi})(2,270 \text{ in.}^3)(1 \text{ ft/12 in.}) \leq 1.6(50 \text{ ksi})(2,040 \text{ in.}^3)(1 \text{ ft/12 in.})$$
$$= 9,460 \text{ kip-ft} < 13,600 \text{ kip-ft}$$
$$= 9,460 \text{ kip-ft}$$

$$M_{yc} = F_y S_{xc} \qquad \text{(Spec. Eq. F4-4)}$$
$$= (50 \text{ ksi})(2,040 \text{ kip-in.})(1 \text{ ft/12 in.})$$
$$= 8,500 \text{ kip-ft}$$

$$h_c = h$$
$$= 62 \text{ in.}$$

$$\lambda = \frac{h_c}{t_w}$$
$$= \frac{62 \text{ in.}}{\tfrac{1}{2} \text{ in.}}$$
$$= 124 > \lambda_{pw} = 90.6; \text{ therefore use AISC } \textit{Specification} \text{ Equation F4-9b}$$

$$R_{pc} = \frac{M_p}{M_{yc}} - \left(\frac{M_p}{M_{yc}} - 1\right)\left(\frac{\lambda - \lambda_{pw}}{\lambda_{rw} - \lambda_{pw}}\right) \leq \frac{M_p}{M_{yc}} \qquad \text{(Spec. Eq. F4-9b)}$$
$$= \frac{9,460 \text{ kip-ft}}{8,500 \text{ kip-ft}} - \left(\frac{9,460 \text{ kip-ft}}{8,500 \text{ kip-ft}} - 1\right)\left(\frac{124 - 90.6}{137 - 90.6}\right) \leq \frac{9,460 \text{ kip-ft}}{8,500 \text{ kip-ft}}$$
$$= 1.03 < 1.11$$
$$= 1.03$$

The nominal flexural strength is calculated as:

$$M_n = R_{pc} M_{yc}$$ (*Spec*. Eq. F4-1)
$$= (1.03)(8{,}500 \text{ kip-ft})$$
$$= 8{,}760 \text{ kip-ft}$$

From AISC *Specification* Section F4.1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(8{,}760 \text{ kip-ft})$ $= 7{,}880 \text{ kip-ft} > 6{,}910 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{8{,}760 \text{ kip-ft}}{1.67}$ $= 5{,}250 \text{ kip-ft} > 5{,}090 \text{ kip-ft}$ **o.k.**

Lateral-Torsional Buckling

The middle unbraced lengths control by inspection. For bracing at quarter points,

$$L_b = (12.5 \text{ ft})(12 \text{ in./ft})$$
$$= 150 \text{ in.}$$

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}}$$ (*Spec*. Eq. F4-12)
$$= \frac{(62 \text{ in.})(\tfrac{1}{2} \text{ in.})}{(14 \text{ in.})(2 \text{ in.})}$$
$$= 1.11$$

$$r_t = \frac{b_{fc}}{\sqrt{12\left(1 + \frac{1}{6} a_w\right)}}$$ (*Spec*. Eq. F4-11)
$$= \frac{14.0 \text{ in.}}{\sqrt{12\left[1 + \left(\frac{1.11}{6}\right)\right]}}$$
$$= 3.71 \text{ in.}$$

From AISC *Specification* Equation F4-7:

$$L_p = 1.1 r_t \sqrt{\frac{E}{F_y}}$$ (*Spec*. Eq. F4-7)
$$= 1.1(3.71 \text{ in.})\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 98.3 < 150 \text{ in.; therefore, lateral-torsional buckling applies}$$

From AISC *Specification* Section F4.2(c)(3):

$$\frac{S_{xt}}{S_{xc}} = \frac{2{,}040 \text{ in.}^3}{2{,}040 \text{ in.}^3}$$
$$= 1.00 > 0.7; \text{ therefore, AISC } \textit{Specification} \text{ Equation F4-6a applies}$$

$$F_L = 0.7F_y \qquad \text{(}\textit{Spec.}\text{ Eq. F4-6a)}$$
$$= 0.7(50 \text{ ksi})$$
$$= 35.0 \text{ ksi}$$

From AISC *Specification* Equation F4-8:

$$L_r = 1.95 r_t \frac{E}{F_L}\sqrt{\frac{J}{S_{xc}h_o} + \sqrt{\left(\frac{J}{S_{xc}h_o}\right)^2 + 6.76\left(\frac{F_L}{E}\right)^2}} \qquad \text{(}\textit{Spec.}\text{ Eq. F4-8)}$$
$$= 1.95(3.71 \text{ in.})\left(\frac{29{,}000 \text{ ksi}}{35.0 \text{ ksi}}\right)\sqrt{\frac{77.3 \text{ in.}^4}{(2{,}040 \text{ in.}^3)(64.0 \text{ in.})} + \sqrt{\left[\frac{77.3 \text{ in.}^4}{(2{,}040 \text{ in.}^3)(64.0 \text{ in.})}\right]^2 + 6.76\left(\frac{35.0 \text{ ksi}}{29{,}000 \text{ ksi}}\right)^2}}$$
$$= 369 \text{ in.}$$

$L_p < L_b \le L_r$; therefore, use AISC *Specification* Equation F4-2

The lateral-torsional buckling modification factor is determined by solving for the moment in the beam using statics. Note: The following solution uses LRFD load combinations. Using ASD load combinations will give approximately the same solution for C_b.

$$M_{max} = 6{,}910 \text{ kip-ft}$$
$$M_A = 4{,}350 \text{ kip-ft}$$
$$M_B = 5{,}210 \text{ kip-ft}$$
$$M_C = 6{,}060 \text{ kip-ft}$$

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \qquad \text{(}\textit{Spec.}\text{ Eq. F1-1)}$$
$$= \frac{12.5(6{,}910 \text{ kip-ft})}{2.5(6{,}910 \text{ kip-ft}) + 3(4{,}350 \text{ kip-ft}) + 4(5{,}210 \text{ kip-ft}) + 3(6{,}060 \text{ kip-ft})}$$
$$= 1.25$$

The nominal flexural strength is calculated as:

$$M_n = C_b\left[R_{pc}M_{yc} - (R_{pc}M_{yc} - F_L S_{xc})\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le R_{pc}M_{yc} \qquad \text{(}\textit{Spec.}\text{ Eq. F4-2)}$$
$$= 1.25\left\{8{,}760 \text{ kip-ft} - \left[8{,}760 \text{ kip-ft} - (35.0 \text{ ksi})(2{,}040 \text{ in.}^3)(1 \text{ ft/12 in.})\right]\left(\frac{150 \text{ in.} - 98.3 \text{ in.}}{369 \text{ in.} - 98.3 \text{ in.}}\right)\right\} \le 8{,}760 \text{ kip-ft}$$
$$= 10{,}300 \text{ kip-ft} > 8{,}760 \text{ kip-ft}$$
$$= 8{,}760 \text{ kip-ft}$$

From AISC *Specification* Section F4.2, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(8{,}760 \text{ kip-ft})$ $= 7{,}880 \text{ kip-ft} > 6{,}910 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{8{,}760 \text{ kip-ft}}{1.67}$ $= 5{,}250 \text{ kip-ft} > 5{,}090 \text{ kip-ft}$ **o.k.**

Compression Flange Local Buckling

From AISC *Specification* Section F4.3(a), this limit state does not apply because the flanges are compact.

Tension Flange Yielding

From AISC *Specification* Section F4.4(a), because $S_{xt} = S_{xc}$, this limit state does not apply.

Nominal Shear Strength

Determine the nominal shear strength without tension field action, using AISC *Specification* Section G2.1. For built-up I-shaped members, determine C_{v1} and k_v from AISC *Specification* Section G2.1(b).

$$\frac{a}{h} = \frac{(25.0 \text{ ft})(12 \text{ in./ft}) - \tfrac{1}{2} \text{ in.}}{62 \text{ in.}}$$
$$= 4.83 > 3.0$$

From AISC *Specification* Section G2.1(b)(2):

$$k_v = 5.34$$

$$1.10\sqrt{\frac{k_v E}{F_y}} = 1.10\sqrt{\frac{5.34(29{,}000 \text{ ksi})}{50 \text{ ksi}}}$$
$$= 61.2 < h/t_w = 124; \text{ therefore, AISC } \textit{Specification} \text{ Equation G2-4 applies}$$

$$C_{v1} = \frac{1.10\sqrt{k_v E/F_y}}{h/t_w} \qquad \text{(Spec. Eq. G2-4)}$$
$$= \frac{61.2}{124}$$
$$= 0.494$$

The nominal shear strength is calculated as follows:

$$V_n = 0.6 F_y A_w C_{v1} \qquad \text{(Spec. Eq. G2-1)}$$
$$= 0.6(50 \text{ ksi})(66 \text{ in.})(\tfrac{1}{2} \text{ in.})(0.494)$$
$$= 489 \text{ kips}$$

From AISC *Specification* Section G.1, the available shear strength is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(489 \text{ kips})$ $= 440 \text{ kips} > 281 \text{ kips}$ **o.k.**	$\Omega_v = 1.67$ $\dfrac{V_n}{\Omega_v} = \dfrac{489 \text{ kips}}{1.67}$ $= 293 \text{ kips} > 207 \text{ kips}$ **o.k.**

Flange-to-Web Fillet Weld—Continuous Weld

Calculate the required shear flow using VQ/I_x because the stress distribution is linearly elastic away from midspan.

$$Q = A\bar{y}$$
$$= b_f t_f \left(\frac{h}{2} + \frac{t_f}{2}\right)$$
$$= (14 \text{ in.})(2 \text{ in.})\left(\frac{62 \text{ in.}}{2} + \frac{2 \text{ in.}}{2}\right)$$
$$= 896 \text{ in.}^3$$

LRFD	ASD
$R_u = \dfrac{V_u Q}{I_x}$ $= \dfrac{(281 \text{ kips})(896 \text{ in.}^3)}{67{,}300 \text{ in.}^4}$ $= 3.74 \text{ kip/in.}$	$R_a = \dfrac{V_a Q}{I_x}$ $= \dfrac{(207 \text{ kips})(896 \text{ in.}^3)}{67{,}300 \text{ in.}^4}$ $= 2.76 \text{ kip/in.}$

From AISC *Specification* Table J2.4, the minimum fillet weld size that can be used on the ½-in.-thick web is:

$$w_{min} = \tfrac{3}{16} \text{ in.}$$

From AISC *Manual* Part 8, the required fillet weld size is:

LRFD	ASD
$D_{req} = \dfrac{R_u}{1.392(2 \text{ sides})}$ (from *Manual* Eq. 8-2a) $= \dfrac{3.74 \text{ kip/in.}}{1.392(2 \text{ sides})}$ $= 1.34 \text{ sixteenths} < 3 \text{ sixteenths}$ Use $w = \tfrac{3}{16}$ in.	$D_{req} = \dfrac{R_a}{0.928(2 \text{ sides})}$ (from *Manual* Eq. 8-2b) $= \dfrac{2.76 \text{ kip/in.}}{0.928(2 \text{ sides})}$ $= 1.49 \text{ sixteenths} < 3 \text{ sixteenths}$ Use $w = \tfrac{3}{16}$ in.

From AISC *Specification* Equation J2-2, the available shear rupture strength of the web in kip/in. is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi F_{nBM} A_{BM}$ $= \phi 0.60 F_u t_w$ $= 0.75(0.60)(65\text{ ksi})(\frac{1}{2}\text{ in.})$ $= 14.6\text{ kip/in.} > 3.74\text{ kip/in.}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{F_{nBM} A_{BM}}{\Omega}$ $= \dfrac{0.60 F_u t_w}{\Omega}$ $= \dfrac{0.60(65\text{ ksi})(\frac{1}{2}\text{ in.})}{2.00}$ $= 9.75\text{ kip/in.} > 2.76\text{ kip/in.}$ **o.k.**

Flange-to-Web Fillet Weld—Intermittent Weld

The two sided intermittent weld is designed using the minimum fillet weld size determined previously, $w_{min} = \frac{3}{16}$ in., and spaced at 12 in. center-to-center.

LRFD	ASD
$R_u = \phi R_n$ (from *Manual* Eq. 8-2a) $= 1.392D(2\text{ sides})\left(\dfrac{l_{req}}{s}\right)$ Solving for l_{req}, $l_{req} = \dfrac{R_u s}{1.392D(2\text{ sides})}$ $= \dfrac{(3.74\text{ kip-in.})(12\text{ in.})}{1.392(3\text{ sixteenth})(2\text{ sides})}$ $= 5.37\text{ in.}$ Use l = 6 in. at 12 in. o.c.	$R_u = \dfrac{R_n}{\Omega}$ (from *Manual* Eq. 8-2b) $= 0.928D(2\text{ sides})\left(\dfrac{l_{req}}{s}\right)$ Solving for l_{req}, $l_{req} = \dfrac{R_u s}{0.928D(2\text{ sides})}$ $= \dfrac{(2.76\text{ kip-in.})(12\text{ in.})}{0.928(3\text{ sixteenth})(2\text{ sides})}$ $= 5.95\text{ in.}$ Use l = 6 in. at 12 in. o.c.

The limitations for a intermittent fillet weld are checked using AISC *Specification* Section J2.2b(e):

$l \geq 4D$

$6\text{ in.} \geq 4(\frac{3}{16}\text{ in.})$

$6\text{ in.} > 0.75\text{ in.}$ **o.k.**

$l \geq 1\frac{1}{2}\text{ in.}$

$6\text{ in.} > 1\frac{1}{2}\text{ in.}$ **o.k.**

CHAPTER F DESIGN EXAMPLE REFERENCES

AISI (2016), *North American Specification for the Design of Cold-Formed Steel Structural Members*, ANSI/AISI Standard S100, American Iron and Steel Institute, Washington D.C.

Seaburg, P.A. and Carter, C.J. (1997), *Torsional Analysis of Structural Steel Members*, Design Guide 9, AISC, Chicago, IL.

Chapter G
Design of Members for Shear

INTRODUCTION

This *Specification* chapter addresses webs of singly or doubly symmetric members subject to shear in the plane of the web, single angles and HSS subject to shear, and shear in the weak direction of singly or doubly symmetric shapes.

G1. GENERAL PROVISIONS

The design shear strength, $\phi_v V_n$, and the allowable shear strength, V_n/Ω_v, are determined as follows:

V_n = nominal shear strength based on shear yielding or shear buckling
ϕ_v = 0.90 (LRFD)
Ω_v = 1.67 (ASD)

Exception: For all current ASTM A6, W, S and HP shapes except W44×230, W40×149, W36×135, W33×118, W30×90, W24×55, W16×26 and W12×14 for F_y = 50 ksi:

ϕ_v = 1.00 (LRFD)
Ω_v = 1.50 (ASD)

Strong axis shear values are tabulated for W-shapes in AISC *Manual* Tables 3-2, 3-6 and 6-2, for S-shapes in AISC *Manual* Table 3-7, for C-shapes in AISC *Manual* Table 3-8, and for MC-shapes in AISC *Manual* Table 3-9. Strong axis shear values are tabulated for rectangular HSS, round HSS and pipe in Part IV. Weak axis shear values for W-shapes, S-shapes, C-shapes and MC-shapes, and shear values for angles, rectangular HSS and box members are not tabulated.

G2. I-SHAPED MEMBERS AND CHANNELS

This section includes provisions for shear strength of webs without the use of tension field action and for interior web panels considering tension field action. Provisions for the design of transverse stiffeners are also included in Section G2.

As indicated in the User Note of this section, virtually all W, S and HP shapes are not subject to shear buckling and are also eligible for the more liberal safety and resistance factors, ϕ_v = 1.00 (LRFD) and Ω_v = 1.50 (ASD). This is presented in Example G.1 for a W-shape. A channel shear strength design is presented in Example G.2. A built-up girder with a thin web and transverse stiffeners is presented in Example G.8.

G3. SINGLE ANGLES AND TEES

A single angle example is illustrated in Example G.3.

G4. RECTANGULAR HSS, BOX SECTIONS, AND OTHER SINGLY AND DOUBLY SYMMETRIC MEMBERS

The shear height for HSS, h, is taken as the clear distance between the flanges less the inside corner radius on each side. If the corner radii are unknown, h shall be taken as the corresponding outside dimension minus 3 times the design thickness. A rectangular HSS example is provided in Example G.4.

G5. ROUND HSS

For all round HSS of ordinary length listed in the AISC *Manual*, F_{cr} can be taken as 0.6F_y in AISC *Specification* Equation G5-1. A round HSS example is illustrated in Example G.5.

G6. WEAK AXIS SHEAR IN DOUBLY SYMMETRIC AND SINGLY SYMMETRIC SHAPES

For examples of weak axis shear, see Example G.6 and Example G.7.

G7. BEAMS AND GIRDERS WITH WEB OPENINGS

For a beam and girder with web openings example, see AISC Design Guide 2, *Design of Steel and Composite Beams with Web Openings* (Darwin, 1990).

EXAMPLE G.1A W-SHAPE IN STRONG AXIS SHEAR

Given:

Using AISC *Manual* tables, determine the available shear strength and adequacy of an ASTM A992 W24×62 with end shears of 48 kips from dead load and 145 kips from live load.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
$V_u = 1.2(48 \text{ kips}) + 1.6(145 \text{ kips})$ $= 290 \text{ kips}$	$V_a = 48 \text{ kips} + 145 \text{ kips}$ $= 193 \text{ kips}$

From AISC *Manual* Table 3-2, the available shear strength is:

LRFD	ASD
$\phi_v V_n = 306 \text{ kips} > 290 \text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega_v} = 204 \text{ kips} > 193 \text{ kips}$ **o.k.**

EXAMPLE G.1B W-SHAPE IN STRONG AXIS SHEAR

Given:

The available shear strength of the W-shape in Example G.1A was easily determined using tabulated values in the AISC *Manual*. This example demonstrates the calculation of the available strength by directly applying the provisions of the AISC *Specification*.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W24×62
$d = 23.7$ in.
$t_w = 0.430$ in.

Nominal Shear Strength

Except for very few sections, which are listed in the User Note, AISC *Specification* Section G2.1(a) is applicable to the I-shaped beams published in the AISC *Manual* for F_y = 50 ksi. The W-shape sections that do not meet the criteria of AISC *Specification* Section G2.1(a) are indicated with footnote "v" in Tables 1-1, 3-2 and 6-2.

$C_{v1} = 1.0$ (*Spec*. Eq. G2-2)

From AISC *Specification* Section G2.1, area of the web, A_w, is determined as follows:

$$\begin{aligned} A_w &= dt_w \\ &= (23.7\text{ in.})(0.430\text{ in.}) \\ &= 10.2\text{ in.}^2 \end{aligned}$$

From AISC *Specification* Section G2.1, the nominal shear strength is:

$$\begin{aligned} V_n &= 0.6F_y A_w C_{v1} \\ &= 0.6(50\text{ ksi})(10.2\text{ in.}^2)(1.0) \\ &= 306\text{ kips} \end{aligned}$$ (*Spec*. Eq. G2-1)

Available Shear Strength

From AISC *Specification* Section G2.1, the available shear strength is:

LRFD	ASD
$\phi_v = 1.00$ $\phi_v V_n = 1.00(306\text{ kips})$ $= 306\text{ kips}$	$\Omega_v = 1.50$ $\dfrac{V_n}{\Omega_v} = \dfrac{306\text{ kips}}{1.50}$ $= 204\text{ kips}$

EXAMPLE G.2A CHANNEL IN STRONG AXIS SHEAR

Given:

Using AISC *Manual* tables, verify the available shear strength and adequacy of an ASTM A36 C15×33.9 channel with end shears of 17.5 kips from dead load and 52.5 kips from live load.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
$V_u = 1.2(17.5 \text{ kips}) + 1.6(52.5 \text{ kips})$ $= 105 \text{ kips}$	$V_a = 17.5 \text{ kips} + 52.5 \text{ kips}$ $= 70.0 \text{ kips}$

From AISC *Manual* Table 3-8, the available shear strength is:

LRFD	ASD
$\phi_v V_n = 117 \text{ kips} > 105 \text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega_v} = 77.6 \text{ kips} > 70.0 \text{ kips}$ **o.k.**

EXAMPLE G.2B CHANNEL IN STRONG AXIS SHEAR

Given:

The available shear strength of the channel in Example G.2A was easily determined using tabulated values in the AISC *Manual*. This example demonstrates the calculation of the available strength by directly applying the provisions of the AISC *Specification*.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-5, the geometric properties are as follows:

C15×33.9
d = 15.0 in.
t_w = 0.400 in.

Nominal Shear Strength

All ASTM A36 channels listed in the AISC *Manual* have $h/t_w \le 1.10\sqrt{k_v E / F_y}$; therefore,

$C_{v1} = 1.0$ (*Spec*. Eq. G2-3)

From AISC *Specification* Section G2.1, the area of the web, A_w, is determined as follows:

$$
\begin{aligned}
A_w &= dt_w \\
&= (15.0 \text{ in.})(0.400 \text{ in.}) \\
&= 6.00 \text{ in.}^2
\end{aligned}
$$

From AISC *Specification* Section G2.1, the nominal shear strength is:

$$
\begin{aligned}
V_n &= 0.6F_y A_w C_{v1} \\
&= 0.6(36 \text{ ksi})(6.00 \text{ in.}^2)(1.0) \\
&= 130 \text{ kips}
\end{aligned}
$$
(*Spec*. Eq. G2-1)

Available Shear Strength

Because AISC *Specification* Section G2.1(a) does not apply for channels, the values of ϕ_v = 1.00 (LRFD) and Ω_v = 1.50 (ASD) may not be used. Instead ϕ_v = 0.90 (LRFD) and Ω_v = 1.67 (ASD) from AISC *Specification* Section G1(a) must be used.

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(130\text{ kips})$ $= 117\text{ kips}$	$\Omega_v = 1.67$ $\dfrac{V_n}{\Omega_v} = \dfrac{130\text{ kips}}{1.67}$ $= 77.8\text{ kips}$

EXAMPLE G.3 ANGLE IN SHEAR

Given:

Determine the available shear strength and adequacy of an ASTM A36 L5×3×¼ (long leg vertical) with end shears of 3.5 kips from dead load and 10.5 kips from live load.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-7, the geometric properties are as follows:

L5×3×¼
b = 5.00 in.
t = ¼ in.

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
$V_u = 1.2(3.5\text{ kips}) + 1.6(10.5\text{ kips})$ $= 21.0\text{ kips}$	$V_a = 3.5\text{ kips} + 10.5\text{ kips}$ $= 14.0\text{ kips}$

Nominal Shear Strength

Note: There are no tables in the AISC *Manual* for angles in shear, but the nominal shear strength can be calculated according to AISC *Specification* Section G3, as follows:

From AISC *Specification* Section G3:

$k_v = 1.2$

Determine C_{v2} from AISC *Specification* Section G2.2.

$$\frac{h}{t_w} = \frac{b}{t}$$
$$= \frac{5.00\text{ in.}}{¼\text{ in.}}$$
$$= 20.0$$

$$1.10\sqrt{\frac{k_v E}{F_y}} = 1.10\sqrt{\frac{1.2(29{,}000\text{ ksi})}{36\text{ ksi}}}$$
$$= 34.2 > 20.0; \text{ therefore, use AISC } \textit{Specification} \text{ Equation G2-9}$$

$C_{v2} = 1.0$ (*Spec*. Eq. G2-9)

From AISC *Specification* Section G3, the nominal shear strength is:

$$V_n = 0.6F_y btC_{v2} \quad \text{(Spec. Eq. G3-1)}$$
$$= 0.6(36 \text{ ksi})(5.00 \text{ in.})(\tfrac{1}{4} \text{ in.})(1.0)$$
$$= 27.0 \text{ kips}$$

Available Shear Strength

From AISC *Specification* Section G1, the available shear strength is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(27.0 \text{ kips})$ $= 24.3 \text{ kips} > 21.0 \text{ kips}$ **o.k.**	$\Omega_v = 1.67$ $\dfrac{V_n}{\Omega_v} = \dfrac{27.0 \text{ kips}}{1.67}$ $= 16.2 \text{ kips} > 14.0 \text{ kips}$ **o.k.**

EXAMPLE G.4 RECTANGULAR HSS IN SHEAR

Given:

Determine the available shear strength by directly applying the provisions of the AISC *Specification* for an ASTM A500 Grade C HSS6×4×⅜ (long leg vertical) beam with end shears of 11 kips from dead load and 33 kips from live load.

Note: There are tables in Part IV of this document that provide the shear strength of square and rectangular HSS shapes.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade C, rectangular
$F_y = 50$ ksi
$F_u = 62$ ksi

From AISC *Manual* Table 1-11, the geometric properties are as follows:

HSS6×4×⅜
$H = 6.00$ in.
$B = 4.00$ in.
$t = 0.349$ in.

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
$V_u = 1.2(11 \text{ kips}) + 1.6(33 \text{ kips})$ $= 66.0 \text{ kips}$	$V_a = 11 \text{ kips} + 33 \text{ kips}$ $= 44.0 \text{ kips}$

Nominal Shear Strength

The nominal shear strength can be determined from AISC *Specification* Section G4 as follows:

The web shear buckling strength coefficient, C_{v2}, is found using AISC *Specification* Section G2.2 with $h/t_w = h/t$ and $k_v = 5$.

From AISC *Specification* Section G4, if the exact radius is unknown, h shall be taken as the corresponding outside dimension minus three times the design thickness.

$$\begin{aligned} h &= H - 3t \\ &= 6.00 \text{ in.} - 3(0.349 \text{ in.}) \\ &= 4.95 \text{ in.} \end{aligned}$$

$$\begin{aligned} \frac{h}{t} &= \frac{4.95 \text{ in.}}{0.349 \text{ in.}} \\ &= 14.2 \end{aligned}$$

$$\begin{aligned} 1.10\sqrt{\frac{k_v E}{F_y}} &= 1.10\sqrt{\frac{5(29{,}000 \text{ ksi})}{50 \text{ ksi}}} \\ &= 59.2 > 14.2; \text{ therefore use AISC } \textit{Specification} \text{ Equation G2-9} \end{aligned}$$

$C_{v2} = 1.0$ (*Spec*. Eq. G2-9)

Note: Most standard HSS sections listed in the AISC *Manual* have $C_{v2} = 1.0$ at $F_y \le 50$ ksi.

Calculate A_w.

$$\begin{aligned} A_w &= 2ht \\ &= 2(4.95 \text{ in.})(0.349 \text{ in.}) \\ &= 3.46 \text{ in.}^2 \end{aligned}$$

Calculate V_n.

$$\begin{aligned} V_n &= 0.6F_y A_w C_{v2} \\ &= 0.6(50 \text{ ksi})(3.46 \text{ in.}^2)(1.0) \\ &= 104 \text{ kips} \end{aligned}$$ (*Spec*. Eq. G4-1)

Available Shear Strength

From AISC *Specification* Section G1, the available shear strength is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(104 \text{ kips})$ $= 93.6 \text{ kips} > 66.0 \text{ kips}$ **o.k.**	$\Omega_v = 1.67$ $\dfrac{V_n}{\Omega_v} = \dfrac{104 \text{ kips}}{1.67}$ $= 62.3 \text{ kips} > 44.0 \text{ kips}$ **o.k.**

EXAMPLE G.5 ROUND HSS IN SHEAR

Given:

Determine the available shear strength by directly applying the provisions of the AISC *Specification* for an ASTM A500 Grade C round HSS16.000×0.375 beam spanning 32 ft with end shears of 30 kips from uniform dead load and 90 kips from uniform live load.

Note: There are tables in Part IV of this document that provide the shear strength of round HSS shapes.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade C, round HSS
$F_y = 46$ ksi
$F_u = 62$ ksi

From AISC *Manual* Table 1-13, the geometric properties are as follows:

HSS16.000×0.375
$A = 17.2\text{ in.}^2$
$D/t = 45.8$

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
$V_u = 1.2(30\text{ kips}) + 1.6(90\text{ kips})$ $= 180\text{ kips}$	$V_a = 30\text{ kips} + 90\text{ kips}$ $= 120\text{ kips}$

Nominal Shear Strength

The nominal strength can be determined from AISC *Specification* Section G5, as follows:

Using AISC *Specification* Section G5, calculate F_{cr} as the larger of:

$$F_{cr} = \frac{1.60E}{\sqrt{\frac{L_v}{D}}\left(\frac{D}{t}\right)^{\frac{5}{4}}} \qquad (Spec.\text{ Eq. G5-2a})$$

and

$$F_{cr} = \frac{0.78E}{\left(\frac{D}{t}\right)^{\frac{3}{2}}}, \text{ but not to exceed } 0.6F_y \qquad (Spec.\text{ Eq. G5-2b})$$

where L_v is taken as the distance from maximum shear force to zero; in this example, half the span.

$$L_v = 0.5(32\text{ ft})(12\text{ in./ft}) = 192\text{ in.}$$

$$F_{cr} = \frac{1.60E}{\sqrt{\frac{L_v}{D}}\left(\frac{D}{t}\right)^{\frac{5}{4}}}$$ (*Spec*. Eq. G5-2a)

$$= \frac{1.60(29{,}000 \text{ ksi})}{\sqrt{\frac{192 \text{ in.}}{16.0 \text{ in.}}}(45.8)^{5/4}}$$

$$= 112 \text{ ksi}$$

$$F_{cr} = \frac{0.78E}{\left(\frac{D}{t}\right)^{\frac{3}{2}}}$$ (*Spec*. Eq. G5-2b)

$$= \frac{0.78(29{,}000 \text{ ksi})}{(45.8)^{3/2}}$$

$$= 73.0 \text{ ksi}$$

The maximum value of F_{cr} permitted is,

$$F_{cr} = 0.6F_y$$

$$= 0.6(46 \text{ ksi})$$

$$= 27.6 \text{ ksi}$$ **controls**

Note: AISC *Specification* Equations G5-2a and G5-2b will not normally control for the sections published in the AISC *Manual* except when high strength steel is used or the span is unusually long.

Calculate V_n using AISC *Specification* Section G5.

$$V_n = \frac{F_{cr}A_g}{2}$$ (*Spec*. Eq. G5-1)

$$= \frac{(27.6 \text{ ksi})(17.2 \text{ in.}^2)}{2}$$

$$= 237 \text{ kips}$$

Available Shear Strength

From AISC *Specification* Section G1, the available shear strength is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(237 \text{ kips})$ $= 213 \text{ kips} > 180 \text{ kips}$ **o.k.**	$\Omega_v = 1.67$ $\frac{V_n}{\Omega_v} = \frac{237 \text{ kips}}{1.67}$ $= 142 \text{ kips} > 120 \text{ kips}$ **o.k.**

EXAMPLE G.6 DOUBLY SYMMETRIC SHAPE IN WEAK AXIS SHEAR

Given:

Verify the available shear strength and adequacy of an ASTM A992 W21×48 beam with end shears of 20.0 kips from dead load and 60.0 kips from live load in the weak direction.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W21×48
$b_f = 8.14$ in.
$t_f = 0.430$ in.

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
$V_u = 1.2(20.0 \text{ kips}) + 1.6(60.0 \text{ kips})$ $= 120 \text{ kips}$	$V_a = 20.0 \text{ kips} + 60.0 \text{ kips}$ $= 80.0 \text{ kips}$

Nominal Shear Strength

From AISC *Specification* Section G6, for weak axis shear, use AISC *Specification* Equation G6-1.

Calculate C_{v2} using AISC *Specification* Section G2.2 with $h/t_w = b_f/2t_f$ and $k_v = 1.2$.

$$\begin{aligned}\frac{h}{t_w} &= \frac{b_f}{2t_f} \\ &= \frac{8.14 \text{ in.}}{2(0.430 \text{ in.})} \\ &= 9.47\end{aligned}$$

$$\begin{aligned}1.10\sqrt{\frac{k_v E}{F_y}} &= 1.10\sqrt{\frac{1.2(29{,}000 \text{ ksi})}{50 \text{ ksi}}} \\ &= 29.0 > 9.47\end{aligned}$$

Therefore, use AISC *Specification* Equation G2-9:

$$C_{v2} = 1.0$$

Note: From the User Note in AISC *Specification* Section G6, $C_{v2} = 1.0$ for all ASTM A6 W-, S-, M- and HP-shapes when $F_y \le 70$ ksi.

Calculate V_n. (Multiply the flange area by two to account for both shear resisting elements.)

$$V_n = 0.6F_y b_f t_f C_{v2}(2) \quad \text{(from } \textit{Spec}\text{. Eq. G6-1)}$$
$$= 0.6(50 \text{ ksi})(8.14 \text{ in.})(0.430 \text{ in.})(1.0)(2)$$
$$= 210 \text{ kips}$$

Available Shear Strength

From AISC *Specification* Section G1, the available shear strength is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(210 \text{ kips})$ $= 189 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\Omega_v = 1.67$ $\dfrac{V_n}{\Omega_v} = \dfrac{210 \text{ kips}}{1.67}$ $= 126 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

EXAMPLE G.7 SINGLY SYMMETRIC SHAPE IN WEAK AXIS SHEAR

Given:

Verify the available shear strength and adequacy of an ASTM A36 C9×20 channel with end shears of 5 kips from dead load and 15 kips from live load in the weak direction.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-5, the geometric properties are as follows:

C9×20
b_f = 2.65 in.
t_f = 0.413 in.

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
$V_u = 1.2(5\text{ kips}) + 1.6(15\text{ kips})$ $= 30.0\text{ kips}$	$V_u = 5\text{ kips} + 15\text{ kips}$ $= 20.0\text{ kips}$

Nominal Shear Strength

Note: There are no AISC *Manual* tables for weak-axis shear in channel sections, but the available strength can be determined from AISC *Specification* Section G6.

Calculate C_{v2} using AISC *Specification* Section G2.2 with $h/t_w = b_f/t_f$ and k_v = 1.2.

$$\frac{h}{t_w} = \frac{b_f}{t_f}$$
$$= \frac{2.65\text{ in.}}{0.413\text{ in.}}$$
$$= 6.42$$

$$1.10\sqrt{\frac{k_v E}{F_y}} = 1.10\sqrt{\frac{1.2(29{,}000\text{ ksi})}{36\text{ ksi}}}$$
$$= 34.2 > 6.42$$

Therefore, use AISC *Specification* Equation G2-9:

$$C_{v2} = 1.0$$

Calculate V_n. (Multiply the flange area by two to account for both shear resisting elements.)

$$V_n = 0.6F_y b_f t_f C_{v2}(2) \qquad \text{(from } \textit{Spec}\text{. Eq. G6-1)}$$
$$= 0.6(36 \text{ ksi})(2.65 \text{ in.})(0.413 \text{ in.})(1.0)(2)$$
$$= 47.3 \text{ kips}$$

Available Shear Strength

From AISC *Specification* Section G1, the available shear strength is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(47.3 \text{ kips})$ $= 42.6 \text{ kips} > 30.0 \text{ kips}$ **o.k.**	$\Omega_v = 1.67$ $\dfrac{V_n}{\Omega_v} = \dfrac{47.3 \text{ kips}}{1.67}$ $= 28.3 \text{ kips} > 20.0 \text{ kips}$ **o.k.**

EXAMPLE G.8A BUILT-UP GIRDER WITH TRANSVERSE STIFFENERS

Given:

Determine the available shear strength of a built-up I-shaped girder for the span and loading as shown in Figure G.8A. The girder is ASTM A36 material and 36 in. deep with 16-in. × 1½-in. flanges and a 5⁄16-in.-thick web. The compression flange is continuously braced. Determine if the member has sufficient available shear strength to support the end shear, without and with tension field action. Use transverse stiffeners, as required.

Note: This built-up girder was purposely selected with a thin web in order to illustrate the design of transverse stiffeners. A more conventionally proportioned plate girder may have at least a ½-in.-thick web and slightly smaller flanges.

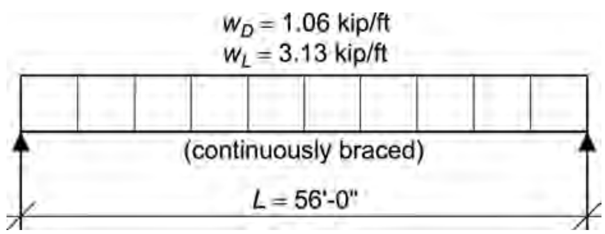


Fig. G.8A. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

The geometric properties are as follows:

Built-up girder
t_w = 5⁄16 in.
d = 36.0 in.
$b_{ft} = b_{fc}$ = 16.0 in.
t_f = 1½ in.
h = 33.0 in.

From Chapter 2 of ASCE/SEI 7, the required shear strength at the support is:

LRFD	ASD
$w_u = 1.2(1.06 \text{ kip/ft}) + 1.6(3.13 \text{ kip/ft})$ $= 6.28 \text{ kip/ft}$ $V_u = \dfrac{w_u L}{2}$ $= \dfrac{(6.28 \text{ kip/ft})(56 \text{ ft})}{2}$ $= 176 \text{ kips}$	$w_a = 1.06 \text{ kip/ft} + 3.13 \text{ kip/ft}$ $= 4.19 \text{ kip/ft}$ $V_a = \dfrac{w_a L}{2}$ $= \dfrac{(4.19 \text{ kip/ft})(56 \text{ ft})}{2}$ $= 117 \text{ kips}$

Stiffener Requirement Check

From AISC *Specification* Section G2.1:

$$A_w = dt_w$$
$$= (36.0 \text{ in.})(\tfrac{5}{16} \text{ in.})$$
$$= 11.3 \text{ in.}^2$$

For webs without transverse stiffeners, k_v = 5.34 from AISC *Specification* Section G2.1(b)(2)(i).

$$\frac{h}{t_w} = \frac{33.0 \text{ in.}}{\tfrac{5}{16} \text{ in.}}$$
$$= 106$$

$$1.10\sqrt{\frac{k_v E}{F_y}} = 1.10\sqrt{\frac{(5.34)(29{,}000 \text{ ksi})}{36 \text{ ksi}}}$$
$$= 72.1 < 106$$

Therefore, use AISC *Specification* Equation G2-4:

$$C_{v1} = \frac{1.10\sqrt{k_v E/F_y}}{h/t_w} \qquad \text{(Spec. Eq. G2-4)}$$
$$= \frac{72.1}{106}$$
$$= 0.680$$

Calculate V_n.

$$V_n = 0.6 F_y A_w C_{v1} \qquad \text{(Spec. Eq. G2-1)}$$
$$= 0.6(36 \text{ ksi})(11.3 \text{ in.}^2)(0.680)$$
$$= 166 \text{ kips}$$

From AISC *Specification* Section G1, the available shear strength without stiffeners is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(166 \text{ kips})$ $= 149 \text{ kips} < 176 \text{ kips}$ **n.g.** **Therefore, stiffeners are required.**	$\Omega_v = 1.67$ $\frac{V_n}{\Omega_v} = \frac{166 \text{ kips}}{1.67}$ $= 99.4 \text{ kips} < 117 \text{ kips}$ **n.g.** **Therefore, stiffeners are required.**

AISC *Manual* Tables 3-16a and 3-16b can be used to select the stiffener spacing needed to develop the required stress in the web.

Stiffener Spacing for End Panel

Tension field action is not permitted for end panels, therefore use AISC *Manual* Table 3-16a.

LRFD	ASD
Use $V_u = \phi_v V_n$ to determine the required stress in the web by dividing by the web area. $\dfrac{\phi_v V_n}{A_w} = \dfrac{V_u}{A_w}$ $= \dfrac{176 \text{ kips}}{11.3 \text{ in.}^2}$ $= 15.6 \text{ ksi}$	Use $V_a = V_n / \Omega_v$ to determine the required stress in the web by dividing by the web area. $\dfrac{V_n}{\Omega_v A_w} = \dfrac{V_a}{A_w}$ $= \dfrac{117 \text{ kips}}{11.3 \text{ in.}^2}$ $= 10.4 \text{ ksi}$

Use Table 3-16a from the AISC *Manual* to select the required stiffener ratio a/h based on the h/t_w ratio of the girder and the required stress. Interpolate and follow an available stress curve, $\phi_v V_n/A_w$= 15.6 ksi for LRFD, $V_n/\Omega_v A_w$ = 10.4 ksi for ASD, until it intersects the horizontal line for an h/t_w value of 106. Project down from this intersection and approximate the value for a/h as 1.40 from the axis across the bottom. Because h = 33.0 in., stiffeners are required at (1.40)(33.0 in.) = 46.2 in. maximum. Conservatively, use a 42-in. spacing.

Stiffener Spacing for the Second Panel

From AISC *Specification* Section G2.2, tension field action is allowed because the second panel is an interior web panel. However, a web panel aspect ratio, a/h, must not exceed three. The required shear strength at the start of the second panel, 42 in. from the end, is:

LRFD	ASD
$V_u = 176 \text{ kips} - (6.28 \text{ kip/ft})(42.0 \text{ in.})(1 \text{ ft/12 in.})$ $= 154 \text{ kips}$	$V_a = 117 \text{ kips} - (4.19 \text{ kip/ft})(42.0 \text{ in.})(1 \text{ ft/12 in.})$ $= 102 \text{ kips}$

From AISC *Specification* Section G1, the available shear strength without stiffeners is:

LRFD	ASD
$\phi_v = 0.90$ From previous calculations, $\phi_v V_n = 149 \text{ kips} < 154 \text{ kips}$ **n.g.** **Therefore, additional stiffeners are required.** Use $V_u = \phi_v V_n$ to determine the required stress in the web by dividing by the web area. $\dfrac{\phi_v V_n}{A_w} = \dfrac{V_u}{A_w}$ $= \dfrac{154 \text{ kips}}{11.3 \text{ in.}^2}$ $= 13.6 \text{ ksi}$	$\Omega_v = 1.67$ From previous calculations, $\dfrac{V_n}{\Omega_v} = 99.4 \text{ kips} < 102 \text{ kips}$ **n.g.** **Therefore, additional stiffeners are required.** Use $V_a = V_n / \Omega_v$ to determine the required stress in the web by dividing by the web area. $\dfrac{V_n}{\Omega_v A_w} = \dfrac{V_a}{A_w}$ $= \dfrac{102 \text{ kips}}{11.3 \text{ in.}^2}$ $= 9.03 \text{ ksi}$

Table 3-16b from the AISC *Manual*, including tension field action, may be used to select the required stiffener ratio a/h based on the h/t_w ratio of the girder and the required stress, provided that the limitations of $2A_w / (A_{fc} + A_{ft}) \le 2.5$, $h/b_{fc} \le 6.0$, and $h/b_{ft} \le 6.0$ are met.

$$\frac{2A_w}{A_{fc}+A_{ft}} = \frac{2\left(11.3\text{ in.}^2\right)}{(16.0\text{ in.})(1\tfrac{1}{2}\text{ in.})+(16.0\text{ in.})(1\tfrac{1}{2}\text{ in.})}$$
$$= 0.471 < 2.5 \quad \textbf{o.k.}$$

$$\frac{h}{b_{fc}} = \frac{h}{b_{ft}}$$
$$= \frac{33.0\text{ in.}}{16.0\text{ in.}}$$
$$= 2.06 < 6.0 \quad \textbf{o.k.}$$

The limitations have been met. Table 3-16b may be used.

Interpolate and follow an available stress curve, $\phi_v V_n/A_w$ = 13.6 ksi for LRFD, $V_n/\Omega_v A_w$ = 9.03 ksi for ASD, until it intersects the horizontal line for an h/t_w value of 106. Because the available stress does not intersect the h/t_w value of 106, the maximum value of 3.0 for a/h may be used. Because h = 33.0 in., an additional stiffener is required at (3.0)(33.0 in.) = 99.0 in. maximum from the previous one. Conservatively, 90.0 in. spacing may be used.

Stiffener Spacing for the Third Panel

From AISC *Specification* Section G2.2, tension field action is allowed because the next panel is not an end panel.

The required shear strength at the start of the third panel, 132 in. from the end is:

LRFD	ASD
$V_u = 176\text{ kips} - (6.28\text{ kip/ft})(132\text{ in.})(1\text{ ft/12 in.})$ $= 107\text{ kips}$	$V_a = 117\text{ kips} - (4.19\text{ kip/ft})(132\text{ in.})(1\text{ ft/12 in.})$ $= 70.9\text{ kips}$

From AISC *Specification* Section G1, the available shear strength without stiffeners is:

LRFD	ASD
$\phi_v = 0.90$ From previous calculations, $\phi_v V_n = 149\text{ kips} > 107\text{ kips}$ **o.k.** **Therefore, additional stiffeners are not required.**	$\Omega_v = 1.67$ From previous calculations, $\frac{V_n}{\Omega_v} = 99.4\text{ kips} > 70.9\text{ kips}$ **o.k.** **Therefore, additional stiffeners are not required.**

The six tables in the AISC *Manual*, 3-16a, 3-16b, 3-16c, 3-17a, 3-17b and 3-17c, are useful because they permit a direct solution for the required stiffener spacing. Alternatively, you can select a stiffener spacing and check the resulting strength, although this process is likely to be iterative. In Example G.8B, the stiffener spacings used are taken from this example.

EXAMPLE G.8B BUILT-UP GIRDER WITH TRANSVERSE STIFFENERS

Given:

Verify the available shear strength and adequacy of the stiffener spacings from Example G.8A, which were easily determined from the tabulated values of the AISC *Manual*, by directly applying the provisions of the AISC *Specification*. Stiffeners are spaced at 42 in. in the first panel and 90 in. in the second panel.

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From Example G.8A, the required shear strength at the support is:

LRFD	ASD
$V_u = 176$ kips	$V_a = 117$ kips

Shear Strength of End Panel

The web plate bucking coefficient, k_v, is determined from AISC *Specification* Equation G2-5.

$$\frac{h}{t_w} = \frac{33.0\text{ in.}}{5/16\text{ in.}} = 106$$

$$k_v = 5 + \frac{5}{(a/h)^2} \qquad \text{(Spec. Eq. G2-5)}$$

$$= 5 + \frac{5}{(42.0\text{ in.}/33.0\text{ in.})^2} = 8.09$$

$$1.10\sqrt{\frac{k_v E}{F_y}} = 1.10\sqrt{\frac{8.09(29{,}000\text{ ksi})}{36\text{ ksi}}} = 88.8 < 106$$

Therefore, use AISC *Specification* Equation G2-4.

$$C_{v1} = \frac{1.10\sqrt{k_v E/F_y}}{h/t_w} \qquad \text{(Spec. Eq. G2-4)}$$

$$= \frac{88.8}{106} = 0.838$$

Calculate V_n.

From Example G.8A:

A_w = 11.3 in.^{2}

$$V_n = 0.6F_y A_w C_{v1} \quad \text{(}Spec\text{. Eq. G2-1)}$$
$$= 0.6(36 \text{ ksi})(11.3 \text{ in.}^2)(0.838)$$
$$= 205 \text{ kips}$$

From AISC *Specification* Section G1, the available shear strength for the end panel is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(205 \text{ kips})$ $= 185 \text{ kips} > 176 \text{ kips}$ **o.k.**	$\Omega_v = 1.67$ $\dfrac{V_n}{\Omega_v} = \dfrac{205 \text{ kips}}{1.67}$ $= 123 \text{ kips} > 117 \text{ kips}$ **o.k.**

Shear Strength of the Second Panel

From Example G.8A, the required shear strength at the start of the second panel is:

LRFD	ASD
$V_u = 154$ kips	$V_a = 102$ kips

The web plate bucking coefficient, k_v, is determined from AISC *Specification* Equation G2-5.

$$k_v = 5 + \frac{5}{(a/h)^2} \quad \text{(}Spec\text{. Eq. G2-5)}$$
$$= 5 + \frac{5}{(90.0 \text{ in.}/33.0 \text{ in.})^2}$$
$$= 5.67$$

$$1.37\sqrt{\frac{k_v E}{F_y}} = 1.37\sqrt{\frac{5.67(29{,}000 \text{ ksi})}{36 \text{ ksi}}}$$
$$= 92.6 < 106$$

Therefore, use AISC *Specification* Equation G2-11 to calculate C_{v2}.

$$C_{v2} = \frac{1.51 k_v E}{(h/t_w)^2 F_y} \quad \text{(}Spec\text{. Eq. G2-11)}$$
$$= \frac{1.51(5.67)(29{,}000 \text{ ksi})}{(106)^2(36 \text{ ksi})}$$
$$= 0.614$$

The limitations of AISC *Specification* Section G2.2(b)(1) are checked as follows:

$$\frac{2A_w}{A_{fc}+A_{ft}} = \frac{2\left(11.3\text{ in.}^2\right)}{(16.0\text{ in.})(1\tfrac{1}{2}\text{ in.})+(16.0\text{ in.})(1\tfrac{1}{2}\text{ in.})}$$
$$= 0.471 < 2.5$$

$$\frac{h}{b_{fc}} = \frac{h}{b_{ft}}$$
$$= \frac{33.0\text{ in.}}{16.0\text{ in.}}$$
$$= 2.06 < 6.0$$

Because $2A_w / (A_{fc} + A_{ft}) \le 2.5$, $h/b_{fc} \le 6.0$, and $h/b_{ft} \le 6.0$, use AISC *Specification* Equation G2-7 with a = 90.0 in..

$$V_n = 0.6F_yA_w\left[C_{v2}+\frac{1-C_{v2}}{1.15\sqrt{1+(a/h)^2}}\right] \quad \text{(\textit{Spec}. Eq. G2-7)}$$
$$= 0.6(36\text{ ksi})\left(11.3\text{ in.}^2\right)\left[0.614+\frac{1-0.614}{1.15\sqrt{1+\left(\frac{90.0\text{ in.}}{33.0\text{ in.}}\right)^2}}\right]$$
$$= 178\text{ kips}$$

From AISC *Specification* Section G1, the available shear strength for the second panel is:

LRFD	ASD
$\phi_v = 0.90$	$\Omega_v = 1.67$
$\phi_vV_n = 0.90(178\text{ kips})$ $= 160\text{ kips} > 154\text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega_v} = \dfrac{178\text{ kips}}{1.67}$ $= 107\text{ kips} > 102\text{ kips}$ **o.k.**

CHAPTER G DESIGN EXAMPLE REFERENCES

Darwin, D. (1990), *Steel and Composite Beams with Web Openings,* Design Guide 2, AISC, Chicago, IL.

Chapter H
Design of Members for Combined Forces and Torsion

For all interaction equations in AISC *Specification* Chapter H, the required forces and moments must include second-order effects, as required by Chapter C of the AISC *Specification*. ASD users of the 1989 AISC *Specification* are accustomed to using an interaction equation that includes a partial second-order amplification. Second-order effects are now addressed in the analysis and are not included in these interaction equations.

EXAMPLE H.1A W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING ABOUT BOTH AXES (BRACED FRAME)

Given:

Using Table IV-5 (located in this document), determine if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes P-δ effects. The unbraced length is 14 ft and the member has pinned ends.

LRFD	ASD
$P_u = 400$ kips $M_{ux} = 250$ kip-ft $M_{uy} = 80.0$ kip-ft	$P_a = 267$ kips $M_{ax} = 167$ kip-ft $M_{ay} = 53.3$ kip-ft

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

The effective length of the member is:

$$\begin{aligned} L_{cx} &= L_{cy} \\ &= KL \\ &= 1.0(14\text{ ft}) \\ &= 14.0\text{ ft} \end{aligned}$$

For $L_c = 14$ ft, the combined strength parameters from Table IV-5 are:

LRFD	ASD
$p = \dfrac{0.887}{10^3\text{ kips}}$	$p = \dfrac{1.33}{10^3\text{ kips}}$
$b_x = \dfrac{1.38}{10^3\text{ kip-ft}}$	$b_x = \dfrac{2.08}{10^3\text{ kip-ft}}$
$b_y = \dfrac{2.85}{10^3\text{ kip-ft}}$	$b_y = \dfrac{4.29}{10^3\text{ kip-ft}}$
Check P_r/P_c limit for AISC *Specification* Equation H1-1a.	Check P_r/P_c limit for AISC *Specification* Equation H1-1a.
$\dfrac{P_u}{\phi_c P_n} = pP_u = \left(\dfrac{0.887}{10^3\text{ kips}}\right)(400\text{ kips}) = 0.355$	$\dfrac{P_a}{P_n/\Omega_c} = pP_a = \left(\dfrac{1.33}{10^3\text{ kips}}\right)(267\text{ kips}) = 0.355$

LRFD	ASD
Because $pP_u \geq 0.2$, $pP_u + b_x M_{ux} + b_y M_{uy} \leq 1.0$ (from Part IV, Eq. IV-8) $= 0.355 + \left(\dfrac{1.38}{10^3 \text{ kip-ft}}\right)(250 \text{ kip-ft})$ $+ \left(\dfrac{2.85}{10^3 \text{ kip-ft}}\right)(80.0 \text{ kip-ft}) \leq 1.0$ $= 0.928 < 1.0$ **o.k.**	Because $pP_a \geq 0.2$, $pP_a + b_x M_{ax} + b_y M_{ay} \leq 1.0$ (from Part IV, Eq. IV-8) $= 0.355 + \left(\dfrac{2.08}{10^3 \text{ kip-ft}}\right)(167 \text{ kip-ft})$ $+ \left(\dfrac{4.29}{10^3 \text{ kip-ft}}\right)(53.3 \text{ kip-ft}) \leq 1.0$ $= 0.931 < 1.0$ **o.k.**

Table IV-5 simplifies the calculation of AISC *Specification* Equations H1-1a and H1-1b. A direct application of these equations is shown in Example H.1B.

EXAMPLE H.1B W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING MOMENT ABOUT BOTH AXES (BRACED FRAME)

Given:

Using AISC *Manual* tables to determine the available compressive and flexural strengths, determine if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes P-δ effects. The unbraced length is 14 ft and the member has pinned ends.

LRFD	ASD
$P_u = 400$ kips $M_{ux} = 250$ kip-ft $M_{uy} = 80$ kip-ft	$P_a = 267$ kips $M_{ax} = 167$ kip-ft $M_{ay} = 53.3$ kip-ft

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

The effective length of the member is:

$$\begin{aligned} L_{cx} &= L_{cy} \\ &= KL \\ &= 1.0(14 \text{ ft}) \\ &= 14.0 \text{ ft} \end{aligned}$$

For $L_c = 14.0$ ft, the available axial and flexural strengths from AISC *Manual* Table 6-2 are:

LRFD	ASD
$P_c = \phi_c P_n$ $= 1{,}130$ kips	$P_c = \dfrac{P_n}{\Omega_c}$ $= 750$ kips
$M_{cx} = \phi_b M_{nx}$ $= 642$ kip-ft	$M_{cx} = \dfrac{M_{nx}}{\Omega_b}$ $= 427$ kip-ft
$M_{cy} = \phi_b M_{ny}$ $= 311$ kip-ft	$M_{cy} = \dfrac{M_{ny}}{\Omega_b}$ $= 207$ kip-ft
$\dfrac{P_u}{\phi_c P_n} = \dfrac{400 \text{ kips}}{1{,}130 \text{ kips}}$ $= 0.354$	$\dfrac{P_a}{P_n / \Omega_c} = \dfrac{267 \text{ kips}}{750 \text{ kips}}$ $= 0.356$

LRFD	ASD
Because $\dfrac{P_u}{\phi_c P_n} \geq 0.2,$	Because $\dfrac{P_a}{P_n / \Omega_c} \geq 0.2,$
$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$ (*Spec*. Eq. H1-1a)	$\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$ (*Spec*. Eq. H1-1a)
$= \dfrac{400 \text{ kips}}{1{,}130 \text{ kips}} + \dfrac{8}{9}\left(\dfrac{250 \text{ kip-ft}}{642 \text{ kip-ft}} + \dfrac{80.0 \text{ kip-ft}}{311 \text{ kip-ft}}\right) \leq 1.0$	$= \dfrac{267 \text{ kips}}{750 \text{ kips}} + \dfrac{8}{9}\left(\dfrac{167 \text{ kip-ft}}{427 \text{ kip-ft}} + \dfrac{53.3 \text{ kip-ft}}{207 \text{ kip-ft}}\right)$
$= 0.928 < 1.0$ **o.k.**	$= 0.932 < 1.0$ **o.k.**

EXAMPLE H.2 W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING MOMENT ABOUT BOTH AXES (BY AISC *SPECIFICATION* SECTION H2)

Given:

Using AISC *Specification* Section H2, determine if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes P-δ effects. The unbraced length is 14 ft and the member has pinned ends. This example is included primarily to illustrate the use of AISC *Specification* Section H2.

LRFD	ASD
P_u = 360 kips M_{ux} = 250 kip-ft M_{uy} = 80 kip-ft	P_a = 240 kips M_{ax} = 167 kip-ft M_{ay} = 53.3 kip-ft

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W14×99
A = 29.1 in.^{2}
S_x = 157 in.^{3}
S_y = 55.2 in.^{3}

The required flexural and axial stresses are:

LRFD	ASD
$f_{ra} = \frac{P_u}{A} = \frac{360\text{ kips}}{29.1\text{ in.}^2} = 12.4\text{ ksi}$	$f_{ra} = \frac{P_a}{A} = \frac{240\text{ kips}}{29.1\text{ in.}^2} = 8.25\text{ ksi}$
$f_{rbx} = \frac{M_{ux}}{S_x} = \frac{(250\text{ kip-ft})(12\text{ in./ft})}{157\text{ in.}^3} = 19.1\text{ ksi}$	$f_{rbx} = \frac{M_{ax}}{S_x} = \frac{(167\text{ kip-ft})(12\text{ in./ft})}{157\text{ in.}^3} = 12.8\text{ ksi}$
$f_{rby} = \frac{M_{uy}}{S_y} = \frac{(80\text{ kip-ft})(12\text{ in./ft})}{55.2\text{ in.}^3} = 17.4\text{ ksi}$	$f_{rby} = \frac{M_{ay}}{S_y} = \frac{(53.3\text{ kip-ft})(12\text{ in./ft})}{55.2\text{ in.}^3} = 11.6\text{ ksi}$

The effective length of the member is:

$$\begin{aligned} L_{cx} &= L_{cy} \\ &= KL \\ &= 1.0(14 \text{ ft}) \\ &= 14.0 \text{ ft} \end{aligned}$$

For L_c = 14.0 ft, calculate the available axial and flexural stresses using the available strengths from AISC *Manual* Table 6-2.

LRFD	ASD
$F_{ca} = \phi_c F_{cr}$ $= \dfrac{\phi_c P_n}{A}$ $= \dfrac{1{,}130 \text{ kips}}{29.1 \text{ in.}^2}$ $= 38.8 \text{ ksi}$	$F_{ca} = \dfrac{F_{cr}}{\Omega_c}$ $= \dfrac{P_n}{\Omega_c A}$ $= \dfrac{750 \text{ kips}}{29.1 \text{ in.}^2}$ $= 25.8 \text{ ksi}$
$F_{cbx} = \dfrac{\phi_b M_{nx}}{S_x}$ $= \dfrac{(642 \text{ kip-ft})(12 \text{ in./ft})}{157 \text{ in.}^3}$ $= 49.1 \text{ ksi}$	$F_{cbx} = \dfrac{M_{nx}}{\Omega_b S_x}$ $= \dfrac{(427 \text{ kip-ft})(12 \text{ in./ft})}{157 \text{ in.}^3}$ $= 32.6 \text{ ksi}$
$F_{cby} = \dfrac{\phi_b M_{ny}}{S_y}$ $= \dfrac{(311 \text{ kip-ft})(12 \text{ in./ft})}{55.2 \text{ in.}^3}$ $= 67.6 \text{ ksi}$	$F_{cby} = \dfrac{M_{ny}}{\Omega_b S_y}$ $= \dfrac{(207 \text{ kip-ft})(12 \text{ in./ft})}{55.2 \text{ in.}^3}$ $= 45.0 \text{ ksi}$

As shown in the LRFD calculation of F_{cby} in the preceding text, the available flexural stresses can exceed the yield stress in cases where the available strength is governed by yielding and the yielding strength is calculated using the plastic section modulus.

Combined Stress Ratio

From AISC *Specification* Section H2, check the combined stress ratios as follows:

LRFD	ASD
$\left\lvert \dfrac{f_{ra}}{F_{ca}} + \dfrac{f_{rbx}}{F_{cbx}} + \dfrac{f_{rby}}{F_{cby}} \right\rvert \leq 1.0$ (from *Spec*. Eq. H2-1)	$\left\lvert \dfrac{f_{ra}}{F_{ca}} + \dfrac{f_{rbx}}{F_{cbx}} + \dfrac{f_{rby}}{F_{cby}} \right\rvert \leq 1.0$ (from *Spec*. Eq. H2-1)
$\left\lvert \dfrac{12.4 \text{ ksi}}{38.8 \text{ ksi}} + \dfrac{19.1 \text{ ksi}}{49.1 \text{ ksi}} + \dfrac{17.4 \text{ ksi}}{67.6 \text{ ksi}} \right\rvert = 0.966 < 1.0$ **o.k.**	$\left\lvert \dfrac{8.25 \text{ ksi}}{25.8 \text{ ksi}} + \dfrac{12.8 \text{ ksi}}{32.6 \text{ ksi}} + \dfrac{11.6 \text{ ksi}}{45.0 \text{ ksi}} \right\rvert = 0.970 < 1.0$ **o.k.**

A comparison of these results with those from Example H.1B shows that AISC *Specification* Equation H1-1a will produce less conservative results than AISC *Specification* Equation H2-1 when its use is permitted.

Note: This check is made at a point on the cross section (extreme fiber, in this example). The designer must therefore determine which point on the cross section is critical, or check multiple points if the critical point cannot be readily determined.

EXAMPLE H.3 W-SHAPE SUBJECT TO COMBINED AXIAL TENSION AND FLEXURE

Given:

Select an ASTM A992 W-shape with a 14-in.-nominal-depth to carry forces of 29 kips from dead load and 87 kips from live load in axial tension, as well as the following moments due to uniformly distributed loads:

$$M_{xD} = 32 \text{ kip-ft}$$
$$M_{xL} = 96 \text{ kip-ft}$$

$$M_{yD} = 11.3 \text{ kip-ft}$$
$$M_{yL} = 33.8 \text{ kip-ft}$$

The unbraced length is 30 ft and the ends are pinned. Assume the connections are made with no holes.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From ASCE/SEI 7, Chapter 2, the required strengths are:

LRFD	ASD
$P_u = 1.2(29 \text{ kips}) + 1.6(87 \text{ kips})$ $= 174 \text{ kips}$	$P_a = 29 \text{ kips} + 87 \text{ kips}$ $= 116 \text{ kips}$
$M_{ux} = 1.2(32 \text{ kip-ft}) + 1.6(96 \text{ kip-ft})$ $= 192 \text{ kip-ft}$	$M_{ax} = 32 \text{ kip-ft} + 96 \text{ kip-ft}$ $= 128 \text{ kip-ft}$
$M_{uy} = 1.2(11.3 \text{ kip-ft}) + 1.6(33.8 \text{ kip-ft})$ $= 67.6 \text{ kip-ft}$	$M_{ay} = 11.3 \text{ kip-ft} + 33.8 \text{ kip-ft}$ $= 45.1 \text{ kip-ft}$

Try a W14×82.

From AISC *Manual* Tables 1-1 and 3-2, the properties are as follows:

W14×82
A_g = 24.0 in.^{2}
S_x = 123 in.^{3}
Z_x = 139 in.^{3}
S_y = 29.3 in.^{3}
Z_y = 44.8 in.^{3}
I_y = 148 in.^{4}
L_p = 8.76 ft
L_r = 33.2 ft

Nominal Tensile Strength

From AISC *Specification* Section D2(a), the nominal tensile strength due to tensile yielding in the gross section is:

$$P_n = F_y A_g \quad \text{(Spec. Eq. D2-1)}$$
$$= (50 \text{ ksi})(24.0 \text{ in.}^2)$$
$$= 1{,}200 \text{ kips}$$

Note that for a member with holes, the rupture strength of the member would also have to be computed using AISC *Specification* Equation D2-2.

Nominal Flexural Strength for Bending About the Major Axis

Yielding

From AISC *Specification* Section F2.1, the nominal flexural strength due to yielding (plastic moment) is:

$$M_{nx} = M_p = F_y Z_x \quad \text{(Spec. Eq. F2-1)}$$
$$= (50 \text{ ksi})(139 \text{ in.}^3)$$
$$= 6{,}950 \text{ kip-in.}$$

Lateral-Torsional Buckling

From AISC *Specification* Section F2.2, the nominal flexural strength due to lateral-torsional buckling is determined as follows:

Because $L_p < L_b \le L_r$, i.e., 8.76 ft < 30 ft < 33.2 ft, AISC *Specification* Equation F2-2 applies.

Lateral-Torsional Buckling Modification Factor, C_b

From AISC *Manual* Table 3-1, C_b = 1.14, without considering the beneficial effects of the tension force. However, per AISC *Specification* Section H1.2, C_b may be modified because the column is in axial tension concurrently with flexure.

$$P_{ey} = \frac{\pi^2 E I_y}{L_b^2}$$
$$= \frac{\pi^2 (29{,}000 \text{ ksi})(148 \text{ in.}^4)}{[(30 \text{ ft})(12.0 \text{ in./ft})]^2}$$
$$= 327 \text{ kips}$$

LRFD	ASD
$\sqrt{1+\frac{\alpha P_u}{P_{ey}}} = \sqrt{1+\frac{1.0(174 \text{ kips})}{327 \text{ kips}}}$ $= 1.24$	$\sqrt{1+\frac{\alpha P_a}{P_{ey}}} = \sqrt{1+\frac{1.6(116 \text{ kips})}{327 \text{ kips}}}$ $= 1.25$

$$C_b = 1.24(1.14)$$
$$= 1.41$$

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad (\textit{Spec}.\text{ Eq. F2-2})$$

$$= 1.41\left\{6{,}950 \text{ kip-in.} - \left[6{,}950 \text{ kip-in.} - 0.7(50 \text{ ksi})(123 \text{ in.}^3)\right]\left(\frac{30 \text{ ft} - 8.76 \text{ ft}}{33.2 \text{ ft} - 8.76 \text{ ft}}\right)\right\} \le 6{,}950 \text{ kip-in.}$$

$$= 6{,}560 \text{ kip-in. or } 547 \text{ kip-ft} \quad \textbf{controls}$$

Local Buckling

Per AISC *Manual* Table 1-1, the cross section is compact at F_y = 50 ksi; therefore, the local buckling limit state does not apply.

Nominal Flexural Strength for Bending About the Minor Axis and the Interaction of Flexure and Tension

Because a W14×82 has compact flanges, only the limit state of yielding applies for bending about the minor axis.

$$M_{ny} = M_p = F_yZ_y \le 1.6F_yS_y \qquad (\textit{Spec}.\text{ Eq. F6-1})$$

$$= (50 \text{ ksi})(44.8 \text{ in.}^3) \le 1.6(50 \text{ ksi})(29.3 \text{ in.}^3)$$

$$= 2{,}240 \text{ kip-in.} < 2{,}340 \text{ kip-in.}$$

$$= 2{,}240 \text{ kip-in. or } 187 \text{ kip-ft}$$

Available Strength

From AISC *Specification* Sections D2 and F1, the available strengths are:

LRFD	ASD
$\phi_b = \phi_t = 0.90$	$\Omega_b = \Omega_t = 1.67$
$P_c = \phi_t P_n$ $= 0.90(1{,}200 \text{ kips})$ $= 1{,}080 \text{ kips}$	$P_c = \dfrac{P_n}{\Omega_t}$ $= \dfrac{1{,}200 \text{ kips}}{1.67}$ $= 719 \text{ kips}$
$M_{cx} = \phi_b M_{nx}$ $= 0.90(547 \text{ kip-ft})$ $= 492 \text{ kip-ft}$	$M_{cx} = \dfrac{M_{nx}}{\Omega_b}$ $= \dfrac{547 \text{ kip-ft}}{1.67}$ $= 328 \text{ kip-ft}$
$M_{cy} = \phi_b M_{ny}$ $= 0.90(187 \text{ kip-ft})$ $= 168 \text{ kip-ft}$	$M_{cy} = \dfrac{M_{ny}}{\Omega_b}$ $= \dfrac{187 \text{ kip-ft}}{1.67}$ $= 112 \text{ kip-ft}$

Interaction of Tension and Flexure

Check limit for AISC *Specification* Equation H1-1a.

LRFD	ASD
$\dfrac{P_r}{P_c} = \dfrac{P_u}{\phi_t P_n}$ $= \dfrac{174 \text{ kips}}{1{,}080 \text{ kips}}$ $= 0.161 < 0.2$ Because $\dfrac{P_r}{P_c} < 0.2,$ $\dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \le 1.0$ (*Spec*. Eq. H1-1b) $= \dfrac{174 \text{ kips}}{2(1{,}080 \text{ kips})} + \dfrac{192 \text{ kip-ft}}{492 \text{ kip-ft}} + \dfrac{67.6 \text{ kip-ft}}{168 \text{ kip-ft}} \le 1.0$ $= 0.873 < 1.0$ **o.k.**	$\dfrac{P_r}{P_c} = \dfrac{P_a}{P_n / \Omega_t}$ $= \dfrac{116 \text{ kips}}{719 \text{ kips}}$ $= 0.161 < 0.2$ Because $\dfrac{P_r}{P_c} < 0.2,$ $\dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \le 1.0$ (*Spec*. Eq. H1-1b) $= \dfrac{116 \text{ kips}}{2(719 \text{ kips})} + \dfrac{128 \text{ kip-ft}}{328 \text{ kip-ft}} + \dfrac{45.1 \text{ kip-ft}}{112 \text{ kip-ft}} \le 1.0$ $= 0.874 < 1.0$ **o.k.**

EXAMPLE H.4 W-SHAPE SUBJECT TO COMBINED AXIAL COMPRESSION AND FLEXURE

Given:

Select an ASTM A992 W-shape with a 10-in.-nominal-depth to carry axial compression forces of 5 kips from dead load and 15 kips from live load. The unbraced length is 14 ft and the ends are pinned. The member also has the following required moment strengths due to uniformly distributed loads, not including second-order effects:

$M_{xD} = 15$ kip-ft
$M_{xL} = 45$ kip-ft

$M_{yD} = 2$ kip-ft
$M_{yL} = 6$ kip-ft

The member is not subject to sidesway (no lateral translation).

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From Chapter 2 of ASCE/SEI 7, the required strength (not considering second-order effects) is:

LRFD	ASD
$P_u = 1.2(5 \text{ kips}) + 1.6(15 \text{ kips})$ $= 30.0 \text{ kips}$ $M_{ux} = 1.2(15 \text{ kip-ft}) + 1.6(45 \text{ kip-ft})$ $= 90.0 \text{ kip-ft}$ $M_{uy} = 1.2(2 \text{ kip-ft}) + 1.6(6 \text{ kip-ft})$ $= 12.0 \text{ kip-ft}$	$P_a = 5 \text{ kips} + 15 \text{ kips}$ $= 20.0 \text{ kips}$ $M_{ax} = 15 \text{ kip-ft} + 45 \text{ kip-ft}$ $= 60.0 \text{ kip-ft}$ $M_{ay} = 2 \text{ kip-ft} + 6 \text{ kip-ft}$ $= 8.00 \text{ kip-ft}$

Try a W10×33.

From AISC *Manual* Tables 1-1 and 3-2, the properties are as follows:

W10×33
$A = 9.71$ in.^{2}
$S_x = 35.0$ in.^{3}
$Z_x = 38.8$ in.^{3}
$I_x = 171$ in.^{4}
$r_x = 4.19$ in.
$S_y = 9.20$ in.^{3}
$Z_y = 14.0$ in.^{3}
$I_y = 36.6$ in.^{4}
$r_y = 1.94$ in.
$L_p = 6.85$ ft
$L_r = 21.8$ ft

Available Axial Strength

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0. Because $L_c = KL_x = KL_y$ = 14.0 ft and $r_x > r_y$, the y-y axis will govern.

From AISC *Manual* Table 6-2, the available axial strength is:

LRFD	ASD
$P_c = \phi_c P_n$ $= 253 \text{ kips}$	$P_c = \dfrac{P_n}{\Omega_c}$ $= 168 \text{ kips}$

Required Flexural Strength (including second-order amplification)

Use the approximate method of second-order analysis procedure from AISC *Specification* Appendix 8. Because the member is not subject to sidesway, only P-δ amplifiers need to be added.

$$B_1 = \frac{C_m}{1 - \alpha P_r / P_{e1}} \geq 1$$ (*Spec*. Eq. A-8-3)

where C_m is conservatively taken per AISC *Specification* A-8.2.1(b):

C_m = 1.0

The x-x axis flexural magnifier is:

$$P_{e1x} = \frac{\pi^2 EI_x}{(L_{c1x})^2}$$ (from *Spec*. Eq. A-8-5)

$$= \frac{\pi^2 (29{,}000 \text{ ksi})(171 \text{ in.}^4)}{\left[(14 \text{ ft})(12 \text{ in./ft})\right]^2}$$

$$= 1{,}730 \text{ kips}$$

LRFD	ASD
$\alpha = 1.0$ $B_{1x} = \dfrac{C_m}{1 - \alpha P_r / P_{e1x}} \geq 1.0$ $= \dfrac{1.0}{1 - 1.0(30 \text{ kips}/1{,}730 \text{ kips})} \geq 1.0$ $= 1.02$ $M_{ux} = 1.02(90 \text{ kip-ft})$ $= 91.8 \text{ kip-ft}$	$\alpha = 1.6$ $B_{1x} = \dfrac{C_m}{1 - \alpha P_r / P_{e1x}} \geq 1.0$ $= \dfrac{1.0}{1 - 1.6(20 \text{ kips}/1{,}730 \text{ kips})} \geq 1.0$ $= 1.02$ $M_{ax} = 1.02(60 \text{ kip-ft})$ $= 61.2 \text{ kip-ft}$

The y-y axis flexural magnifier is:

$$P_{e1y} = \frac{\pi^2 EI_y}{(L_{c1y})^2} \qquad \text{(modified } \textit{Spec.} \text{ Eq. A-8-5)}$$

$$= \frac{\pi^2 (29{,}000 \text{ ksi})(36.6 \text{ in.}^4)}{[(14 \text{ ft})(12 \text{ in./ft})]^2}$$

$$= 371 \text{ kips}$$

LRFD	ASD
$\alpha = 1.0$ $B_{1y} = \dfrac{C_m}{1 - \alpha P_r / P_{e1y}} \geq 1.0$ $= \dfrac{1.0}{1 - 1.0(30 \text{ kips} / 371 \text{ kips})} \geq 1.0$ $= 1.09$ $M_{uy} = 1.09(12 \text{ kip-ft})$ $= 13.1 \text{ kip-ft}$	$\alpha = 1.6$ $B_{1y} = \dfrac{C_m}{1 - \alpha P_r / P_{e1y}} \geq 1.0$ $= \dfrac{1.0}{1 - 1.6(20 \text{ kips} / 371 \text{ kips})} \geq 1.0$ $= 1.09$ $M_{ay} = 1.09(8 \text{ kip-ft})$ $= 8.72 \text{ kip-ft}$

Nominal Flexural Strength about the Major Axis

Yielding

$$M_{nx} = M_p = F_y Z_x \qquad (\textit{Spec.} \text{ Eq. F2-1})$$

$$= (50 \text{ ksi})(38.8 \text{ in.}^3)$$

$$= 1{,}940 \text{ kip-in.}$$

Lateral-Torsional Buckling

Because $L_p < L_b \leq L_r$, i.e., 6.85 ft < 14.0 ft < 21.8 ft, AISC *Specification* Equation F2-2 applies.

From AISC *Manual* Table 3-1, $C_b = 1.14$

$$M_{nx} = C_b \left[M_p - (M_p - 0.7 F_y S_x) \left(\frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p \qquad (\textit{Spec.} \text{ Eq. F2-2})$$

$$= 1.14 \left\{ 1{,}940 \text{ kip-in.} - \left[1{,}940 \text{ kip-in.} - 0.7(50 \text{ ksi})(35.0 \text{ in.}^3) \right] \left(\frac{14 \text{ ft} - 6.85 \text{ ft}}{21.8 \text{ ft} - 6.85 \text{ ft}} \right) \right\}$$

$$= 1{,}820 \text{ kip-in.} < 1{,}940 \text{ kip-in.}$$

$$= 1{,}820 \text{ kip-in. or } 152 \text{ kip-ft} \quad \textbf{controls}$$

Local Buckling

Per AISC *Manual* Table 1-1, the member is compact for $F_y = 50$ ksi, so the local buckling limit state does not apply.

Nominal Flexural Strength about the Minor Axis

Determine the nominal flexural strength for bending about the minor axis from AISC *Specification* Section F6. Because a W10×33 has compact flanges, only the yielding limit state applies.

From AISC *Specification* Section F1:

$$M_{nx} = M_p = F_y Z_x \leq 1.6 F_y S_y$$ (*Spec*. Eq. F6-1)

$$= (50 \text{ ksi})(14.0 \text{ in.}^3) \leq 1.6(50 \text{ ksi})(9.20 \text{ in.}^3)$$

$$= 700 \text{ kip-in.} < 736 \text{ kip-in.}$$

$$= 700 \text{ kip-in. or } 58.3 \text{ kip-ft}$$

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $M_{cx} = \phi_b M_{nx}$ $= 0.90(152 \text{ kip-ft})$ $= 137 \text{ kip-ft}$ $M_{cy} = \phi_b M_{ny}$ $= 0.90(58.3 \text{ kip-ft})$ $= 52.5 \text{ kip-ft}$	$\Omega_b = 1.67$ $M_{cx} = \dfrac{M_{nx}}{\Omega_b}$ $= \dfrac{152 \text{ kip-ft}}{1.67}$ $= 91.0 \text{ kip-ft}$ $M_{cy} = \dfrac{M_{ny}}{\Omega_b}$ $= \dfrac{58.3 \text{ kip-ft}}{1.67}$ $= 34.9 \text{ kip-ft}$

Check limit for AISC *Specification* Equations H1-1a and H1-1b.

LRFD	ASD
$\dfrac{P_r}{P_c} = \dfrac{P_u}{\phi_c P_n}$ $= \dfrac{30 \text{ kips}}{253 \text{ kips}}$ $= 0.119 < 0.2$ Because $\dfrac{P_r}{P_c} < 0.2,$ $\dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$ (*Spec*. Eq. H1-1b) $= \dfrac{30 \text{ kips}}{2(253 \text{ kips})} + \left(\dfrac{91.8 \text{ kip-ft}}{137 \text{ kip-ft}} + \dfrac{13.1 \text{ kip-ft}}{52.5 \text{ kip-ft}}\right) \leq 1.0$ $= 0.979 < 1.0$ **o.k.**	$\dfrac{P_r}{P_c} = \dfrac{P_a}{P_n / \Omega_c}$ $= \dfrac{20 \text{ kips}}{168 \text{ kips}}$ $= 0.119 < 0.2$ Because $\dfrac{P_r}{P_c} < 0.2,$ $\dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$ (*Spec*. Eq. H1-1b) $= \dfrac{20 \text{ kips}}{2(168 \text{ kips})} + \left(\dfrac{61.2 \text{ kip-ft}}{91.0 \text{ kip-ft}} + \dfrac{8.72 \text{ kip-ft}}{34.9 \text{ kip-ft}}\right)$ $= 0.982 < 1.0$ **o.k.**

EXAMPLE H.5A RECTANGULAR HSS TORSIONAL STRENGTH

Given:

Determine the available torsional strength of an ASTM A500, Grade C, HSS6×4×¼.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade C
$F_y = 50$ ksi
$F_u = 62$ ksi

From AISC *Manual* Table 1-11, the geometric properties are as follows:

HSS6×4×¼
$t = 0.233$ in.
$b/t = 14.2$
$h/t = 22.8$
$C = 10.1$ in.^{3}

The available torsional strength for rectangular HSS is stipulated in AISC *Specification* Section H3.1. The critical stress, F_{cr}, is determined from AISC *Specification* Section H3.1(b).

Because $h/t > b/t$, h/t governs.

$$2.45\sqrt{\frac{E}{F_y}} = 2.45\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 59.0 > 22.8; \text{ therefore, use AISC } \textit{Specification} \text{ Equation H3-3 to determine } F_{cr}$$

$$F_{cr} = 0.6F_y \qquad \text{(\textit{Spec}. Eq. H3-3)}$$
$$= 0.6(50 \text{ ksi})$$
$$= 30.0 \text{ ksi}$$

The nominal torsional strength is:

$$T_n = F_{cr}C \qquad \text{(\textit{Spec}. Eq. H3-1)}$$
$$= (30.0 \text{ ksi})(10.1 \text{ in.}^3)$$
$$= 303 \text{ kip-in.}$$

From AISC *Specification* Section H3.1, the available torsional strength is:

LRFD	ASD
$\phi_T = 0.90$ $\phi_T T_n = 0.90(303 \text{ kip-in.})$ $= 273$ kip-in.	$\Omega_T = 1.67$ $\frac{T_n}{\Omega_T} = \frac{303 \text{ kip-in.}}{1.67}$ $= 181$ kip-in.

Note: For more complete guidance on designing for torsion, see AISC Design Guide 9, *Torsional Analysis of Structural Steel Members* (Seaburg and Carter, 1997).

EXAMPLE H.5B ROUND HSS TORSIONAL STRENGTH

Given:

Determine the available torsional strength of an ASTM A500, Grade C, HSS5.000×0.250 that is 14 ft long.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade C
F_y = 46 ksi
F_u = 62 ksi

From AISC *Manual* Table 1-13, the geometric properties are as follows:

HSS5.000×0.250
D = 5.00 in.
t = 0.233 in.
D/t = 21.5
C = 7.95 in.^{3}

The available torsional strength for round HSS is stipulated in AISC *Specification* Section H3.1.The critical stress, F_{cr}, is determined from AISC *Specification* Section H3.1(a).

Calculate the critical stress as the larger of:

$$F_{cr} = \frac{1.23E}{\sqrt{\frac{L}{D}}\left(\frac{D}{t}\right)^{5/4}} \qquad \text{(Spec. Eq. H3-2a)}$$

$$= \frac{1.23(29{,}000\text{ ksi})}{\sqrt{\frac{(14\text{ ft})(12\text{ in./ft})}{5.00\text{ in.}}}(21.5)^{5/4}}$$

$$= 133\text{ ksi}$$

and

$$F_{cr} = \frac{0.60E}{\left(\frac{D}{t}\right)^{3/2}} \qquad \text{(Spec. Eq. H3-2b)}$$

$$= \frac{0.60(29{,}000\text{ ksi})}{(21.5)^{3/2}}$$

$$= 175\text{ ksi}$$

However, F_{cr} shall not exceed the following:

$$0.6F_y = 0.6(46\text{ ksi}) = 27.6\text{ ksi}$$

Therefore, $F_{cr} = 27.6$ ksi.

The nominal torsional strength is:

$$T_n = F_{cr}C \quad \text{(Spec. Eq. H3-1)}$$
$$= (27.6 \text{ ksi})(7.95 \text{ in.}^3)$$
$$= 219 \text{ kip-in.}$$

From AISC *Specification* Section H3.1, the available torsional strength is:

LRFD	ASD
$\phi_T = 0.90$ $\phi_T T_n = 0.90(219 \text{ kip-in.})$ $= 197 \text{ kip-in.}$	$\Omega_T = 1.67$ $\dfrac{T_n}{\Omega_T} = \dfrac{219 \text{ kip-in.}}{1.67}$ $= 131 \text{ kip-in.}$

Note: For more complete guidance on designing for torsion, see AISC Design Guide 9, *Torsional Analysis of Structural Steel Members* (Seaburg and Carter, 1997).

EXAMPLE H.5C RECTANGULAR HSS COMBINED TORSIONAL AND FLEXURAL STRENGTH

Given:

Verify the strength of an ASTM A500, Grade C, HSS6×4×¼ loaded as shown. The beam is simply supported and is torsionally fixed at the ends. Bending is about the strong axis.

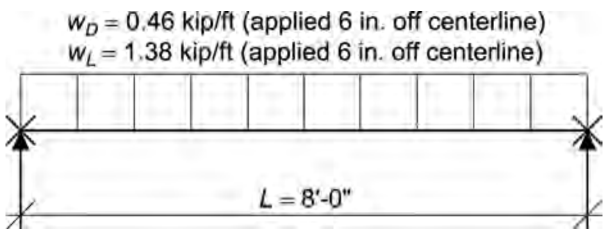


Fig. H.5C. Beam loading and bracing diagram.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade C
F_y = 50 ksi
F_u = 62 ksi

From AISC *Manual* Table 1-11, the geometric properties are as follows:

HSS6×4×¼
t = 0.233 in.
A_g = 4.30 in.^{2}
b/t = 14.2
h/t = 22.8
r_y = 1.61 in.
Z_x = 8.53 in.^{3}
J = 23.6 in.^{4}

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$w_u = 1.2(0.46\text{ kip/ft}) + 1.6(1.38\text{ kip/ft})$ $= 2.76\text{ kip/ft}$	$w_a = 0.46\text{ kip/ft} + 1.38\text{ kip/ft}$ $= 1.84\text{ kip/ft}$

Calculate the maximum shear (at the supports) using AISC *Manual* Table 3-23, Case 1.

LRFD	ASD
$V_r = V_u$ $= \dfrac{w_u L}{2}$ $= \dfrac{(2.76\text{ kip/ft})(8\text{ ft})}{2}$ $= 11.0\text{ kips}$	$V_r = V_a$ $= \dfrac{w_a L}{2}$ $= \dfrac{(1.84\text{ kip/ft})(8\text{ ft})}{2}$ $= 7.36\text{ kips}$

Calculate the maximum torsion (at the supports).

LRFD	ASD
$T_r = T_u$ $= \dfrac{w_u Le}{2}$ $= \dfrac{(2.76\text{ kip/ft})(8\text{ ft})(6\text{ in.})}{2}$ $= 66.2\text{ kip-in.}$	$T_r = T_a$ $= \dfrac{w_a Le}{2}$ $= \dfrac{(1.84\text{ kip/ft})(8\text{ ft})(6\text{ in.})}{2}$ $= 44.2\text{ kip-in.}$

Available Shear Strength

Determine the available shear strength from AISC *Specification* Section G4. Using the provisions given in AISC *Specification* Section B4.1b(d), determine the web depth, *d*, as follows:

$$\begin{aligned} h &= 6.00\text{ in.} - 3(0.233\text{ in.}) \\ &= 5.30\text{ in.} \end{aligned}$$

From AISC *Specification* Section G4:

$$\begin{aligned} A_w &= 2ht \\ &= 2(5.30\text{ in.})(0.233\text{ in.}) \\ &= 2.47\text{ in.}^2 \end{aligned}$$

$$k_v = 5$$

The web shear buckling coefficient is determined from AISC *Specification* Section G2.2.

$$\begin{aligned} 1.10\sqrt{\frac{k_v E}{F_y}} &= 1.10\sqrt{\frac{5(29{,}000\text{ ksi})}{50\text{ ksi}}} \\ &= 59.2 > 22.8;\text{ therefore use AISC } \textit{Specification} \text{ Section G2.2(b)(i)} \end{aligned}$$

$$C_{v2} = 1.0 \qquad (\textit{Spec}.\text{ Eq. G2-9})$$

The nominal shear strength from AISC *Specification* Section G4 is:

$$\begin{aligned} V_n &= 0.6F_y A_w C_2 \qquad (\textit{Spec}.\text{ Eq. G4-1}) \\ &= 0.6(50\text{ ksi})(2.47\text{ in.}^2)(1.0) \\ &= 74.1\text{ kips} \end{aligned}$$

From AISC *Specification* Section G1, the available shear strength is:

LRFD	ASD
$\phi_v = 0.90$ $V_c = \phi_v V_n$ $= 0.90(74.1 \text{ kips})$ $= 66.7 \text{ kips}$	$\Omega_v = 1.67$ $V_c = \dfrac{V_n}{\Omega_v}$ $= \dfrac{74.1 \text{ kips}}{1.67}$ $= 44.4 \text{ kips}$

Available Flexural Strength

The available flexural strength is determined from AISC *Specification* Section F7 for rectangular HSS. For the limit state of flexural yielding, the nominal flexural strength is:

$$\begin{aligned} M_n &= M_p \\ &= F_y Z_x \\ &= (50 \text{ ksi})(8.53 \text{ in.}^3) \\ &= 427 \text{ kip-in.} \end{aligned}$$ (*Spec*. Eq. F7-1)

Determine if the limit state of flange local buckling applies as follows:

$$\begin{aligned} \lambda &= \frac{b}{t} \\ &= 14.2 \end{aligned}$$

Determine the flange compact slenderness limit from AISC *Specification* Table B4.1b, Case 17.

$$\begin{aligned} \lambda_p &= 1.12\sqrt{\frac{E}{F_y}} \\ &= 1.12\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}} \\ &= 27.0 \end{aligned}$$

$\lambda < \lambda_p$; therefore, the flange is compact and the flange local buckling limit state does not apply

Determine if the limit state of web local buckling applies as follows:

$$\begin{aligned} \lambda &= \frac{h}{t} \\ &= 22.8 \end{aligned}$$

Determine the web compact slenderness limit from AISC *Specification* Table B4.1b, Case 19.

$$\begin{aligned} \lambda_p &= 2.42\sqrt{\frac{E}{F_y}} \\ &= 2.42\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}} \\ &= 58.3 \end{aligned}$$

$\lambda < \lambda_p$; therefore, the web is compact and the web local buckling limit state does not apply

Determine if lateral-torsional buckling applies as follows:

$$L_p = 0.13Er_y \frac{\sqrt{JA_g}}{M_p} \qquad \text{(\textit{Spec}. Eq. F7-12)}$$

$$= 0.13(29{,}000 \text{ ksi})(1.61 \text{ in.}) \frac{\sqrt{(23.6 \text{ in.}^4)(4.30 \text{ in.}^2)}}{427 \text{ kip-in.}}$$

$$= 143 \text{ in. or } 11.9 \text{ ft}$$

Since L_b = 8 ft < L_p = 11.9 ft, lateral-torsional buckling is not applicable and M_n = 427 kip-in., controlled by the flexural yielding limit state. From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $M_c = \phi_b M_n$ $= 0.90(427 \text{ kip-in.})$ $= 384 \text{ kip-in.}$	$\Omega_b = 1.67$ $M_c = \dfrac{M_n}{\Omega_b}$ $= \dfrac{427 \text{ kip-in.}}{1.67}$ $= 256 \text{ kip-in.}$

From Example H.5A, the available torsional strength is:

LRFD	ASD
$T_c = \phi_T T_n$ $= 273 \text{ kip-in.}$	$T_c = \dfrac{T_n}{\Omega_T}$ $= 181 \text{ kip-in.}$

Using AISC *Specification* Section H3.2, check combined strength at several locations where $T_r > 0.2T_c$. First check at the supports, which is the point of maximum shear and torsion:

LRFD	ASD
$\dfrac{T_r}{T_c} = \dfrac{66.2 \text{ kip-in.}}{273 \text{ kip-in.}}$ $= 0.242 > 0.2$ Therefore, use AISC *Specification* Equation H3-6: $\left(\dfrac{P_r}{P_c} + \dfrac{M_r}{M_c}\right) + \left(\dfrac{V_r}{V_c} + \dfrac{T_r}{T_c}\right)^2 \leq 1.0$ (*Spec* Eq. H3-6) $= (0+0) + \left(\dfrac{11.0 \text{ kips}}{66.7 \text{ kips}} + \dfrac{66.2 \text{ kip-in.}}{273 \text{ kip-in.}}\right)^2$ $= 0.166 < 1.0$ **o.k.**	$\dfrac{T_r}{T_c} = \dfrac{44.2 \text{ kip-in.}}{181 \text{ kip-in.}}$ $= 0.244 > 0.2$ Therefore, use AISC *Specification* Equation H3-6: $\left(\dfrac{P_r}{P_c} + \dfrac{M_r}{M_c}\right) + \left(\dfrac{V_r}{V_c} + \dfrac{T_r}{T_c}\right)^2 \leq 1.0$ (*Spec* Eq. H3-6) $= (0+0) + \left(\dfrac{7.36 \text{ kips}}{44.4 \text{ kips}} + \dfrac{44.2 \text{ kip-in.}}{181 \text{ kip-in.}}\right)^2$ $= 0.168 < 1.0$ **o.k.**

Check the combined strength near the location where $T_r = 0.2T_c$. This is the location with the largest bending moment required to be considered in the interaction. Calculate the shear and moment at this location, x.

LRFD	ASD
$\dfrac{T_r}{T_c} = 0.20$ Therefore at x: $T_r = 0.20(273 \text{ kip-in.})$ $= 54.6 \text{ kip-in.}$ $x = \dfrac{(T_r \text{ at support}) - (T_r \text{ at } x)}{w_u e}$ $= \dfrac{66.2 \text{ kip-in.} - 54.6 \text{ kip-in.}}{(2.76 \text{ kip/ft})(6 \text{ in.})}$ $= 0.700 \text{ ft}$ $V_r = 11.0 \text{ kips} - (0.700 \text{ ft})(2.76 \text{ kip/ft})$ $= 9.07 \text{ kips}$ $M_r = \dfrac{w_u x}{2}(l - x)$ $= \dfrac{(2.76 \text{ kip/ft})(0.700 \text{ ft})}{2}(8 \text{ ft} - 0.700 \text{ ft})$ $= 7.05 \text{ kip-ft or } 84.6 \text{ kip-in.}$ $\left(\dfrac{P_r}{P_c} + \dfrac{M_r}{M_c}\right) + \left(\dfrac{V_r}{V_c} + \dfrac{T_r}{T_c}\right)^2 \le 1.0$ (*Spec* Eq. H3-6) $= \left(0 + \dfrac{84.6 \text{ kip-in.}}{384 \text{ kip-in.}}\right) + \left(\dfrac{9.07 \text{ kips}}{66.7 \text{ kips}} + 0.20\right)^2$ $= 0.333 < 1.0$ **o.k.**	$\dfrac{T_r}{T_c} = 0.20$ Therefore at x: $T_r = 0.20(181 \text{ kip-in.})$ $= 36.2 \text{ kip-in.}$ $x = \dfrac{(T_r \text{ at support}) - (T_r \text{ at } x)}{w_a e}$ $= \dfrac{44.2 \text{ kip-in.} - 36.2 \text{ kip-in.}}{(1.84 \text{ kip/ft})(6 \text{ in.})}$ $= 0.725 \text{ ft}$ $V_r = 7.36 \text{ kips} - (0.725 \text{ ft})(1.84 \text{ kips/ft})$ $= 6.03 \text{ kips}$ $M_r = \dfrac{w_a x}{2}(l - x)$ $= \dfrac{(1.84 \text{ kip/ft})(0.725 \text{ ft})}{2}(8 \text{ ft} - 0.725 \text{ ft})$ $= 4.85 \text{ kip-ft or } 58.2 \text{ kip-in.}$ $\left(\dfrac{P_r}{P_c} + \dfrac{M_r}{M_c}\right) + \left(\dfrac{V_r}{V_c} + \dfrac{T_r}{T_c}\right)^2 \le 1.0$ (*Spec* Eq. H3-6) $= \left(0 + \dfrac{58.2 \text{ kip-in.}}{256 \text{ kip-in.}}\right) + \left(\dfrac{6.03 \text{ kips}}{44.4 \text{ kips}} + 0.20\right)^2$ $= 0.340 < 1.0$ **o.k.**

Note: The remainder of the beam, where $T_r \le 0.2T_c$, must also be checked to determine if the strength without torsion controls over the interaction with torsion.

EXAMPLE H.6 W-SHAPE TORSIONAL STRENGTH

Given:

As shown in Figure H.6-1, an ASTM A992 W10×49 spans 15 ft and supports concentrated loads at midspan that act at a 6-in. eccentricity with respect to the shear center. Determine the stresses on the cross section, the adequacy of the section to support the loads, and the maximum rotation.

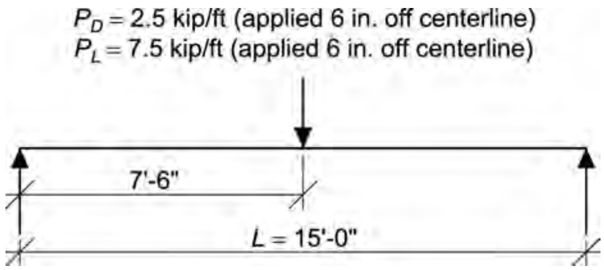


Fig. H.6-1. Beam loading diagram.

The end conditions are assumed to be flexurally pinned and unrestrained for warping torsion. The eccentric load can be resolved into a torsional moment and a load applied through the shear center.

A similar design example appears in AISC Design Guide 9, *Torsional Analysis of Structural Steel Members* (Seaburg and Carter, 1997).

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W10×49
t_w = 0.340 in.
t_f = 0.560 in.
I_x = 272 in.^{4}
S_x = 54.6 in.^{3}
Z_x = 60.4 in.^{3}
J = 1.39 in.^{4}
C_w = 2,070 in.^{6}

From the AISC Shapes Database, the additional torsional properties are as follows:

W10×49
S_{w1} = 33.0 in.^{4}
W_{no} = 23.6 in.^{2}
Q_f = 12.8 in.^{3}
Q_w = 29.8 in.^{3}

From AISC Design Guide 9, the torsional property, a, is calculated as follows:

$$a = \sqrt{\frac{EC_w}{GJ}} \quad \text{(Design Guide 9, Eq. 3.6)}$$

$$= \sqrt{\frac{(29{,}000 \text{ ksi})(2{,}070 \text{ in.}^6)}{(11{,}200 \text{ ksi})(1.39 \text{ in.}^4)}}$$

$$= 62.1 \text{ in.}$$

From ASCE/SEI 7, Chapter 2, and AISC *Manual* Table 3-23, Case 7, the required strengths are:

LRFD	ASD
$P_u = 1.2(2.5 \text{ kips}) + 1.6(7.5 \text{ kips})$ $= 15.0 \text{ kips}$ $V_u = \frac{P_u}{2}$ $= \frac{15.0 \text{ kips}}{2}$ $= 7.50 \text{ kips}$ $M_u = \frac{P_u L}{4}$ $= \frac{(15.0 \text{ kips})(15 \text{ ft})(12 \text{ in./ft})}{4}$ $= 675 \text{ kip-in.}$ $T_u = P_u e$ $= (15.0 \text{ kips})(6 \text{ in.})$ $= 90.0 \text{ kip-in.}$	$P_a = 2.5 \text{ kips} + 7.5 \text{ kips}$ $= 10.0 \text{ kips}$ $V_a = \frac{P_a}{2}$ $= \frac{10.0 \text{ kips}}{2}$ $= 5.00 \text{ kips}$ $M_a = \frac{P_a L}{4}$ $= \frac{(10.0 \text{ kips})(15 \text{ ft})(12 \text{ in./ft})}{4}$ $= 450 \text{ kip-in.}$ $T_a = P_a e$ $= (10.0 \text{ kips})(6 \text{ in.})$ $= 60.0 \text{ kip-in.}$

Normal and Shear Stresses from Flexure

The normal and shear stresses from flexure are determined from AISC Design Guide 9, as follows:

LRFD	ASD
$\sigma_{ub} = \frac{M_u}{S_x}$ (from Design Guide 9, Eq. 4.5) $= \frac{675 \text{ kip-in.}}{54.6 \text{ in.}^3}$ $= 12.4 \text{ ksi}$ (compression at top, tension at bottom) $\tau_{ub\,web} = \frac{V_u Q_w}{I_x t_w}$ (from Design Guide 9, Eq. 4.6) $= \frac{(7.50 \text{ kips})(29.8 \text{ in.}^3)}{(272 \text{ in.}^4)(0.340 \text{ in.})}$ $= 2.42 \text{ ksi}$	$\sigma_{ab} = \frac{M_a}{S_x}$ (from Design Guide 9, Eq. 4.5) $= \frac{450 \text{ kip-in.}}{54.6 \text{ in.}^3}$ $= 8.24 \text{ ksi}$ (compression at top, tension at bottom) $\tau_{ab\,web} = \frac{V_a Q_w}{I_x t_w}$ (from Design Guide 9, Eq. 4.6) $= \frac{(5.00 \text{ kips})(29.8 \text{ in.}^3)}{(272 \text{ in.}^4)(0.340 \text{ in.})}$ $= 1.61 \text{ ksi}$

LRFD	ASD
$\tau_{ub\ flange} = \dfrac{V_u Q_f}{I_x t_f}$ (from Design Guide 9, Eq. 4.6) $= \dfrac{(7.50\text{ kips})(12.8\text{ in.}^3)}{(272\text{ in.}^4)(0.560\text{ in.})}$ $= 0.630\text{ ksi}$	$\tau_{ab\ flange} = \dfrac{V_a Q_f}{I_x t_f}$ (from Design Guide 9, Eq. 4.6) $= \dfrac{(5.00\text{ kips})(12.8\text{ in.}^3)}{(272\text{ in.}^4)(0.560\text{ in.})}$ $= 0.420\text{ ksi}$

Torsional Stresses

The following functions are taken from AISC Design Guide 9, Appendix B, Case 3, with α = 0.5 for the torsional load applied at midspan.

$$\frac{L}{a} = \frac{(15\text{ ft})(12\text{ in./ft})}{62.1\text{ in.}}$$
$$= 2.90$$

Using the graphs in AISC Design Guide 9, Appendix B, select values for θ, θ', θ'' and θ'''.

At midspan (z/l = 0.5):

For θ: $\theta \times \left(\dfrac{GJ}{T_r}\right)\left(\dfrac{1}{l}\right) = +0.09$ Solve for: $\theta = +0.09\dfrac{T_r l}{GJ}$

For θ': $\theta' \times \left(\dfrac{GJ}{T_r}\right) = 0$ Therefore: $\theta' = 0$

For θ'': $\theta'' \times \left(\dfrac{GJ}{T_r}\right)a = -0.44$ Solve for: $\theta'' = -0.44\dfrac{T_r}{GJa}$

For θ''': $\theta''' \times \left(\dfrac{GJ}{T_r}\right)a^2 = -0.50$ Solve for: $\theta''' = -0.50\dfrac{T_r}{GJa^2}$

At the support (z/l = 0):

For θ: $\theta \times \left(\dfrac{GJ}{T_r}\right)\left(\dfrac{1}{l}\right) = 0$ Therefore: $\theta = 0$

For θ': $\theta' \times \left(\dfrac{GJ}{T_r}\right) = +0.28$ Solve for: $\theta' = +0.28\dfrac{T_r}{GJ}$

For θ'': $\theta'' \times \left(\dfrac{GJ}{T_r}\right)a = 0$ Therefore: $\theta'' = 0$

For θ''': $\theta''' \times \left(\dfrac{GJ}{T_r}\right)a^2 = -0.22$ Solve for: $\theta''' = -0.22\dfrac{T_r}{GJa^2}$

In the preceding calculations, note that the applied torque is negative based on the sign convention used in the AISC Design Guide 9 graphs.

Calculate T_r/GJ as follows:

LRFD	ASD
$\dfrac{T_u}{GJ} = \dfrac{-90.0\text{ kip-in.}}{(11{,}200\text{ ksi})(1.39\text{ in.}^4)}$ $= -5.78\times10^{-3}\text{ rad/in.}$	$\dfrac{T_a}{GJ} = \dfrac{-60.0\text{ kip-in.}}{(11{,}200\text{ ksi})(1.39\text{ in.}^4)}$ $= -3.85\times10^{-3}\text{ rad/in.}$

Shear Stresses Due to Pure Torsion

The shear stresses due to pure torsion are determined from AISC Design Guide 9 as follows:

$$\tau_t = Gt\theta' \qquad \text{(Design Guide 9, Eq. 4.1)}$$

LRFD	ASD
At midspan: $\theta' = 0$; therefore $\tau_{ut} = 0$ At the support, for the web: $\tau_{ut} = (11{,}200\text{ ksi})(0.340\text{ in.})(0.28)\left(\dfrac{-5.78\text{ rad}}{10^3\text{ in.}}\right)$ $= -6.16\text{ ksi}$ At the support, for the flange: $\tau_{ut} = (11{,}200\text{ ksi})(0.560\text{ in.})(0.28)\left(\dfrac{-5.78\text{ rad}}{10^3\text{ in.}}\right)$ $= -10.2\text{ ksi}$	At midspan: $\theta' = 0$; therefore $\tau_{at} = 0$ At the support, for the web: $\tau_{at} = (11{,}200\text{ ksi})(0.340\text{ in.})(0.28)\left(\dfrac{-3.85\text{ rad}}{10^3\text{ in.}}\right)$ $= -4.11\text{ ksi}$ At the support, for the flange: $\tau_{at} = (11{,}200\text{ ksi})(0.560\text{ in.})(0.28)\left(\dfrac{-3.85\text{ rad}}{10^3\text{ in.}}\right)$ $= -6.76\text{ ksi}$

Shear Stresses Due to Warping

The shear stresses due to warping are determined from AISC Design Guide 9 as follows:

$$\tau_w = \frac{-ES_{w1}\theta'''}{t_f} \qquad \text{(Design Guide 9, Eq. 4.2a)}$$

LRFD	ASD
At midspan: $\tau_{uw} = \dfrac{(-29{,}000\text{ ksi})(33.0\text{ in.}^4)}{0.560\text{ in.}}\left[\dfrac{-0.50(-5.78\text{ rad})}{(62.1\text{ in.})^2(10^3\text{ in.})}\right]$ $= -1.28\text{ ksi}$ At the support: $\tau_{uw} = \dfrac{(-29{,}000\text{ ksi})(33.0\text{ in.}^4)}{0.560\text{ in.}}\left[\dfrac{-0.22(-5.78\text{ rad})}{(62.1\text{ in.})^2(10^3\text{ in.})}\right]$ $= -0.563\text{ ksi}$	At midspan: $\tau_{aw} = \dfrac{(-29{,}000\text{ ksi})(33.0\text{ in.}^4)}{0.560\text{ in.}}\left[\dfrac{-0.50(-3.85\text{ rad})}{(62.1\text{ in.})^2(10^3\text{ in.})}\right]$ $= -0.853\text{ ksi}$ At the support: $\tau_{aw} = \dfrac{(-29{,}000\text{ ksi})(33.0\text{ in.}^4)}{0.560\text{ in.}}\left[\dfrac{-0.22(-3.85\text{ rad})}{(62.1\text{ in.})^2(10^3\text{ in.})}\right]$ $= -0.375\text{ ksi}$

Normal Stresses Due to Warping

The normal stresses due to warping are determined from AISC Design Guide 9 as follows:

$$\sigma_w = EW_{no}\theta''$$ (Design Guide 9, Eq. 4.3a)

LRFD	ASD
At midspan: $\sigma_{uw} = (29{,}000\text{ ksi})(23.6\text{ in.}^2)\left[\dfrac{-0.44(-5.78\text{ rad})}{(62.1\text{ in.})(10^3\text{ in.})}\right]$ $= 28.0\text{ ksi}$ At the support: Because $\theta'' = 0$, $\sigma_{uw} = 0$.	At midspan: $\sigma_{aw} = (29{,}000\text{ ksi})(23.6\text{ in.}^2)\left[\dfrac{-0.44(-3.85\text{ rad})}{(62.1\text{ in.})(10^3\text{ in.})}\right]$ $= 18.7\text{ ksi}$ At the support: Because $\theta'' = 0$, $\sigma_{aw} = 0$.

Combined Stresses

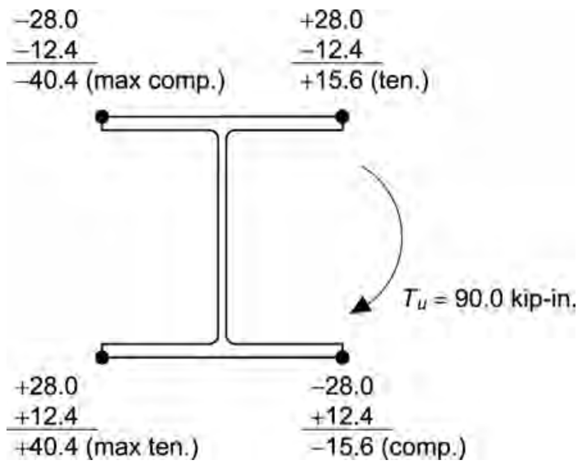
The stresses are summarized in Tables H.6-1A and H.6-1B and shown in Figure H.6-2.

Table H.6-1A
Summary of Stresses Due to Flexure and Torsion (LRFD), ksi

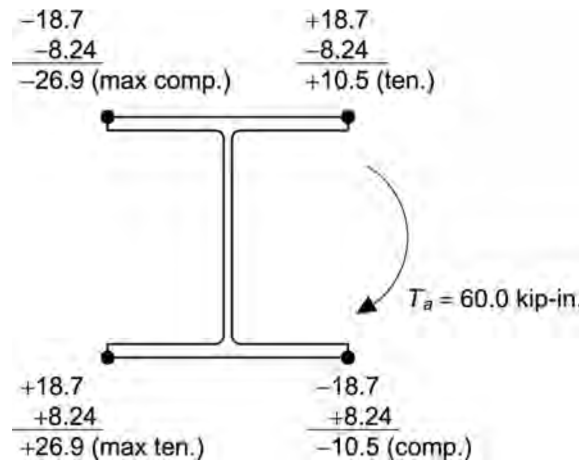
Location	Normal Stress			Shear Stress			
	σ_{uw}	σ_{ub}	f_{un}	τ_{ut}	τ_{uw}	τ_{ub}	f_{uv}
Midspan							
Flange	±28.0	±12.4	±40.4	0	–1.28	±0.630	–1.91
Web	–	–	–	0	–	±2.42	±2.42
Support							
Flange	0	0	0	–10.2	–0.563	±0.630	–11.4
Web	–	–	–	–6.16	–	±2.42	–8.58
Maximum			±40.4				–11.4

Table H.6-1B
Summary of Stresses Due to Flexure and Torsion (ASD), ksi

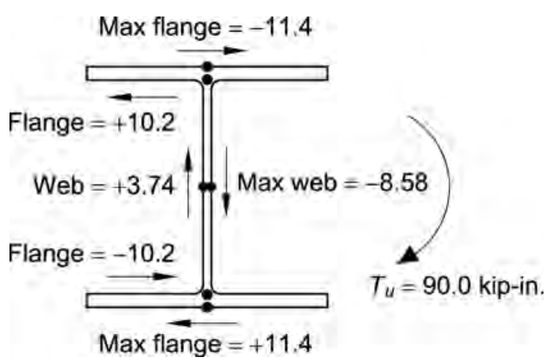
Location	Normal Stress			Shear Stress			
	σ_{aw}	σ_{ab}	f_{an}	τ_{at}	τ_{aw}	τ_{ab}	f_{av}
Midspan							
Flange	±18.7	±8.24	±26.9	0	–0.853	±0.420	–1.27
Web	–	–	–	0	–	±1.61	±1.61
Support							
Flange	0	0	0	–6.76	–0.375	±0.420	–7.56
Web	–	–	–	–4.11	–	±1.61	–5.72
Maximum			±26.9				–7.56



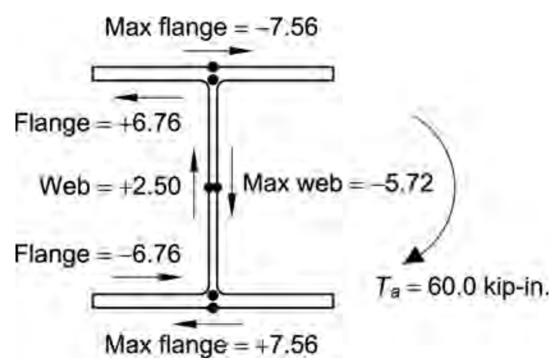
(a) Normal stresses due to flexure and torsion at midspan—LRFD



(b) Normal stresses due to flexure and torsion at midspan—ASD



(c) Shear stresses due to flexure and torsion at support—LRFD



(d) Shear stresses due to flexure and torsion at support—ASD

Fig. H.6-2. Stresses due to flexure and torsion.

LRFD	ASD
The maximum normal stress due to flexure and torsion occurs at the edge of the flange at midspan and is equal to 40.4 ksi. The maximum shear stress due to flexure and torsion occurs in the middle of the flange at the support and is equal to 11.4 ksi.	The maximum normal stress due to flexure and torsion occurs at the edge of the flange at midspan and is equal to 26.9 ksi. The maximum shear stress due to flexure and torsion occurs in the middle of the flange at the support and is equal to 7.56 ksi.

Available Torsional Strength

The available torsional strength is the lowest value determined for the limit states of yielding under normal stress, shear yielding under shear stress, or buckling in accordance with AISC *Specification* Section H3.3. The nominal torsional strength due to the limit states of yielding under normal stress and shear yielding under shear stress are compared to the applicable buckling limit states.

Buckling

For the buckling limit state, lateral-torsional buckling and local buckling must be evaluated. The nominal torsional strength due to the limit state of lateral-torsional buckling is determined as follows.

C_b = 1.32 from AISC *Manual* Table 3-1.

Compute F_n for a W10×49 using values from AISC *Manual* Table 3-10 with L_b = 15 ft and C_b = 1.0.

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 204$ kips $F_n = F_{cr}$ (*Spec*. Eq. H3-9) $= C_b\left(\frac{M_n}{S_x}\right)$ $= 1.32\left[\frac{(204\text{ kip-ft})(12\text{ in./ft})}{0.90(54.6\text{ in.}^3)}\right]$ $= 65.8$ ksi	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = 136$ kip-ft $F_n = F_{cr}$ (*Spec*. Eq. H3-9) $= C_b\left(\frac{M_n}{S_x}\right)$ $= 1.32\left[\frac{1.67(136\text{ kip-ft})(12\text{ in./ft})}{(54.6\text{ in.}^3)}\right]$ $= 65.9$ ksi

The limit state of local buckling does not apply because a W10×49 is compact in flexure per the user note in AISC *Specification* Section F2.

Yielding Under Normal Stress

The nominal torsional strength due to the limit state of yielding under normal stress is determined as follows:

$$F_n = F_y \quad \text{(\textit{Spec}. Eq. H3-7)}$$
$$= 50 \text{ ksi}$$

Therefore, the limit state of yielding under normal stress controls over buckling. The available torsional strength for yielding under normal stress is determined as follows, from AISC *Specification* Section H3:

LRFD	ASD
$\phi_T = 0.90$ $\phi_T F_n = 0.90(50\text{ ksi})$ $= 45.0\text{ ksi} > 40.4\text{ ksi}$ **o.k.**	$\Omega_T = 1.67$ $\frac{F_n}{\Omega_T} = \frac{50\text{ ksi}}{1.67}$ $= 29.9\text{ ksi} > 26.9\text{ ksi}$ **o.k.**

Shear Yielding Under Shear Stress

The nominal torsional strength due to the limit state of shear yielding under shear stress is:

$$F_n = 0.6F_y \quad \text{(\textit{Spec}. Eq. H3-8)}$$
$$= 0.6(50 \text{ ksi})$$
$$= 30.0 \text{ ksi}$$

The limit state of shear yielding under shear stress controls over buckling. The available torsional strength for shear yielding under shear stress is determined as follows, from AISC *Specification* Section H3:

LRFD	ASD
$\phi_T = 0.90$ $\phi_T F_n = 0.90(30 \text{ ksi})$ $= 27.0 \text{ ksi} > 11.4 \text{ ksi}$ **o.k.**	$\Omega_T = 1.67$ $\dfrac{F_n}{\Omega_T} = \dfrac{30 \text{ ksi}}{1.67}$ $= 18.0 \text{ ksi} > 7.56 \text{ ksi}$ **o.k.**

Maximum Rotation at Service Load

The maximum rotation occurs at midspan. The service load torque is:

$$
\begin{aligned}
T &= Pe \\
&= -(2.50 \text{ kips} + 7.50 \text{ kips})(6 \text{ in.}) \\
&= -60.0 \text{ kip-in.}
\end{aligned}
$$

As determined previously from AISC Design Guide 9, Appendix B, Case 3 with $\alpha = 0.5$, the maximum rotation is:

$$
\begin{aligned}
\theta &= +0.09\frac{Tl}{GJ} \\
&= \frac{0.09(-60.0 \text{ kip-in.})(15 \text{ ft})(12 \text{ in./ft})}{(11{,}200 \text{ ksi})(1.39 \text{ in.}^4)} \\
&= -0.0624 \text{ rad or } -3.58^\circ
\end{aligned}
$$

See AISC Design Guide 9, *Torsional Analysis of Structural Steel Members,* for additional guidance.

CHAPTER H DESIGN EXAMPLE REFERENCES

Seaburg, P.A. and Carter, C.J. (1997), *Torsional Analysis of Structural Steel Members,* Design Guide 9, AISC, Chicago, IL.

Chapter I
Design of Composite Members

I1. GENERAL PROVISIONS

Design, detailing, and material properties related to the concrete and steel reinforcing portions of composite members are governed by ACI 318 (ACI 318, 2014) as modified with composite-specific provisions by the AISC *Specification*.

The available strength of composite sections may be calculated by one of four methods: the plastic stress distribution method, the strain-compatibility method, the elastic stress distribution method, or the effective stress-strain method. The composite design tables in Part IV of this document are based on the plastic stress distribution method.

Filled composite sections are classified for local buckling according to the slenderness of the compression steel elements as illustrated in AISC *Specification* Tables I1.1a and I1.1b, and Examples I.4, I.6 and I.7. Local buckling effects do not need to be considered for encased composite members.

Terminology used within the Examples for filled composite section geometry is illustrated in Figure I-1.

I2. AXIAL FORCE

The available compressive strength of a composite member is based on a summation of the strengths of all of the components of the column with reductions applied for member slenderness and local buckling effects where applicable.

For tension members, the concrete tensile strength is ignored and only the strength of the steel member and properly connected reinforcing is permitted to be used in the calculation of available tensile strength.

The available compressive strengths for filled composite sections are given in Part IV of this document and reflect the requirements given in AISC *Specification* Sections I1.4 and I2.2. The design of filled composite compression and tension members is presented in Examples I.4 and I.5, respectively.

The design of encased composite compression and tension members is presented in Examples I.9 and I.10, respectively. There are no tables in the AISC *Manual* for the design of these members.

Note that the AISC *Specification* stipulates that the available compressive strength need not be less than that specified for the bare steel member.

I3. FLEXURE

The design of typical composite beams with steel anchors is illustrated in Examples I.1 and I.2. AISC *Manual* Table 3-19 provides available flexural strengths for composite W-shape beams, Table 3-20 provides lower-bound moments of inertia for plastic composite sections, and Table 3-21 provides shear strengths of steel headed stud anchors utilized for composite action in composite beams.

The design of filled composite members for flexure is illustrated within Examples I.6 and I.7, and the design of encased composite members for flexure is illustrated within Example I.11.

I4. SHEAR

For composite beams with formed steel deck, the available shear strength is based upon the properties of the steel section alone in accordance with AISC *Specification* Chapter G as illustrated in Examples I.1 and I.2.

For filled and encased composite members, either the shear strength of the steel section alone, the steel section plus the reinforcing steel, or the reinforced concrete alone are permitted to be used in the calculation of available shear strength. The calculation of shear strength for filled composite members is illustrated within Examples I.6 and I.7 and for encased composite members within Example I.11.

I5. COMBINED FLEXURE AND AXIAL FORCE

Design for combined axial force and flexure may be accomplished using either the strain compatibility method or the plastic-distribution method. Several different procedures for employing the plastic-distribution method are outlined in the Commentary, and each of these procedures is demonstrated for filled composite members in Example I.6 and for encased composite members in Example I.11. Interaction calculations for noncompact and slender filled composite members are illustrated in Example I.7.

To assist in developing the interaction curves illustrated within the design examples, a series of equations is provided in AISC *Manual* Part 6, Tables 6-3a, 6-3b, 6-4 and 6-5. These equations define selected points on the interaction curve, without consideration of slenderness effects. Specific cases are outlined and the applicability of the equations to a cross section that differs should be carefully considered. As an example, the equations in AISC *Manual* Table 6-3a are appropriate for the case of side bars located at the centerline, but not for other side bar locations. In contrast, these equations are appropriate for any amount of reinforcing at the extreme reinforcing bar location. In AISC *Manual* Table 6-3b the equations are appropriate only for the case of four reinforcing bars at the corners of the encased section. When design cases deviate from those presented the appropriate interaction equations can be derived from first principles.

I6. LOAD TRANSFER

The AISC *Specification* provides several requirements to ensure that the concrete and steel portions of the section act together. These requirements address both force allocation—how much of the applied loads are resisted by the steel versus the reinforced concrete; and force transfer mechanisms—how the force is transferred between the two materials. These requirements are illustrated in Example I.3 for filled composite members and Example I.8 for encased composite members.

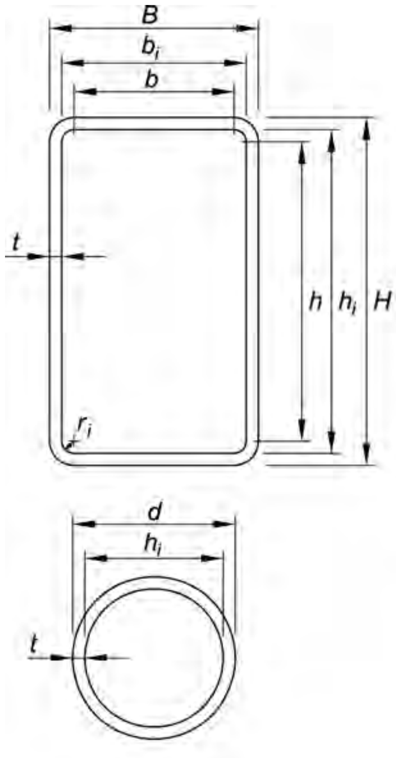
I7. COMPOSITE DIAPHRAGMS AND COLLECTOR BEAMS

The Commentary provides guidance on design methodologies for both composite diaphragms and composite collector beams.

I8. STEEL ANCHORS

AISC *Specification* Section I8 addresses the strength of steel anchors in composite beams and in composite components. Examples I.1 and I.2 illustrates the design of composite beams with steel headed stud anchors.

The application of steel anchors in composite component provisions have strict limitations as summarized in the User Note provided at the beginning of AISC *Specification* Section I8.3. These provisions do not apply to typical composite beam designs nor do they apply to hybrid construction where the steel and concrete do not resist loads together via composite action such as in embed plates. The most common application for these provisions is for the transfer of longitudinal shear within the load introduction length of composite columns as demonstrated in Example I.8. The application of these provisions to an isolated anchor within an applicable composite system is illustrated in Example I.12.



B = Overall width of section parallel to the axis of bending, in.
H = Overall height of section perpendicular to the axis of bending, in.
b = Width of stiffened compression element, in.
= $B - 3t$ per AISC *Specification* Section B4.1b(d)
b_i = Inside width of section, in.
= $B - 2t$
d = Outside diameter of round HSS, in.
h = Width of stiffened compression element, in.
= $H - 3t$ per AISC *Specification* Section B4.1b(d)
h_i = Inside diameter of round HSS, in.
= $d - 2t$
h_i = Inside height of section, in.
= $H - 2t$
r_i = $0.75t$ for b/t and h/t, in.
r_i = $1.0t$ for all area, section modulus, and moment of inertia calculations, in.
t = $0.93t_{nom}$, in.

Fig. I-1. Terminology used for filled members.

EXAMPLE I.1 COMPOSITE BEAM DESIGN

Given:

A typical bay of a composite floor system is illustrated in Figure I.1-1. Select an appropriate ASTM A992 W-shaped beam and determine the required number of ¾-in.-diameter steel headed stud anchors. The beam will not be shored during construction.

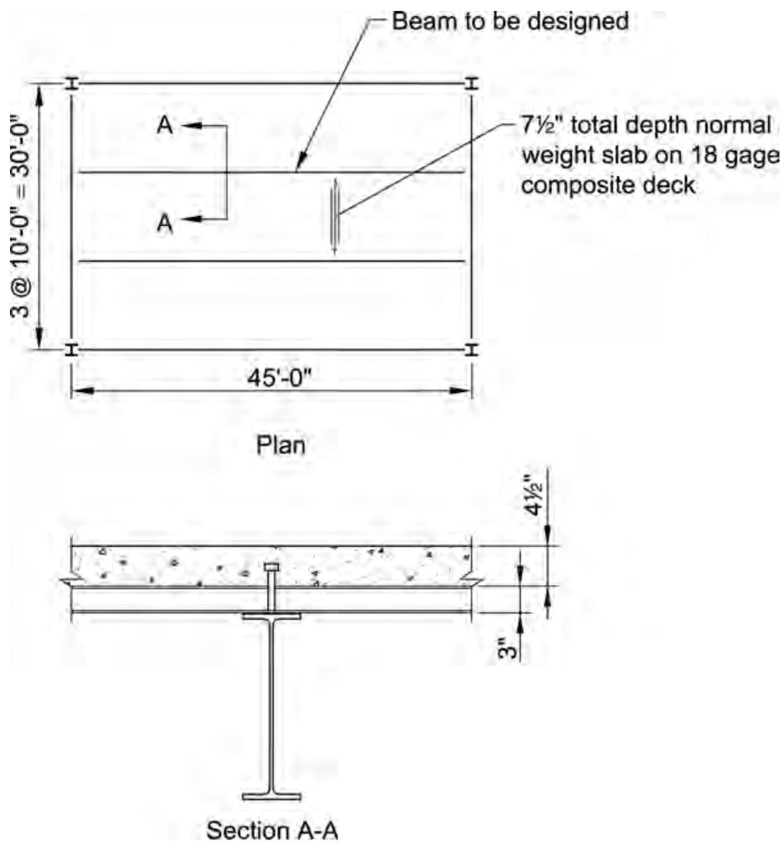


Fig. I.1-1. Composite bay and beam section.

To achieve a two-hour fire rating without the application of spray applied fire protection material to the composite deck, 4½ in. of normal weight (145 lb/ft^3) concrete will be placed above the top of the deck. The concrete has a specified compressive strength, f'_c = 4 ksi.

Applied loads are given in the following:

Dead Loads:
Pre-composite:
Slab = 75 lb/ft^2 (in accordance with metal deck manufacturer's data)
Self-weight = 5 lb/ft^2 (assumed uniform load to account for beam weight)

Composite (applied after composite action has been achieved):
Miscellaneous = 10 lb/ft^2 (HVAC, ceiling, floor covering, etc.)

Live Loads:
Pre-composite:
Construction = 25 lb/ft^2 (temporary loads during concrete placement)

Composite (applied after composite action has been achieved):
Non-reducible = 100 lb/ft^2 (assembly occupancy)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Applied Loads

For slabs that are to be placed at a constant elevation, AISC Design Guide 3 (West and Fisher, 2003) recommends an additional 10% of the nominal slab weight be applied to account for concrete ponding due to deflections resulting from the wet weight of the concrete during placement. For the slab under consideration, this would result in an additional load of 8 lb/ft^2; however, for this design the slab will be placed at a constant thickness, and thus, no additional weight for concrete ponding is required.

For pre-composite construction live loading, 25 lb/ft^2 will be applied in accordance with recommendations from *Design Loads on Structures During Construction,* ASCE/SEI 37 (ASCE, 2014), for a light duty operational class that includes concrete transport and placement by hose and finishing with hand tools.

Composite Deck and Anchor Requirements

Check composite deck and anchor requirements stipulated in AISC *Specification* Sections I1.3, I3.2c and I8.

1. Concrete Strength: $3 \text{ ksi} \leq f_c' \leq 10 \text{ ksi}$ (for normal weight concrete) (*Spec.* Section I1.3)
   $f_c' = 4 \text{ ksi}$ **o.k.**

2. Rib height: $h_r \leq 3 \text{ in.}$ (*Spec.* Section I3.2c)
   $h_r = 3 \text{ in.}$ **o.k.**

3. Average rib width: $w_r \geq 2 \text{ in.}$ (*Spec.* Section I3.2c)
   $w_r = 6 \text{ in.}$ (from deck manufacturer's literature) **o.k.**

4. Use steel headed stud anchors ¾ in. or less in diameter. (*Spec.* Section I8.1)

   Use ¾-in.-diameter steel anchors per problem statement. **o.k.**

5. Steel headed stud anchor diameter: $d_{sa} \leq 2.5t_f$ (*Spec.* Section I8.1)

   In accordance with AISC *Specification* Section I8.1, this limit only applies if steel headed stud anchors are not welded to the flange directly over the web. The ¾-in.-diameter anchors will be placed in pairs transverse to the web in some locations, thus this limit must be satisfied. Select a beam size with a minimum flange thickness of 0.300 in., as determined in the following:

$$
\begin{aligned}
t_f &\geq \frac{d_{sa}}{2.5} \\
&\geq \frac{\text{¾ in.}}{2.5} \\
&\geq 0.300 \text{ in.}
\end{aligned}
$$

6. In accordance with AISC *Specification* I3.2c, steel headed stud anchors, after installation, shall extend not less than 1½ in. above the top of the steel deck. A minimum anchor length of 4½ in. is required to meet this requirement for 3 in. deep deck. From steel headed stud anchor manufacturer's data, a standard stock length of 4⅞ in. is selected. Using a ⅜-in. length reduction to account for burn off during anchor installation through the deck yields a final installed length of 4½ in.

7. Minimum length of stud anchors $= 4d_{sa}$ (*Spec*. Section I8.2)

   $4\tfrac{1}{2}\text{ in.} > 4(\tfrac{3}{4}\text{ in.}) = 3.00\text{ in.}$ **o.k.**

8. In accordance with AISC *Specification* Section I3.2c, there shall be at least ½ in. of specified concrete cover above the top of the headed stud anchors.

   As discussed in AISC *Specification* Commentary to Section I3.2c, it is advisable to provide greater than ½ in. minimum cover to assure anchors are not exposed in the final condition, particularly for intentionally cambered beams.

   $7\tfrac{1}{2}\text{ in.} - 4\tfrac{1}{2}\text{ in.} = 3.00\text{ in.} > \tfrac{1}{2}\text{ in.}$ **o.k.**

9. In accordance with AISC *Specification* Section I3.2c, slab thickness above steel deck shall not be less than 2 in.

   $4\tfrac{1}{2}\text{ in.} > 2\text{ in.}$ **o.k.**

Design for Pre-Composite Condition

Construction (Pre-Composite) Loads

The beam is uniformly loaded by its tributary width as follows:

$$\begin{aligned} w_D &= \left[(10\text{ ft})\left(75\text{ lb/ft}^2 + 5\text{ lb/ft}^2\right)\right](1\text{ kip}/1{,}000\text{ lb}) \\ &= 0.800\text{ kip/ft} \end{aligned}$$

$$\begin{aligned} w_L &= \left[(10\text{ ft})\left(25\text{ lb/ft}^2\right)\right](1\text{ kip}/1{,}000\text{ lb}) \\ &= 0.250\text{ kip/ft} \end{aligned}$$

Construction (Pre-Composite) Flexural Strength

From ASCE/SEI 7, Chapter 2, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.800\text{ kip/ft}) + 1.6(0.250\text{ kip/ft})$ $= 1.36\text{ kip/ft}$ $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(1.36\text{ kip/ft})(45\text{ ft})^2}{8}$ $= 344\text{ kip-ft}$	$w_a = 0.800\text{ kip/ft} + 0.250\text{ kip/ft}$ $= 1.05\text{ kip/ft}$ $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(1.05\text{ kip/ft})(45\text{ ft})^2}{8}$ $= 266\text{ kip-ft}$

Beam Selection

Assume that attachment of the deck perpendicular to the beam provides adequate bracing to the compression flange during construction, thus the beam can develop its full plastic moment capacity. The required plastic section modulus, Z_x, is determined as follows, from AISC *Specification* Equation F2-1:

LRFD	ASD
$\phi_b = 0.90$ $Z_{x,min} = \dfrac{M_u}{\phi_b F_y}$ $= \dfrac{(344 \text{ kip-ft})(12 \text{ in./ft})}{0.90(50 \text{ ksi})}$ $= 91.7 \text{ in.}^3$	$\Omega_b = 1.67$ $Z_{x,min} = \dfrac{\Omega_b M_a}{F_y}$ $= \dfrac{1.67(266 \text{ kip-ft})(12 \text{ in./ft})}{50 \text{ ksi}}$ $= 107 \text{ in.}^3$

From AISC *Manual* Table 3-2, select a W21×50 with a Z_x value of 110 in.^3

Note that for the member size chosen, the self-weight on a pounds per square foot basis is $50 \text{ plf}/10 \text{ ft} = 5.00 \text{ psf}$; thus the initial self-weight assumption is adequate.

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W21×50
$A = 14.7 \text{ in.}^2$
$t_f = 0.535 \text{ in.}$
$h/t_w = 49.4$
$I_x = 984 \text{ in.}^4$

Pre-Composite Deflections

AISC Design Guide 3 (West and Fisher, 2003) recommends deflections due to concrete plus self-weight not exceed the minimum of L/360 or 1.0 in.

From AISC *Manual* Table 3-23, Case 1:

$$\Delta_{nc} = \frac{5w_D L^4}{384EI}$$

Substituting for the moment of inertia of the non-composite section, $I = 984 \text{ in.}^4$, yields a dead load deflection of:

$$\Delta_{nc} = \frac{5(0.800 \text{ kip/ft})(1 \text{ ft/12 in.})\left[(45 \text{ ft})(12 \text{ in./ft})\right]^4}{384(29{,}000 \text{ ksi})(984 \text{ in.}^4)}$$
$$= 2.59 \text{ in.}$$
$$= L/208 > L/360 \quad \textbf{n.g.}$$

Pre-composite deflections exceed the recommended limit. One possible solution is to increase the member size. A second solution is to induce camber into the member. For this example, the second solution is selected, and the beam will be cambered to reduce the net pre-composite deflections.

Reducing the estimated simple span deflections to 80% of the calculated value to reflect the partial restraint of the end connections as recommended in AISC Design Guide 3 yields a camber of:

$$\begin{aligned}\text{Camber} &= 0.8(2.59\text{ in.})\\ &= 2.07\text{ in.}\end{aligned}$$

Rounding down to the nearest ¼-in. increment yields a specified camber of 2 in.

Select a W21×50 with 2 in. of camber.

Design for Composite Condition

Required Flexural Strength

Using tributary area calculations, the total uniform loads (including pre-composite dead loads in addition to dead and live loads applied after composite action has been achieved) are determined as:

$$\begin{aligned}w_D &= \left[(10\text{ ft})\left(75\text{ lb/ft}^2 + 5\text{ lb/ft}^2 + 10\text{ lb/ft}^2\right)\right](1\text{ kip}/1{,}000\text{ lb})\\ &= 0.900\text{ kip/ft}\end{aligned}$$

$$\begin{aligned}w_L &= \left[(10\text{ ft})\left(100\text{ lb/ft}^2\right)\right](1\text{ kip}/1{,}000\text{ lb})\\ &= 1.00\text{ kip/ft}\end{aligned}$$

From ASCE/SEI 7, Chapter 2, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.900\text{ kip/ft}) + 1.6(1.00\text{ kip/ft})$ $= 2.68\text{ kip/ft}$ $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(2.68\text{ kip/ft})(45\text{ ft})^2}{8}$ $= 678\text{ kip-ft}$	$w_a = 0.900\text{ kip/ft} + 1.00\text{ kip/ft}$ $= 1.90\text{ kip/ft}$ $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(1.90\text{ kip/ft})(45\text{ ft})^2}{8}$ $= 481\text{ kip-ft}$

Determine effective width, b

The effective width of the concrete slab is the sum of the effective widths to each side of the beam centerline as determined by the minimum value of the three widths set forth in AISC *Specification* Section I3.1a:

1. one-eighth of the beam span, center-to-center of supports

$$\frac{45\text{ ft}}{8}(2\text{ sides}) = 11.3\text{ ft}$$

2. one-half the distance to the centerline of the adjacent beam

$$\frac{10\text{ ft}}{2}(2\text{ sides}) = 10.0\text{ ft} \quad \textbf{controls}$$

3. distance to the edge of the slab

   The latter is not applicable for an interior member.

Available Flexural Strength

According to AISC *Specification* Section I13.2a, the nominal flexural strength shall be determined from the plastic stress distribution on the composite section when $h/t_w \le 3.76\sqrt{E/F_y}$.

$$
\begin{aligned}
49.4 &\le 3.76\sqrt{(29{,}000 \text{ ksi})/(50 \text{ ksi})} \\
&< 90.6
\end{aligned}
$$

Therefore, use the plastic stress distribution to determine the nominal flexural strength.

According to the User Note in AISC *Specification* Section I13.2a, this check is generally unnecessary as all current W-shapes satisfy this limit for $F_y \le 70$ ksi.

Flexural strength can be determined using AISC *Manual* Table 3-19 or calculated directly using the provisions of AISC *Specification* Chapter I. This design example illustrates the use of the *Manual* table only. For an illustration of the direct calculation procedure, refer to Design Example I.2.

To utilize AISC *Manual* Table 3-19, the distance from the compressive concrete flange force to beam top flange, $Y2$, must first be determined as illustrated by *Manual* Figure 3-3. Fifty percent composite action [$\Sigma Q_n \approx 0.50(A_s F_y)$] is used to calculate a trial value of the compression block depth, a_{trial}, for determining $Y2$ as follows:

$$
\begin{aligned}
a_{trial} &= \frac{\sum Q_n}{0.85 f'_c b} \\
&= \frac{0.50(A_s F_y)}{0.85 f'_c b} \\
&= \frac{0.50(14.7 \text{ in.}^2)(50 \text{ ksi})}{0.85(4 \text{ ksi})(10 \text{ ft})(12 \text{ in./ft})} \\
&= 0.90 \text{ in.} \rightarrow \text{say } 1.00 \text{ in.}
\end{aligned}
$$

(from *Manual* Eq. 3-7)

Note that a trial value of $a = 1$ in. is a common starting point in many design problems.

$$
Y2 = Y_{con} - \frac{a_{trial}}{2}
$$

(from *Manual*. Eq 3-6)

where

Y_{con} = distance from top of steel beam to top of slab, in.
= 7.50 in.

$$
\begin{aligned}
Y2 &= 7.50 \text{ in.} - \frac{1 \text{ in.}}{2} \\
&= 7.00 \text{ in.}
\end{aligned}
$$

Enter AISC *Manual* Table 3-19 with the required strength and $Y2 = 7.00$ in. to select a plastic neutral axis location for the W21×50 that provides sufficient available strength.

Selecting PNA location 5 (BFL) with $\sum Q_n = 386$ kips provides a flexural strength of:

LRFD	ASD
$\phi_b M_n = 769\text{ kip-ft} > 678\text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = 512\text{ kip-ft} > 481\text{ kip-ft}$ **o.k.**

Based on the available flexural strength provided in Table 3-19, the required PNA location for ASD and LRFD design methodologies differ. This discrepancy is due to the live to dead load ratio in this example, which is not equal to the ratio of 3 at which ASD and LRFD design methodologies produce equivalent results as discussed in AISC *Specification* Commentary Section B3.2. The selected PNA location 5 is acceptable for ASD design, and more conservative for LRFD design.

The actual value for the compression block depth, a, is determined as follows:

$$a = \frac{\sum Q_n}{0.85 f_c' b} \qquad \text{(\textit{Manual} Eq. 3-7)}$$

$$= \frac{386\text{ kips}}{0.85(4\text{ ksi})(10\text{ ft})(12\text{ in./ft})}$$

$$= 0.946\text{ in.} < a_{trial} = 1.00\text{ in.} \quad \textbf{o.k.}$$

Live Load Deflection

Deflections due to live load applied after composite action has been achieved will be limited to $L/360$ under the design live load as required by Table 1604.3 of the *International Building Code* (IBC) (ICC, 2015), or 1 in. using a 50% reduction in design live load as recommended by AISC Design Guide 3.

Deflections for composite members may be determined using the lower bound moment of inertia provided by *Specification* Commentary Equation C-I3-1 and tabulated in AISC *Manual* Table 3-20. The *Specification* Commentary also provides an alternate method for determining deflections of a composite member through the calculation of an effective moment of inertia. This design example illustrates the use of the *Manual* table. For an illustration of the direct calculation procedure for each method, refer to Design Example I.2.

Entering Table 3-20, for a W21×50 with PNA location 5 and $Y2$ = 7.00 in., provides a lower bound moment of inertia of $I_{LB} = 2{,}520\text{ in.}^4$

Inserting I_{LB} into AISC *Manual* Table 3-23, Case 1, to determine the live load deflection under the full design live load for comparison to the IBC limit yields:

$$\Delta_c = \frac{5 w_L L^4}{384 E I_{LB}}$$

$$= \frac{5(1.00\text{ kip/ft})(1\text{ ft/12 in.})\left[(45\text{ ft})(12\text{ in./ft})\right]^4}{384(29{,}000\text{ ksi})\left(2{,}520\text{ in.}^4\right)}$$

$$= 1.26\text{ in.}$$

$$= L/429 < L/360 \quad \textbf{o.k.}$$

Performing the same check with 50% of the design live load for comparison to the AISC Design Guide 3 limit yields:

$$\Delta_c = 0.50(1.26\text{ in.})$$
$$= 0.630\text{ in.} < 1\text{ in.} \quad \textbf{o.k.}$$

Steel Anchor Strength

Steel headed stud anchor strengths are tabulated in AISC *Manual* Table 3-21 for typical conditions. Conservatively assuming that all anchors are placed in the weak position, the strength for ¾-in.-diameter anchors in normal weight concrete with $f_c' = 4$ ksi and deck oriented perpendicular to the beam is:

1 anchor per rib: $Q_n = 17.2$ kips/anchor
2 anchors per rib: $Q_n = 14.6$ kips/anchor

Number and Spacing of Anchors

Deck flutes are spaced at 12 in. on center according to the deck manufacturer's literature. The minimum number of deck flutes along each half of the 45-ft-long beam, assuming the first flute begins a maximum of 12 in. from the support line at each end, is:

$$n_{flutes} = n_{spaces} + 1$$
$$= \frac{45\text{ ft} - 2(12\text{ in.})(1\text{ ft}/12\text{ in.})}{2(1\text{ ft per space})} + 1$$
$$= 22.5 \rightarrow \text{say 22 flutes}$$

According to AISC *Specification* Section I8.2c, the number of steel headed stud anchors required between the section of maximum bending moment and the nearest point of zero moment is determined by dividing the required horizontal shear, $\sum Q_n$, by the nominal shear strength per anchor, Q_n. Assuming one anchor per flute:

$$n_{anchors} = \frac{\sum Q_n}{Q_n}$$
$$= \frac{386\text{ kips}}{17.2\text{ kips/anchor}}$$
$$= 22.4 \rightarrow \text{place 23 anchors on each side of the beam centerline}$$

As the number of anchors exceeds the number of available flutes by one, place two anchors in the first flute. The revised horizontal shear capacity of the anchors taking into account the reduced strength for two anchors in one flute is:

$$\sum Q_n = 2(14.6\text{ kips}) + 21(17.2\text{ kips})$$
$$= 390\text{ kips} > 386\text{ kips} \quad \textbf{o.k.}$$

Steel Anchor Ductility Check

As discussed in AISC *Specification* Commentary to Section I3.2d, beams are not susceptible to connector failure due to insufficient deformation capacity if they meet one or more of the following conditions:

1. Beams with span not exceeding 30 ft;
2. Beams with a degree of composite action of at least 50%; or

3. Beams with an average nominal shear connector capacity of at least 16 kips per foot along their span, corresponding to a ¾-in.-diameter steel headed stud anchor placed at 12 in. spacing on average.

The span is 45 ft, which exceeds the 30 ft limit. The percent composite action is:

$$\begin{aligned}\frac{\sum Q_n}{\min\left\{0.85 f_c' A_c, F_y A_s\right\}} &= \frac{390\text{ kips}}{\min\left\{0.85(4\text{ ksi})(10\text{ ft})(12\text{ in./ft})(4.5\text{ in.}),\ (50\text{ ksi})(14.7\text{ in.}^2)\right\}}(100)\\ &= \frac{390\text{ kips}}{735\text{ kips}}(100)\\ &= 53.1\%\end{aligned}$$

which exceeds the minimum degree of composite action of 50%. The average shear connector capacity is:

$$\frac{(42\text{ anchors})(17.2\text{ kips/anchor})+(4\text{ anchors})(14.6\text{ kips/anchor})}{45\text{ ft}} = 17.4\text{ kip/ft}$$

which exceeds the minimum capacity of 16 kips per foot. Since at least one of the conditions has been met (in fact, two have been met), the shear connectors meet the ductility requirements. The final anchor pattern chosen is illustrated in Figure I.1-2.

Review steel headed stud anchor spacing requirements of AISC *Specification* Sections I8.2d and I3.2c.

1. Maximum anchor spacing along beam [Section I8.2d(e)]:

$$\begin{aligned}8t_{slab} &= 8(7.50\text{ in.})\\ &= 60.0\text{ in.}\end{aligned}$$

   or

$$36\text{ in.}$$

   The maximum anchor spacing permitted is 36 in.

$$36\text{ in.} > 12\text{ in.} \quad \textbf{o.k.}$$

2. Minimum anchor spacing along beam [Section I8.2d(d)]:

$$\begin{aligned}4d_{sa} &= 4(¾\text{ in.})\\ &= 3.00\text{ in.} < 12\text{ in.} \quad \textbf{o.k.}\end{aligned}$$

3. Minimum transverse spacing between anchor pairs [Section I8.2d(d)]:

$$\begin{aligned}4d_{sa} &= 4(¾\text{ in.})\\ &= 3.00\text{ in.} \le 3.00\text{ in.} \quad \textbf{o.k.}\end{aligned}$$

4. Minimum distance to free edge in the direction of the horizontal shear force:

   AISC *Specification* Section I8.2d requires that the distance from the center of an anchor to a free edge in the direction of the shear force be a minimum of 8 in. for normal weight concrete slabs.

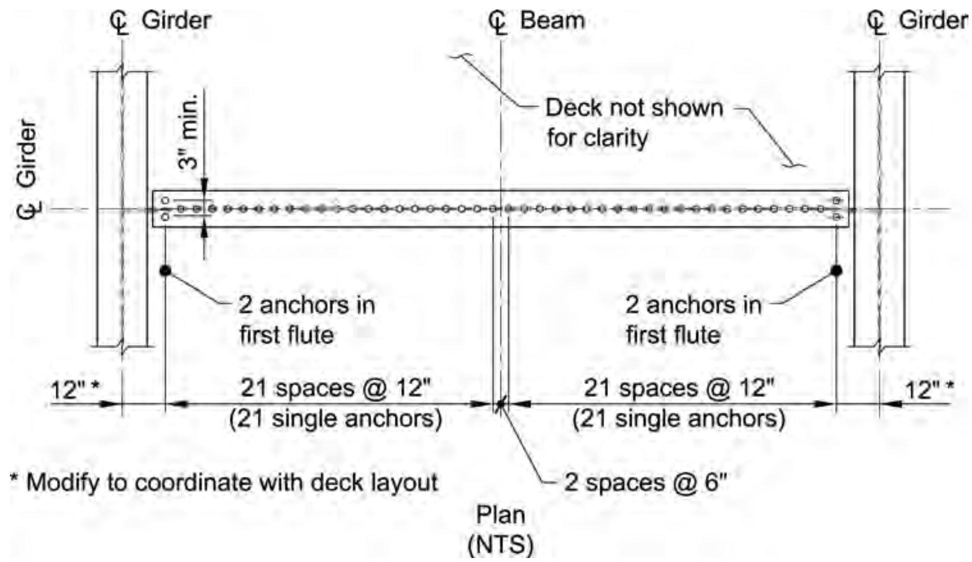


Fig. I.1-2. Steel headed stud anchor layout.

5. Maximum spacing of deck attachment:

   AISC *Specification* Section I3.2c.1(d) requires that steel deck be anchored to all supporting members at a maximum spacing of 18 in. The stud anchors are welded through the metal deck at a maximum spacing of 12 inches in this example, thus this limit is met without the need for additional puddle welds or mechanical fasteners.

Available Shear Strength

According to AISC *Specification* Section I4.2, the beam should be assessed for available shear strength as a bare steel beam using the provisions of Chapter G.

Applying the loads previously determined for the governing ASCE/SEI 7 load combinations and using available shear strengths from AISC *Manual* Table 3-2 for a W21×50 yields the following:

LRFD	ASD
$V_u = \dfrac{w_u L}{2}$ $= \dfrac{(2.68 \text{ kips/ft})(45 \text{ ft})}{2}$ $= 60.3 \text{ kips}$ $\phi_v V_n = 237 \text{ kips} > 60.3 \text{ kips}$ **o.k.**	$V_a = \dfrac{w_a L}{2}$ $= \dfrac{(1.90 \text{ kips/ft})(45 \text{ ft})}{2}$ $= 42.8 \text{ kips}$ $\dfrac{V_n}{\Omega_v} = 158 \text{ kips} > 42.8 \text{ kips}$ **o.k.**

Serviceability

Depending on the intended use of this bay, vibrations might need to be considered. Refer to AISC Design Guide 11 (Murray et al., 2016) for additional information.

Summary

From Figure I.1-2, the total number of stud anchors used is equal to (2)(2 + 21) = 46. A plan layout illustrating the final beam design is provided in Figure I.1-3. A W21×50 with 2 in. of camber and 46, ¾-in.-diameter by 4⅞-in.-long steel headed stud anchors is adequate to resist the imposed loads.

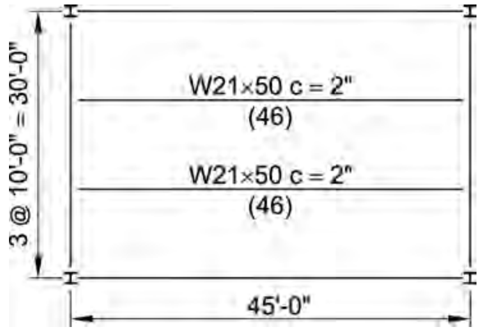


Fig. I.1-3. Revised plan.

EXAMPLE I.2 COMPOSITE GIRDER DESIGN

Given:

Two typical bays of a composite floor system are illustrated in Figure I.2-1. Select an appropriate ASTM A992 W-shaped girder and determine the required number of steel headed stud anchors. The girder will not be shored during construction. Use steel headed stud anchors made from ASTM A108 material, with F_u = 65 ksi.

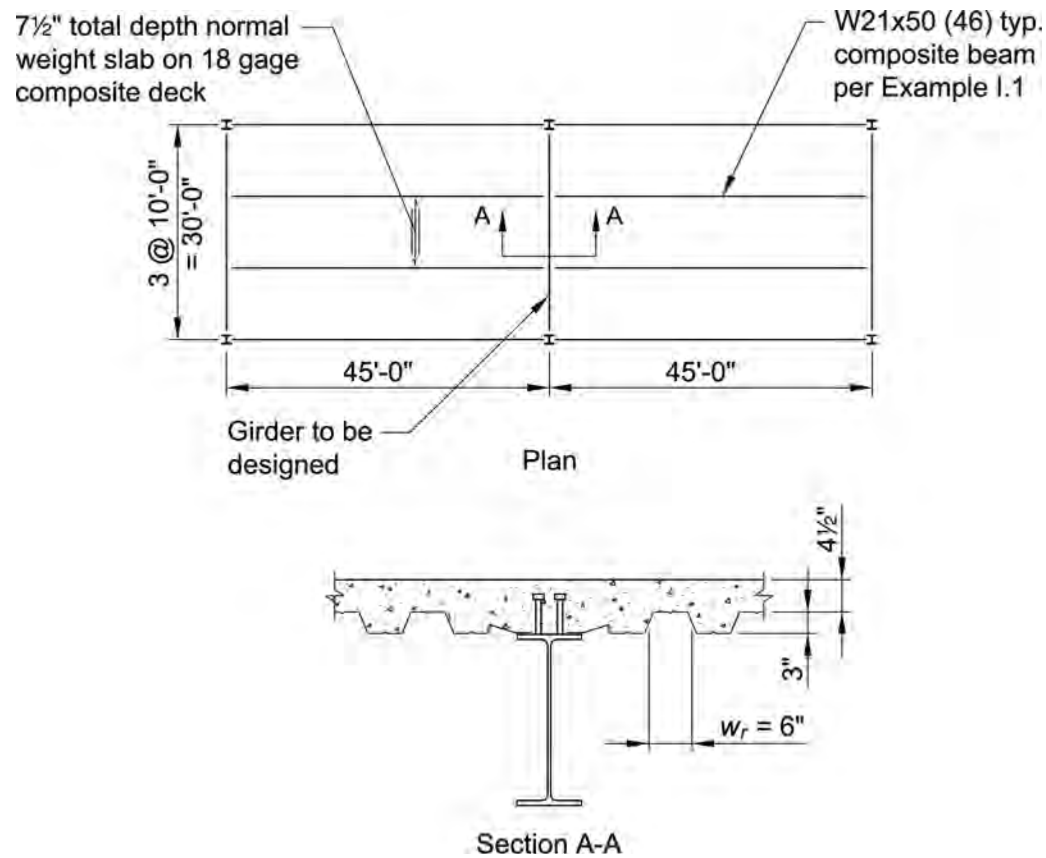


Fig. I.2-1. Composite bay and girder section.

To achieve a two-hour fire rating without the application of spray applied fire protection material to the composite deck, 4½ in. of normal weight (145 lb/ft^3) concrete will be placed above the top of the deck. The concrete has a specified compressive strength, $f_c' = 4$ ksi.

Applied loads are given in the following:

Dead Loads:
Pre-composite:
Slab = 75 lb/ft^2 (in accordance with metal deck manufacturer's data)
Self-weight = 80 lb/ft (trial girder weight)
= 50 lb/ft (beam weight from Design Example I.1)

Composite (applied after composite action has been achieved):
Miscellaneous = 10 lb/ft^2 (HVAC, ceiling, floor covering, etc.)

Live Loads:
Pre-composite:
Construction = 25 lb/ft^2 (temporary loads during concrete placement)

Composite (applied after composite action has been achieved):
Non-reducible = 100 lb/ft^2 (assembly occupancy)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Applied Loads

For slabs that are to be placed at a constant elevation, AISC Design Guide 3 (West and Fisher, 2003) recommends an additional 10% of the nominal slab weight be applied to account for concrete ponding due to deflections resulting from the wet weight of the concrete during placement. For the slab under consideration, this would result in an additional load of 8 lb/ft^2; however, for this design the slab will be placed at a constant thickness, and thus, no additional weight for concrete ponding is required.

For pre-composite construction live loading, 25 lb/ft^2 will be applied in accordance with recommendations from *Design Loads on Structures During Construction,* ASCE/SEI 37 (ASCE, 2014), for a light duty operational class that includes concrete transport and placement by hose and finishing with hand tools.

Composite Deck and Anchor Requirements

Check composite deck and anchor requirements stipulated in AISC *Specification* Sections I1.3, I3.2c and I8.

1. Concrete strength: $3 \text{ ksi} \leq f_c' \leq 10 \text{ ksi}$ (for normal weight concrete) (*Spec.* Section I1.3)

   $f_c' = 4 \text{ ksi}$ **o.k.**

2. Rib height: $h_r \leq 3 \text{ in.}$ (*Spec.* Section I3.2c)

   $h_r = 3 \text{ in.}$ **o.k.**

3. Average rib width: $w_r \geq 2 \text{ in.}$ (*Spec.* Section I3.2c)

   $w_r = 6$ in. (See Figure I.2-1) **o.k.**

4. Use steel headed stud anchors ¾ in. or less in diameter. (*Spec.* Section I8.1)

   Select ¾-in.-diameter steel anchors. **o.k.**

5. Steel headed stud anchor diameter: $d_{sa} \leq 2.5t_f$ (*Spec.* Section I8.1)

   In accordance with AISC *Specification* Section I8.1, this limit only applies if steel headed stud anchors are not welded to the flange directly over the web. The ¾-in.-diameter anchors will be attached in a staggered pattern, thus this limit must be satisfied. Select a girder size with a minimum flange thickness of 0.300 in., as determined in the following:

$$
\begin{aligned}
t_f &\geq \frac{d_{sa}}{2.5} \\
&\geq \frac{\text{¾ in.}}{2.5} \\
&\geq 0.300 \text{ in.}
\end{aligned}
$$

6. In accordance with AISC *Specification* I3.2c, steel headed stud anchors, after installation, shall extend not less than 1½ in. above the top of the steel deck. A minimum anchor length of 4½ in. is required to meet this requirement for 3-in.-deep deck. From steel headed stud anchor manufacturer's data, a standard stock length of 4⅞ in. is selected. Using a 3⁄16-in. length reduction to account for burn off during anchor installation directly to the girder flange yields a final installed length of 4 11⁄16 in.

   4 11⁄16 in. > 4½ in. **o.k.**

7. Minimum length of stud anchors = $4d_{sa}$ (*Spec.* Section I8.2)

   4 11⁄16 in. > 4(¾ in.) = 3.00 in. **o.k.**

8. In accordance with AISC *Specification* Section I3.2c, there shall be at least ½ in. of specified concrete cover above the top of the headed stud anchors.

   As discussed in the *Specification* Commentary to Section I3.2c, it is advisable to provide greater than ½-in. minimum cover to assure anchors are not exposed in the final condition.

   $7\frac{1}{2}\text{ in.} - 4\frac{11}{16}\text{ in.} = 2\frac{13}{16}\text{ in.} > \frac{1}{2}\text{ in.}$ **o.k.**

9. In accordance with AISC *Specification* Section I3.2c, slab thickness above steel deck shall not be less than 2 in.

   $4\frac{1}{2}\text{ in.} > 2\text{ in.}$ **o.k.**

Design for Pre-Composite Condition

Construction (Pre-Composite) Loads

The girder will be loaded at third points by the supported beams. Determine point loads using tributary areas.

$$\begin{aligned} P_D &= \left[(45\text{ ft})(10\text{ ft})\left(75\text{ lb/ft}^2\right)+(45\text{ ft})(50\text{ lb/ft})\right](1\text{ kip/1,000 lb}) \\ &= 36.0\text{ kips} \end{aligned}$$

$$\begin{aligned} P_L &= \left[(45\text{ ft})(10\text{ ft})\left(25\text{ lb/ft}^2\right)\right](1\text{ kip/1,000 lb}) \\ &= 11.3\text{ kips} \end{aligned}$$

Construction (Pre-Composite) Flexural Strength

From ASCE/SEI 7, Chapter 2, the required flexural strength is:

LRFD	ASD
$P_u = 1.2(36.0\text{ kips})+1.6(11.3\text{ kips})$ $= 61.3\text{ kips}$ $w_u = 1.2(80\text{ lb/ft})(1\text{ kip/1,000 lb})$ $= 0.0960\text{ kip/ft}$	$P_a = 36.0\text{ kips}+11.3\text{ kips}$ $= 47.3\text{ kips}$ $w_a = (80\text{ lb/ft})(1\text{ kip/1,000 lb})$ $= 0.0800\text{ kip/ft}$

LRFD	ASD
$M_u = P_u a + \dfrac{w_u L^2}{8}$ $= (61.3 \text{ kips})(10 \text{ ft}) + \dfrac{(0.0960 \text{ kip/ft})(30 \text{ ft})^2}{8}$ $= 624 \text{ kip-ft}$	$M_a = P_a a + \dfrac{w_a L^2}{8}$ $= (47.3 \text{ kips})(10 \text{ ft}) + \dfrac{(0.0800 \text{ kip/ft})(30 \text{ ft})^2}{8}$ $= 482 \text{ kip-ft}$

Girder Selection

Based on the required flexural strength under construction loading, a trial member can be selected utilizing AISC *Manual* Table 3-2. For the purposes of this example, the unbraced length of the girder prior to hardening of the concrete is taken as the distance between supported beams (one-third of the girder length).

Try a W24×76

$$L_b = 10 \text{ ft}$$
$$L_p = 6.78 \text{ ft}$$
$$L_r = 19.5 \text{ ft}$$

LRFD	ASD
$\phi_b BF = 22.6 \text{ kips}$ $\phi_b M_{px} = 750 \text{ kip-ft}$ $\phi_b M_{rx} = 462 \text{ kip-ft}$	$BF/\Omega_b = 15.1 \text{ kips}$ $M_{px}/\Omega_b = 499 \text{ kip-ft}$ $M_{rx}/\Omega_b = 307 \text{ kip-ft}$

Because $L_p < L_b < L_r$, use AISC *Manual* Equations 3-4a and 3-4b with $C_b = 1.0$ within the center girder segment in accordance with AISC *Manual* Table 3-1:

LRFD	ASD
From AISC *Manual* Equation 3-4a: $\phi_b M_n = C_b \left[\phi_b M_{px} - \phi_b BF(L_b - L_p)\right] \leq \phi_b M_{px}$ $= 1.0[750 \text{ kip-ft} - (22.6 \text{ kips})(10 \text{ ft} - 6.78 \text{ ft})]$ $\leq 750 \text{ kip-ft}$ $= 677 \text{ kip-ft} < 750 \text{ kip-ft}$ $= 677 \text{ kip-ft}$ $\phi_b M_n \geq M_u$ $677 \text{ kip-ft} > 624 \text{ kip-ft}$ **o.k.**	From AISC *Manual* Equation 3-4b: $\dfrac{M_n}{\Omega_b} = C_b \left[\dfrac{M_{px}}{\Omega_b} - \dfrac{BF}{\Omega_b}(L_b - L_p)\right] \leq \dfrac{M_{px}}{\Omega_b}$ $= 1.0[499 \text{ kip-ft} - (15.1 \text{ kips})(10 \text{ ft} - 6.78 \text{ ft})]$ $\leq 499 \text{ kip-ft}$ $= 450 \text{ kip-ft} < 499 \text{ kip-ft}$ $= 450 \text{ kip-ft}$ $\dfrac{M_n}{\Omega_b} \geq M_a$ $450 \text{ kip-ft} < 482 \text{ kip-ft}$ **n.g.**

For this example, the relatively low live load to dead load ratio results in a lighter member when LRFD methodology is employed. When ASD methodology is employed, a heavier member is required, and it can be shown that a W24×84 is adequate for pre-composite flexural strength. This example uses a W24×76 member to illustrate the determination of flexural strength of the composite section using both LRFD and ASD methodologies; however, this is done for comparison purposes only, and calculations for a W24×84 would be required to provide a satisfactory ASD design. Calculations for the heavier section are not shown as they would essentially be a duplication of the calculations provided for the W24×76 member.

Note that for the member size chosen, 76 lb/ft < 80 lb/ft, thus the initial weight assumption is adequate.

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W24×76
A = 22.4 in.^{2}
h/t_w = 49.0
I_x = 2,100 in.^{4}
b_f = 8.99 in.
t_f = 0.680 in.
d = 23.9 in.

Pre-Composite Deflections

AISC Design Guide 3 (West and Fisher, 2003) recommends deflections due to concrete plus self-weight not exceed the minimum of L/360 or 1.0 in.

From the superposition of AISC *Manual* Table 3-23, Cases 1 and 9:

$$\Delta_{nc} = \frac{23 P_D L^3}{648 EI} + \frac{5 w_D L^4}{384 EI}$$

Substituting for the moment of inertia of the non-composite section, I = 2,100 in.^{4}, yields a dead load deflection of:

$$\begin{aligned}\Delta_{nc} &= \frac{23(36.0 \text{ kips})\left[(30 \text{ ft})(12 \text{ in./ft})\right]^3}{648(29{,}000 \text{ ksi})(2{,}100 \text{ in.}^4)} + \frac{5(0.0760 \text{ kip/ft})(1 \text{ ft/12 in.})\left[(30 \text{ ft})(12 \text{ in./ft})\right]^4}{384(29{,}000 \text{ ksi})(2{,}100 \text{ in.}^4)} \\ &= 1.00 \text{ in.} \\ &= L/360 \quad \textbf{o.k.}\end{aligned}$$

Pre-composite deflections barely meet the recommended value. Although technically acceptable, judgment leads one to consider ways to minimize pre-composite deflections. One possible solution is to increase the member size. A second solution is to introduce camber into the member. For this example, the second solution is selected, and the girder will be cambered to reduce pre-composite deflections.

Reducing the estimated simple span deflections to 80% of the calculated value to reflect the partial restraint of the end connections as recommended in AISC Design Guide 3 yields a camber of:

$$\begin{aligned}\text{Camber} &= 0.80(1.00 \text{ in.}) \\ &= 0.800 \text{ in.}\end{aligned}$$

Rounding down to the nearest ¼-in. increment yields a specified camber of ¾ in.

Select a W24×76 with ¾ in. of camber.

Design for Composite Flexural Strength

Required Flexural Strength

Using tributary area calculations, the total applied point loads (including pre-composite dead loads in addition to dead and live loads applied after composite action has been achieved) are determined as:

$$P_D = \left[(45\text{ ft})(10\text{ ft})\left(75\text{ lb/ft}^2 + 10\text{ lb/ft}^2\right) + (45\text{ ft})(50\text{ lb/ft})\right](1\text{ kip/1,000 lb})$$
$$= 40.5\text{ kips}$$

$$P_L = \left[(45\text{ ft})(10\text{ ft})\left(100\text{ lb/ft}^2\right)\right](1\text{ kip/1,000 lb})$$
$$= 45.0\text{ kips}$$

The required flexural strength diagram is illustrated by Figure I.2-2:

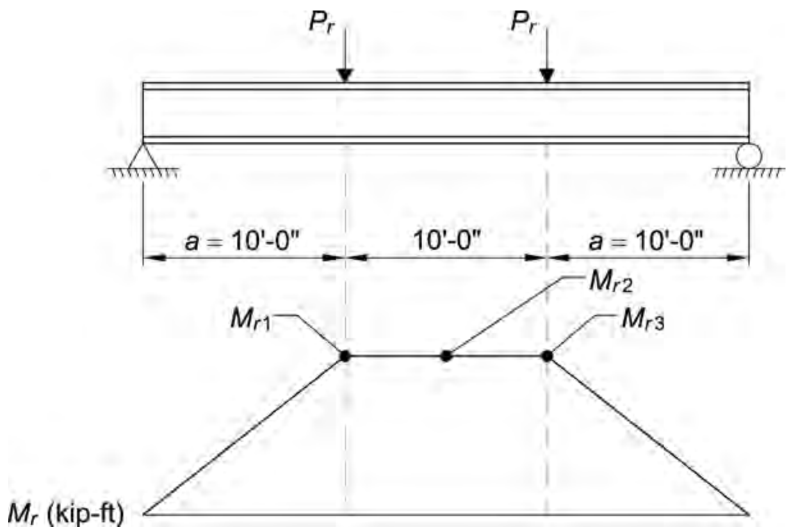


Fig. I.2-2. Required flexural strength.

From ASCE/SEI 7, Chapter 2, the required flexural strength is:

LRFD	ASD
$P_r = P_u$ $= 1.2(40.5\text{ kips}) + 1.6(45.0\text{ kips})$ $= 121\text{ kips}$ $w_u = 1.2(0.0760\text{ kip/ft})$ $= 0.0912$ kip/ft (from self weight of W24×76)	$P_r = P_a$ $= 40.5\text{ kips} + 45.0\text{ kips}$ $= 85.5\text{ kips}$ $w_a = 0.0760$ kip/ft (from self weight of W24×76)

LRFD	ASD
From AISC *Manual* Table 3-23, Case 1 and 9: $M_{u1} = M_{u3}$ $= P_u a + \dfrac{w_u a}{2}(L-a)$ $= (121\text{ kips})(10\text{ ft})$ $+ \dfrac{(0.0912\text{ kip/ft})(10\text{ ft})}{2}(30\text{ ft} - 10\text{ ft})$ $= 1{,}220\text{ kip-ft}$	From AISC *Manual* Table 3-23, Case 1 and 9: $M_{a1} = M_{a3}$ $= P_a a + \dfrac{w_a a}{2}(L-a)$ $= (85.5\text{ kips})(10\text{ ft})$ $+ \dfrac{(0.0760\text{ kip/ft})(10\text{ ft})}{2}(30\text{ ft} - 10\text{ ft})$ $= 863\text{ kip-ft}$

LRFD	ASD
$M_{u2} = P_u a + \frac{w_u L^2}{8}$ $= (121 \text{ kips})(10 \text{ ft}) + \frac{(0.0912 \text{ kip/ft})(30 \text{ ft})^2}{8}$ $= 1,220 \text{ kip-ft}$	$M_{a2} = P_a a + \frac{w_a L^2}{8}$ $= (85.5 \text{ kips})(10 \text{ ft}) + \frac{(0.0760 \text{ kip/ft})(30 \text{ ft})^2}{8}$ $= 864 \text{ kip-ft}$

Determine Effective Width, b

The effective width of the concrete slab is the sum of the effective widths to each side of the beam centerline as determined by the minimum value of the three conditions set forth in AISC *Specification* Section I3.1a:

1. one-eighth of the girder span center-to-center of supports

$$\left(\frac{30 \text{ ft}}{8}\right)(2 \text{ sides}) = 7.50 \text{ ft} \quad \textbf{controls}$$

2. one-half the distance to the centerline of the adjacent girder

$$\left(\frac{45 \text{ ft}}{2}\right)(2 \text{ sides}) = 45.0 \text{ ft}$$

3. distance to the edge of the slab

   The latter is not applicable for an interior member.

Available Flexural Strength

According to AISC *Specification* Section I3.2a, the nominal flexural strength shall be determined from the plastic stress distribution on the composite section when $h/t_w \leq 3.76\sqrt{E/F_y}$.

$$49.0 \leq 3.76\sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}}$$
$$< 90.6$$

Therefore, use the plastic stress distribution to determine the nominal flexural strength.

According to the User Note in AISC *Specification* Section I3.2a, this check is generally unnecessary as all current W-shapes satisfy this limit for $F_y \leq 70$ ksi.

AISC *Manual* Table 3-19 can be used to facilitate the calculation of flexural strength for composite beams. Alternately, the available flexural strength can be determined directly using the provisions of AISC *Specification* Chapter I. Both methods will be illustrated for comparison in the following calculations.

Method 1: AISC Manual

To utilize AISC *Manual* Table 3-19, the distance from the compressive concrete flange force to beam top flange, *Y*2, must first be determined as illustrated by *Manual* Figure 3-3. Fifty percent composite action [$\Sigma Q_n \approx 0.50(A_s F_y)$] is used to calculate a trial value of the compression block depth, a_{trial}, for determining *Y*2 as follows:

$$a_{trial} = \frac{\sum Q_n}{0.85 f'_c b}$$ (from *Manual* Eq. 3-7)

$$= \frac{0.50\left(A_s F_y\right)}{0.85 f'_c b}$$

$$= \frac{0.50\left(22.4 \text{ in.}^2\right)\left(50 \text{ ksi}\right)}{0.85\left(4 \text{ ksi}\right)\left(7.50 \text{ ft}\right)\left(12 \text{ in./ft}\right)}$$

$$= 1.83 \text{ in.}$$

$$Y2 = Y_{con} - \frac{a_{trial}}{2}$$ (from *Manual*. Eq. 3-6)

where

$$Y_{con} = \text{distance from top of steel beam to top of slab}$$
$$= 7.50 \text{ in.}$$

$$Y2 = 7.50 \text{ in.} - \frac{1.83 \text{ in.}}{2}$$
$$= 6.59 \text{ in.}$$

Enter AISC *Manual* Table 3-19 with the required strength and $Y2 = 6.59$ in. to select a plastic neutral axis location for the W24×76 that provides sufficient available strength. Based on the available flexural strength provided in Table 3-19, the required PNA location for ASD and LRFD design methodologies differ. This discrepancy is due to the live-to-dead load ratio in this example, which is not equal to the ratio of 3 at which ASD and LRFD design methodologies produce equivalent results as discussed in AISC *Specification* Commentary Section B3.2.

Selecting PNA location 5 (BFL) with $\sum Q_n = 509$ kips provides a flexural strength of:

LRFD	ASD
$\phi_b M_n = 1,240 \text{ kip-ft} > 1,220 \text{ kip-ft}$ **o.k.**	$\frac{M_n}{\Omega_b} = 823 \text{ kip-ft} < 864 \text{ kip-ft}$ **n.g.**

The selected PNA location 5 is acceptable for LRFD design, but inadequate for ASD design. For ASD design, it can be shown that a W24×76 is adequate if a higher composite percentage of approximately 60% is employed. However, as discussed previously, this beam size is not adequate for construction loading and a larger section is necessary when designing utilizing ASD.

The actual value for the compression block depth, *a*, for the chosen PNA location is determined as follows:

$$a = \frac{\sum Q_n}{0.85 f'_c b}$$ (*Manual* Eq. 3-7)

$$= \frac{509 \text{ kips}}{0.85\left(4 \text{ ksi}\right)\left(7.50 \text{ ft}\right)\left(12 \text{ in./ft}\right)}$$

$$= 1.66 \text{ in.} < a_{trial} = 1.83 \text{ in.}$$ **o.k. for LRFD design**

Method 2: Direct Calculation

According to AISC *Specification* Commentary Section I3.2a, the number and strength of steel headed stud anchors will govern the compressive force, C, for a partially composite beam. The composite percentage is based on the minimum of the limit states of concrete crushing and steel yielding as follows:

1. Concrete crushing

A_c = Area of concrete slab within effective width. Assume that the deck profile is 50% void and 50% concrete fill.

$$= b_{eff}\,(4\tfrac{1}{2}\text{ in.}) + (b_{eff}/2)(3\text{ in.})$$

$$= (7.50\text{ ft})(12\text{ in./ft})(4\tfrac{1}{2}\text{ in.}) + \left[\frac{(7.50\text{ ft})(12\text{ in./ft})}{2}\right](3\text{ in.})$$

$$= 540\text{ in.}^2$$

$$C = 0.85 f_c' A_c \qquad (Spec.\text{ Comm. Eq. C-I3-7})$$

$$= 0.85(4\text{ ksi})(540\text{ in.}^2)$$

$$= 1{,}840\text{ kips}$$

2. Steel yielding

$$C = A_s F_y \qquad (Spec.\text{ Comm. Eq. C-I3-6})$$

$$= (22.4\text{ in.}^2)(50\text{ ksi})$$

$$= 1{,}120\text{ kips}$$

3. Shear transfer

Fifty percent is used as a trial percentage of composite action as follows:

$$C = \Sigma Q_n \qquad (Spec.\text{ Comm. Eq. C-I3-8})$$

$$= 50\%\left(\min\begin{Bmatrix}1{,}840\text{ kips}\\1{,}120\text{ kips}\end{Bmatrix}\right)$$

$$= 560\text{ kips to achieve 50\% composite action}$$

Location of the Plastic Neutral Axis

The plastic neutral axis (PNA) is located by determining the axis above and below which the sum of horizontal forces is equal. This concept is illustrated in Figure I.2-3, assuming the trial PNA location is within the top flange of the girder.

$$\Sigma F_{above\ PNA} = \Sigma F_{below\ PNA}$$

$$C + x b_f F_y = (A_s - b_f x) F_y$$

Solving for x:

$$x = \frac{A_s F_y - C}{2b_f F_y}$$
$$= \frac{\left(22.4 \text{ in.}^2\right)\left(50 \text{ ksi}\right) - 560 \text{ kips}}{2\left(8.99 \text{ in.}\right)\left(50 \text{ ksi}\right)}$$
$$= 0.623 \text{ in.} < t_f = 0.680 \text{ in.; therefore, the PNA is in the flange}$$

Determine the nominal moment resistance of the composite section following the procedure in AISC *Specification* Commentary Section I3.2a, as illustrated in Figure C-I3.3.

$$a = \frac{C}{0.85 f_c' b} \quad \text{(\textit{Spec}. Comm. Eq. C-I3-9)}$$
$$= \frac{560 \text{ kips}}{0.85\left(4 \text{ ksi}\right)\left(7.50 \text{ ft}\right)\left(12 \text{ in./ft}\right)}$$
$$= 1.83 \text{ in.} < 4.50 \text{ in. (above top of deck)}$$

$$d_1 = t_{slab} - \frac{a}{2}$$
$$= 7.50 \text{ in.} - \frac{1.83 \text{ in.}}{2}$$
$$= 6.59 \text{ in.}$$

$$d_2 = \frac{x}{2}$$
$$= \frac{0.623 \text{ in.}}{2}$$
$$= 0.312 \text{ in.}$$

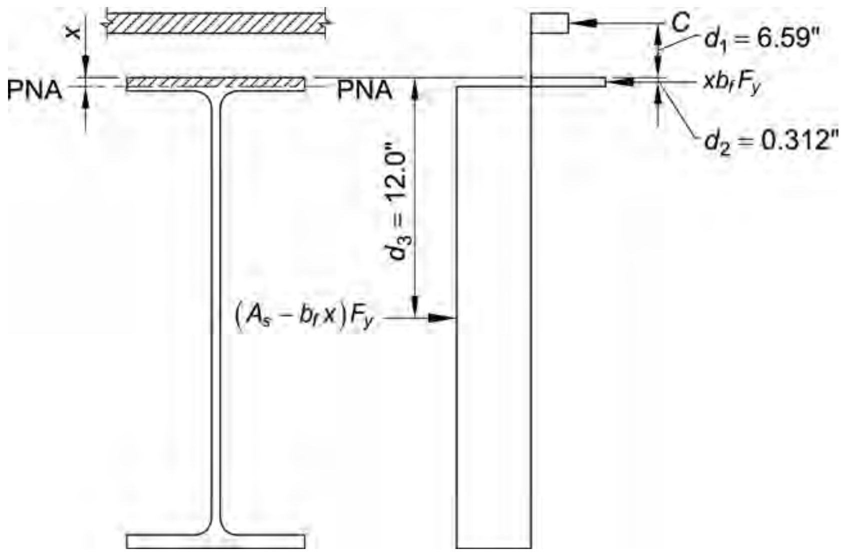


Fig. I.2-3. Plastic neutral axis location.

$$d_3 = \frac{d}{2}$$
$$= \frac{23.9 \text{ in.}}{2}$$
$$= 12.0 \text{ in.}$$

$$P_y = A_s F_y$$
$$= \left(22.4 \text{ in.}^2\right)\left(50 \text{ ksi}\right)$$
$$= 1{,}120 \text{ kips}$$

$$M_n = C\left(d_1 + d_2\right) + P_y\left(d_3 - d_2\right)$$ (*Spec*. Comm. Eq. C-I3-10)
$$= \left(560 \text{ kips}\right)\left(6.59 \text{ in.} + 0.312 \text{ in.}\right) + \left(1{,}120 \text{ kips}\right)\left(12.0 \text{ in.} - 0.312 \text{ in.}\right)$$
$$= 17{,}000 \text{ kip-in. or } 1{,}420 \text{ kip-ft}$$

Note that Equation C-I3-10 is based on the summation of moments about the centroid of the compression force in the steel; however, the same answer may be obtained by summing moments about any arbitrary point.

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90\left(1{,}420 \text{ kip-ft}\right)$ $= 1{,}280 \text{ kip-ft} > 1{,}220 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{1{,}420 \text{ kip-ft}}{1.67}$ $= 850 \text{ kip-ft} < 864 \text{ kip-ft}$ **n.g.**

As was determined previously using the Manual Tables, a W24×76 with 50% composite action is acceptable when LRFD methodology is employed, while for ASD design the beam is inadequate at this level of composite action.

Continue with the design using a W24×76 with 50% composite action.

Steel Anchor Strength

Steel headed stud anchor strengths are tabulated in AISC *Manual* Table 3-21 for typical conditions and may be calculated according to AISC *Specification* Section I8.2a as follows:

$$A_{sa} = \frac{\pi d_{sa}^2}{4}$$
$$= \frac{\pi\left(3/4 \text{ in.}\right)^2}{4}$$
$$= 0.442 \text{ in.}^2$$

$$f'_c = 4 \text{ ksi}$$

$$E_c = w_c^{1.5}\sqrt{f'_c}$$
$$= \left(145 \text{ lb/ft}^3\right)^{1.5}\sqrt{4 \text{ ksi}}$$
$$= 3{,}490 \text{ ksi}$$

$R_g = 1.0$, stud anchors welded directly to the steel shape within the slab haunch

$R_p = 0.75$, stud anchors welded directly to the steel shape

$F_u = 65$ ksi

$$Q_n = 0.5A_{sa}\sqrt{f_c'E_c} \le R_g R_p A_{sa} F_u \qquad (Spec.\ \text{Eq. I8-1})$$
$$= (0.5)\left(0.442 \text{ in.}^2\right)\sqrt{(4 \text{ ksi})(3{,}490 \text{ ksi})} \le (1.0)(0.75)\left(0.442 \text{ in.}^2\right)(65 \text{ ksi})$$
$$= 26.1 \text{ kips} > 21.5 \text{ kips}$$

Use Q_n = 21.5 kips.

Number and Spacing of Anchors

According to AISC *Specification* Section I8.2c, the number of steel headed stud anchors required between any concentrated load and the nearest point of zero moment shall be sufficient to develop the maximum moment required at the concentrated load point.

From Figure I.2-2 the moment at the concentrated load points, M_{r1} and M_{r3}, is approximately equal to the maximum beam moment, M_{r2}. The number of anchors between the beam ends and the point loads should therefore be adequate to develop the required compressive force associated with the maximum moment, C, previously determined to be 560 kips.

$$N_{anchors} = \frac{\sum Q_n}{Q_n}$$
$$= \frac{C}{Q_n}$$
$$= \frac{560 \text{ kips}}{21.5 \text{ kips/anchor}}$$
$$= 26 \text{ anchors from each end to concentrated load points}$$

In accordance with AISC *Specification* Section I8.2d, anchors between point loads should be spaced at a maximum of:

$8t_{slab} = 60.0$ in.

or 36 in. **controls**

For beams with deck running parallel to the span such as the one under consideration, spacing of the stud anchors is independent of the flute spacing of the deck. Single anchors can therefore be spaced as needed along the beam length provided a minimum longitudinal spacing of six anchor diameters in accordance with AISC *Specification* Section I8.2d is maintained. Anchors can also be placed in aligned or staggered pairs provided a minimum transverse spacing of four stud diameters = 3 in. is maintained. For this design, it was chosen to use pairs of anchors along each end of the girder to meet strength requirements and single anchors along the center section of the girder to meet maximum spacing requirements as illustrated in Figure I.2-4.

AISC *Specification* Section I8.2d requires that the distance from the center of an anchor to a free edge in the direction of the shear force be a minimum of 8 in. for normal weight concrete slabs. For simply-supported composite beams this provision could apply to the distance between the slab edge and the first anchor at each end of the beam. Assuming the slab edge is coincident to the centerline of support, Figure I.2-4 illustrates an acceptable edge distance of 9 in., though in this case the column flange would prevent breakout and negate the need for this check. The slab edge is often uniformly supported by a column flange or pour stop in typical composite construction thus preventing the possibility of a concrete breakout failure and nullifying the edge distance requirement as discussed in AISC *Specification* Commentary Section I8.3.

For this example, the minimum number of headed stud anchors required to meet the maximum spacing limit previously calculated is used within the middle third of the girder span. Note also that AISC *Specification* Section I3.2c.1(d) requires that steel deck be anchored to all supporting members at a maximum spacing of 18 in. Additionally, *Standard for Composite Steel Floor Deck-Slabs,* ANSI/SDI C1.0-2011 (SDI, 2011), requires deck attachment at an average of 12 in. but no more than 18 in.

From the previous discussion and Figure I.2-4, the total number of stud anchors used is equal to $(13)(2)+3+(13)(2)=55$. A plan layout illustrating the final girder design is provided in Figure I.2-5.

Steel Anchor Ductility Check

As discussed in AISC *Specification* Commentary Section I3.2d, beams are not susceptible to connector failure due to insufficient deformation capacity if they meet one or more of the following conditions:

(1) Beams with span not exceeding 30 ft;
(2) Beams with a degree of composite action of at least 50%; or

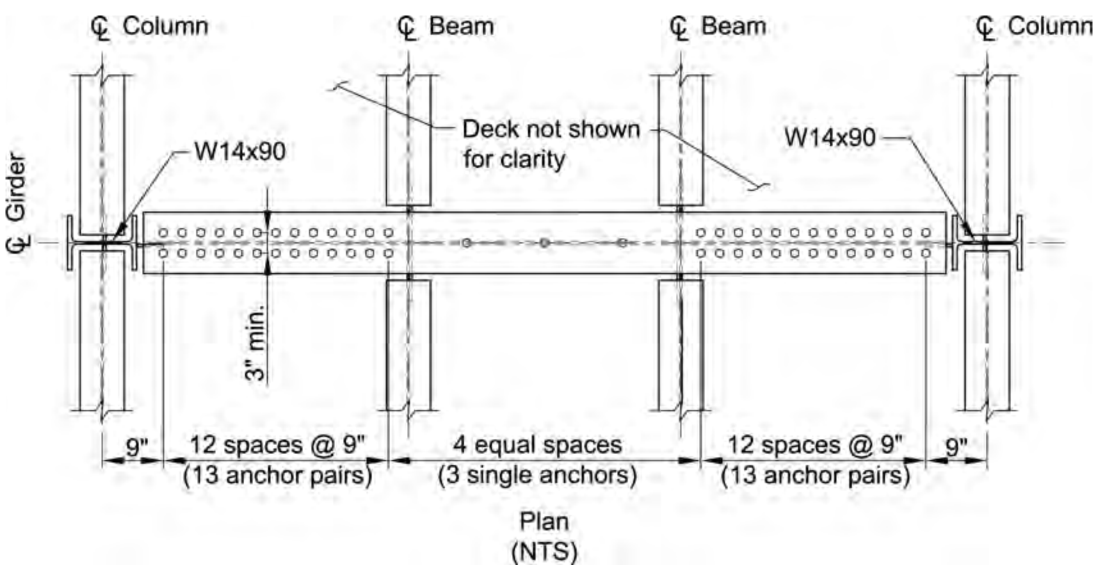


Fig. I.2-4. Steel headed stud anchor layout.

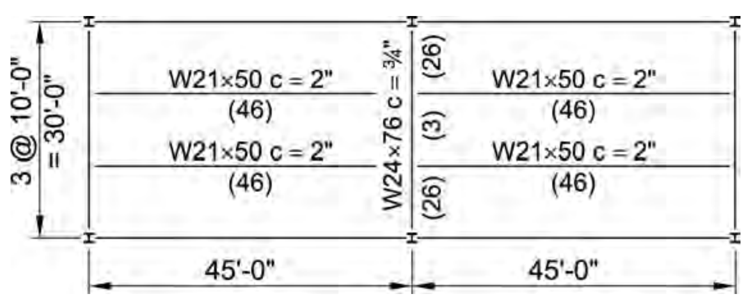


Fig. I.2-5. Revised plan.

(3) Beams with an average nominal shear connector capacity of at least 16 kips per foot along their span, corresponding to a ¾-in.-diameter steel headed stud anchor placed at 12-in. spacing on average.

The span is 30 ft, which meets the 30 ft limit. The percent composite action is:

$$\frac{\Sigma Q_n}{\min\left\{0.85 f_c' A_c, F_y A_s\right\}} = \frac{560 \text{ kips}}{\min\left\{0.85(4 \text{ ksi})\left(540 \text{ in.}^2\right),\ (50 \text{ ksi})\left(22.4 \text{ in.}^2\right)\right\}}$$
$$= \frac{560 \text{ kips}}{1{,}120 \text{ kips}}(100)$$
$$= 50.0\%$$

which meets the minimum degree of composite action of 50%. The average shear connector capacity is:

$$\frac{(55 \text{ anchors})(21.5 \text{ kips/anchor})}{30 \text{ ft}} = 39.4 \text{ kip/ft}$$

which exceeds the minimum capacity of 16 kips per foot. Because at least one of the conditions has been met (in fact, all three have been met), the shear connectors meet the ductility requirements.

Live Load Deflection Criteria

Deflections due to live load applied after composite action has been achieved will be limited to $L/360$ under the design live load as required by Table 1604.3 of the *International Building Code* (IBC) (ICC, 2015), or 1 in. using a 50% reduction in design live load as recommended by AISC Design Guide 3.

Deflections for composite members may be determined using the lower bound moment of inertia provided in AISC *Specification* Commentary Equation C-I3-1 and tabulated in AISC *Manual* Table 3-20. The *Specification* Commentary also provides an alternate method for determining deflections through the calculation of an effective moment of inertia. Both methods are acceptable and are illustrated in the following calculations for comparison purposes:

Method 1: Calculation of the lower bound moment of inertia, I_{LB}

$$I_{LB} = I_x + A_s\left(Y_{ENA} - d_3\right)^2 + \left(\frac{\Sigma Q_n}{F_y}\right)\left(2d_3 + d_1 - Y_{ENA}\right)^2 \qquad \text{(}\textit{Spec}\text{. Comm. Eq. C-I3-1)}$$

Variables d_1 and d_3 in AISC *Specification* Commentary Equation C-I3-1 are determined using the same procedure previously illustrated for calculating nominal moment resistance. However, for the determination of I_{LB} the nominal strength of steel anchors is calculated between the point of maximum positive moment and the point of zero moment as opposed to between the concentrated load and point of zero moment used previously. The maximum moment is located at the center of the span and it can be seen from Figure I.2-4 that 27 anchors are located between the midpoint of the beam and each end.

$$\begin{aligned}\Sigma Q_n &= (27\text{ anchors})(21.5\text{ kips/anchor})\\ &= 581\text{ kips}\end{aligned}$$

$$\begin{aligned}a &= \frac{C}{0.85 f_c' b} \qquad \text{(}\textit{Spec}\text{. Eq. C-I3-9)}\\ &= \frac{\Sigma Q_n}{0.85 f_c' b}\\ &= \frac{581\text{ kips}}{0.85(4\text{ ksi})(7.50\text{ ft})(12\text{ in./ft})}\\ &= 1.90\text{ in.}\end{aligned}$$

$$\begin{aligned}d_1 &= t_{slab} - \frac{a}{2}\\ &= 7.50\text{ in.} - \frac{1.90\text{ in.}}{2}\\ &= 6.55\text{ in.}\end{aligned}$$

$$\begin{aligned}x &= \frac{A_s F_y - \Sigma Q_n}{2 b_f F_y}\\ &= \frac{\left(22.4\text{ in.}^2\right)(50\text{ ksi}) - 581\text{ kips}}{2(8.99\text{ in.})(50\text{ ksi})}\\ &= 0.600\text{ in.} < t_f = 0.680\text{ in.; therefore, the PNA is within the flange}\end{aligned}$$

$$\begin{aligned}d_3 &= \frac{d}{2}\\ &= \frac{23.9\text{ in.}}{2}\\ &= 12.0\text{ in.}\end{aligned}$$

The distance from the top of the steel section to the elastic neutral axis, Y_{ENA}, for use in Equation C-I3-1 is calculated using the procedure provided in AISC *Specification* Commentary Section I3.2 as follows:

$$Y_{ENA} = \frac{A_s d_3 + \left(\frac{\Sigma Q_n}{F_y}\right)(2d_3 + d_1)}{A_s + \left(\frac{\Sigma Q_n}{F_y}\right)}$$ (*Spec*. Comm. Eq. C-I3-2)

$$= \frac{(22.4 \text{ in.}^2)(12.0 \text{ in.}) + \left(\frac{581 \text{ kips}}{50 \text{ ksi}}\right)[2(12.0 \text{ in.}) + 6.55 \text{ in.}]}{22.4 \text{ in.}^2 + \left(\frac{581 \text{ kips}}{50 \text{ ksi}}\right)}$$

$$= 18.3 \text{ in.}$$

Substituting these values into AISC *Specification* Commentary Equation C-I3-1 yields the following lower bound moment of inertia:

$$I_{LB} = 2{,}100 \text{ in.}^4 + (22.4 \text{ in.}^2)(18.3 \text{ in.} - 12.0 \text{ in.})^2 + \left(\frac{581 \text{ kips}}{50 \text{ ksi}}\right)[2(12.0 \text{ in.}) + 6.55 \text{ in.} - 18.3 \text{ in.}]^2$$

$$= 4{,}730 \text{ in.}^4$$

Alternately, this value can be determined directly from AISC *Manual* Table 3-20 as illustrated in Design Example I.1.

Method 2: Calculation of the equivalent moment of inertia, I_{equiv}

An alternate procedure for determining a moment of inertia for the deflection calculation of the composite section is presented in AISC *Specification* Commentary Section I3.2 and in the following:

Determine the transformed moment of inertia, I_{tr}

The effective width of the concrete below the top of the deck may be approximated with the deck profile resulting in a 50% effective width as depicted in Figure I.2-6. The effective width, b_{eff} = (7.50 ft)(12 in./ft) = 90.0 in.

Transformed slab widths are calculated as follows:

$$n = \frac{E_s}{E_c}$$

$$= \frac{29{,}000 \text{ ksi}}{3{,}490 \text{ ksi}}$$

$$= 8.31$$

$$b_{tr1} = \frac{b_{eff}}{n}$$

$$= \frac{90.0 \text{ in.}}{8.31}$$

$$= 10.8 \text{ in.}$$

$$b_{tr2} = \frac{0.5b_{eff}}{n}$$
$$= \frac{0.5(90.0 \text{ in.})}{8.31}$$
$$= 5.42 \text{ in.}$$

The transformed model is illustrated in Figure I.2-7.

Determine the elastic neutral axis of the transformed section (assuming fully composite action) and calculate the transformed moment of inertia using the information provided in Table I.2-1 and Figure I.2-7. For this problem, a trial location for the elastic neutral axis (ENA) is assumed to be within the depth of the composite deck.

Table I.2-1. Properties for Elastic Neutral Axis Determination of Transformed Section

Part	A, in.^{2}	y, in.	I, in.^{4}
A_1	48.6	$2.25 + x$	82.0
A_2	$5.42x$	$x/2$	$0.452x^3$
W24×76	22.4	$x - 15.0$	2,100

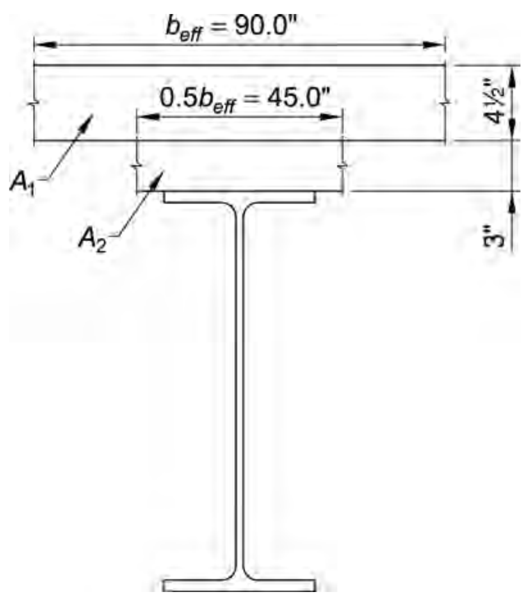


Fig. I.2-6. Effective concrete width.

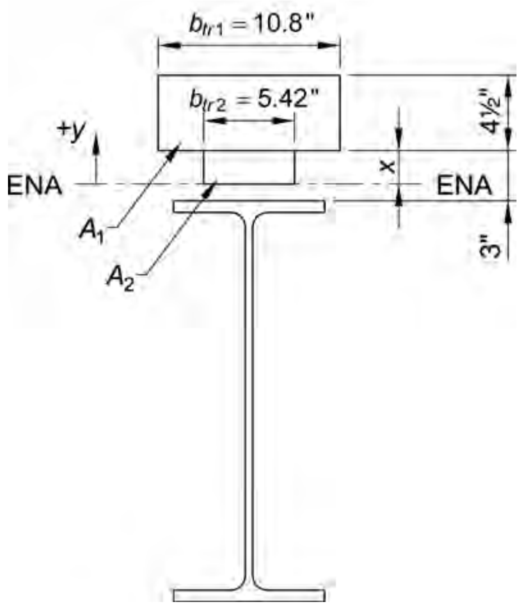


Fig. I.2-7. Transformed area model.

ΣA_y about elastic neutral axis = 0

$$\left(48.6\text{ in.}^2\right)\left(2.25\text{ in.}+x\right)+\left(5.42\text{ in.}\right)\left(\frac{x^2}{2}\right)+\left(22.4\text{ in.}^2\right)\left(x-15.0\text{ in.}\right)=0$$

Solving for *x*:

$$x = 2.88\text{ in.}$$

Verify trial location:

$2.88\text{ in.} < h_r = 3\text{ in.}$; therefore, the elastic neutral axis is within the composite deck

Utilizing the parallel axis theorem and substituting for *x* yields:

$$\begin{aligned} I_{tr} &= \Sigma I + \Sigma A y^2 \\ &= 82.0\text{ in.}^4 + \left(0.452\text{ in.}\right)\left(2.88\text{ in.}\right)^3 + 2{,}100\text{ in.}^4 + \left(48.6\text{ in.}^2\right)\left(2.25\text{ in.}+2.88\text{ in.}\right)^2 + \left(15.6\text{ in.}^2\right)\left(\frac{2.88\text{ in.}}{2}\right)^2 \\ &\quad + \left(22.4\text{ in.}^2\right)\left(2.88\text{ in.}-15.0\text{ in.}\right)^2 \\ &= 6{,}800\text{ in.}^4 \end{aligned}$$

Determine the equivalent moment of inertia, I_{equiv}

$\Sigma Q_n = 581$ kips (previously determined in Method 1)

C_f = compression force for fully composite beam previously determined to be controlled by $A_s F_y = 1{,}120$ kips

$$I_{equiv} = I_s + \sqrt{(\Sigma Q_n / C_f)}(I_{tr} - I_s)$$ (*Spec*. Comm. Eq. C-I3-3)

$$= 2{,}100 \text{ in.}^4 + \sqrt{(581 \text{ kips})/(1{,}120 \text{ kips})}\left(6{,}800 \text{ in.}^4 - 2{,}100 \text{ in.}^4\right)$$

$$= 5{,}490 \text{ in.}^4$$

Comparison of Methods and Final Deflection Calculation

I_{LB} was determined to be 4,730 in.^{4} and I_{equiv} was determined to be 5,490 in.^{4} I_{LB} will be used for the remainder of this example.

From AISC *Manual* Table 3-23, Case 9:

$$\Delta_{LL} = \frac{23P_L L^3}{648EI_{LB}}$$

$$= \frac{23(45.0 \text{ kips})\left[(30 \text{ ft})(12 \text{ in./ft})\right]^3}{648(29{,}000 \text{ ksi})\left(4{,}730 \text{ in.}^4\right)}$$

$$= 0.543 \text{ in.} < 1.00 \text{ in. (for AISC Design Guide 3 limit)} \quad \textbf{o.k.}$$

(50% reduction in design live load as allowed by Design Guide 3 was not necessary to meet this limit)

$$= L/662 < L/360 \text{ (for IBC 2015 Table 1604.3 limit)} \quad \textbf{o.k.}$$

Available Shear Strength

According to AISC *Specification* Section I4.2, the girder should be assessed for available shear strength as a bare steel beam using the provisions of Chapter G.

Applying the loads previously determined for the governing load combination of ASCE/SEI 7 and obtaining available shear strengths from AISC *Manual* Table 3-2 for a W24×76 yields the following:

LRFD	ASD
$V_u = 121 \text{ kips} + (0.0912 \text{ kip/ft})\left(\frac{30 \text{ ft}}{2}\right)$ $= 122 \text{ kips}$ $\phi_v V_n = 315 \text{ kips} > 122 \text{ kips}$ **o.k.**	$V_a = 85.5 \text{ kips} + (0.0760 \text{ kip/ft})\left(\frac{30 \text{ ft}}{2}\right)$ $= 86.6 \text{ kips}$ $\frac{V_n}{\Omega_v} = 210 \text{ kips} > 86.6 \text{ kips}$ **o.k.**

Serviceability

Depending on the intended use of this bay, vibrations might need to be considered. See AISC Design Guide 11 (Murray et al., 2016) for additional information.

It has been observed that cracking of composite slabs can occur over girder lines. The addition of top reinforcing steel transverse to the girder span will aid in mitigating this effect.

Summary

Using LRFD design methodology, it has been determined that a W24×76 with ¾ in. of camber and 55, ¾-in.-diameter by 4⅞-in.-long steel headed stud anchors as depicted in Figure I.2-4, is adequate for the imposed loads and deflection criteria. Using ASD design methodology, a W24×84 with a steel headed stud anchor layout determined using a procedure analogous to the one demonstrated in this example would be required.

EXAMPLE I.3 FILLED COMPOSITE MEMBER FORCE ALLOCATION AND LOAD TRANSFER

Given:

Refer to Figure I.3-1.

Part I: For each loading condition (a) through (c) determine the required longitudinal shear force, V_r', to be transferred between the steel section and concrete fill.

Part II: For loading condition (a), investigate the force transfer mechanisms of direct bearing, shear connection, and direct bond interaction.

The composite member consists of an ASTM A500, Grade C, HSS with normal weight (145 lb/ft^3) concrete fill having a specified concrete compressive strength, f_c' = 5 ksi. Use ASTM A36 material for the bearing plate.

Applied loading, P_r, for each condition illustrated in Figure I.3-1 is composed of the following nominal loads:

P_D = 32 kips
P_L = 84 kips

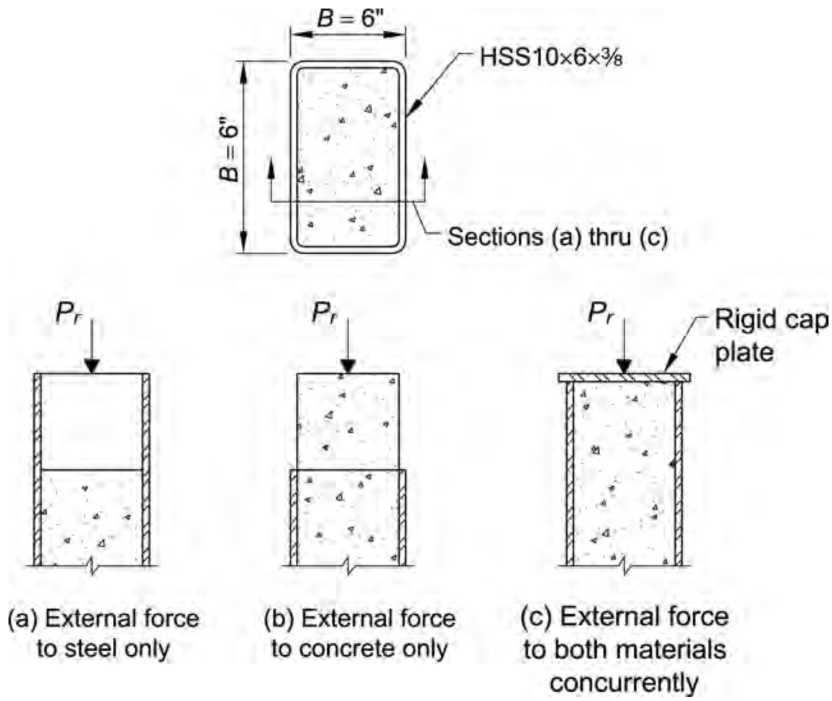


Fig. I.3-1. Filled composite member in compression.

Solution:

Part I—Force Allocation

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade C
F_y = 50 ksi
F_u = 62 ksi

From AISC *Manual* Table 1-11 and Figure I.3-1, the geometric properties are as follows:

HSS10×6×⅜
A_s = 10.4 in.^{2}
H = 10.0 in.
B = 6.00 in.
t_{nom} = ⅜ in. (nominal wall thickness)
t = 0.349 in. (design wall thickness in accordance with AISC *Specification* Section B4.2)
h/t = 25.7
b/t = 14.2

Calculate the concrete area using geometry compatible with that used in the calculation of the steel area in AISC *Manual* Table 1-11 (taking into account the design wall thickness and an outside corner radii of two times the design wall thickness in accordance with AISC *Manual* Part 1), as follows:

$$\begin{aligned} h_i &= H - 2t \\ &= 10.0 \text{ in.} - 2(0.349 \text{ in.}) \\ &= 9.30 \text{ in.} \end{aligned}$$

$$\begin{aligned} b_i &= B - 2t \\ &= 6.00 \text{ in.} - 2(0.349 \text{ in.}) \\ &= 5.30 \text{ in.} \end{aligned}$$

$$\begin{aligned} A_c &= b_i h_i - t^2(4 - \pi) \\ &= (5.30 \text{ in.})(9.30 \text{ in.}) - (0.349)^2(4 - \pi) \\ &= 49.2 \text{ in.}^2 \end{aligned}$$

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_r = P_u$ $= 1.2(32 \text{ kips}) + 1.6(84 \text{ kips})$ $= 173 \text{ kips}$	$P_r = P_a$ $= 32 \text{ kips} + 84 \text{ kips}$ $= 116 \text{ kips}$

Composite Section Strength for Force Allocation

In order to determine the composite section strength for force allocation, the member is first classified as compact, noncompact or slender in accordance with AISC *Specification* Table I1.1a.

Governing Width-to-Thickness Ratio

$$\lambda = \frac{h}{t}$$
$$= 25.7$$

The limiting width-to-thickness ratio for a compact compression steel element in a composite member subject to axial compression is:

$$\lambda_p = 2.26\sqrt{\frac{E}{F_y}}$$ (*Spec*. Table I1.1a)

$$= 2.26\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 54.4 > 25.7; \text{ therefore the HSS wall is compact}$$

The nominal axial compressive strength without consideration of length effects, P_{no}, used for force allocation calculations is therefore determined as:

$$P_{no} = P_p$$ (*Spec*. Eq. I2-9a)

$$P_p = F_y A_s + C_2 f_c'\left(A_c + A_{sr}\frac{E_s}{E_c}\right)$$ (*Spec*. Eq. I2-9b)

where

C_2 = 0.85 for rectangular sections
A_{sr} = 0 in.^{2} when no reinforcing steel is present within the HSS

$$P_{no} = F_y A_s + C_2 f_c'\left(A_c + A_{sr}\frac{E_s}{E_c}\right)$$
$$= (50 \text{ ksi})(10.4 \text{ in.}^2) + 0.85(5 \text{ ksi})(49.2 \text{ in.}^2 + 0 \text{ in.}^2)$$
$$= 729 \text{ kips}$$

Transfer Force for Condition (a)

Refer to Figure I.3-1(a). For this condition, the entire external force is applied to the steel section only, and the provisions of AISC *Specification* Section I6.2a apply.

$$V_r' = P_r\left(1 - \frac{F_y A_s}{P_{no}}\right)$$ (*Spec*. Eq. I6-1)

$$= P_r\left[1 - \frac{(50 \text{ ksi})(10.4 \text{ in.}^2)}{729 \text{ kips}}\right]$$
$$= 0.287 P_r$$

LRFD	ASD
$V_r' = 0.287(173 \text{ kips})$ $= 49.7 \text{ kips}$	$V_r' = 0.287(116 \text{ kips})$ $= 33.3 \text{ kips}$

Transfer Force for Condition (b)

Refer to Figure I.3-1(b). For this condition, the entire external force is applied to the concrete fill only, and the provisions of AISC *Specification* Section I6.2b apply.

$$V_r' = P_r\left(\frac{F_y A_s}{P_{no}}\right) \qquad (\textit{Spec}. \text{ Eq. I6-2a})$$

$$= P_r\left[\frac{(50\text{ ksi})(10.4\text{ in.}^2)}{729\text{ kips}}\right]$$

$$= 0.713P_r$$

LRFD	ASD
$V_r' = 0.713(173\text{ kips})$ $= 123\text{ kips}$	$V_r' = 0.713(116\text{ kips})$ $= 82.7\text{ kips}$

Transfer Force for Condition (c)

Refer to Figure I.3-1(c). For this condition, external force is applied to the steel section and concrete fill concurrently, and the provisions of AISC *Specification* Section I6.2c apply.

AISC *Specification* Commentary Section I6.2 states that when loads are applied to both the steel section and concrete fill concurrently, V_r' can be taken as the difference in magnitudes between the portion of the external force applied directly to the steel section and that required by Equation I6-2a and b. Using the plastic distribution approach employed in AISC *Specification* Equations I6-1 and I6-2a, this concept can be written in equation form as follows:

$$V_r' = \left|P_{rs} - P_r\left(\frac{A_s F_y}{P_{no}}\right)\right| \qquad (\text{Eq. 1})$$

where

P_{rs} = portion of external force applied directly to the steel section, kips

Note that this example assumes the external force imparts compression on the composite element as illustrated in Figure I.3-1. If the external force would impart tension on the composite element, consult the AISC *Specification* Commentary for discussion.

Currently the *Specification* provides no specific requirements for determining the distribution of the applied force for the determination of P_{rs}, so it is left to engineering judgment. For a bearing plate condition such as the one represented in Figure I.3-1(c), one possible method for determining the distribution of applied forces is to use an elastic distribution based on the material axial stiffness ratios as follows:

$$E_c = w_c^{1.5}\sqrt{f_c'}$$

$$= (145\text{ lb/ft}^3)^{1.5}\sqrt{5\text{ ksi}}$$

$$= 3{,}900\text{ ksi}$$

$$P_{rs} = \left(\frac{E_s A_s}{E_s A_s + E_c A_c}\right) P_r$$
$$= \left[\frac{(29{,}000 \text{ ksi})(10.4 \text{ in.}^2)}{(29{,}000 \text{ ksi})(10.4 \text{ in.}^2) + (3{,}900 \text{ ksi})(49.2 \text{ in.}^2)}\right] P_r$$
$$= 0.611 P_r$$

Substituting the results into Equation 1 yields:

$$V_r' = \left| 0.611 P_r - P_r \left(\frac{A_s F_y}{P_{no}}\right) \right|$$
$$= \left| 0.611 P_r - P_r \left[\frac{(10.4 \text{ in.}^2)(50 \text{ ksi})}{729 \text{ kips}}\right] \right|$$
$$= 0.102 P_r$$

LRFD	ASD
$V_r' = 0.102(173 \text{ kips})$ $= 17.6 \text{ kips}$	$V_r' = 0.102(116 \text{ kips})$ $= 11.8 \text{ kips}$

An alternate approach would be the use of a plastic distribution method whereby the load is partitioned to each material in accordance with their contribution to the composite section strength given in Equation I2-9b. This method eliminates the need for longitudinal shear transfer provided the local bearing strength of the concrete and steel are adequate to resist the forces resulting from this distribution.

Additional Discussion

- The design and detailing of the connections required to deliver external forces to the composite member should be performed according to the applicable sections of AISC *Specification* Chapters J and K. Note that for checking bearing strength on concrete confined by a steel HSS or box member, the $\sqrt{A_2 / A_1}$ term in Equation J8-2 may be taken as 2.0 according to the User Note in *Specification* Section I6.2.

- The connection cases illustrated by Figure I.3-1 are idealized conditions representative of the mechanics of actual connections. For instance, a standard shear connection welded to the face of an HSS column is an example of a condition where all external force is applied directly to the steel section only. Note that the connection configuration can also impact the strength of the force transfer mechanism as illustrated in Part II of this example.

Solution:

Part II—Load Transfer

The required longitudinal force to be transferred, V_r', determined in Part I condition (a) will be used to investigate the three applicable force transfer mechanisms of AISC *Specification* Section I6.3: direct bearing, shear connection, and direct bond interaction. As indicated in the *Specification*, these force transfer mechanisms may not be superimposed; however, the mechanism providing the greatest nominal strength may be used.

Direct Bearing

Trial Layout of Bearing Plate

For investigating the direct bearing load transfer mechanism, the external force is delivered directly to the HSS section by standard shear connections on each side of the member as illustrated in Figure I.3-2. One method for utilizing direct bearing in this instance is through the use of an internal bearing plate. Given the small clearance within the HSS section under consideration, internal access for welding is limited to the open ends of the HSS; therefore, the HSS section will be spliced at the bearing plate location. Additionally, it is a practical consideration that no more than 50% of the internal width of the HSS section be obstructed by the bearing plate in order to facilitate concrete placement. It is essential that concrete mix proportions and installation of concrete fill produce full bearing above and below the projecting plate. Based on these considerations, the trial bearing plate layout depicted in Figure I.3-2 was selected using an internal plate protrusion, L_p, of 1.0 in.

Location of Bearing Plate

The bearing plate is placed within the load introduction length discussed in AISC *Specification* Section I6.4b. The load introduction length is defined as two times the minimum transverse dimension of the HSS both above and below the load transfer region. The load transfer region is defined in *Specification* Commentary Section I6.4 as the depth of the connection. For the configuration under consideration, the bearing plate should be located within 2(B = 6 in.) = 12 in. of the bottom of the shear connection. From Figure I.3-2, the location of the bearing plate is 6 in. from the bottom of the shear connection and is therefore adequate.

Available Strength for the Limit State of Direct Bearing

The contact area between the bearing plate and concrete, A_1, may be determined as follows:

$$A_1 = A_c - (b_i - 2L_p)(h_i - 2L_p) \qquad \text{(Eq. 2)}$$

where

L_p = typical protrusion of bearing plate inside HSS
= 1.0 in.

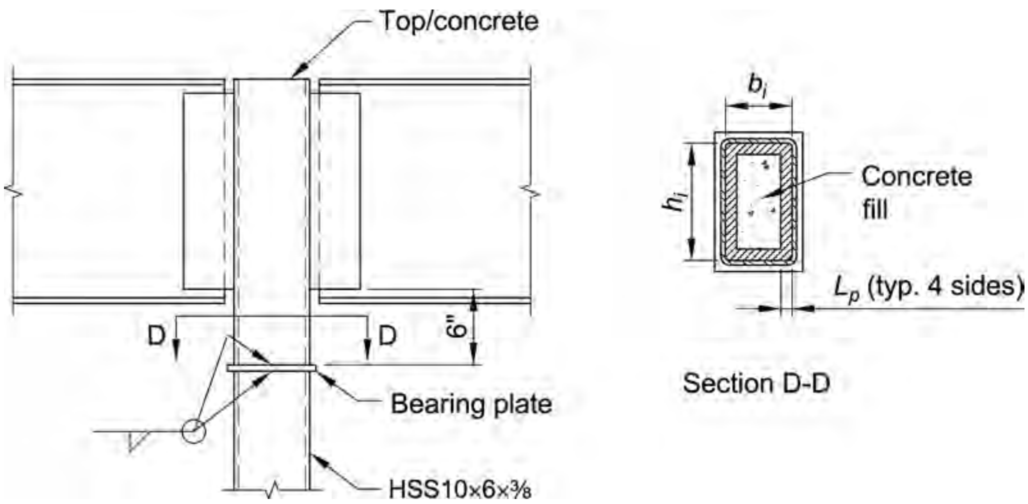


Fig. I.3-2. Internal bearing plate configuration.

Substituting for the appropriate geometric properties previously determined in Part I into Equation 2 yields:

$$A_1 = 49.2 \text{ in.}^2 - [5.30 \text{ in.} - 2(1.0 \text{ in.})][9.30 \text{ in.} - 2(1.0 \text{ in.})]$$
$$= 25.1 \text{ in.}^2$$

The available strength for the direct bearing force transfer mechanism is:

$$R_n = 1.7 f_c' A_1 \qquad (Spec. \text{ Eq. I6-3})$$

LRFD	ASD
$\phi_B = 0.65$ $\phi_B R_n = 0.65(1.7)(5 \text{ ksi})(25.1 \text{ in.}^2)$ $= 139 \text{ kips} > V_r' = 49.7 \text{ kips}$ **o.k.**	$\Omega_B = 2.31$ $\frac{R_n}{\Omega_B} = \frac{1.7(5 \text{ ksi})(25.1 \text{ in.}^2)}{2.31}$ $= 92.4 \text{ kips} > V_r' = 33.3 \text{ kips}$ **o.k.**

Required Thickness of Internal Bearing Plate

There are several methods available for determining the bearing plate thickness. For round HSS sections with circular bearing plate openings, a closed-form elastic solution such as those found in *Roark's Formulas for Stress and Strain* (Young and Budynas, 2002) may be used. Alternately, the use of computational methods such as finite element analysis may be employed.

For this example, yield line theory can be employed to determine a plastic collapse mechanism of the plate. In this case, the walls of the HSS lack sufficient stiffness and strength to develop plastic hinges at the perimeter of the bearing plate. Utilizing only the plate material located within the HSS walls, and ignoring the HSS corner radii, the yield line pattern is as depicted in Figure I.3-3.

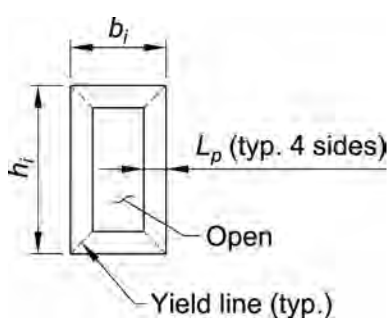


Fig. I.3-3. Yield line pattern.

Utilizing the results of the yield line analysis with $F_y = 36$ ksi plate material, the plate thickness may be determined as follows:

LRFD	ASD
$\phi = 0.90$ $$t_p = \sqrt{\frac{w_u}{2\phi F_y}\left[L_p\left(b_i + h_i\right) - \frac{8L_p^2}{3}\right]}$$ where w_u = bearing pressure on plate determined using LRFD load combinations $= \dfrac{V_r'}{A_1}$ $= \dfrac{49.7 \text{ kips}}{25.1 \text{ in.}^2}$ $= 1.98 \text{ ksi}$ $$t_p = \sqrt{\left[\frac{1.98 \text{ ksi}}{2(0.90)(36 \text{ ksi})}\right] \times \left[(1.0 \text{ in.})(5.30 \text{ in.} + 9.30 \text{ in.}) - \frac{8(1.0 \text{ in.})^2}{3}\right]}$$ $= 0.604$ in.	$\Omega = 1.67$ $$t_p = \sqrt{\frac{\Omega w_a}{2F_y}\left[L_p\left(b_i + h_i\right) - \frac{8L_p^2}{3}\right]}$$ where w_a = bearing pressure on plate determined using ASD load combinations $= \dfrac{V_r'}{A_1}$ $= \dfrac{33.3 \text{ kips}}{25.1 \text{ in.}^2}$ $= 1.33 \text{ ksi}$ $$t_p = \sqrt{\left[\frac{(1.67)(1.33 \text{ ksi})}{2(36 \text{ ksi})}\right] \times \left[(1.0 \text{ in.})(5.30 \text{ in.} + 9.30 \text{ in.}) - \frac{8(1.0 \text{ in.})^2}{3}\right]}$$ $= 0.607$ in.

Thus, select a ¾-in.-thick bearing plate.

Splice Weld

The HSS is in compression due to the imposed loads, therefore the splice weld indicated in Figure I.3-2 is sized according to the minimum weld size requirements of Chapter J. Should uplift or flexure be applied in other loading conditions, the splice should be designed to resist these forces using the applicable provisions of AISC *Specification* Chapters J and K.

Shear Connection

Shear connection involves the use of steel headed stud or channel anchors placed within the HSS section to transfer the required longitudinal shear force. The use of the shear connection mechanism for force transfer in filled HSS is usually limited to large HSS sections and built-up box shapes, and is not practical for the composite member in question. Consultation with the fabricator regarding their specific capabilities is recommended to determine the feasibility of shear connection for HSS and box members. Should shear connection be a feasible load transfer mechanism, AISC *Specification* Section I6.3b in conjunction with the steel anchors in composite component provisions of Section I8.3 apply.

Direct Bond Interaction

The use of direct bond interaction for load transfer is limited to filled HSS and depends upon the location of the load transfer point within the length of the member being considered (end or interior) as well as the number of faces to which load is being transferred.

From AISC *Specification* Section I6.3c, the nominal bond strength for a rectangular section is:

$$R_n = p_b L_{in} F_{in} \quad \text{(Spec. Eq. I6-5)}$$

where

p_b = perimeter of the steel-concrete bond interface within the composite cross section, in.

$$= (2)(10.0\text{ in.} + 6.00\text{ in.}) - (8)\left[(2)(0.349\text{ in.})\right] + (4)\left[\frac{\pi(0.349\text{ in.})}{2}\right]$$

$$= 28.6\text{ in.}$$

L_{in} = load introduction length, determined in accordance with AISC *Specification* Section I6.4

$$= 2\left[\min(B, H)\right]$$
$$= 2(6.00\text{ in.})$$
$$= 12.0\text{ in.}$$

$$F_{in} = \frac{12t}{H^2} \le 0.1\text{, ksi (for a rectangular cross section)}$$
$$= \frac{12(0.349\text{ in.})}{(10.0\text{ in.})^2} \le 0.1\text{ ksi}$$
$$= 0.0419\text{ ksi}$$

For the design of this load transfer mechanism, two possible cases will be considered:

Case 1: End Condition—Load Transferred to Member from Four Sides Simultaneously

For this case the member is loaded at an end condition (the composite member only extends to one side of the point of force transfer). Force is applied to all four sides of the section simultaneously thus allowing the full perimeter of the section to be mobilized for bond strength.

From AISC *Specification* Equation I6-5:

LRFD	ASD
$\phi = 0.50$ $\phi R_n = \phi p_b L_{in} F_{in}$ $= 0.50(28.6\text{ in.})(12.0\text{ in.})(0.0419\text{ ksi})$ $= 7.19\text{ kips} < V_r' = 49.7\text{ kips}$ **n.g.**	$\Omega = 3.00$ $\dfrac{R_n}{\Omega} = \dfrac{p_b L_{in} F_{in}}{\Omega}$ $= \dfrac{(28.6\text{ in.})(12.0\text{ in.})(0.0419\text{ ksi})}{3.00}$ $= 4.79\text{ kips} < V_r' = 33.3\text{ kips}$ **n.g.**

Bond strength is inadequate and another force transfer mechanism such as direct bearing must be used to meet the load transfer provisions of AISC *Specification* Section I6.

Alternately, the detail could be revised so that the external force is applied to both the steel section and concrete fill concurrently as schematically illustrated in Figure I.3-1(c). Comparing bond strength to the load transfer requirements for concurrent loading determined in Part I of this example yields:

LRFD	ASD
$\phi = 0.50$ $\phi R_n = 7.19 \text{ kips} < V_r' = 17.6 \text{ kips}$ **n.g.**	$\Omega = 3.00$ $\dfrac{R_n}{\Omega} = 4.79 \text{ kips} < V_r' = 11.8 \text{ kips}$ **n.g.**

Bond strength remains inadequate and another force transfer mechanism such as direct bearing must be used to meet the load transfer provisions of AISC *Specification* Section I6.

Case 2: Interior Condition—Load Transferred to Three Faces

For this case the composite member is loaded from three sides away from the end of the member (the composite member extends to both sides of the point of load transfer) as indicated in Figure I.3-4.

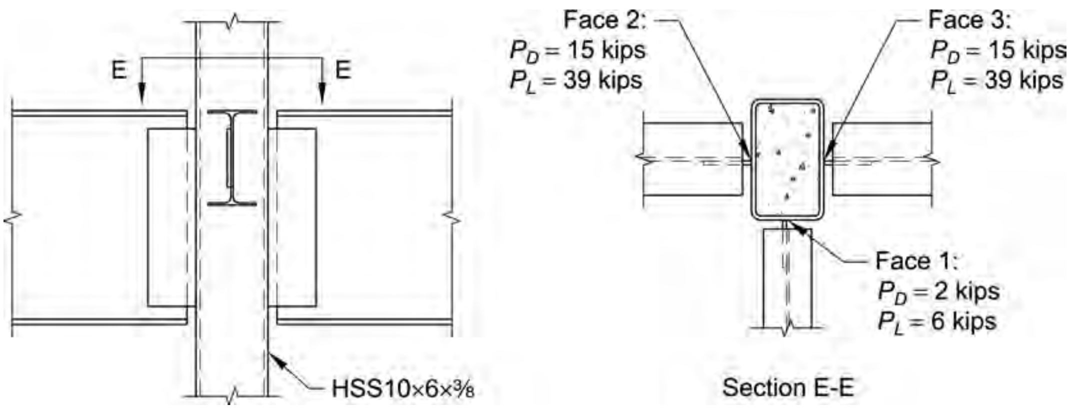


Fig. I.3-4. Case 2 load transfer.

Longitudinal shear forces to be transferred at each face of the HSS are calculated using the relationship to external forces determined in Part I of the example for condition (a) shown in Figure I.3-1, and the applicable ASCE/SEI 7 load combinations as follows:

LRFD	ASD
Face 1: $P_{r1} = P_u$ $= 1.2(2 \text{ kips}) + 1.6(6 \text{ kips})$ $= 12.0 \text{ kips}$ $V'_{r1} = 0.287P_{r1}$ $= 0.287(12.0 \text{ kips})$ $= 3.44 \text{ kips}$	Face 1: $P_{r1} = P_a$ $= 2 \text{ kips} + 6 \text{ kips}$ $= 8.00 \text{ kips}$ $V'_{r1} = 0.287P_{r1}$ $= 0.287(8.00 \text{ kips})$ $= 2.30 \text{ kips}$

LRFD	ASD
Faces 2 and 3: $P_{r2-3} = P_u$ $= 1.2(15 \text{ kips}) + 1.6(39 \text{ kips})$ $= 80.4 \text{ kips}$ $V'_{r2-3} = 0.287P_{r2-3}$ $= 0.287(80.4 \text{ kips})$ $= 23.1 \text{ kips}$	Faces 2 and 3: $P_{r2-3} = P_u$ $= 15 \text{ kips} + 39 \text{ kips}$ $= 54.0 \text{ kips}$ $V'_{r2-3} = 0.287P_{r2-3}$ $= 0.287(54.0 \text{ kips})$ $= 15.5 \text{ kips}$

Load transfer at each face of the section is checked separately for the longitudinal shear at that face using Equation I6-5 as follows:

LRFD	ASD
$\phi = 0.50$ Face 1: $p_b = 6.00 \text{ in.} - (2 \text{ corners})(2)(0.349 \text{ in.})$ $= 4.60 \text{ in.}$ $\phi R_{n1} = 0.50(4.60 \text{ in.})(12.0 \text{ in.})(0.0419 \text{ ksi})$ $= 1.16 \text{ kips} < V'_{r1} = 3.44 \text{ kips}$ **n.g.** Faces 2 and 3: $p_b = 10.0 \text{ in.} - (2 \text{ corners})(2)(0.349 \text{ in.})$ $= 8.60 \text{ in.}$ $\phi R_{n2-3} = 0.50(8.60 \text{ in.})(12.0 \text{ in.})(0.0419 \text{ ksi})$ $= 2.16 \text{ kips} < V'_{r2-3} = 23.1 \text{ kips}$ **n.g.**	$\Omega = 3.00$ Face 1: $p_b = 6.00 \text{ in.} - (2 \text{ corners})(2)(0.349 \text{ in.})$ $= 4.60 \text{ in.}$ $\dfrac{R_{n1}}{\Omega} = \dfrac{(4.60 \text{ in.})(12.0 \text{ in.})(0.0419 \text{ ksi})}{3.00}$ $= 0.771 \text{ kip} < V'_{r1} = 2.30 \text{ kips}$ **n.g.** Faces 2 and 3: $p_b = 10.0 \text{ in.} - (2 \text{ corners})(2)(0.349 \text{ in.})$ $= 8.60 \text{ in.}$ $\dfrac{R_{n2-3}}{\Omega} = \dfrac{(8.60 \text{ in.})(12.0 \text{ in.})(0.0419 \text{ ksi})}{3.00}$ $= 1.44 \text{ kips} < V'_{r2-3} = 15.5 \text{ kips}$ **n.g.**

The calculations indicate that the bond strength is inadequate for all faces, thus an alternate means of load transfer such as the use of internal bearing plates as demonstrated previously in this example is necessary.

As demonstrated by this example, direct bond interaction provides limited available strength for transfer of longitudinal shears and is generally only acceptable for lightly loaded columns or columns with low shear transfer requirements such as those with loads applied to both concrete fill and steel encasement simultaneously.

EXAMPLE I.4 FILLED COMPOSITE MEMBER IN AXIAL COMPRESSION

Given:

Determine if the filled composite member illustrated in Figure I.4-1 is adequate for the indicated dead and live loads. Table IV-1B in Part IV will be used in this example.

The composite member consists of an ASTM A500 Grade C HSS with normal weight (145 lb/ft^3) concrete fill having a specified concrete compressive strength, $f_c' = 5$ ksi.

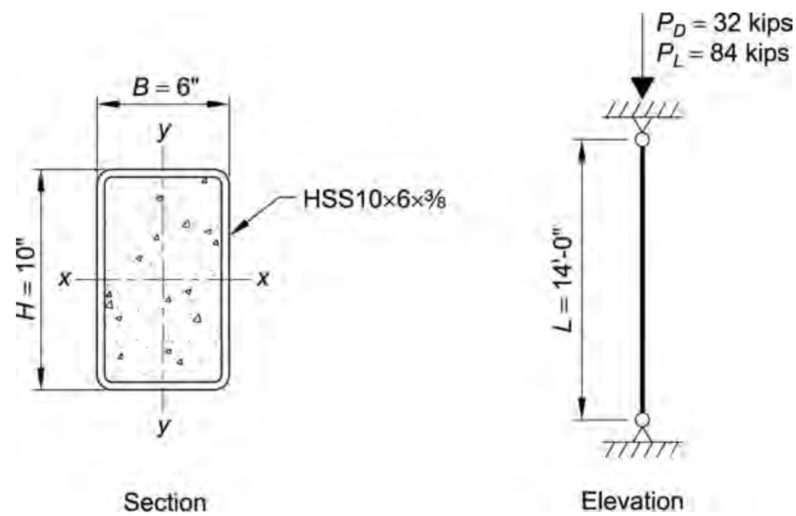


Fig. I.4-1. Filled composite member section and applied loading.

Solution:

From AISC *Manual* Table 2-4, the material properties are:

ASTM A500 Grade C
$F_y = 50$ ksi
$F_u = 62$ ksi

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_r = P_u$ $= 1.2(32\text{ kips}) + 1.6(84\text{ kips})$ $= 173\text{ kips}$	$P_r = P_a$ $= 32\text{ kips} + 84\text{ kips}$ $= 116\text{ kips}$

Method 1: AISC Tables

The most direct method of calculating the available compressive strength is through the use of Table IV-1B (Part IV of this document). A K factor of 1.0 is used for a pin-ended member. Because the unbraced length is the same in both the *x-x* and *y-y* directions, and I_x exceeds I_y, *y-y* axis buckling will govern.

Entering Table IV-1B with $L_{cy} = KL_y = 14$ ft yields:

LRFD	ASD
$\phi_c P_n = 368 \text{ kips} > 173 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 245 \text{ kips} > 116 \text{ kips}$ **o.k.**

Method 2: AISC Specification Calculations

As an alternate to using Table IV-1B, the available compressive strength can be calculated directly using the provisions of AISC *Specification* Chapter I.

From AISC *Manual* Table 1-11 and Figure I.4-1, the geometric properties of an HSS10×6×⅜ are as follows:

A_s = 10.4 in.^{2}
H = 10.0 in.
B = 6.00 in.
t_{nom} = ⅜ in. (nominal wall thickness)
t = 0.349 in. (design wall thickness)
h/t = 25.7
b/t = 14.2
I_{sx} = 137 in.^{4}
I_{sy} = 61.8 in.^{4}

As shown in Figure I.1-1, internal clear distances are determined as:

$$\begin{aligned} h_i &= H - 2t \\ &= 10.0 \text{ in.} - 2(0.349 \text{ in.}) \\ &= 9.30 \text{ in.} \end{aligned}$$

$$\begin{aligned} b_i &= B - 2t \\ &= 6.00 \text{ in.} - 2(0.349 \text{ in.}) \\ &= 5.30 \text{ in.} \end{aligned}$$

From Design Example I.3, the area of concrete, A_c, equals 49.2 in.^{2} The steel and concrete areas can be used to calculate the gross cross-sectional area as follows:

$$\begin{aligned} A_g &= A_s + A_c \\ &= 10.4 \text{ in.}^2 + 49.2 \text{ in.}^2 \\ &= 59.6 \text{ in.}^2 \end{aligned}$$

Calculate the concrete moment of inertia using geometry compatible with that used in the calculation of the steel area in AISC *Manual* Table 1-11 (taking into account the design wall thickness and corner radii of two times the design wall thickness in accordance with AISC *Manual* Part 1), the following equations may be used, based on the terminology given in Figure I-1 in the introduction to these examples:

For bending about the *x-x* axis:

$$I_{cx} = \frac{(B-4t)h_i^3}{12} + \frac{t(H-4t)^3}{6} + \frac{(9\pi^2-64)t^4}{36\pi} + \pi t^2\left(\frac{H-4t}{2} + \frac{4t}{3\pi}\right)^2$$

$$= \frac{[6.00\text{ in.} - 4(0.349\text{ in.})](9.30\text{ in.})^3}{12} + \frac{(0.349\text{ in.})[10.0\text{ in.} - 4(0.349\text{ in.})]^3}{6} + \frac{(9\pi^2-64)(0.349\text{ in.})^4}{36\pi}$$

$$+ \pi(0.349\text{ in.})^2\left[\frac{10.0\text{ in.} - 4(0.349\text{ in.})}{2} + \frac{4(0.349\text{ in.})}{3\pi}\right]^2$$

$$= 353\text{ in.}^4$$

For bending about the *y-y* axis:

$$I_{cy} = \frac{(H-4t)b_i^3}{12} + \frac{t(B-4t)^3}{6} + \frac{(9\pi^2-64)t^4}{36\pi} + \pi t^2\left(\frac{B-4t}{2} + \frac{4t}{3\pi}\right)^2$$

$$= \frac{[10.0\text{ in.} - 4(0.349\text{ in.})](5.30\text{ in.})^3}{12} + \frac{(0.349\text{ in.})[6.00\text{ in.} - 4(0.349\text{ in.})]^3}{6} + \frac{(9\pi^2-64)(0.349\text{ in.})^4}{36\pi}$$

$$+ \pi(0.349\text{ in.})^2\left[\frac{6.00\text{ in.} - 4(0.349\text{ in.})}{2} + \frac{4(0.349\text{ in.})}{3\pi}\right]^2$$

$$= 115\text{ in.}^4$$

Limitations of AISC Specification Sections I1.3 and I2.2a

(1) Concrete Strength: $3\text{ ksi} \le f_c' \le 10\text{ ksi}$

$f_c' = 5\text{ ksi}$ **o.k.**

(2) Specified minimum yield stress of structural steel: $F_y \le 75\text{ ksi}$

$F_y = 50\text{ ksi}$ **o.k.**

(3) Cross-sectional area of steel section: $A_s \ge 0.01A_g$

$$10.4\text{ in.}^2 \ge (0.01)(59.6\text{ in.}^2)$$
$$> 0.596\text{ in.}^2 \quad \textbf{o.k.}$$

There are no minimum longitudinal reinforcement requirements in the AISC *Specification* within filled composite members; therefore, the area of reinforcing bars, A_{sr}, for this example is zero.

Classify Section for Local Buckling

In order to determine the strength of the composite section subject to axial compression, the member is first classified as compact, noncompact or slender in accordance with AISC *Specification* Table I1.1a.

$$\lambda_p = 2.26\sqrt{\frac{E}{F_y}}$$
$$= 2.26\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 54.4$$

$$\lambda_{controlling} = \max\begin{Bmatrix} h/t = 25.7 \\ b/t = 14.2 \end{Bmatrix}$$
$$= 25.7$$

$\lambda_{controlling} \le \lambda_p$; therefore, the section is compact

Available Compressive Strength

The nominal axial compressive strength for compact sections without consideration of length effects, P_{no}, is determined from AISC *Specification* Section I2.2b as:

$$P_{no} = P_p \qquad \text{(Spec. Eq. I2-9a)}$$

$$P_p = F_y A_s + C_2 f_c'\left(A_c + A_{sr}\frac{E_s}{E_c}\right) \qquad \text{(Spec. Eq. I2-9b)}$$

where

C_2 = 0.85 for rectangular sections

$$P_{no} = (50\text{ ksi})(10.4\text{ in.}^2) + 0.85(5\text{ ksi})(49.2\text{ in.}^2 + 0.0\text{ in.}^2)$$
$$= 729\text{ kips}$$

Because the unbraced length is the same in both the x-x and y-y directions, the column will buckle about the weaker y-y axis (the axis having the lower moment of inertia). I_{cy} and I_{sy} will therefore be used for calculation of length effects in accordance with AISC *Specification* Sections I2.2b and I2.1b as follows:

$$C_3 = 0.45 + 3\left(\frac{A_s + A_{sr}}{A_g}\right) \le 0.9 \qquad \text{(Spec. Eq. I2-13)}$$
$$= 0.45 + 3\left(\frac{10.4\text{ in.}^2 + 0.0\text{ in.}^2}{59.6\text{ in.}^2}\right) \le 0.9$$
$$= 0.973 > 0.9$$
$$= 0.9$$

$$E_c = w_c^{1.5}\sqrt{f_c'}$$
$$= (145\text{ lb/ft}^3)^{1.5}\sqrt{5\text{ ksi}}$$
$$= 3{,}900\text{ ksi}$$

$$EI_{eff} = E_s I_{sy} + E_s I_{sr} + C_3 E_c I_{cy} \qquad \text{(from Spec. Eq. I2-12)}$$
$$= (29{,}000\text{ ksi})(61.8\text{ in.}^4) + 0\text{ kip-in.}^2 + 0.9(3{,}900\text{ ksi})(115\text{ in.}^4)$$
$$= 2{,}200{,}000\text{ kip-in.}^2$$

$$P_e = \pi^2 (EI_{eff})/(L_c)^2 \qquad \text{(Spec. Eq. I2-5)}$$

where $L_c = KL$ and K = 1.0 for a pin-ended member

$$P_e = \frac{\pi^2\left(2{,}200{,}000 \text{ kip-in.}^2\right)}{\left[(1.0)(14 \text{ ft})(12 \text{ in./ft})\right]^2}$$
$$= 769 \text{ kips}$$

$$\frac{P_{no}}{P_e} = \frac{729 \text{ kips}}{769 \text{ kips}}$$
$$= 0.948 < 2.25$$

Therefore, use AISC *Specification* Equation I2-2.

$$P_n = P_{no}\left(0.658^{\frac{P_{no}}{P_e}}\right) \qquad (Spec.\text{ Eq. I2-2})$$
$$= (729 \text{ kips})(0.658)^{0.948}$$
$$= 490 \text{ kips}$$

Check adequacy of the composite column for the required axial compressive strength:

LRFD	ASD
$\phi_c = 0.75$ $\phi_c P_n = 0.75(490 \text{ kips})$ $= 368 \text{ kips} > 173 \text{ kips}$ **o.k.**	$\Omega_c = 2.00$ $\frac{P_n}{\Omega_c} = \frac{490 \text{ kips}}{2.00}$ $= 245 \text{ kips} > 116 \text{ kips}$ **o.k.**

The values match those tabulated in Table IV-1B.

Available Compressive Strength of Bare Steel Section

Due to the differences in resistance and safety factors between composite and noncomposite column provisions, it is possible to calculate a lower available compressive strength for a composite column than one would calculate for the corresponding bare steel section. However, in accordance with AISC *Specification* Section I2.2b, the available compressive strength need not be less than that calculated for the bare steel member in accordance with Chapter E.

From AISC *Manual* Table 4-3, for an HSS10×6×⅜, KL_y = 14 ft:

LRFD	ASD
$\phi_c P_n = 331 \text{ kips} < 368 \text{ kips}$	$\frac{P_n}{\Omega_c} = 220 \text{ kips} < 245 \text{ kips}$

Thus, the composite section strength controls and is adequate for the required axial compressive strength as previously demonstrated.

Force Allocation and Load Transfer

Load transfer calculations for external axial forces should be performed in accordance with AISC *Specification* Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions is provided in Design Example I.3.

EXAMPLE I.5 FILLED COMPOSITE MEMBER IN AXIAL TENSION

Given:

Determine if the filled composite member illustrated in Figure I.5-1 is adequate for the indicated dead load compression and wind load tension. The entire load is applied to the steel section.

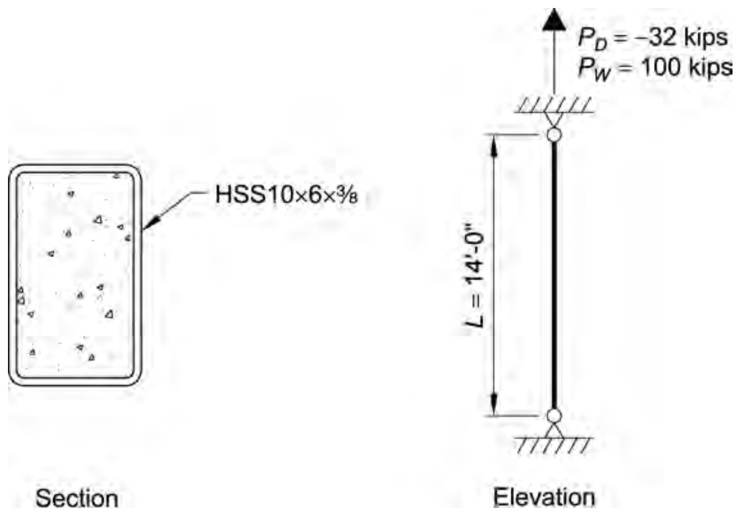


Fig. I.5-1. Filled composite member section and applied loading.

The composite member consists of an ASTM A500, Grade C, HSS with normal weight (145 lb/ft^3) concrete fill having a specified concrete compressive strength, $f_c' = 5$ ksi.

Solution:

From AISC *Manual* Table 2-4, the material properties are:

ASTM A500 Grade C
$F_y = 50$ ksi
$F_u = 62$ ksi

From AISC *Manual* Table 1-11, the geometric properties are as follows:

HSS10×6×3⁄8
$A_s = 10.4$ in.^{2}

There are no minimum requirements for longitudinal reinforcement in the AISC *Specification*; therefore, it is common industry practice to use filled shapes without longitudinal reinforcement, thus $A_{sr} = 0$.

From ASCE/SEI 7, Chapter 2, the required compressive strength is (taking compression as negative and tension as positive):

LRFD	ASD
Governing Uplift Load Combination $= 0.9D + 1.0W$ $P_r = P_u$ $= 0.9(-32 \text{ kips}) + 1.0(100 \text{ kips})$ $= 71.2$ kips	Governing Uplift Load Combination $= 0.6D + 0.6W$ $P_r = P_a$ $= 0.6(-32 \text{ kips}) + 0.6(100 \text{ kips})$ $= 40.8$ kips

Available Tensile Strength

Available tensile strength for a filled composite member is determined in accordance with AISC *Specification* Section I2.2c.

$$P_n = A_s F_y + A_{sr} F_{ysr}$$ (*Spec*. Eq. I2-14)

$$= \left(10.4 \text{ in.}^2\right)\left(50 \text{ ksi}\right) + \left(0 \text{ in.}^2\right)\left(60 \text{ ksi}\right)$$

$$= 520 \text{ kips}$$

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n = 0.90\left(520 \text{ kips}\right)$ $= 468 \text{ kips} > 71.2 \text{ kips}$ **o.k.**	$\Omega_t = 1.67$ $\dfrac{P_n}{\Omega_t} = \dfrac{520 \text{ kips}}{1.67}$ $= 311 \text{ kips} > 40.8 \text{ kips}$ **o.k.**

For filled composite HSS members with no internal longitudinal reinforcing, the values for available tensile strength may also be taken directly from AISC *Manual* Table 5-4. The values calculated here match those for the limit state of yielding shown in Table 5-4.

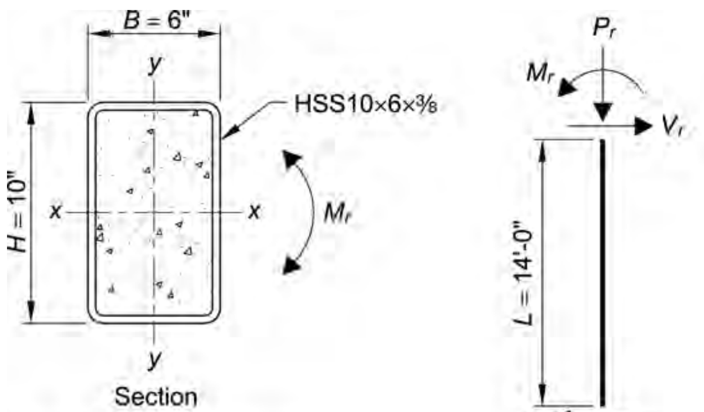
Force Allocation and Load Transfer

Load transfer calculations are not required for filled composite members in axial tension that do not contain longitudinal reinforcement, such as the one under investigation, as only the steel section resists tension.

EXAMPLE I.6 FILLED COMPOSITE MEMBER IN COMBINED AXIAL COMPRESSION, FLEXURE AND SHEAR

Given:

Using AISC design tables, determine if the filled composite member illustrated in Figure I.6-1 is adequate for the indicated axial forces, shears and moments that have been determined in accordance with the direct analysis method of AISC *Specification* Chapter C for the controlling ASCE/SEI 7 load combinations.



	LRFD	ASD
P_r (kips)	129	98.2
M_r (kip-ft)	120	54
V_r (kips)	17.1	10.3

Fig. I.6-1. Filled composite member section and member forces.

The composite member consists of an ASTM A500, Grade C, HSS with normal weight (145 lb/ft^3) concrete fill having a specified concrete compressive strength, f'_c = 5 ksi.

Solution:

From AISC *Manual* Table 2-4, the material properties are:

ASTM A500 Grade C
F_y = 50 ksi
F_u = 62 ksi

From AISC *Manual* Table 1-11 and Figure I.6-1, the geometric properties are as follows:

HSS10×6×⅜
H = 10.0 in.
B = 6.00 in.
t_{nom} = ⅜ in. (nominal wall thickness)
t = 0.349 in. (design wall thickness)
h/t = 25.7
b/t = 14.2
A_s = 10.4 in.^{2}
I_{sx} = 137 in.^{4}
I_{sy} = 61.8 in.^{4}
Z_{sx} = 33.8 in.^{3}

Additional geometric properties used for composite design are determined in Design Examples I.3 and I.4 as follows:

h_i = 9.30 in. clear distance between HSS walls (longer side)
b_i = 5.30 in. clear distance between HSS walls (shorter side)
A_c = 49.2 in.^{2} cross-sectional area of concrete fill
A_g = 59.6 in.^{2} gross cross-sectional area of composite member
A_{sr} = 0 in.^{2} area of longitudinal reinforcement
E_c = 3,900 ksi modulus of elasticity of concrete
I_{cx} = 353 in.^{4} moment of inertia of concrete fill about the *x*-*x* axis
I_{cy} = 115 in.^{4} moment of inertia of concrete fill about the *y*-*y* axis

Limitations of AISC Specification Sections I1.3 and I2.2a

(1) Concrete Strength: $3 \text{ ksi} \le f_c' \le 10 \text{ ksi}$

$f_c' = 5 \text{ ksi}$ **o.k.**

(2) Specified minimum yield stress of structural steel: $F_y \le 75 \text{ ksi}$

$F_y = 50 \text{ ksi}$ **o.k.**

(3) Cross-sectional area of steel section: $A_s \ge 0.01A_g$

$$10.4 \text{ in.}^2 \ge (0.01)\left(59.6 \text{ in.}^2\right)$$
$$> 0.596 \text{ in.}^2 \quad \textbf{o.k.}$$

Classify Section for Local Buckling

The composite member in question was shown to be compact for pure compression in Example I.4 in accordance with AISC *Specification* Table I1.1a. The section must also be classified for local buckling due to flexure in accordance with *Specification* Table I1.1b; however, since the limits for members subject to flexure are equal to or less stringent than those for members subject to compression, the member is compact for flexure.

Interaction of Axial Force and Flexure

The interaction between axial forces and flexure in composite members is governed by AISC *Specification* Section I5 which, for compact members, permits the use of the methods of Section I1.2 with the option to use the interaction equations of Section H1.1.

The strain compatibility method is a generalized approach that allows for the construction of an interaction diagram based upon the same concepts used for reinforced concrete design. Application of the strain compatibility method is required for irregular/nonsymmetrical sections, and its general application may be found in reinforced concrete design texts and will not be discussed further here.

Plastic stress distribution methods are discussed in AISC *Specification* Commentary Section I5 which provides three acceptable procedures for compact filled members. The first procedure, Method 1, invokes the interaction equations of Section H1. The second procedure, Method 2, involves the construction of a piecewise-linear interaction curve using the plastic strength equations provided in AISC *Manual* Table 6-4. The third procedure, Method 2—Simplified, is a reduction of the piecewise-linear interaction curve that allows for the use of less conservative interaction equations than those presented in Chapter H (refer to AISC *Specification* Commentary Figure C-I5.3).

For this design example, each of the three applicable plastic stress distribution procedures are reviewed and compared.

Method 1: Interaction Equations of Section H1

The most direct and conservative method of assessing interaction effects is through the use of the interaction equations of AISC *Specification* Section H1. For HSS shapes, both the available compressive and flexural strengths can be determined from Table IV-1B (included in Part IV of this document). In accordance with the direct analysis method, a K factor of 1 is used. Because the unbraced length is the same in both the x-x and y-y directions, and I_x exceeds I_y, y-y axis buckling will govern for the compressive strength. Flexural strength is determined for the x-x axis to resist the applied moment about this axis indicated in Figure I.6-1.

Entering Table IV-1B with L_{cy} = 14 ft yields:

LRFD	ASD
$\phi_c P_n = 368 \text{ kips}$ $\phi_b M_{nx} = 141 \text{ kip-ft}$ $\dfrac{P_r}{P_c} = \dfrac{P_u}{\phi_c P_n}$ $= \dfrac{129 \text{ kips}}{368 \text{ kips}}$ $= 0.351 > 0.2$ Therefore, use AISC *Specification* Equation H1-1a. $\dfrac{P_u}{\phi_c P_n} + \dfrac{8}{9}\left(\dfrac{M_u}{\phi_b M_n}\right) \le 1.0$ (from *Spec.* Eq. H1-1a) $\dfrac{129 \text{ kips}}{368 \text{ kips}} + \dfrac{8}{9}\left(\dfrac{120 \text{ kip-ft}}{141 \text{ kip-ft}}\right) \le 1.0$ $1.11 > 1.0$ **n.g.**	$P_n/\Omega_c = 245 \text{ kips}$ $M_{nx}/\Omega_b = 93.5 \text{ kip-ft}$ $\dfrac{P_r}{P_c} = \dfrac{P_a}{P_n/\Omega_c}$ $= \dfrac{98.2 \text{ kips}}{245 \text{ kips}}$ $= 0.401 > 0.2$ Therefore, use AISC *Specification* Equation H1-1a. $\dfrac{P_a}{P_n/\Omega_c} + \dfrac{8}{9}\left(\dfrac{M_a}{M_n/\Omega_b}\right) \le 1.0$ (from *Spec.* Eq. H1-1a) $\dfrac{98.2 \text{ kips}}{245 \text{ kips}} + \dfrac{8}{9}\left(\dfrac{54 \text{ kip-ft}}{93.5 \text{ kip-ft}}\right) \le 1.0$ $0.914 < 1.0$ **o.k.**

Using LRFD methodology, Method 1 indicates that the section is inadequate for the applied loads. The designer can elect to choose a new section that passes the interaction check or re-analyze the current section using a less conservative design method such as Method 2. The use of Method 2 is illustrated in the following section. Using ASD methodology, Method 1 indicates that the section is adequate for the applied loads.

Method 2: Interaction Curves from the Plastic Stress Distribution Model

The procedure for creating an interaction curve using the plastic stress distribution model is illustrated graphically in Figure I.6-2.

Referencing Figure I.6-2, the nominal strength interaction surface A, B, C, D, E is first determined using the equations provided in AISC *Manual* Table 6-4. This curve is representative of the short column member strength without consideration of length effects. A slenderness reduction factor, λ, is then calculated and applied to each point to create surface A', B', C', D', E'. The appropriate resistance or safety factors are then applied to create the design surface A'', B'', C'', D'', E''. Finally, the required axial and flexural strengths from the applicable load combinations of ASCE/SEI 7 are plotted on the design surface, and the member is acceptable for the applied loading if all points fall within the design surface. These steps are illustrated in detail by the following calculations.

Step 1: Construct nominal strength interaction surface A, B, C, D, E without length effects

Using the equations provided in AISC *Manual* Table 6-4 for bending about the x-x axis yields:

Point A (pure axial compression):

$$P_A = F_y A_s + 0.85 f_c' A_c$$
$$= (50 \text{ ksi})(10.4 \text{ in.}^2) + 0.85(5 \text{ ksi})(49.2 \text{ in.}^2)$$
$$= 729 \text{ kips}$$

$$M_A = 0 \text{ kip-ft}$$

Point D (maximum nominal moment strength):

$$P_D = \frac{0.85 f_c' A_c}{2}$$
$$= \frac{0.85(5 \text{ ksi})(49.2 \text{ in.}^2)}{2}$$
$$= 105 \text{ kips}$$

$$Z_{sx} = 33.8 \text{ in.}^3$$

$$r_i = t$$
$$= 0.349 \text{ in.}$$

$$Z_c = \frac{b_i h_i^2}{4} - 0.429 r_i^2 h_i + 0.192 r_i^3$$
$$= \frac{(5.30 \text{ in.})(9.30 \text{ in.})^2}{4} - 0.429(0.349 \text{ in.})^2(9.30 \text{ in.}) + 0.192(0.349 \text{ in.})^3$$
$$= 114 \text{ in.}^3$$

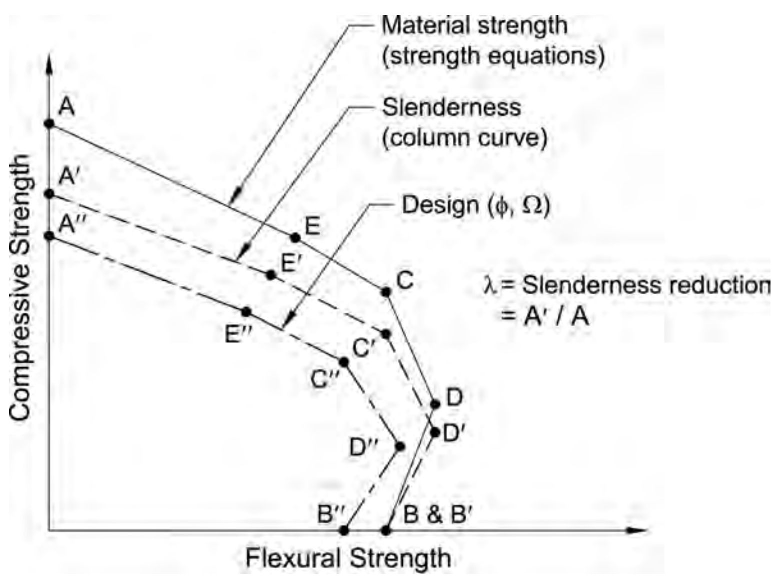


Fig. I.6-2. Interaction diagram for composite beam-column—Method 2.

$$M_D = F_y Z_{sx} + \frac{0.85 f_c' Z_c}{2}$$

$$= \left[(50 \text{ ksi})(33.8 \text{ in.}^3) + \frac{0.85(5 \text{ ksi})(114 \text{ in.}^3)}{2} \right] \left(\frac{1}{12 \text{ in./ft}} \right)$$

$$= 161 \text{ kip-ft}$$

Point B (pure flexure):

$$P_B = 0 \text{ kips}$$

$$h_n = \frac{0.85 f_c' A_c}{2(0.85 f_c' b_i + 4F_y t)} \leq \frac{h_i}{2}$$

$$= \frac{0.85(5 \text{ ksi})(49.2 \text{ in.}^2)}{2[0.85(5 \text{ ksi})(5.30 \text{ in.}) + 4(50 \text{ ksi})(0.349 \text{ in.})]} \leq \frac{9.30 \text{ in.}}{2}$$

$$= 1.13 \text{ in.} < 4.65 \text{ in.}$$

$$= 1.13 \text{ in.}$$

$$Z_{sn} = 2th_n^2$$

$$= 2(0.349 \text{ in.})(1.13 \text{ in.})^2$$

$$= 0.891 \text{ in.}^3$$

$$Z_{cn} = b_i h_n^2$$

$$= (5.30 \text{ in.})(1.13 \text{ in.})^2$$

$$= 6.77 \text{ in.}^3$$

$$M_B = M_D - F_y Z_{sn} - 0.85 f_c' \left(\frac{Z_{cn}}{2} \right)$$

$$= 161 \text{ kip-ft} - (50 \text{ ksi})(0.891 \text{ in.}^3) \left(\frac{1}{12 \text{ in./ft}} \right) - 0.85(5 \text{ ksi}) \left(\frac{6.77 \text{ in.}^3}{2} \right) \left(\frac{1}{12 \text{ in./ft}} \right)$$

$$= 156 \text{ kip-ft}$$

Point C (intermediate point):

$$P_C = 0.85 f_c' A_c$$

$$= 0.85(5 \text{ ksi})(49.2 \text{ in.}^2)$$

$$= 209 \text{ kips}$$

$$M_C = M_B$$

$$= 156 \text{ kip-ft}$$

Point E (optional):

Point E is an optional point that helps better define the interaction curve.

$$h_E = \frac{h_n}{2} + \frac{H}{4} \quad \text{where } h_n = 1.13 \text{ in. from Point B}$$
$$= \frac{1.13 \text{ in.}}{2} + \frac{10.0 \text{ in.}}{4}$$
$$= 3.07 \text{ in.}$$

$$P_E = \frac{0.85 f_c' A_c}{2} + 0.85 f_c' b_i h_E + 4 F_y t h_E$$
$$= \frac{0.85(5 \text{ ksi})(49.2 \text{ in.}^2)}{2} + 0.85(5 \text{ ksi})(5.30 \text{ in.})(3.07 \text{ in.}) + 4(50 \text{ ksi})(0.349 \text{ in.})(3.07 \text{ in.})$$
$$= 388 \text{ kips}$$

$$Z_{cE} = b_i h_E^2$$
$$= (5.30 \text{ in.})(3.07 \text{ in.})^2$$
$$= 50.0 \text{ in.}^3$$

$$Z_{sE} = 2 t h_E^2$$
$$= 2(0.349 \text{ in.})(3.07 \text{ in.})^2$$
$$= 6.58 \text{ in.}^3$$

$$M_E = M_D - F_y Z_{sE} - \frac{0.85 f_c' Z_{cE}}{2}$$
$$= 161 \text{ kip-ft} - (50 \text{ ksi})(6.58 \text{ in.}^3)\left(\frac{1}{12 \text{ in./ft}}\right) - \left[\frac{0.85(5 \text{ ksi})(50.0 \text{ in.}^3)}{2}\right]\left(\frac{1}{12 \text{ in./ft}}\right)$$
$$= 125 \text{ kip-ft}$$

The calculated points are plotted to construct the nominal strength interaction surface without length effects as depicted in Figure I.6-3.

Step 2: Construct nominal strength interaction surface A′, B′, C′, D′, E′ with length effects

The slenderness reduction factor, λ, is calculated for Point A using AISC *Specification* Section I2.2 in accordance with *Specification* Commentary Section I5.

$$P_{no} = P_A$$
$$= 729 \text{ kips}$$

$$C_3 = 0.45 + 3\left(\frac{A_s + A_{sr}}{A_g}\right) \le 0.9 \qquad \text{(Spec. Eq. I2-13)}$$
$$= 0.45 + 3\left(\frac{10.4 \text{ in.}^2 + 0 \text{ in.}^2}{59.6 \text{ in.}^2}\right) \le 0.9$$
$$= 0.973 > 0.9$$
$$= 0.9$$

$$EI_{eff} = E_s I_{sy} + E_s I_{sr} + C_3 E_c I_{cy}$$ (from *Spec*. Eq. I2-12)

$$= (29{,}000 \text{ ksi})(61.8 \text{ in.}^4) + 0 + 0.9(3{,}900 \text{ ksi})(115 \text{ in.}^4)$$

$$= 2{,}200{,}000 \text{ ksi}$$

$P_e = \pi^2 (EI_{eff})/(L_c)^2$, where $L_c = KL$ and $K = 1.0$ in accordance with the direct analysis method (*Spec*. Eq. I2-5)

$$= \frac{\pi^2 (2{,}200{,}000 \text{ ksi})}{[(14 \text{ ft})(12 \text{ in./ft})]^2}$$

$$= 769 \text{ kips}$$

$$\frac{P_{no}}{P_e} = \frac{729 \text{ kips}}{769 \text{ kips}}$$

$$= 0.948 < 2.25$$

Use AISC *Specification* Equation I2-2.

$$P_n = P_{no}\left(0.658^{\frac{P_{no}}{P_e}}\right)$$ (*Spec*. Eq. I2-2)

$$= (729 \text{ kips})(0.658)^{0.948}$$

$$= 490 \text{ kips}$$

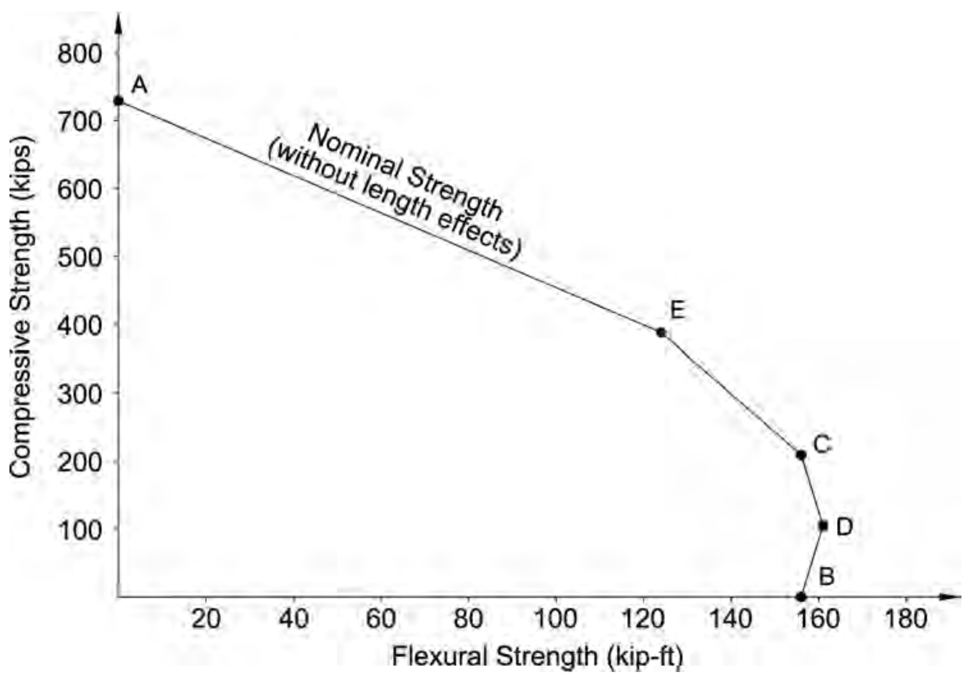


Fig. I.6-3. Nominal strength interaction surface without length effects.

From AISC *Specification* Commentary Section I5:

$$\lambda = \frac{P_n}{P_{no}}$$
$$= \frac{490 \text{ kips}}{729 \text{ kips}}$$
$$= 0.672$$

In accordance with AISC *Specification* Commentary Section I5, the same slenderness reduction is applied to each of the remaining points on the interaction surface as follows:

$$P_{A'} = \lambda P_A$$
$$= 0.672(729 \text{ kips})$$
$$= 490 \text{ kips}$$

$$P_{B'} = \lambda P_B$$
$$= 0.672(0 \text{ kip})$$
$$= 0 \text{ kip}$$

$$P_{C'} = \lambda P_C$$
$$= 0.672(209 \text{ kips})$$
$$= 140 \text{ kips}$$

$$P_{D'} = \lambda P_D$$
$$= 0.672(105 \text{ kips})$$
$$= 70.6 \text{ kips}$$

$$P_{E'} = \lambda P_E$$
$$= 0.672(388 \text{ kips})$$
$$= 261 \text{ kips}$$

The modified axial strength values are plotted with the flexural strength values previously calculated to construct the nominal strength interaction surface including length effects. These values are superimposed on the nominal strength surface not including length effects for comparison purposes in Figure I.6-4.

Step 3: Construct design interaction surface A″, B″, C″, D″, E″ and verify member adequacy

The final step in the Method 2 procedure is to reduce the interaction surface for design using the appropriate resistance or safety factors.

LRFD	ASD
Design compressive strength: $\phi_c = 0.75$ $P_{X''} = \phi_c P_{X'}$, where X = A, B, C, D or E	Allowable compressive strength: $\Omega_c = 2.00$ $P_{X''} = P_{X'} / \Omega_c$, where X = A, B, C, D or E

LRFD	ASD
$P_{A''} = 0.75(490 \text{ kips})$ $= 368 \text{ kips}$	$P_{A''} = 490 \text{ kips}/2.00$ $= 245 \text{ kips}$
$P_{B''} = 0.75(0 \text{ kip})$ $= 0 \text{ kip}$	$P_{B''} = 0 \text{ kip}/2.00$ $= 0 \text{ kip}$
$P_{C''} = 0.75(140 \text{ kips})$ $= 105 \text{ kips}$	$P_{C''} = 140 \text{ kips}/2.00$ $= 70.0 \text{ kips}$
$P_{D''} = 0.75(70.6 \text{ kips})$ $= 53.0 \text{ kips}$	$P_{D''} = 70.6 \text{ kips}/2.00$ $= 35.3 \text{ kips}$
$P_{E''} = 0.75(261 \text{ kips})$ $= 196 \text{ kips}$	$P_{E''} = 261 \text{ kips}/2.00$ $= 131 \text{ kips}$

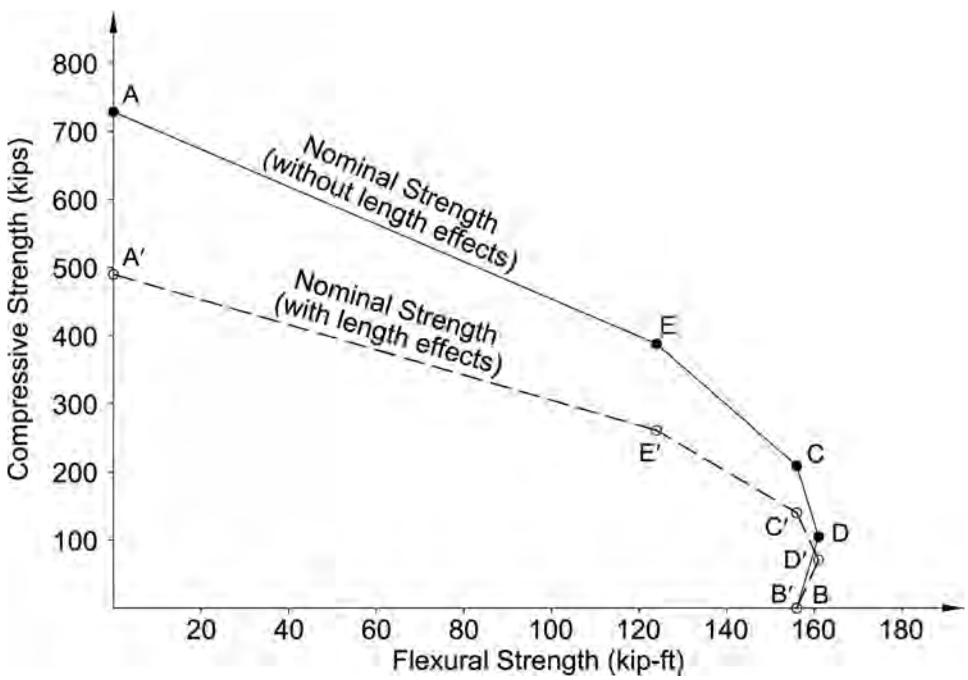


Fig. I.6-4. Nominal strength interaction surfaces (with and without length effects).

LRFD	ASD
Design flexural strength: $\phi_b = 0.90$ $M_{X''} = \phi_b M_X$, where X = A, B, C, D or E $M_{A''} = 0.90(0 \text{ kip-ft})$ $= 0 \text{ kip-ft}$ $M_{B''} = 0.90(156 \text{ kip-ft})$ $= 140 \text{ kip-ft}$ $M_{C''} = 0.90(156 \text{ kip-ft})$ $= 140 \text{ kip-ft}$ $M_{D''} = 0.90(161 \text{ kip-ft})$ $= 145 \text{ kip-ft}$ $M_{E''} = 0.90(124 \text{ kip-ft})$ $= 112 \text{ kip-ft}$	Allowable flexural strength: $\Omega_b = 1.67$ $M_{X''} = M_X / \Omega_b$, where X = A, B, C, D or E $M_{A''} = 0 \text{ kip-ft}/1.67$ $= 0 \text{ kip-ft}$ $M_{B''} = 156 \text{ kip-ft}/1.67$ $= 93.4 \text{ kip-ft}$ $M_{C''} = 156 \text{ kip-ft}/1.67$ $= 93.4 \text{ kip-ft}$ $M_{D''} = 161 \text{ kip-ft}/1.67$ $= 96.4 \text{ kip-ft}$ $M_{E''} = 124 \text{ kip-ft}/1.67$ $= 74.3 \text{ kip-ft}$

The available strength values for each design method can now be plotted. These values are superimposed on the nominal strength surfaces (with and without length effects) previously calculated for comparison purposes in Figure I.6-5.

By plotting the required axial and flexural strength values determined for the governing load combinations on the available strength surfaces indicated in Figure I.6-5, it can be seen that both ASD (M_a, P_a) and LRFD (M_u, P_u) points lie within their respective design surfaces. The member in question is therefore adequate for the applied loads.

Designers should carefully review the proximity of the available strength values in relation to point D'' on Figure I.6-5 as it is possible for point D'' to fall outside of the nominal strength curve, thus resulting in an unsafe design. This possibility is discussed further in AISC *Specification* Commentary Section I5 and is avoided through the use of Method 2—Simplified as illustrated in the following section.

Method 2: Simplified

The simplified version of Method 2 involves the removal of points D″ and E″ from the Method 2 interaction surface leaving only points A″, B″ and C″ as illustrated in the comparison of the two methods in Figure I.6-6.

Reducing the number of interaction points allows for a bilinear interaction check defined by AISC *Specification* Commentary Equations C-I5-1a and C-I5-1b to be performed. Using the available strength values previously calculated in conjunction with the Commentary equations, interaction ratios are determined as follows:

LRFD	ASD
$P_r = P_u$ $= 129 \text{ kips}$	$P_r = P_a$ $= 98.2 \text{ kips}$
$P_r \geq P_{C''}$ $\geq 105 \text{ kips}$	$P_r \geq P_{C''}$ $\geq 70.0 \text{ kips}$
Therefore, use AISC *Specification* Commentary Equation C-I5-1b.	Therefore, use AISC *Specification* Commentary Equation C-I5-1b.
$\dfrac{P_r - P_C}{P_A - P_C} + \dfrac{M_r}{M_C} \leq 1.0$ (from *Spec*. Eq. C-I5-1b)	$\dfrac{P_r - P_C}{P_A - P_C} + \dfrac{M_r}{M_C} \leq 1.0$ (from *Spec*. Eq. C-I5-1b)
which for LRFD equals:	which for ASD equals:
$\dfrac{P_u - P_{C''}}{P_{A''} - P_{C''}} + \dfrac{M_u}{M_{C''}} \leq 1.0$ $\dfrac{129 \text{ kips} - 105 \text{ kips}}{368 \text{ kips} - 105 \text{ kips}} + \dfrac{120 \text{ kip-ft}}{140 \text{ kip-ft}} \leq 1.0$ $0.948 < 1.0$ **o.k.**	$\dfrac{P_a - P_{C''}}{P_{A''} - P_{C''}} + \dfrac{M_a}{M_{C''}} \leq 1.0$ $\dfrac{98.2 \text{ kips} - 70.0 \text{ kips}}{245 \text{ kips} - 70.0 \text{ kips}} + \dfrac{54 \text{ kip-ft}}{93.4 \text{ kip-ft}} \leq 1.0$ $0.739 < 1.0$ **o.k.**

Thus, the member is adequate for the applied loads.

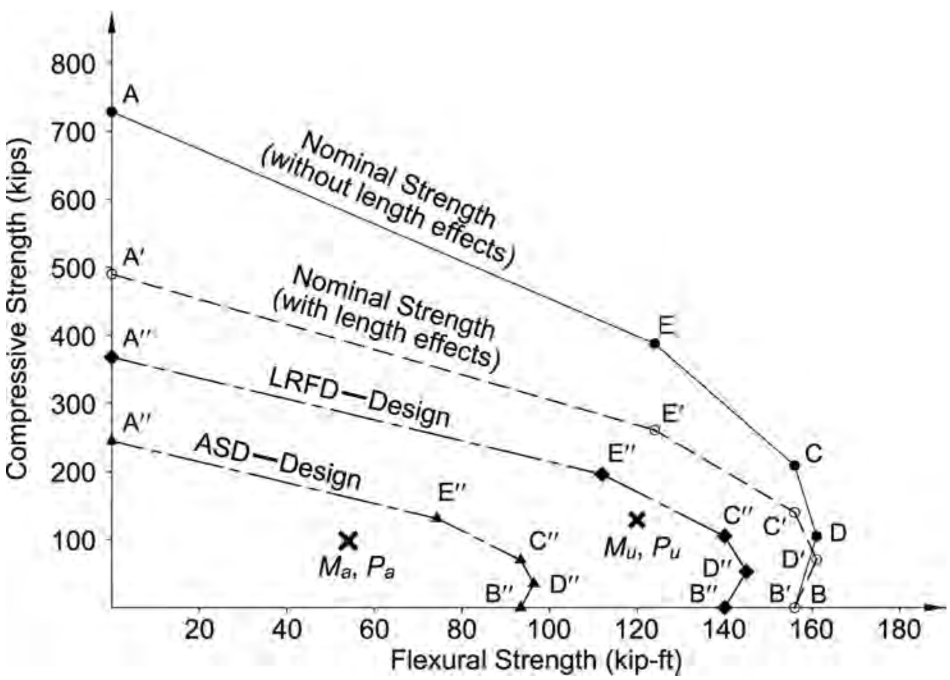


Fig. I.6-5. Available and nominal interaction surfaces.

Comparison of Methods

The composite member was found to be inadequate using Method 1—Chapter H interaction equations, but was found to be adequate using both Method 2 and Method 2—Simplified procedures. A comparison between the methods is most easily made by overlaying the design curves from each method as illustrated in Figure I.6-7 for LRFD design.

From Figure I.6-7, the conservative nature of the Chapter H interaction equations can be seen. Method 2 provides the highest available strength; however, the Method 2—Simplified procedure also provides a good representation of the complete design curve. By using the Part IV design tables to determine the available strength of the composite member in compression and flexure (Points A'' and B'' respectively), the modest additional effort required to calculate the available compressive strength at Point C'' can result in appreciable gains in member strength when using Method 2—Simplified as opposed to Method 1.

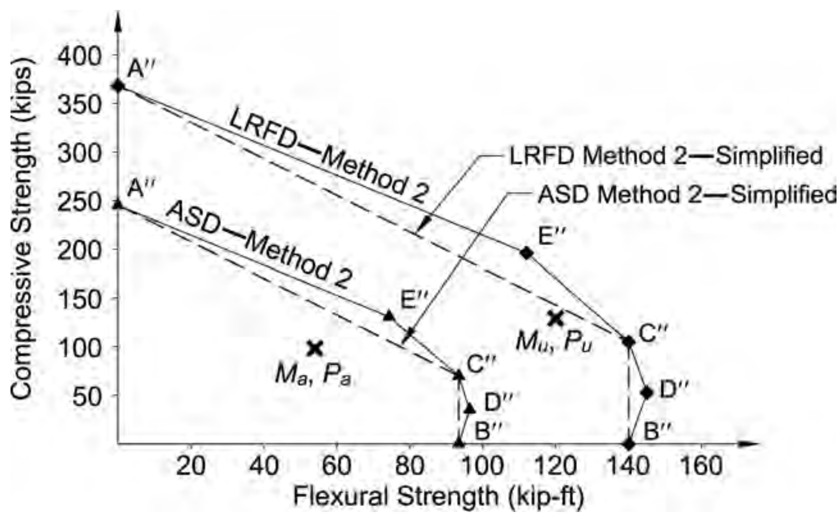


Fig. I.6-6. Comparison of Method 2 and Method 2—Simplified.

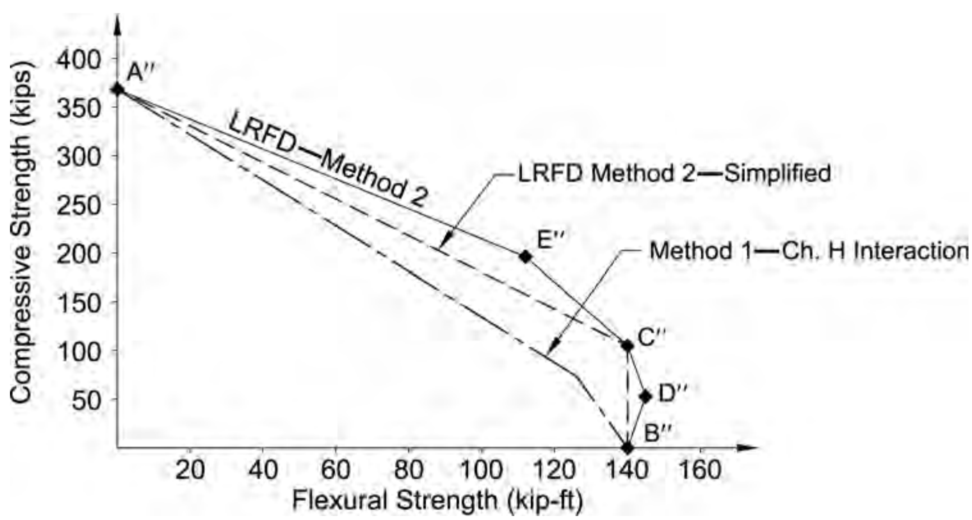


Fig. I.6-7. Comparison of interaction methods (LRFD).

Available Shear Strength

AISC *Specification* Section I4.1 provides three methods for determining the available shear strength of a filled composite member: available shear strength of the steel section alone in accordance with Chapter G; available shear strength of the reinforced concrete portion alone per ACI 318 (ACI 318, 2014); or available shear strength of the steel section plus the reinforcing steel ignoring the contribution of the concrete. The available shear strength will be determined using the first two methods because there is no reinforcing steel provided in this example.

Available Shear Strength of Steel Section

The nominal shear strength, V_n, of rectangular HSS members is determined using the provisions of AISC *Specification* Section G4. The web shear coefficient, C_{v2}, is determined from AISC *Specification* Section G2.2 with, $h/t_w = h/t$ and $k_v = 5$.

$$\begin{aligned}1.10\sqrt{k_v E/F_y} &= 1.10\sqrt{\frac{(5)(29{,}000\text{ ksi})}{50\text{ ksi}}}\\ &= 59.2 > h/t = 25.7\end{aligned}$$

Use AISC *Specification* Equation G2-9.

$$C_{v2} = 1.0 \qquad \text{(Spec. Eq. G2-9)}$$

The nominal shear strength is calculated as:

$$\begin{aligned}h &= H - 3t\\ &= 10.0\text{ in.} - 3(0.349\text{ in.})\\ &= 8.95\text{ in.}\end{aligned}$$

$$\begin{aligned}A_w &= 2ht\\ &= 2(8.95\text{ in.})(0.349\text{ in.})\\ &= 6.25\text{ in.}^2\end{aligned}$$

$$\begin{aligned}V_n &= 0.6F_y A_w C_{v2} \qquad \text{(Spec. Eq. G4-1)}\\ &= 0.6(50\text{ ksi})(6.25\text{ in.}^2)(1.0)\\ &= 188\text{ kips}\end{aligned}$$

The available shear strength of the steel section is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(188\text{ kips})$ $= 169\text{ kips} > 17.1\text{ kips}$ **o.k.**	$\Omega_v = 1.67$ $\dfrac{V_n}{\Omega_v} = \dfrac{188\text{ kips}}{1.67}$ $= 113\text{ kips} > 10.3\text{ kips}$ **o.k.**

Available Shear Strength of the Reinforced Concrete

The available shear strength of the steel section alone has been shown to be sufficient, but the available shear strength of the concrete will be calculated for demonstration purposes. Considering that the member does not have longitudinal reinforcing, the method of shear strength calculation involving reinforced concrete is not valid; however, the design shear strength of the plain concrete using ACI 318, Chapter 14, can be determined as follows:

ϕ = 0.60 for plain concrete design from ACI 318 Section 21.2.1
λ = 1.0 for normal weight concrete from ACI 318 Section 19.2.4.2

$$V_n = \left(\frac{4}{3}\right)\lambda\sqrt{f_c'}b_w h \qquad \text{(ACI 318 Section 14.5.5.1)}$$

$$b_w = b_i$$
$$h = h_i$$

$$V_n = \left(\frac{4}{3}\right)(1.0)\sqrt{5{,}000 \text{ psi}}\,(5.30 \text{ in.})(9.30 \text{ in.})\left(\frac{1 \text{ kip}}{1{,}000 \text{ lb}}\right)$$
$$= 4.65 \text{ kips}$$

$$\phi V_n = 0.60(4.65 \text{ kips}) \qquad \text{(ACI 318 Section 14.5.1.1)}$$
$$= 2.79 \text{ kips} < 17.1 \text{ kips} \quad \textbf{n.g.}$$

As can be seen from this calculation, the shear resistance provided by plain concrete is small and the strength of the steel section alone is generally sufficient.

Force Allocation and Load Transfer

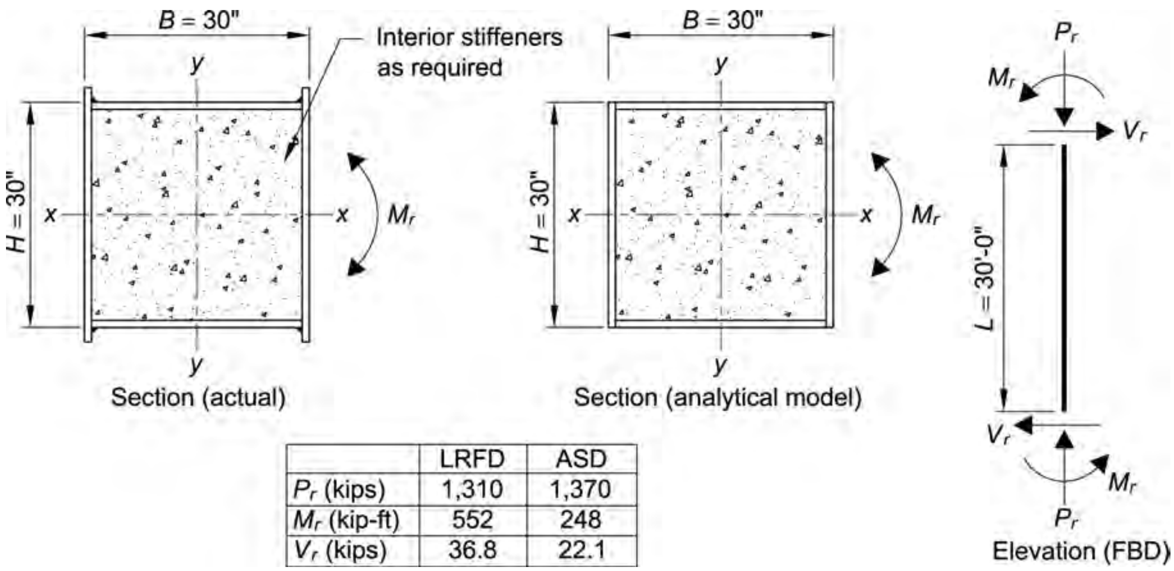
Load transfer calculations for applied axial forces should be performed in accordance with AISC *Specification* Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions is provided in Design Example I.3.

TOC Back

EXAMPLE I.7 FILLED COMPOSITE BOX COLUMN WITH NONCOMPACT/SLENDER ELEMENTS

Given:

Determine the required ASTM A36 plate thickness of the filled composite box column illustrated in Figure I.7-1 to resist the indicated axial forces, shears and moments that have been determined in accordance with the direct analysis method of AISC *Specification* Chapter C for the controlling ASCE/SEI 7 load combinations. The core is composed of normal weight (145 lb/ft^3) concrete fill having a specified concrete compressive strength, f'_c = 7 ksi.



	LRFD	ASD
P_r (kips)	1,310	1,370
M_r (kip-ft)	552	248
V_r (kips)	36.8	22.1

Fig. I.7-1. Composite box column section and member forces.

Solution:

From AISC *Manual* Table 2-5, the material properties are:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

Trial Size 1 (Noncompact)

For ease of calculation the contribution of the plate extensions to the member strength will be ignored as illustrated by the analytical model in Figure I.7-1.

Trial Plate Thickness and Geometric Section Properties of the Composite Member

Select a trial plate thickness, t, of ⅜ in. Note that the design wall thickness reduction of AISC *Specification* Section B4.2 applies only to electric-resistance-welded HSS members and does not apply to built-up sections such as the one under consideration.

The calculated geometric properties of the 30 in. by 30 in. steel box column are:

$$B = 30 \text{ in.}$$
$$H = 30 \text{ in.}$$
$$A_g = 900 \text{ in.}^2$$
$$A_c = 856 \text{ in.}^2$$
$$A_s = 44.4 \text{ in.}^2$$

$$\begin{aligned} b_i &= B - 2t \\ &= 30 \text{ in.} - 2\left(\tfrac{3}{8} \text{ in.}\right) \\ &= 29.2 \text{ in.} \end{aligned}$$

$$\begin{aligned} h_i &= H - 2t \\ &= 30 \text{ in.} - 2\left(\tfrac{3}{8} \text{ in.}\right) \\ &= 29.2 \text{ in.} \end{aligned}$$

$$\begin{aligned} E_c &= w_c^{1.5}\sqrt{f_c'} \\ &= \left(145 \text{ lb/ft}^3\right)^{1.5}\sqrt{7 \text{ ksi}} \\ &= 4{,}620 \text{ ksi} \end{aligned}$$

$$\begin{aligned} I_{gx} &= \frac{BH^3}{12} \\ &= \frac{(30 \text{ in.})(30 \text{ in.})^3}{12} \\ &= 67{,}500 \text{ in.}^4 \end{aligned}$$

$$\begin{aligned} I_{cx} &= \frac{b_i h_i^3}{12} \\ &= \frac{(29.2 \text{ in.})(29.2 \text{ in.})^3}{12} \\ &= 60{,}600 \text{ in.}^4 \end{aligned}$$

$$\begin{aligned} I_{sx} &= I_{gx} - I_{cx} \\ &= 67{,}500 \text{ in.}^4 - 60{,}600 \text{ in.}^4 \\ &= 6{,}900 \text{ in.}^4 \end{aligned}$$

Limitations of AISC Specification Sections I1.3 and I2.2a

(1) Concrete Strength: $3 \text{ ksi} \le f_c' \le 10 \text{ ksi}$

$f_c' = 7 \text{ ksi}$ **o.k.**

(2) Specified minimum yield stress of structural steel: $F_y \le 75 \text{ ksi}$

$F_y = 36 \text{ ksi}$ **o.k.**

(3) Cross-sectional area of steel section: $A_s \geq 0.01A_g$

$$44.4 \text{ in.}^2 \geq (0.01)(900 \text{ in.}^2)$$
$$> 9.00 \text{ in.}^2 \quad \textbf{o.k.}$$

Classify Section for Local Buckling

Classification of the section for local buckling is performed in accordance with AISC *Specification* Table I1.1a for compression and Table I1.1b for flexure. As noted in *Specification* Section I1.4, the definitions of width, depth and thickness used in the evaluation of slenderness are provided in Section B4.1b.

For box columns, the widths of the stiffened compression elements used for slenderness checks, b and h, are equal to the clear distances between the column walls, b_i and h_i. The slenderness ratios are determined as follows:

$$\lambda = \frac{b_i}{t} = \frac{h_i}{t}$$
$$= \frac{29.2 \text{ in.}}{3/8 \text{ in.}}$$
$$= 77.9$$

Classify section for local buckling in steel elements subject to axial compression from AISC *Specification* Table I1.1a:

$$\lambda_p = 2.26\sqrt{\frac{E}{F_y}}$$
$$= 2.26\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$
$$= 64.1$$

$$\lambda_r = 3.00\sqrt{\frac{E}{F_y}}$$
$$= 3.00\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$
$$= 85.1$$

$\lambda_p \leq \lambda \leq \lambda_r$; therefore, the section is noncompact for compression

According to AISC *Specification* Section I1.4, if any side of the section in question is noncompact or slender, then the entire section is treated as noncompact or slender. For the square section under investigation; however, this distinction is unnecessary as all sides are equal in length.

Classification of the section for local buckling in elements subject to flexure is performed in accordance with AISC *Specification* Table I1.1b. Note that flanges and webs are treated separately; however, for the case of a square section only the most stringent limitations, those of the flange, need be applied. Noting that the flange limitations for bending are the same as those for compression,

$\lambda_p \leq \lambda \leq \lambda_r$; therefore, the section is noncompact for flexure

Available Compressive Strength

Compressive strength for noncompact filled composite members is determined in accordance with AISC *Specification* Section I2.2b(b).

$$P_p = F_y A_s + C_2 f_c'\left(A_c + A_{sr}\frac{E_s}{E_c}\right), \text{ where } C_2 = 0.85 \text{ for rectangular sections} \qquad (Spec.\ \text{Eq. I2-9b})$$

$$= (36\text{ ksi})(44.4\text{ in.}^2) + 0.85(7\text{ ksi})(856\text{ in.}^2 + 0\text{ in.}^2)$$

$$= 6{,}690\text{ kips}$$

$$P_y = F_y A_s + 0.7 f_c'\left(A_c + A_{sr}\frac{E_s}{E_c}\right) \qquad (Spec.\ \text{Eq. I2-9d})$$

$$= (36\text{ ksi})(44.4\text{ in.}^2) + 0.7(7\text{ ksi})(856\text{ in.}^2 + 0\text{ in.}^2)$$

$$= 5{,}790\text{ kips}$$

$$P_{no} = P_p - \frac{P_p - P_y}{(\lambda_r - \lambda_p)^2}(\lambda - \lambda_p)^2 \qquad (Spec.\ \text{Eq. I2-9c})$$

$$= 6{,}690\text{ kips} - \frac{6{,}690\text{ kips} - 5{,}790\text{ kips}}{(85.1 - 64.1)^2}(77.9 - 64.1)^2$$

$$= 6{,}300\text{ kips}$$

$$C_3 = 0.45 + 3\left(\frac{A_s + A_{sr}}{A_g}\right) \le 0.9 \qquad (Spec.\ \text{Eq. I2-13})$$

$$= 0.45 + 3\left(\frac{44.4\text{ in.}^2 + 0\text{ in.}^2}{900\text{ in.}^2}\right) \le 0.9$$

$$= 0.598 < 0.9$$

$$= 0.598$$

$$EI_{eff} = E_s I_s + E_s I_{sr} + C_3 E_c I_c \qquad (Spec.\ \text{Eq. I2-12})$$

$$= (29{,}000\text{ ksi})(6{,}900\text{ in.}^4) + 0.0\text{ kip-in.}^2 + 0.598(4{,}620\text{ ksi})(60{,}600\text{ in.}^4)$$

$$= 368{,}000{,}000\text{ kip-in.}^2$$

$$P_e = \pi^2(EI_{eff})/(L_c)^2, \text{ where } L_c = KL \text{ and } K = 1.0 \text{ in accordance with the direct analysis method} \qquad (Spec.\ \text{Eq. I2-5})$$

$$= \frac{\pi^2(368{,}000{,}000\text{ kip-in.}^2)}{[(30\text{ ft})(12\text{ in./ft})]^2}$$

$$= 28{,}000\text{ kips}$$

$$\frac{P_{no}}{P_e} = \frac{6{,}300\text{ kips}}{28{,}000\text{ kips}}$$

$$= 0.225 < 2.25$$

Therefore, use AISC *Specification* Equation I2-2.

$$P_n = P_{no}\left(0.658^{\frac{P_{no}}{P_e}}\right) \qquad (\textit{Spec}.\ \text{Eq. I2-2})$$

$$= (6{,}300\text{ kips})(0.658)^{0.225}$$
$$= 5{,}730\text{ kips}$$

According to AISC *Specification* Section I2.2b, the compression strength need not be less than that specified for the bare steel member as determined by *Specification* Chapter E. It can be shown that the compression strength of the bare steel for this section is equal to 955 kips, thus the strength of the composite section controls.

The available compressive strength is:

LRFD	ASD
$\phi_c = 0.75$ $\phi_c P_n = 0.75(5{,}730\text{ kips})$ $= 4{,}300\text{ kips}$	$\Omega_c = 2.00$ $\dfrac{P_n}{\Omega_c} = \dfrac{5{,}730\text{ kips}}{2.00}$ $= 2{,}870\text{ kips}$

Available Flexural Strength

Flexural strength of noncompact filled composite members is determined in accordance with AISC *Specification* Section I3.4b(b):

$$M_n = M_p - (M_p - M_y)\frac{(\lambda - \lambda_p)}{(\lambda_r - \lambda_p)} \qquad (\textit{Spec}.\ \text{Eq. I3-3b})$$

In order to utilize Equation I3-3b, both the plastic moment strength of the section, M_p, and the yield moment strength of the section, M_y, must be calculated.

Plastic Moment Strength

The first step in determining the available flexural strength of a noncompact section is to calculate the moment corresponding to the plastic stress distribution over the composite cross section, M_p. This concept is illustrated graphically in AISC *Specification* Commentary Figure C-I3.7(a) and follows the force distribution depicted in Figure I.7-2 and detailed in Table I.7-1.

Table I.7-1. Plastic Moment Equations		
Component	**Force**	**Moment Arm**
Compression in steel flange	$C_1 = b_i t_f F_y$	$y_{C1} = a_p - \frac{t_f}{2}$
Compression in concrete	$C_2 = 0.85 f'_c \left(a_p - t_f\right) b_i$	$y_{C2} = \frac{a_p - t_f}{2}$
Compression in steel web	$C_3 = a_p 2 t_w F_y$	$y_{C3} = \frac{a_p}{2}$
Tension in steel web	$T_1 = \left(H - a_p\right) 2 t_w F_y$	$y_{T1} = \frac{H - a_p}{2}$
Tension in steel flange	$T_2 = b_i t_f F_y$	$y_{T2} = H - a_p - \frac{t_f}{2}$

where:

$$a_p = \frac{2 F_y H t_w + 0.85 f'_c b_i t_f}{4 t_w F_y + 0.85 f'_c b_i}$$

$$M_p = \sum \left(\text{force}\right)\left(\text{moment arm}\right)$$

Using the equations provided in Table I.7-1 for the section in question results in the following:

$$a_p = \frac{2\left(36\text{ ksi}\right)\left(30\text{ in.}\right)\left(\tfrac{3}{8}\text{ in.}\right) + 0.85\left(7\text{ ksi}\right)\left(29.2\text{ in.}\right)\left(\tfrac{3}{8}\text{ in.}\right)}{4\left(\tfrac{3}{8}\text{ in.}\right)\left(36\text{ ksi}\right) + 0.85\left(7\text{ ksi}\right)\left(29.2\text{ in.}\right)}$$
$$= 3.84\text{ in.}$$

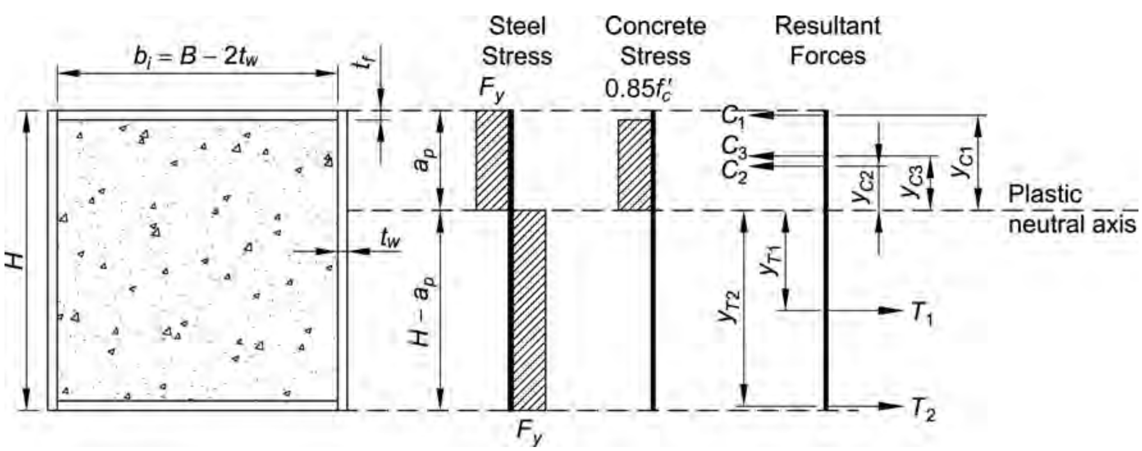


Figure I.7-2. Plastic moment stress blocks and force distribution.

Force	Moment Arm	Force × Moment Arm
$C_1 = (29.2\text{ in.})(\tfrac{3}{8}\text{ in.})(36\text{ ksi})$ $= 394\text{ kips}$	$y_{C1} = 3.84\text{ in.} - \dfrac{\tfrac{3}{8}\text{ in.}}{2}$ $= 3.65\text{ in.}$	$C_1 y_{C1} = 1{,}440\text{ kip-in.}$
$C_2 = 0.85(7\text{ ksi})(3.84\text{ in.} - \tfrac{3}{8}\text{ in.})(29.2\text{ in.})$ $= 602\text{ kips}$	$y_{C2} = \dfrac{3.84\text{ in.} - \tfrac{3}{8}\text{ in.}}{2}$ $= 1.73\text{ in.}$	$C_2 y_{C2} = 1{,}040\text{ kip-in.}$
$C_3 = (3.84\text{ in.})(2)(\tfrac{3}{8}\text{ in.})(36\text{ ksi})$ $= 104\text{ kips}$	$y_{C3} = \dfrac{3.84\text{ in.}}{2}$ $= 1.92\text{ in.}$	$C_3 y_{C3} = 200\text{ kip-in.}$
$T_1 = (30\text{ in.} - 3.84\text{ in.})(2)(\tfrac{3}{8}\text{ in.})(36\text{ ksi})$ $= 706\text{ kips}$	$y_{T1} = \dfrac{30\text{ in.} - 3.84\text{ in.}}{2}$ $= 13.1\text{ in.}$	$T_1 y_{T1} = 9{,}250\text{ kip-in.}$
$T_2 = (29.2\text{ in.})(\tfrac{3}{8}\text{ in.})(36\text{ ksi})$ $= 394\text{ kips}$	$y_{T2} = 30\text{ in.} - 3.84\text{ in.} - \dfrac{\tfrac{3}{8}\text{ in.}}{2}$ $= 26.0\text{ in.}$	$T_2 y_{T2} = 10{,}200\text{ kip-in.}$
$M_p = \sum(\text{force})(\text{moment arm})$ $= \dfrac{1{,}440\text{ kip-in.} + 1{,}040\text{ kip-in.} + 200\text{ kip-in.} + 9{,}250\text{ kip-in.} + 10{,}200\text{ kip-in.}}{12\text{ in./ft}}$ $= 1{,}840\text{ kip-ft}$		

Yield Moment Strength

The next step in determining the available flexural strength of a noncompact filled member is to determine the yield moment strength. The yield moment is defined in AISC *Specification* Section I3.4b(b) as the moment corresponding to first yield of the compression flange calculated using a linear elastic stress distribution with a maximum concrete compressive stress of $0.7 f_c'$. This concept is illustrated diagrammatically in *Specification* Commentary Figure C-I3.7(b) and follows the force distribution depicted in Figure I.7-3 and detailed in Table I.7-2.

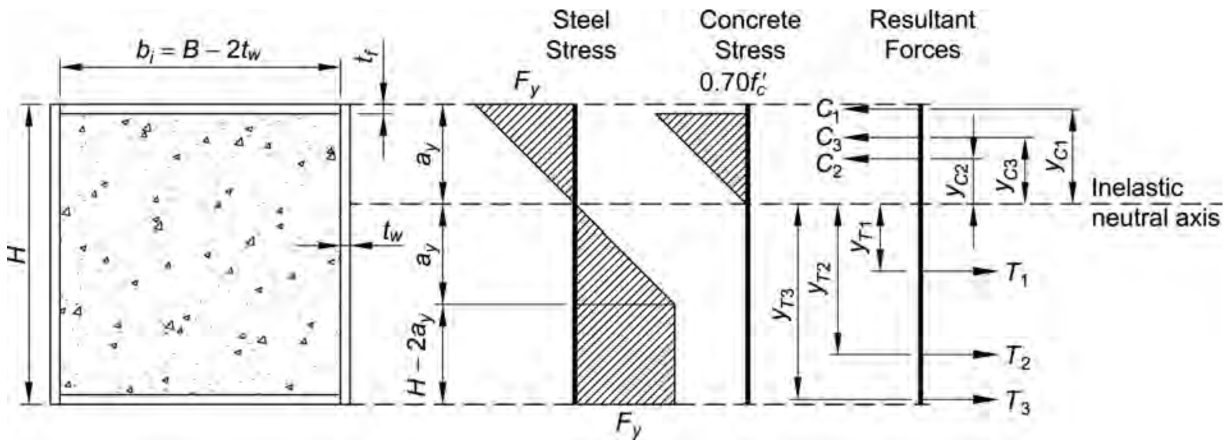


Figure I.7-3. Yield moment stress blocks and force distribution.

Table I.7-2. Yield Moment Equations

Component	Force	Moment Arm
Compression in steel flange	$C_1 = b_i t_f F_y$	$y_{C1} = a_y - \dfrac{t_f}{2}$
Compression in concrete	$C_2 = 0.35 f'_c (a_y - t_f) b_i$	$y_{C2} = \dfrac{2(a_y - t_f)}{3}$
Compression in steel web	$C_3 = a_y 2t_w 0.5F_y$	$y_{C3} = \dfrac{2a_y}{3}$
Tension in steel web	$T_1 = a_y 2t_w 0.5F_y$ $T_2 = (H - 2a_y) 2t_w F_y$	$y_{T1} = \dfrac{2a_y}{3}$ $y_{T2} = \dfrac{H}{2}$
Tension in steel flange	$T_3 = b_i t_f F_y$	$y_{T3} = H - a_y - \dfrac{t_f}{2}$

where

$$a_y = \frac{2F_y H t_w + 0.35 f'_c b_i t_f}{4t_w F_y + 0.35 f'_c b_i}$$

$$M_y = \sum(\text{force})(\text{moment arm})$$

Using the equations provided in Table I.7-2 for the section in question results in the following:

$$a_y = \frac{2(36 \text{ ksi})(30 \text{ in.})(3/8 \text{ in.}) + 0.35(7 \text{ ksi})(29.2 \text{ in.})(3/8 \text{ in.})}{4(3/8 \text{ in.})(36 \text{ ksi}) + 0.35(7 \text{ ksi})(29.2 \text{ in.})}$$
$$= 6.66 \text{ in.}$$

Force	Moment Arm	Force × Moment Arm
$C_1 = (29.2 \text{ in.})(3/8 \text{ in.})(36 \text{ ksi})$ $= 394 \text{ kips}$	$y_{C1} = 6.66 \text{ in.} - \dfrac{3/8 \text{ in.}}{2}$ $= 6.47 \text{ in.}$	$C_1 y_{C1} = 2{,}550 \text{ kip-in.}$
$C_2 = 0.35(7 \text{ ksi})(6.66 \text{ in.} - 3/8 \text{ in.})(29.2 \text{ in.})$ $= 450 \text{ kips}$	$y_{C2} = \dfrac{2(6.66 \text{ in.} - 3/8 \text{ in.})}{3}$ $= 4.19 \text{ in.}$	$C_2 y_{C2} = 1{,}890 \text{ kip-in.}$
$C_3 = (6.66 \text{ in.})(2)(3/8 \text{ in.})(0.5)(36 \text{ ksi})$ $= 89.9 \text{ kips}$	$y_{C3} = \dfrac{2(6.66 \text{ in.})}{3}$ $= 4.44 \text{ in.}$	$C_3 y_{C3} = 399 \text{ kip-in.}$
$T_1 = (6.66 \text{ in.})(2)(3/8 \text{ in.})(0.5)(36 \text{ ksi})$ $= 89.9 \text{ kips}$	$y_{T1} = \dfrac{2(6.66 \text{ in.})}{3}$ $= 4.44 \text{ in.}$	$T_1 y_{T1} = 399 \text{ kip-in.}$
$T_2 = [30 \text{ in.} - 2(6.66 \text{ in.})](2)(3/8 \text{ in.})(36 \text{ ksi})$ $= 450 \text{ kips}$	$y_{T2} = \dfrac{30 \text{ in.}}{2}$ $= 15.0 \text{ in.}$	$T_2 y_{T2} = 6{,}750 \text{ kip-in.}$
$T_3 = (29.2 \text{ in.})(3/8 \text{ in.})(36 \text{ ksi})$ $= 394 \text{ kips}$	$y_{T3} = 30 \text{ in.} - 6.66 \text{ in.} - \dfrac{3/8 \text{ in.}}{2}$ $= 23.2 \text{ in.}$	$T_3 y_{T3} = 9{,}140 \text{ kip-in.}$

$$M_y = \sum(\text{force})(\text{moment arm})$$
$$= \frac{2{,}550 \text{ kip-in.} + 1{,}890 \text{ kip-in.} + 399 \text{ kip-in.} + 399 \text{ kip-in.} + 6{,}750 \text{ kip-in.} + 9{,}140 \text{ kip-in.}}{12 \text{ in./ft}}$$
$$= 1{,}760 \text{ kip-ft}$$

Now that both M_p and M_y have been determined, Equation I3-3b may be used in conjunction with the flexural slenderness values previously calculated to determine the nominal flexural strength of the composite section as follows:

$$M_n = M_p - (M_p - M_y)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right) \qquad \text{(Spec. Eq. I3-3b)}$$
$$= 1{,}840 \text{ kip-ft} - (1{,}840 \text{ kip-ft} - 1{,}760 \text{ kip-ft})\left(\frac{77.9 - 64.1}{85.1 - 64.1}\right)$$
$$= 1{,}790 \text{ kip-ft}$$

The available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(1{,}790 \text{ kip-ft})$ $= 1{,}610 \text{ kip-ft}$	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{1{,}790 \text{ kip-ft}}{1.67}$ $= 1{,}070 \text{ kip-ft}$

Interaction of Flexure and Compression

Design of members for combined forces is performed in accordance with AISC *Specification* Section I5. For filled composite members with noncompact or slender sections, interaction may be determined in accordance with Section H1.1 as follows:

LRFD	ASD
$P_u = 1{,}310 \text{ kips}$ $M_u = 552 \text{ kip-ft}$ $\dfrac{P_r}{P_c} = \dfrac{P_u}{\phi_c P_n}$ $= \dfrac{1{,}310 \text{ kips}}{4{,}300 \text{ kips}}$ $= 0.305 > 0.2$ Therefore, use AISC *Specification* Equation H1-1a. $\dfrac{P_u}{\phi_c P_n} + \dfrac{8}{9}\left(\dfrac{M_u}{\phi_b M_n}\right) \le 1.0$ (from *Spec.* Eq. H1-1a) $0.305 + \dfrac{8}{9}\left(\dfrac{552 \text{ kip-ft}}{1{,}610 \text{ kip-ft}}\right) \le 1.0$ $0.610 < 1.0$ **o.k.**	$P_a = 1{,}370 \text{ kips}$ $M_a = 248 \text{ kip-ft}$ $\dfrac{P_r}{P_c} = \dfrac{P_a}{P_n / \Omega_c}$ $= \dfrac{1{,}370 \text{ kips}}{2{,}870 \text{ kips}}$ $= 0.477 > 0.2$ Therefore, use AISC *Specification* Equation H1-1a. $\dfrac{P_a}{P_n / \Omega_c} + \dfrac{8}{9}\left(\dfrac{M_a}{M_n / \Omega_b}\right) \le 1.0$ (from *Spec.* Eq. H1-1a) $0.477 + \dfrac{8}{9}\left(\dfrac{248 \text{ kip-ft}}{1{,}070 \text{ kip-ft}}\right) \le 1.0$ $0.683 < 1.0$ **o.k.**

The composite section is adequate; however, as there is available strength remaining for the trial plate thickness chosen, re-analyze the section to determine the adequacy of a reduced plate thickness.

Trial Size 2 (Slender)

The calculated geometric section properties using a reduced plate thickness of t = ¼ in. are:

$$B = 30 \text{ in.}$$
$$H = 30 \text{ in.}$$
$$A_g = 900 \text{ in.}^2$$
$$A_c = 870 \text{ in.}^2$$
$$A_s = 29.8 \text{ in.}^2$$

$$\begin{aligned} b_i &= B - 2t \\ &= 30 \text{ in.} - 2(\text{¼ in.}) \\ &= 29.5 \text{ in.} \end{aligned}$$

$$\begin{aligned} h_i &= H - 2t \\ &= 30 \text{ in.} - 2(\text{¼ in.}) \\ &= 29.5 \text{ in.} \end{aligned}$$

$$\begin{aligned} E_c &= w_c^{1.5}\sqrt{f_c'} \\ &= \left(145 \text{ lb/ft}^3\right)^{1.5}\sqrt{7 \text{ ksi}} \\ &= 4{,}620 \text{ ksi} \end{aligned}$$

$$\begin{aligned} I_{gx} &= \frac{BH^3}{12} \\ &= \frac{(30 \text{ in.})(30 \text{ in.})^3}{12} \\ &= 67{,}500 \text{ in.}^4 \end{aligned}$$

$$\begin{aligned} I_{cx} &= \frac{b_i h_i^3}{12} \\ &= \frac{(29.5 \text{ in.})(29.5 \text{ in.})^3}{12} \\ &= 63{,}100 \text{ in.}^4 \end{aligned}$$

$$\begin{aligned} I_{sx} &= I_{gx} - I_{cx} \\ &= 67{,}500 \text{ in.}^4 - 63{,}100 \text{ in.}^4 \\ &= 4{,}400 \text{ in.}^4 \end{aligned}$$

Limitations of AISC Specification Sections I1.3 and I2.2a

(1) Concrete Strength: $3 \text{ ksi} \le f_c' \le 10 \text{ ksi}$

$f_c' = 7 \text{ ksi}$ **o.k.**

(2) Specified minimum yield stress of structural steel: $F_y \le 75 \text{ ksi}$

$F_y = 36 \text{ ksi}$ **o.k.**

(3) Cross sectional area of steel section: $A_s \geq 0.01A_g$

$$29.8 \text{ in.}^2 \geq (0.01)\left(900 \text{ in.}^2\right)$$
$$> 9.00 \text{ in.}^2 \quad \textbf{o.k.}$$

Classify Section for Local Buckling

As noted previously, the definitions of width, depth and thickness used in the evaluation of slenderness are provided in AISC *Specification* Section B4.1b.

For a box column, the slenderness ratio is determined as the ratio of clear distance-to-wall thickness:

$$\lambda = \frac{b_i}{t} = \frac{h_i}{t}$$
$$= \frac{29.5 \text{ in.}}{\frac{1}{4} \text{ in.}}$$
$$= 118$$

Classify section for local buckling in steel elements subject to axial compression from AISC *Specification* Table I1.1a. As determined previously, $\lambda_r = 85.1$.

$$\lambda_{max} = 5.00\sqrt{\frac{E}{F_y}}$$
$$= 5.00\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$
$$= 142$$

$\lambda_r \leq \lambda \leq \lambda_{max}$; therefore, the section is slender for compression

Classification of the section for local buckling in elements subject to flexure occurs separately per AISC *Specification* Table I1.1b. Because the flange limitations for bending are the same as those for compression,

$\lambda_r \leq \lambda \leq \lambda_{max}$; therefore, the section is slender for flexure

Available Compressive Strength

Compressive strength for a slender filled member is determined in accordance with AISC *Specification* Section I2.2b(c).

$$F_{cr} = \frac{9E_s}{\left(\frac{b}{t}\right)^2} \qquad \text{(\textit{Spec}. Eq. I2-10)}$$
$$= \frac{9(29{,}000 \text{ ksi})}{(118)^2}$$
$$= 18.7 \text{ ksi}$$

$$P_{no} = F_{cr}A_s + 0.7f_c'\left(A_c + A_{sr}\frac{E_s}{E_c}\right) \qquad (Spec.\ \text{Eq. I2-9e})$$

$$= (18.7\text{ ksi})(29.8\text{ in.}^2) + 0.7(7\text{ ksi})(870\text{ in.}^2 + 0\text{ in.}^2)$$

$$= 4{,}820\text{ kips}$$

$$C_3 = 0.45 + 3\left(\frac{A_s + A_{sr}}{A_g}\right) \le 0.9 \qquad (Spec.\ \text{Eq. I2-13})$$

$$= 0.45 + 3\left(\frac{29.8\text{ in.}^2 + 0\text{ in.}^2}{900\text{ in.}^2}\right) \le 0.9$$

$$= 0.549 < 0.9$$

$$= 0.549$$

$$EI_{eff} = E_sI_s + E_sI_{sr} + C_3E_cI_c \qquad (Spec.\ \text{Eq. I2-12})$$

$$= (29{,}000\text{ ksi})(4{,}400\text{ in.}^4) + 0\text{ kip-in.}^2 + 0.549(4{,}620\text{ ksi})(63{,}100\text{ in.}^4)$$

$$= 288{,}000{,}000\text{ kip-in.}^2$$

$P_e = \pi^2(EI_{eff})/(L_c)^2$, where $L_c = KL$ and $K = 1.0$ in accordance with the direct analysis method (*Spec*. Eq. I2-5)

$$= \frac{\pi^2(288{,}000{,}000\text{ kip-in.}^2)}{\left[(30\text{ ft})(12\text{ in./ft})\right]^2}$$

$$= 21{,}900\text{ kips}$$

$$\frac{P_{no}}{P_e} = \frac{4{,}820\text{ kips}}{21{,}900\text{ kips}}$$

$$= 0.220 < 2.25$$

Therefore, use AISC *Specification* Equation I2-2.

$$P_n = P_{no}\left(0.658^{\frac{P_{no}}{P_e}}\right) \qquad (Spec.\ \text{Eq. I2-2})$$

$$= (4{,}820\text{ kips})(0.658)^{0.220}$$

$$= 4{,}400\text{ kips}$$

According to AISC *Specification* Section I2.2b the compression strength need not be less than that determined for the bare steel member using *Specification* Chapter E. It can be shown that the compression strength of the bare steel for this section is equal to 450 kips, thus the strength of the composite section controls.

The available compressive strength is:

LRFD	ASD
$\phi_c = 0.75$ $\phi_c P_n = 0.75(4{,}400 \text{ kips})$ $= 3{,}300 \text{ kips}$	$\Omega_c = 2.00$ $\dfrac{P_n}{\Omega_c} = \dfrac{4{,}400 \text{ kips}}{2.00}$ $= 2{,}200 \text{ kips}$

Available Flexural Strength

Flexural strength of slender filled composite members is determined in accordance with AISC *Specification* Section I3.4b(c). The nominal flexural strength is determined as the first yield moment, M_{cr}, corresponding to a flange compression stress of F_{cr} using a linear elastic stress distribution with a maximum concrete compressive stress of $0.7 f_c'$. This concept is illustrated diagrammatically in *Specification* Commentary Figure C-I3.7(c) and follows the force distribution depicted in Figure I.7-4 and detailed in Table I.7-3.

Table I.7-3. First Yield Moment Equations

Component	Force	Moment Arm
Compression in steel flange	$C_1 = b_i t_f F_{cr}$	$y_{C1} = a_{cr} - \dfrac{t_f}{2}$
Compression in concrete	$C_2 = 0.35 f_c' (a_{cr} - t_f) b_i$	$y_{C2} = \dfrac{2(a_{cr} - t_f)}{3}$
Compression in steel web	$C_3 = a_{cr} 2t_w 0.5F_{cr}$	$y_{C3} = \dfrac{2a_{cr}}{3}$
Tension in steel web	$T_1 = (H - a_{cr}) 2t_w 0.5F_y$	$y_{T1} = \dfrac{2(H - a_{cr})}{3}$
Tension in steel flange	$T_2 = b_i t_f F_y$	$y_{T2} = H - a_{cr} - \dfrac{t_f}{2}$

where:

$$a_{cr} = \frac{F_y H t_w + (0.35 f_c' + F_y - F_{cr}) b_i t_f}{t_w (F_{cr} + F_y) + 0.35 f_c' b_i}$$

$$M_{cr} = \sum (\text{force})(\text{moment arm})$$

Using the equations provided in Table I.7-3 for the section in question results in the following:

$$a_{cr} = \frac{(36 \text{ ksi})(30 \text{ in.})(\text{¼ in.}) + [0.35(7 \text{ ksi}) + 36 \text{ ksi} - 18.7 \text{ ksi}](29.5 \text{ in.})(\text{¼ in.})}{(\text{¼ in.})(18.7 \text{ ksi} + 36 \text{ ksi}) + 0.35(7 \text{ ksi})(29.5 \text{ in.})}$$
$$= 4.84 \text{ in.}$$

Force	Moment Arm	Force × Moment Arm
$C_1 = (29.5\text{ in.})(\tfrac{1}{4}\text{ in.})(18.7\text{ ksi})$ $= 138\text{ kips}$	$y_{C1} = 4.84\text{ in.} - \dfrac{\tfrac{1}{4}\text{ in.}}{2}$ $= 4.72\text{ in.}$	$C_1 y_{C1} = 651\text{ kip-in.}$
$C_2 = 0.35(7\text{ ksi})(4.84\text{ in.} - \tfrac{1}{4}\text{ in.})(29.5\text{ in.})$ $= 332\text{ kips}$	$y_{C2} = \dfrac{2(4.84\text{ in.} - \tfrac{1}{4}\text{ in.})}{3}$ $= 3.06\text{ in.}$	$C_2 y_{C2} = 1{,}020\text{ kip-in.}$
$C_3 = (4.84\text{ in.})(2)(\tfrac{1}{4}\text{ in.})(0.5)(18.7\text{ ksi})$ $= 22.6\text{ kips}$	$y_{C3} = \dfrac{2(4.84\text{ in.})}{3}$ $= 3.23\text{ in.}$	$C_3 y_{C3} = 73.0\text{ kip-in.}$
$T_1 = (30\text{ in.} - 4.84\text{ in.})(2)(\tfrac{1}{4}\text{ in.})(0.5)(36\text{ ksi})$ $= 226\text{ kips}$	$y_{T1} = \dfrac{2(30\text{ in.} - 4.84\text{ in.})}{3}$ $= 16.8\text{ in.}$	$T_1 y_{T1} = 3{,}800\text{ kip-in.}$
$T_2 = (29.5\text{ in.})(\tfrac{1}{4}\text{ in.})(36\text{ ksi})$ $= 266\text{ kips}$	$y_{T2} = 30\text{ in.} - 4.84\text{ in.} - \dfrac{\tfrac{1}{4}\text{ in.}}{2}$ $= 25.0\text{ in.}$	$T_2 y_{T2} = 6{,}650\text{ kip-in.}$

$$M_{cr} = \sum(\text{force component})(\text{moment arm})$$
$$= \frac{651\text{ kip-in.} + 1{,}020\text{ kip-in.} + 73.0\text{ kip-in.} + 3{,}800\text{ kip-in.} + 6{,}650\text{ kip-in.}}{12\text{ in./ft}}$$
$$= 1{,}020\text{ kip-ft}$$

The available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $M_n = 0.90(1{,}020\text{ kip-ft})$ $= 918\text{ kip-ft}$	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{1{,}020\text{ kip-ft}}{1.67}$ $= 611\text{ kip-ft}$

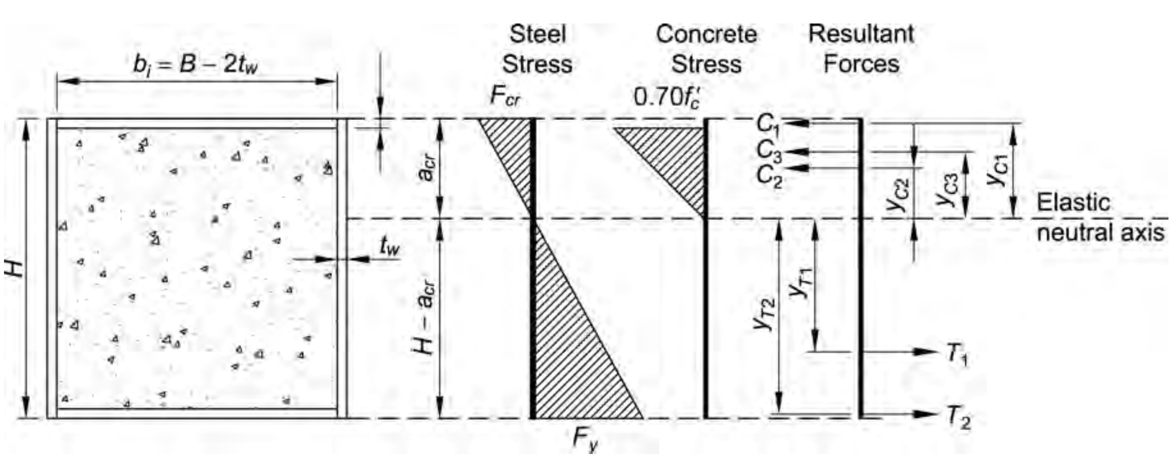


Figure I.7-4. First yield moment stress blocks and force distribution.

Interaction of Flexure and Compression

The interaction of flexure and compression may be determined in accordance with AISC *Specification* Section H1.1 as follows:

LRFD	ASD
$P_u = 1{,}310$ kips $M_u = 552$ kip-ft $\dfrac{P_r}{P_c} = \dfrac{P_u}{\phi_c P_n}$ $= \dfrac{1{,}310 \text{ kips}}{3{,}300 \text{ kips}}$ $= 0.397 > 0.2$ Therefore, use AISC *Specification* Equation H1-1a. $\dfrac{P_u}{\phi_c P_n} + \dfrac{8}{9}\left(\dfrac{M_u}{\phi_b M_n}\right) \le 1.0$ (from *Spec*. Eq. H1-1a) $0.397 + \dfrac{8}{9}\left(\dfrac{552 \text{ kip-ft}}{918 \text{ kip-ft}}\right) \le 1.0$ $0.931 < 1.0$ **o.k.**	$P_a = 1{,}370$ kips $M_a = 248$ kip-ft $\dfrac{P_r}{P_c} = \dfrac{P_a}{P_n / \Omega_c}$ $= \dfrac{1{,}370 \text{ kips}}{2{,}200 \text{ kips}}$ $= 0.622 > 0.2$ Therefore, use AISC *Specification* Equation H1-1a. $\dfrac{P_a}{P_n / \Omega_c} + \dfrac{8}{9}\left(\dfrac{M_a}{M_n / \Omega_c}\right) \le 1.0$ (from *Spec*. Eq. H1-1a) $0.622 + \dfrac{8}{9}\left(\dfrac{248 \text{ kip-ft}}{611 \text{ kip-ft}}\right) \le 1.0$ $0.983 < 1.0$ **o.k.**

Thus, a plate thickness of ¼ in. is adequate.

Note that in addition to the design checks performed for the composite condition, design checks for other load stages should be performed as required by AISC *Specification* Section I1. These checks should take into account the effect of hydrostatic loads from concrete placement as well as the strength of the steel section alone prior to composite action.

Available Shear Strength

According to AISC *Specification* Section I4.1, there are three acceptable methods for determining the available shear strength of the member: available shear strength of the steel section alone in accordance with Chapter G; available shear strength of the reinforced concrete portion alone per ACI 318; or available shear strength of the steel section in addition to the reinforcing steel ignoring the contribution of the concrete. Considering that the member in question does not have longitudinal reinforcing, it is determined by inspection that the shear strength will be controlled by the steel section alone using the provisions of Chapter G.

From AISC *Specification* Section G4, the nominal shear strength, V_n, of box members is determined using AISC *Specification* Equation G4-1 with C_{v2} determined from AISC *Specification* Section G2.2 with $k_v = 5$. As opposed to HSS sections that require the use of a reduced web area to take into account the corner radii, the web area of a box section may be used as follows:

$$
\begin{aligned}
A_w &= 2ht_w, \text{ where } h = \text{clear distance between flanges} \\
&= 2(29.5 \text{ in.})(\text{¼ in.}) \\
&= 14.8 \text{ in.}^2
\end{aligned}
$$

The slenderness value, $h/t_w = h/t$, which is the same as that calculated previously for use in local buckling classification, $\lambda = 118$.

$$1.37\sqrt{k_v E/F_y} = 1.37\sqrt{5\left(\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}\right)}$$
$$= 86.9 < h/t = 118$$

Therefore, use AISC *Specification* Equation G2-11 to calculate C_{v2}.

The web shear coefficient and nominal shear strength are calculated as:

$$C_{v2} = \frac{1.51 k_v E}{(h/t_w)^2 F_y} \qquad \text{(Spec. Eq. G2-11)}$$
$$= \frac{1.51(5)(29{,}000 \text{ ksi})}{(118)^2(36 \text{ ksi})}$$
$$= 0.437$$
$$V_n = 0.6 F_y A_w C_{v2}$$
$$= 0.6(36 \text{ ksi})(14.8 \text{ in.}^2)(0.437) \qquad \text{(Spec. Eq. G4-1)}$$
$$= 140 \text{ kips}$$

The available shear strength is checked as follows:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(140 \text{ kips})$ $= 126 \text{ kips} > 36.8 \text{ kips}$ **o.k.**	$\Omega_v = 1.67$ $\frac{V_n}{\Omega_v} = \frac{140 \text{ kips}}{1.67}$ $= 83.8 \text{ kips} > 22.1 \text{ kips}$ **o.k.**

Force Allocation and Load Transfer

Load transfer calculations for applied axial forces should be performed in accordance with AISC *Specification* Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions is provided in Example I.3.

Summary

It has been determined that a 30 in. × 30 in. composite box column composed of ¼-in.-thick plate is adequate for the imposed loads.

EXAMPLE I.8 ENCASED COMPOSITE MEMBER FORCE ALLOCATION AND LOAD TRANSFER

Given:

Refer to Figure I.8-1.

Part I: For each loading condition (a) through (c), determine the required longitudinal shear force, V_r', to be transferred between the embedded steel section and concrete encasement.

Part II: For loading condition (b), investigate the force transfer mechanisms of direct bearing and shear connection.

The composite member consists of an ASTM A992 W-shape encased by normal weight (145 lb/ft^3) reinforced concrete having a specified concrete compressive strength, $f_c' = 5$ ksi.

Deformed reinforcing bars conform to ASTM A615 with a minimum yield stress, F_{yr}, of 60 ksi.

Applied loading, P_r, for each condition illustrated in Figure I.8-1 is composed of the following loads:

$P_D = 260$ kips
$P_L = 780$ kips

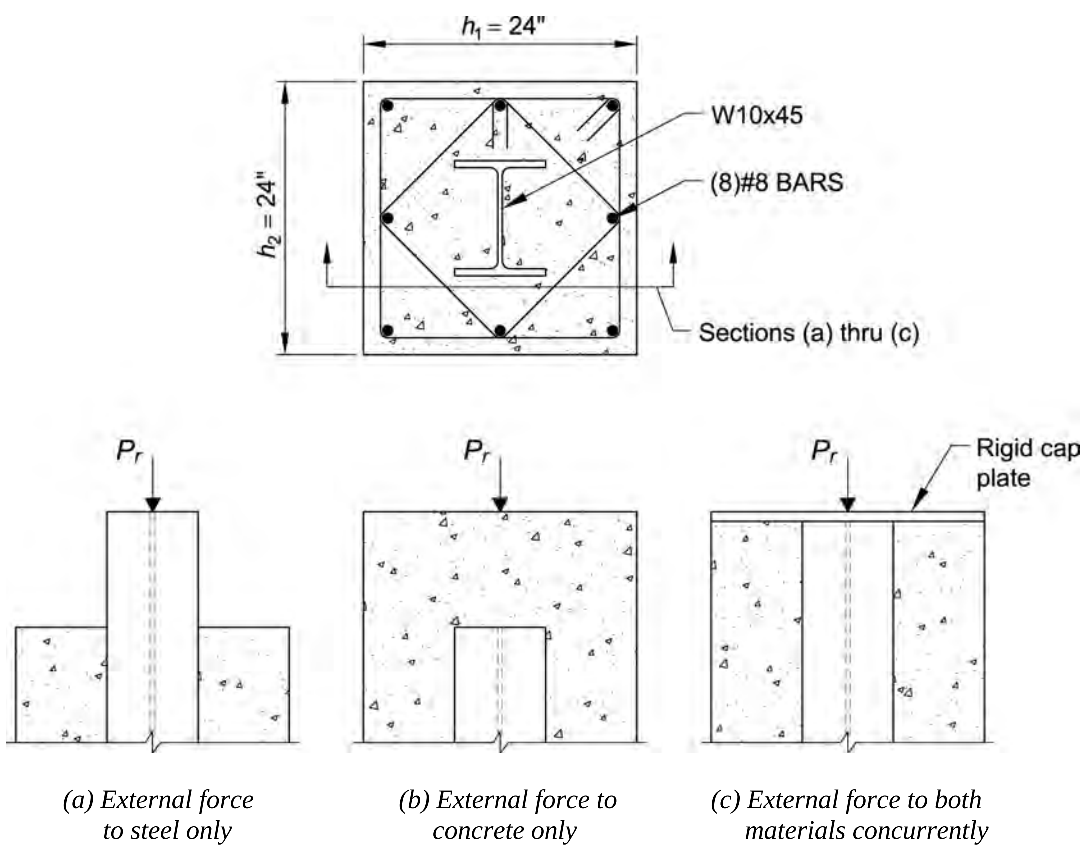


Fig. I.8-1. Encased composite member in compression.

Solution:

Part I—Force Allocation

From AISC *Manual* Table 2-4, the steel material properties are:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1 and Figure I.8-1, the geometric properties of the encased W10×45 are as follows:

$$A_s = 13.3 \text{ in.}^2$$
$$b_f = 8.02 \text{ in.}$$
$$t_f = 0.620 \text{ in.}$$
$$t_w = 0.350 \text{ in.}$$
$$d = 10.1 \text{ in.}$$
$$h_1 = 24 \text{ in.}$$
$$h_2 = 24 \text{ in.}$$

Additional geometric properties of the composite section used for force allocation and load transfer are calculated as follows:

$$\begin{aligned} A_g &= h_1 h_2 \\ &= (24 \text{ in.})(24 \text{ in.}) \\ &= 576 \text{ in.}^2 \end{aligned}$$

$$A_{sri} = 0.79 \text{ in.}^2 \text{ for a No. 8 bar}$$

$$\begin{aligned} A_{sr} &= \sum_{i=1}^{n} A_{sri} \\ &= 8\left(0.79 \text{ in.}^2\right) \\ &= 6.32 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_c &= A_g - A_s - A_{sr} \\ &= 576 \text{ in.}^2 - 13.3 \text{ in.}^2 - 6.32 \text{ in.}^2 \\ &= 556 \text{ in.}^2 \end{aligned}$$

where
A_c = cross-sectional area of concrete encasement, in.^{2}
A_g = gross cross-sectional area of composite section, in.^{2}
A_{sri}= cross-sectional area of reinforcing bar i, in.^{2}
A_{sr} = cross-sectional area of continuous reinforcing bars, in.^{2}
n = number of continuous reinforcing bars in composite section

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_r = P_u$ $= 1.2(260 \text{ kips}) + 1.6(780 \text{ kips})$ $= 1{,}560 \text{ kips}$	$P_r = P_a$ $= 260 \text{ kips} + 780 \text{ kips}$ $= 1{,}040 \text{ kips}$

Composite Section Strength for Force Allocation

In accordance with AISC *Specification* Section I6, force allocation calculations are based on the nominal axial compressive strength of the encased composite member without length effects, P_{no}. This section strength is defined in Section I2.1b as:

$$P_{no} = F_y A_s + F_{ysr} A_{sr} + 0.85 f_c' A_c \qquad (Spec.\ \text{Eq. I2-4})$$

$$= (50 \text{ ksi})(13.3 \text{ in.}^2) + (60 \text{ ksi})(6.32 \text{ in.}^2) + 0.85(5 \text{ ksi})(556 \text{ in.}^2)$$

$$= 3{,}410 \text{ kips}$$

Transfer Force for Condition (a)

Refer to Figure I.8-1(a). For this condition, the entire external force is applied to the steel section only, and the provisions of AISC *Specification* Section I6.2a apply.

$$V_r' = P_r\left(1 - \frac{F_y A_s}{P_{no}}\right) \qquad (Spec.\ \text{Eq. I6-1})$$

$$= P_r\left[1 - \frac{(50 \text{ ksi})(13.3 \text{ in.}^2)}{3{,}410 \text{ kips}}\right]$$

$$= 0.805 P_r$$

LRFD	ASD
$V_r' = 0.805(1{,}560 \text{ kips})$ $= 1{,}260 \text{ kips}$	$V_r' = 0.805(1{,}040 \text{ kips})$ $= 837 \text{ kips}$

Transfer Force for Condition (b)

Refer to Figure I.8-1(b). For this condition, the entire external force is applied to the concrete encasement only, and the provisions of AISC *Specification* Section I6.2b apply.

$$V_r' = P_r\left(\frac{F_y A_s}{P_{no}}\right) \qquad (Spec.\ \text{Eq. I6-2a})$$

$$= P_r\left[\frac{(50 \text{ ksi})(13.3 \text{ in.}^2)}{3{,}410 \text{ kips}}\right]$$

$$= 0.195 P_r$$

LRFD	ASD
$V_r' = 0.195(1{,}560 \text{ kips})$ $= 304 \text{ kips}$	$V_r' = 0.195(1{,}040 \text{ kips})$ $= 203 \text{ kips}$

Transfer Force for Condition (c)

Refer to Figure I.8-1(c). For this condition, external force is applied to the steel section and concrete encasement concurrently, and the provisions of AISC *Specification* Section I6.2c apply.

AISC *Specification* Commentary Section I6.2 states that when loads are applied to both the steel section and concrete encasement concurrently, V_r' can be taken as the difference in magnitudes between the portion of the external force applied directly to the steel section and that required by Equation I6-2a. This concept can be written in equation form as follows:

$$V_r' = \left| P_{rs} - P_r \left(\frac{F_y A_s}{P_{no}} \right) \right| \qquad \text{(Eq. 1)}$$

where

P_{rs} = portion of external force applied directly to the steel section, kips

Currently, the *Specification* provides no specific requirements for determining the distribution of the applied force for the determination of P_{rs}, so it is left to engineering judgment. For a bearing plate condition such as the one represented in Figure I.8-1(c), one possible method for determining the distribution of applied forces is to use an elastic distribution based on the material axial stiffness ratios as follows:

$$\begin{aligned} E_c &= w_c^{1.5} \sqrt{f_c'} \\ &= \left(145 \text{ lb/ft}^3\right)^{1.5} \sqrt{5 \text{ ksi}} \\ &= 3{,}900 \text{ ksi} \end{aligned}$$

$$\begin{aligned} P_{rs} &= \left(\frac{E_s A_s}{E_s A_s + E_c A_c + E_{sr} A_{sr}} \right) P_r \\ &= \left[\frac{(29{,}000 \text{ ksi})\left(13.3 \text{ in.}^2\right)}{(29{,}000 \text{ ksi})\left(13.3 \text{ in.}^2\right) + (3{,}900 \text{ ksi})\left(556 \text{ in.}^2\right) + (29{,}000 \text{ ksi})\left(6.32 \text{ in.}^2\right)} \right] P_r \\ &= 0.141 P_r \end{aligned}$$

Substituting the results into Equation 1 yields:

$$\begin{aligned} V_r' &= \left| 0.141 P_r - P_r \left(\frac{F_y A_s}{P_{no}} \right) \right| \\ &= \left| 0.141 P_r - P_r \left[\frac{(50 \text{ ksi})\left(13.3 \text{ in.}^2\right)}{3{,}410 \text{ kips}} \right] \right| \\ &= 0.0540 P_r \end{aligned}$$

LRFD	ASD
$V_r' = 0.0540(1{,}560 \text{ kips})$ $= 84.2 \text{ kips}$	$V_r' = 0.0540(1{,}040 \text{ kips})$ $= 56.2 \text{ kips}$

An alternate approach would be use of a plastic distribution method whereby the load is partitioned to each material in accordance with their contribution to the composite section strength given in Equation I2-4. This method eliminates the need for longitudinal shear transfer provided the local bearing strength of the concrete and steel are adequate to resist the forces resulting from this distribution.

Additional Discussion

- The design and detailing of the connections required to deliver external forces to the composite member should be performed according to the applicable sections of AISC *Specification* Chapters J and K.
- The connection cases illustrated by Figure I.8-1 are idealized conditions representative of the mechanics of actual connections. For instance, an extended single plate connection welded to the flange of the W10 and extending out beyond the face of concrete to attach to a steel beam is an example of a condition where it may be assumed that all external force is applied directly to the steel section only.

Solution:

Part II—Load Transfer

The required longitudinal force to be transferred, V_r', determined in Part I condition (b) is used to investigate the applicable force transfer mechanisms of AISC *Specification* Section I6.3: direct bearing and shear connection. As indicated in the *Specification*, these force transfer mechanisms may not be superimposed; however, the mechanism providing the greatest nominal strength may be used. Note that direct bond interaction is not applicable to encased composite members as the variability of column sections and connection configurations makes confinement and bond strength more difficult to quantify than in filled HSS.

Direct Bearing

Determine Layout of Bearing Plates

One method of utilizing direct bearing as a load transfer mechanism is through the use of internal bearing plates welded between the flanges of the encased W-shape as indicated in Figure I.8-2.

When using bearing plates in this manner, it is essential that concrete mix proportions and installation techniques produce full bearing at the plates. Where multiple sets of bearing plates are used as illustrated in Figure I.8-2, it is recommended that the minimum spacing between plates be equal to the depth of the encased steel member to enhance constructability and concrete consolidation. For the configuration under consideration, this guideline is met with a plate spacing of $24 \text{ in.} \geq d = 10.1 \text{ in.}$

Bearing plates should be located within the load introduction length given in AISC *Specification* Section I6.4a. The load introduction length is defined as two times the minimum transverse dimension of the composite member both above and below the load transfer region. The load transfer region is defined in *Specification* Commentary Section I6.4 as the depth of the connection. For the connection configuration under consideration, where the majority of the required force is being applied from the concrete column above, the depth of connection is conservatively taken as zero. Because the composite member only extends to one side of the point of force transfer, the bearing plates should be located within $2h_2$ = 48 in. of the top of the composite member as indicated in Figure I.8-2.

Available Strength for the Limit State of Direct Bearing

Assuming two sets of bearing plates are to be used as indicated in Figure I.8-2, the total contact area between the bearing plates and the concrete, A_1, may be determined as follows:

$$a = \frac{b_f - t_w}{2}$$
$$= \frac{8.02\text{ in.} - 0.350\text{ in.}}{2}$$
$$= 3.84\text{ in.}$$

$$b = d - 2t_f$$
$$= 10.1\text{ in.} - 2(0.620\text{ in.})$$
$$= 8.86\text{ in.}$$

c = width of clipped corners
= ¾ in.

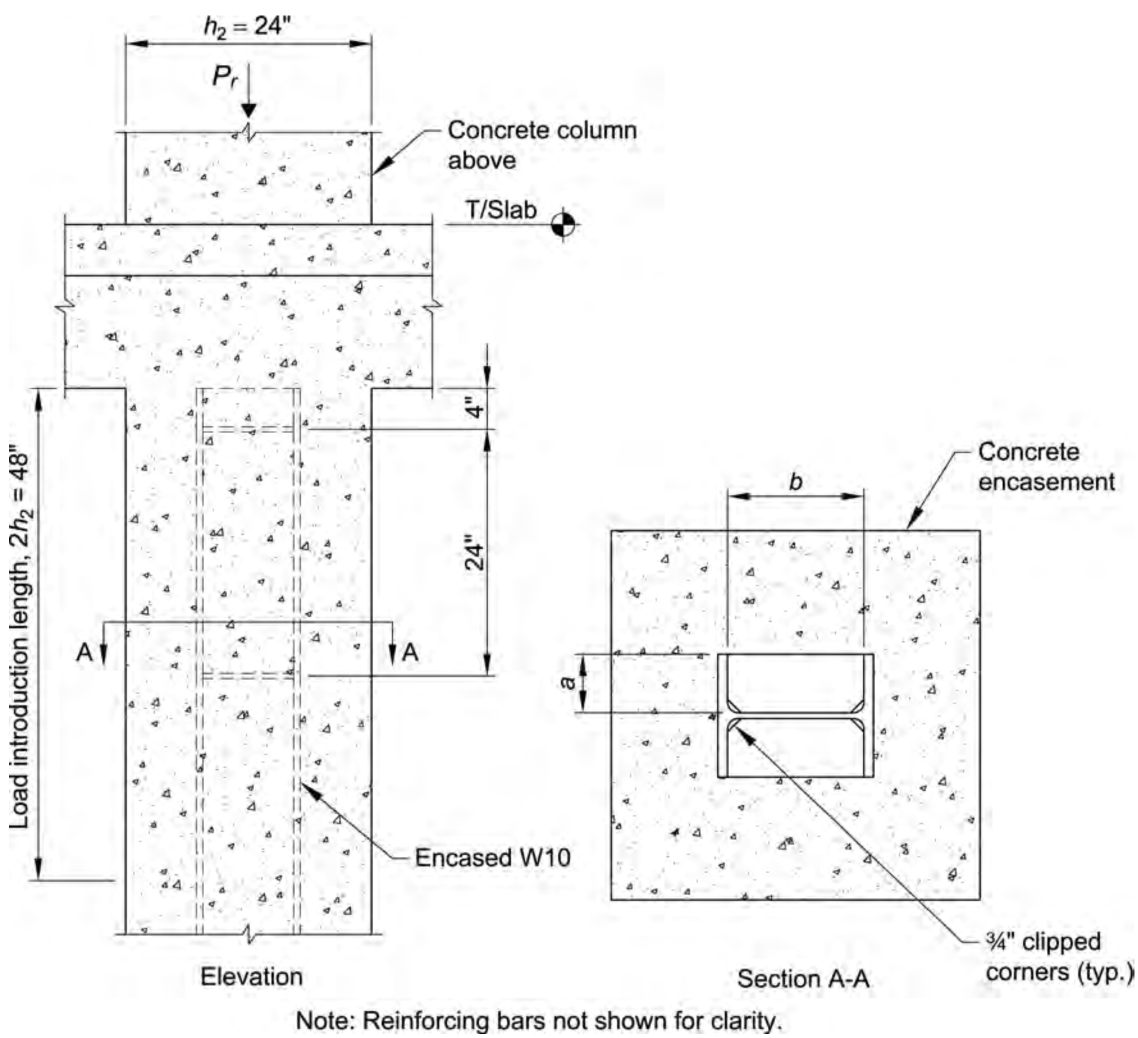


Fig. I.8-2. Composite member with internal bearing plates.

$$A_1 = \left(2ab - 2c^2\right)(\text{number of bearing plate sets})$$
$$= \left[2(3.84 \text{ in.})(8.86 \text{ in.}) - 2(\text{3/4 in.})^2\right](2)$$
$$= 134 \text{ in.}^2$$

The available strength for the direct bearing force transfer mechanism is:

$$R_n = 1.7 f_c' A_1$$
$$= 1.7(5 \text{ ksi})\left(134 \text{ in.}^2\right) \qquad (\textit{Spec}. \text{ Eq. I6-3})$$
$$= 1{,}140 \text{ kips}$$

LRFD	ASD
$\phi_B = 0.65$ $\phi_B R_n = 0.65(1{,}140 \text{ kips})$ $= 741 \text{ kips} > V_r' = 304 \text{ kips}$ **o.k.**	$\Omega_B = 2.31$ $\dfrac{R_n}{\Omega_B} = \dfrac{1{,}140 \text{ kips}}{2.31}$ $= 494 \text{ kips} > V_r' = 203 \text{ kips}$ **o.k.**

Thus, two sets of bearing plates are adequate. From these calculations, it can be seen that one set of bearing plates are adequate for force transfer purposes; however, the use of two sets of bearing plates serves to reduce the bearing plate thickness calculated in the following section.

Required Bearing Plate Thickness

There are several methods available for determining the bearing plate thickness. For rectangular plates supported on three sides, elastic solutions for plate stresses, such as those found in *Roark's Formulas for Stress and Strain* (Young and Budynas, 2002), may be used in conjunction with AISC *Specification* Section F12 for thickness calculations. Alternately, yield line theory or computational methods such as finite element analysis may be employed.

For this example, yield line theory is employed. Results of the yield line analysis depend on an assumption of column flange strength versus bearing plate strength in order to estimate the fixity of the bearing plate to column flange connection. In general, if the thickness of the bearing plate is less than the column flange thickness, fixity and plastic hinging can occur at this interface; otherwise, the use of a pinned condition is conservative. Ignoring the fillets of the W-shape and clipped corners of the bearing plate, the yield line pattern chosen for the fixed condition is depicted in Figure I.8-3. Note that the simplifying assumption of 45° yield lines illustrated in Figure I.8-3 has been shown to provide reasonably accurate results (Park and Gamble, 2000), and that this yield line pattern is only valid where $b \geq 2a$.

The plate thickness using $F_y = 36$ ksi material may be determined as:

LRFD	ASD
$\phi = 0.90$ If $t_p \geq t_f$: $t_p = \sqrt{\dfrac{2a^2 w_u (3b - 2a)}{3\phi F_y (4a + b)}}$	$\Omega = 1.67$ If $t_p \geq t_f$: $t_p = \sqrt{\left(\dfrac{2\Omega}{3F_y}\right)\left[\dfrac{a^2 w_a (3b - 2a)}{(4a + b)}\right]}$

LRFD	ASD
If $t_p < t_f$: $$t_p = \sqrt{\frac{2a^2 w_u (3b-2a)}{3\phi F_y (6a+b)}}$$ where w_u = bearing pressure on plate determined using LRFD load combinations $$= \frac{V_r'}{A_1}$$ $$= \frac{304 \text{ kips}}{134 \text{ in.}^2}$$ $$= 2.27 \text{ ksi}$$ Assuming $t_p \geq t_f$ $$t_p = \sqrt{\frac{2(3.84 \text{ in.})^2 (2.27 \text{ ksi}) \times [3(8.86 \text{ in.}) - 2(3.84 \text{ in.})]}{3(0.90)(36 \text{ ksi})[4(3.84 \text{ in.}) + 8.86 \text{ in.}]}}$$ $$= 0.733 \text{ in.}$$ Select ¾-in. plate. t_p = ¾ in. > t_f = 0.620 in. **assumption o.k.**	If $t_p < t_f$: $$t_p = \sqrt{\left(\frac{2\Omega}{3F_y}\right)\left[\frac{a^2 w_a (3b-2a)}{(6a+b)}\right]}$$ where w_a = bearing pressure on plate determined using ASD load combinations $$= \frac{V_r'}{A_1}$$ $$= \frac{203 \text{ kips}}{134 \text{ in.}^2}$$ $$= 1.51 \text{ ksi}$$ Assuming $t_p \geq t_f$ $$t_p = \sqrt{\frac{2(1.67)(3.84 \text{ in.})^2 (1.51 \text{ ksi}) \times [3(8.86 \text{ in.}) - 2(3.84 \text{ in.})]}{3(36 \text{ ksi})[4(3.84 \text{ in.}) + 8.86 \text{ in.}]}}$$ $$= 0.733 \text{ in.}$$ Select ¾-in. plate t_p = ¾ in. > t_f = 0.620 in. **assumption o.k.**

Thus, select ¾-in.-thick bearing plates.

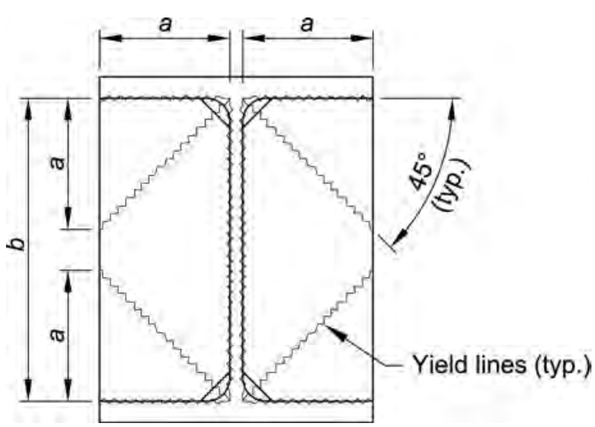


Fig. I.8-3. Internal bearing plate yield line pattern (fixed condition).

Bearing Plate to Encased Steel Member Weld

The bearing plates should be connected to the encased steel member using welds designed in accordance with AISC *Specification* Chapter J to develop the full strength of the plate. For fillet welds, a weld size of $\frac{5}{8}t_p$ will serve to develop the strength of either a 36- or 50-ksi plate as discussed in AISC *Manual* Part 10.

Shear Connection

Shear connection involves the use of steel headed stud or channel anchors placed on at least two faces of the steel shape in a generally symmetric configuration to transfer the required longitudinal shear force. For this example, ¾-in.-diameter × 4³⁄₁₆-in.-long steel headed stud anchors composed of ASTM A108 material are selected. The specified minimum tensile strength, F_u, of ASTM A108 material is 65 ksi.

Available Shear Strength of Steel Headed Stud Anchors

The available shear strength of an individual steel headed stud anchor is determined in accordance with the composite component provisions of AISC *Specification* Section I8.3 as directed by Section I6.3b.

$$Q_{nv} = F_u A_{sa} \qquad \text{(}\textit{Spec}\text{. Eq. I8-3)}$$

$$A_{sa} = \frac{\pi(\text{¾ in.})^2}{4} = 0.442 \text{ in.}^2$$

LRFD	ASD
$\phi_v = 0.65$ $\phi_v Q_{nv} = 0.65(65 \text{ ksi})(0.442 \text{ in.}^2)$ $= 18.7$ kips per steel headed stud anchor	$\Omega_v = 2.31$ $\frac{Q_{nv}}{\Omega_v} = \frac{(65 \text{ ksi})(0.442 \text{ in.}^2)}{2.31}$ $= 12.4$ kips per steel headed stud anchor

Required Number of Steel Headed Stud Anchors

The number of steel headed stud anchors required to transfer the longitudinal shear is calculated as follows:

LRFD	ASD
$n_{anchors} = \frac{V_r'}{\phi_v Q_{nv}}$ $= \frac{304 \text{ kips}}{18.7 \text{ kips}}$ $= 16.3$ steel headed stud anchors	$n_{anchors} = \frac{V_r'}{Q_{nv}/\Omega_v}$ $= \frac{203 \text{ kips}}{12.4 \text{ kips}}$ $= 16.4$ steel headed stud anchors

With anchors placed in pairs on each flange, select 20 anchors to satisfy the symmetry provisions of AISC *Specification* Section I6.4a.

Placement of Steel Headed Stud Anchors

Steel headed stud anchors are placed within the load introduction length in accordance with AISC *Specification* Section I6.4a. Because the composite member only extends to one side of the point of force transfer, the steel anchors are located within $2h_2 = 48$ in. of the top of the composite member.

Placing two anchors on each flange provides four anchors per group, and maximum stud spacing within the load introduction length is determined as:

$$s_{max} = \frac{\text{load introduction length} - \text{distance to first anchor group from upper end of encased shape}}{\left[\dfrac{\text{total number of anchors}}{\text{number of anchors per group}}\right] - 1}$$

$$= \frac{48 \text{ in.} - 6 \text{ in.}}{\left[\dfrac{20 \text{ anchors}}{4 \text{ anchors per group}}\right] - 1}$$

$$= 10.5 \text{ in.}$$

Use 10 in. spacing beginning 6 in. from top of encased member.

In addition to anchors placed within the load introduction length, anchors must also be placed along the remainder of the composite member at a maximum spacing of 32 times the anchor shank diameter = 24 in. in accordance with AISC *Specification* Sections I6.4a and I8.3e.

The chosen anchor layout and spacing is illustrated in Figure I.8-4.

Steel Headed Stud Anchor Detailing Limitations of AISC Specification *Sections I6.4a, I8.1 and I8.3*

Steel headed stud anchor detailing limitations are reviewed in this section with reference to the anchor configuration provided in Figure I.8-4 for anchors having a shank diameter, d_{sa}, of ¾ in. Note that these provisions are specific to the detailing of the anchors themselves and that additional limitations for the structural steel, concrete and reinforcing components of composite members should be reviewed as demonstrated in Design Example I.9.

(1) Anchors must be placed on at least two faces of the steel shape in a generally symmetric configuration:

Anchors are located in pairs on both faces. **o.k.**

(2) Maximum anchor diameter: $d_{sa} \le 2.5(t_f)$

$$\text{¾ in.} < 2.5(0.620 \text{ in.}) = 1.55 \text{ in.} \quad \textbf{o.k.}$$

(3) Minimum steel headed stud anchor height-to-diameter ratio: $h / d_{sa} \ge 5$

The minimum ratio of installed anchor height (base to top of head), h, to shank diameter, d_{sa}, must meet the provisions of AISC *Specification* Section I8.3 as summarized in the User Note table at the end of the section. For shear in normal weight concrete the limiting ratio is five. As previously discussed, a 4³⁄₁₆-in.-long anchor was selected from anchor manufacturer's data. As the h/d_{sa} ratio is based on the installed length, a length reduction for burn off during installation of ³⁄₁₆ in. is taken to yield the final installed length of 4 in.

$$\frac{h}{d_{sa}} = \frac{4 \text{ in.}}{\text{¾ in.}} = 5.33 > 5 \quad \textbf{o.k.}$$

(4) Minimum lateral clear concrete cover = 1½ in.

From AWS D1.1 (AWS, 2015) Figure 7.1, the head diameter of a ¾-in.-diameter stud anchor is equal to 1.25 in.

$$\text{lateral clear cover} = \left(\frac{h_1}{2}\right) - \left(\frac{\text{lateral spacing between anchor centerlines}}{2}\right) - \left(\frac{\text{anchor head diameter}}{2}\right)$$
$$= \left(\frac{24\text{ in.}}{2}\right) - \left(\frac{4\text{ in.}}{2}\right) - \left(\frac{1.25\text{ in.}}{2}\right)$$
$$= 9.38\text{ in.} > 1\tfrac{1}{2}\text{ in.} \quad \textbf{o.k.}$$

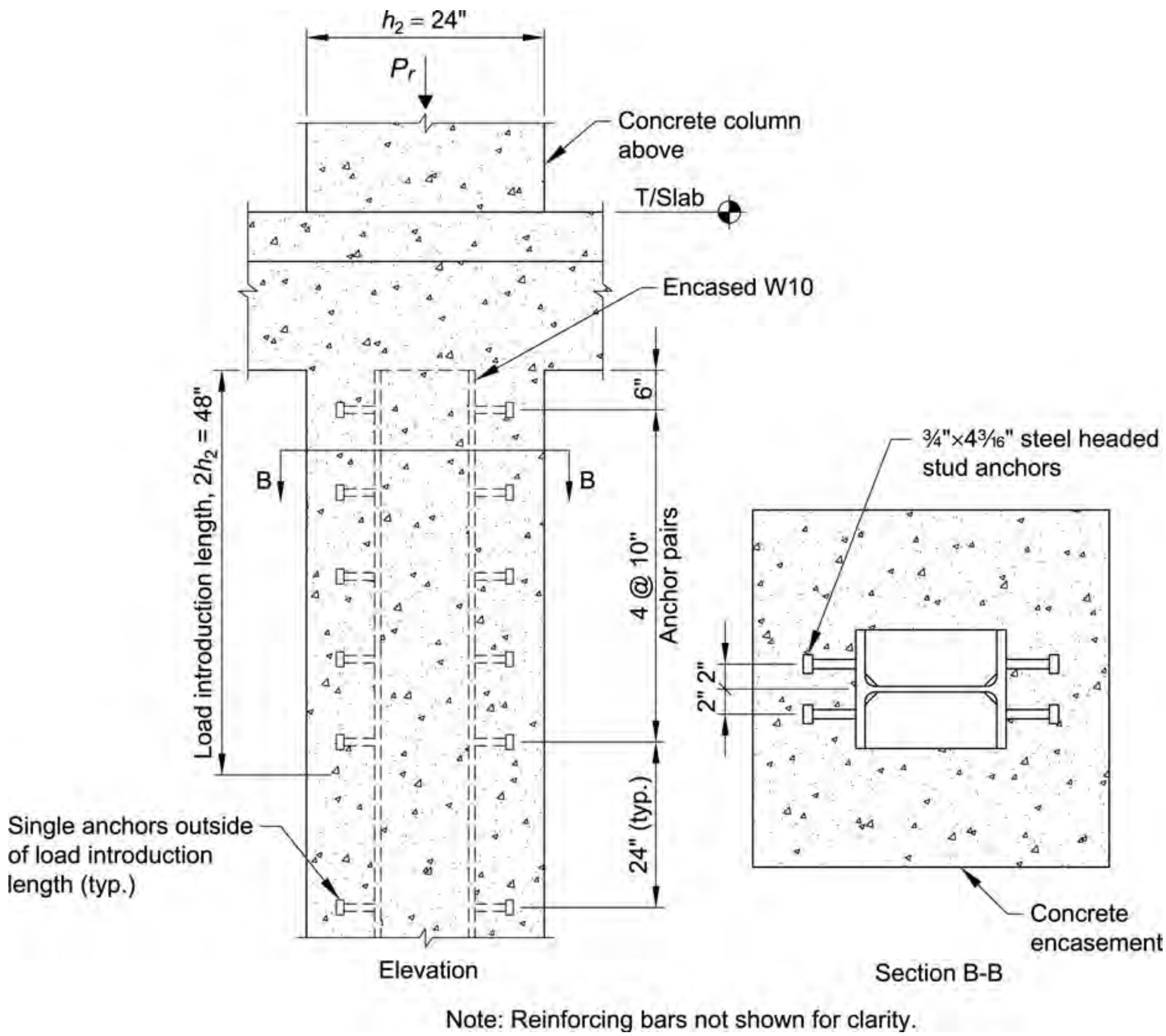


Fig. I.8-4. Composite member with steel anchors.

(5) Minimum anchor spacing:

$$s_{min} = 4d_{sa}$$
$$= 4(\text{¾ in.})$$
$$= 3.00 \text{ in.}$$

In accordance with AISC *Specification* Section I8.3e, this spacing limit applies in any direction.

$$s_{transverse} = 4 \text{ in.} > s_{min} \quad \textbf{o.k.}$$
$$s_{longitudinal} = 10 \text{ in.} > s_{min} \quad \textbf{o.k.}$$

(6) Maximum anchor spacing:

$$s_{max} = 32d_{sa}$$
$$= 32(\text{¾ in.})$$
$$= 24.0 \text{ in.}$$

In accordance with AISC *Specification* Section I6.4a, the spacing limits of Section I8.3e apply to steel anchor spacing both within and outside of the load introduction region.

$$s = 24.0 \text{ in.} \le s_{max} \quad \textbf{o.k.}$$

(7) Clear cover above the top of the steel headed stud anchors:

Minimum clear cover over the top of the steel headed stud anchors is not explicitly specified for steel anchors in composite components; however, in keeping with the intent of AISC *Specification* Section I1.1, it is recommended that the clear cover over the top of the anchor head follow the cover requirements of ACI 318 (ACI 318, 2014) Section 20.6.1. For concrete columns, ACI 318 specifies a clear cover of 1½ in.

$$\text{clear cover above anchor} = \frac{h_2}{2} - \frac{d}{2} - \text{installed anchor length}$$
$$= \frac{24 \text{ in.}}{2} - \frac{10.1 \text{ in.}}{2} - 4 \text{ in.}$$
$$= 2.95 \text{ in.} > 1\tfrac{1}{2} \text{ in.} \quad \textbf{o.k.}$$

Concrete Breakout

AISC *Specification* Section I8.3a states that in order to use Equation I8-3 for shear strength calculations as previously demonstrated, concrete breakout strength in shear must not be an applicable limit state. If concrete breakout is deemed to be an applicable limit state, the *Specification* provides two alternatives: either the concrete breakout strength can be determined explicitly using ACI 318, Chapter 17, in accordance with *Specification* Section I8.3a(b), or anchor reinforcement can be provided to resist the breakout force as discussed in *Specification* Section I8.3a(a).

Determining whether concrete breakout is a viable failure mode is left to the engineer. According to AISC *Specification* Commentary Section I8.3, "it is important that it be deemed by the engineer that a concrete breakout failure mode in shear is directly avoided through having the edges perpendicular to the line of force supported, and the edges parallel to the line of force sufficiently distant that concrete breakout through a side edge is not deemed viable."

For the composite member being designed, no free edge exists in the direction of shear transfer along the length of the column, and concrete breakout in this direction is not an applicable limit state. However, it is still incumbent upon the engineer to review the possibility of concrete breakout through a side edge parallel to the line of force.

One method for explicitly performing this check is through the use of the provisions of ACI 318, Chapter 17, as follows:

ACI 318, Section 17.5.2.1(c), specifies that concrete breakout shall be checked for shear force parallel to the edge of a group of anchors using twice the value for the nominal breakout strength provided by ACI 318, Equation 17.5.2.1b, when the shear force in question acts perpendicular to the edge.

For the composite member being designed, symmetrical concrete breakout planes form to each side of the encased shape, one of which is illustrated in Figure I.8-5.

$\phi = 0.75$ for anchors governed by concrete breakout with supplemental reinforcement (provided by tie reinforcement) in accordance with ACI 318, Section 17.3.3

$$V_{cbg} = 2\left[\frac{A_{Vc}}{A_{Vco}}\Psi_{ec,V}\Psi_{ed,V}\Psi_{c,V}\Psi_{h,V}V_b\right], \text{ for shear force parallel to an edge} \quad \text{(ACI 318, Eq. 17.5.2.1b)}$$

$$\begin{aligned} A_{Vco} &= 4.5(c_{a1})^2 \quad \text{(ACI 318, Eq. 17.5.2.1c)} \\ &= 4.5(10\text{ in.})^2 \\ &= 450\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{Vc} &= (15\text{ in.} + 40\text{ in.} + 15\text{ in.})(24\text{ in.}), \text{ from Figure I.8-5} \\ &= 1{,}680\text{ in.}^2 \end{aligned}$$

$\Psi_{ec,V} = 1.0$ no eccentricity

$\Psi_{ed,V} = 1.0$ in accordance with ACI 318, Section 17.5.2.1(c)

$\Psi_{c,V} = 1.4$ compression-only member assumed uncracked

$\Psi_{h,V} = 1.0$

$$V_b = \left[8\left(\frac{l_e}{d_a}\right)^{0.2}\sqrt{d_a}\right]\lambda_a\sqrt{f'_c}\,(c_{a1})^{1.5} \quad \text{(ACI 318, Eq. 17.5.2.3)}$$

where

$l_e = 4\text{ in.} - \tfrac{3}{8}$-in. anchor head thickness from AWS D1.1, Figure 7.1
$= 3.63\text{ in.}$

$d_a = \tfrac{3}{4}$-in. anchor diameter

$\lambda_a = 1.0\lambda$ from ACI 318, Section 17.2.6, for normal weight concrete

$\lambda = 1.0$ from ACI 318, Table 19.2.4.2, for normal weight concrete

$$V_b = \left[8\left(\frac{3.63\text{ in.}}{3/4\text{ in.}}\right)^{0.2}\sqrt{3/4\text{ in.}}\right](1.0)\frac{\sqrt{5{,}000\text{ psi}}}{1{,}000\text{ lb/kip}}(10\text{ in.})^{1.5}$$
$$= 21.2\text{ kips}$$

$$V_{cbg} = 2\left[\frac{1{,}680\text{ in.}^2}{450\text{ in.}^2}(1.0)(1.0)(1.4)(1.0)(21.2\text{ kips})\right]$$
$$= 222\text{ kips}$$

$$\phi V_{cbg} = 0.75(222\text{ kips})$$
$$= 167\text{ kips per breakout plane}$$

$$\phi V_{cbg} = (2\text{ breakout planes})(167\text{ kips/plane})$$
$$= 334\text{ kips}$$

$\phi V_{cbg} > V_r' = 304$ kips **o.k.**

Thus, concrete breakout along an edge parallel to the direction of the longitudinal shear transfer is not a controlling limit state, and Equation I8-3 is appropriate for determining available anchor strength.

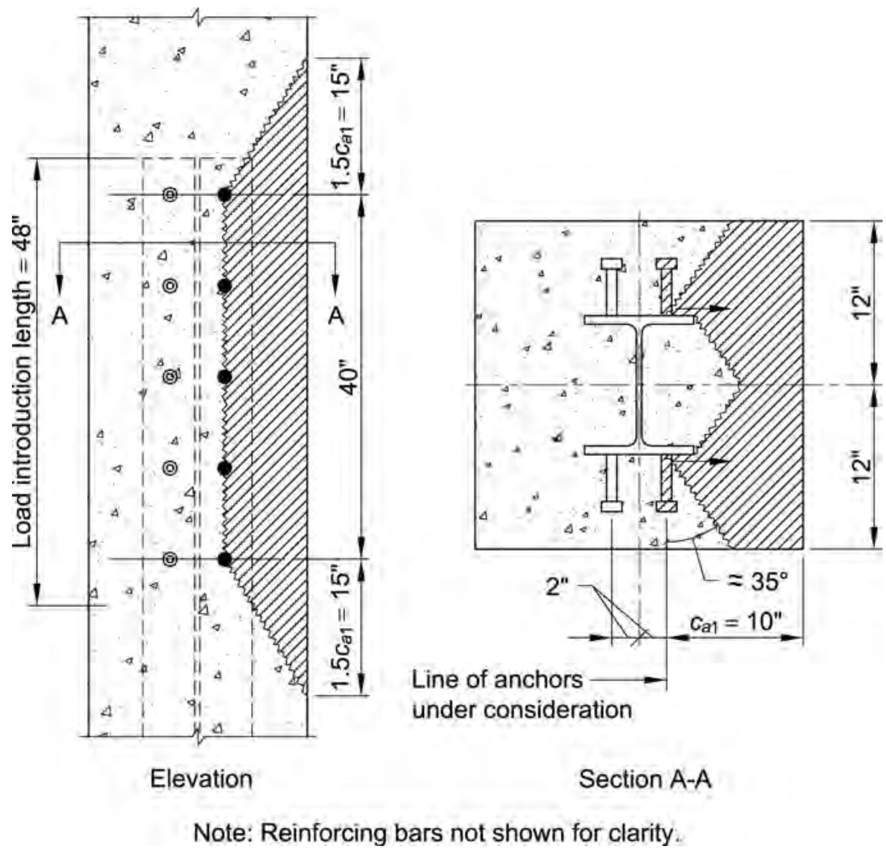


Fig. I.8-5. Concrete breakout check for shear force parallel to an edge.

Encased beam-column members with reinforcing detailed in accordance with the AISC *Specification* have demonstrated adequate confinement in tests to prevent concrete breakout along a parallel edge from occurring; however, it is still incumbent upon the engineer to review the project-specific detailing used for susceptibility to this limit state.

If concrete breakout was determined to be a controlling limit state, transverse reinforcing ties could be analyzed as anchor reinforcement in accordance with AISC *Specification* Section I8.3a(a), and tie spacing through the load introduction length adjusted as required to prevent breakout. Alternately, the steel headed stud anchors could be relocated to the web of the encased member where breakout is prevented by confinement between the column flanges.

EXAMPLE I.9 ENCASED COMPOSITE MEMBER IN AXIAL COMPRESSION

Given:

Determine if the encased composite member illustrated in Figure I.9-1 is adequate for the indicated dead and live loads.

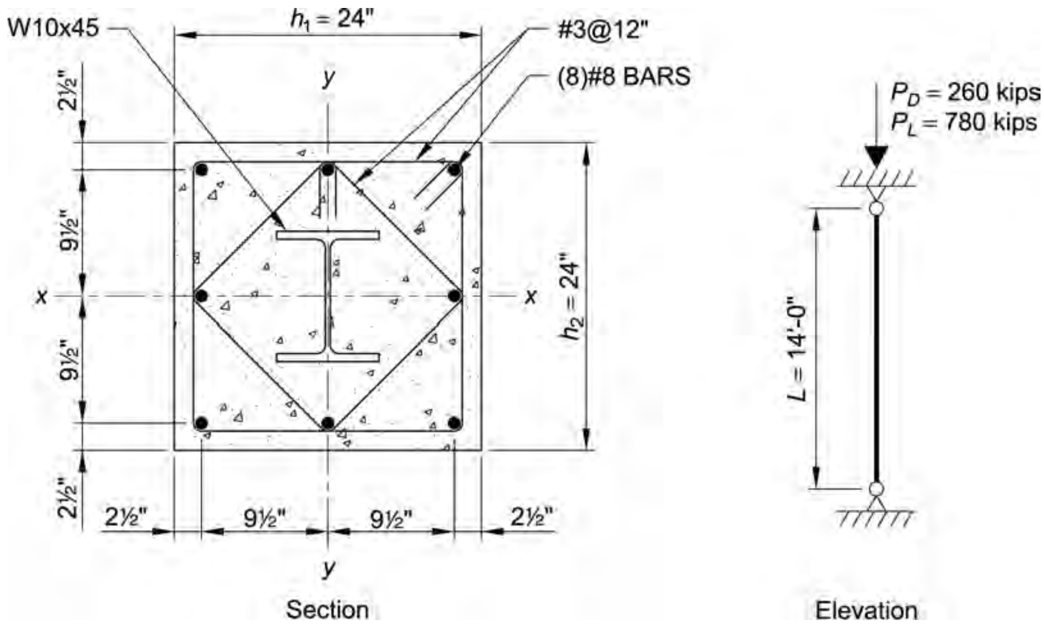


Fig. I.9-1. Encased composite member section and applied loading.

The composite member consists of an ASTM A992 W-shape encased by normal weight (145 lb/ft^3) reinforced concrete having a specified concrete compressive strength, f'_c = 5 ksi.

Deformed reinforcing bars conform to ASTM A615 with a minimum yield stress, F_{yr}, of 60 ksi.

Solution:

From AISC *Manual* Table 2-4, the steel material properties are:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, Figure I.9-1, and Design Example I.8, geometric and material properties of the composite section are:

A_s = 13.3 in.^{2}	b_f = 8.02 in.	t_f = 0.620 in.	d = 10.1 in.
h_1 = 24 in.	h_2 = 24 in.	I_{sx} = 248 in.^{4}	I_{sy} = 53.4 in.^{4}
A_g = 576 in.^{2}	A_{sri}= 0.790 in.^{2}	A_{sr} = 6.32 in.^{2}	A_c = 556 in.^{2}
E_c = 3,900 ksi			

The moment of inertia of the reinforcing bars about the elastic neutral axis of the composite section, I_{sr}, is required for composite member design and is calculated as follows:

$d_b = 1$ in. for the diameter of a No. 8 bar

$$I_{sri} = \frac{\pi d_b^4}{64}$$
$$= \frac{\pi(1\text{ in.})^4}{64}$$
$$= 0.0491\text{ in.}^4$$

$$I_{sr} = \sum_{i=1}^{n} I_{sri} + \sum_{i=1}^{n} A_{sri} e_i^{\,2}$$
$$= 8\left(0.0491\text{ in.}^4\right) + 6\left(0.79\text{ in.}^2\right)(9.50\text{ in.})^2 + 2\left(0.79\text{ in.}^2\right)(0\text{ in.})^2$$
$$= 428\text{ in.}^4$$

where

A_{sri} = cross-sectional area of reinforcing bar i, in.^{2}
I_{sri} = moment of inertia of reinforcing bar i about its elastic neutral axis, in.^{4}
I_{sr} = moment of inertia of the reinforcing bars about the elastic neutral axis of the composite section, in.^{4}
d_b = nominal diameter of reinforcing bar, in.
e_i = eccentricity of reinforcing bar i with respect to the elastic neutral axis of the composite section, in.
n = number of reinforcing bars in composite section

Note that the elastic neutral axis for each direction of the section in question is located at the x-x and y-y axes illustrated in Figure I.9-1, and that the moment of inertia calculated for the longitudinal reinforcement is valid about either axis due to symmetry.

The moment of inertia values for the concrete about each axis are determined as:

$$I_{cx} = I_{gx} - I_{sx} - I_{srx}$$
$$= \frac{(24\text{ in.})^4}{12} - 248\text{ in.}^4 - 428\text{ in.}^4$$
$$= 27{,}000\text{ in.}^4$$

$$I_{cy} = I_{gy} - I_{sy} - I_{sry}$$
$$= \frac{(24\text{ in.})^4}{12} - 53.4\text{ in.}^4 - 428\text{ in.}^4$$
$$= 27{,}200\text{ in.}^4$$

Classify Section for Local Buckling

In accordance with AISC *Specification* Section I1.2, local buckling effects need not be considered for encased composite members, thus all encased sections are treated as compact sections for strength calculations.

Material and Detailing Limitations

According to the User Note at the end of AISC *Specification* Section I1.1, the intent of the *Specification* is to implement the noncomposite detailing provisions of ACI 318 in conjunction with the composite-specific provisions of *Specification* Chapter I. Detailing provisions may be grouped into material related limits, transverse reinforcement provisions, and longitudinal and structural steel reinforcement provisions as illustrated in the following discussion.

Material limits are provided in AISC *Specification* Sections I1.1(b) and I1.3 as follows:

(1) Concrete strength: $3 \text{ ksi} \le f_c' \le 10 \text{ ksi}$

$f_c' = 5 \text{ ksi}$ **o.k.**

(2) Specified minimum yield stress of structural steel: $F_y \le 75 \text{ ksi}$

$F_y = 50 \text{ ksi}$ **o.k.**

(3) Specified minimum yield stress of reinforcing bars: $F_{yr} \le 75 \text{ ksi}$

$F_{yr} = 60 \text{ ksi}$ **o.k.**

Transverse reinforcement limitations are provided in AISC *Specification* Section I1.1(c), I2.1a(b) and ACI 318 as follows:

(1) Tie size and spacing limitations:

The AISC *Specification* requires that either lateral ties or spirals be used for transverse reinforcement. Where lateral ties are used, a minimum of either No. 3 bars spaced at a maximum of 12 in. on center or No. 4 bars or larger spaced at a maximum of 16 in. on center are required.

No. 3 lateral ties at 12 in. o.c. are provided. **o.k.**

Note that AISC *Specification* Section I1.1(a) specifically excludes the composite column provisions of ACI 318, so it is unnecessary to meet the tie reinforcement provisions of ACI 318 when designing composite columns using the provisions of AISC *Specification* Chapter I.

If spirals are used, the requirements of ACI 318 should be met according to the User Note at the end of AISC *Specification* Section I2.1a.

(2) Additional tie size limitation:

No. 4 ties or larger are required where No. 11 or larger bars are used as longitudinal reinforcement in accordance with ACI 318, Section 9.7.6.4.2.

No. 3 lateral ties are provided for No. 8 longitudinal bars. **o.k.**

(3) Maximum tie spacing should not exceed 0.5 times the least column dimension:

$$s_{max} = 0.5\min\begin{Bmatrix} h_1 = 24 \text{ in.} \\ h_2 = 24 \text{ in.} \end{Bmatrix}$$

$$= 12.0 \text{ in.}$$

$$s = 12.0 \text{ in.} \le s_{max} \quad \textbf{o.k.}$$

(4) Concrete cover:

ACI 318, Section 20.6.1.3 contains concrete cover requirements. For concrete not exposed to weather or in contact with ground, the required cover for column ties is 1½ in.

$$\text{cover} = 2.5\text{ in.} - \frac{d_b}{2} - \text{diameter of No. 3 tie}$$
$$= 2.5\text{ in.} - \tfrac{1}{2}\text{ in.} - \tfrac{3}{8}\text{ in.}$$
$$= 1.63\text{ in.} > 1\tfrac{1}{2}\text{ in.} \quad \textbf{o.k.}$$

(5) Provide ties as required for lateral support of longitudinal bars:

AISC *Specification* Commentary Section I2.1a references ACI 318 for additional transverse tie requirements. In accordance with ACI 318, Section 25.7.2.3 and Figure R25.7.2.3a, ties are required to support longitudinal bars located farther than 6 in. clear on each side from a laterally supported bar. For corner bars, support is typically provided by the main perimeter ties. For intermediate bars, Figure I.9-1 illustrates one method for providing support through the use of a diamond-shaped tie.

Longitudinal and structural steel reinforcement limits are provided in AISC *Specification* Sections I1.1, I2.1 and ACI 318 as follows:

(1) Structural steel minimum reinforcement ratio: $A_s/A_g \geq 0.01$

$$\frac{A_s}{A_g} = \frac{13.3\text{ in.}^2}{576\text{ in.}^2} \geq 0.01$$
$$= 0.0231 > 0.01 \quad \textbf{o.k.}$$

An explicit maximum reinforcement ratio for the encased steel shape is not provided in the AISC *Specification*; however, a range of 8 to 12% has been noted in the literature to result in economic composite members for the resistance of gravity loads (Leon and Hajjar, 2008).

(2) Minimum longitudinal reinforcement ratio: $A_{sr}/A_g \geq 0.004$

$$\frac{A_{sr}}{A_g} = \frac{6.32\text{ in.}^2}{576\text{ in.}^2} \geq 0.004$$
$$= 0.0110 > 0.004 \quad \textbf{o.k.}$$

As discussed in AISC *Specification Commentary* Section I2.1a(c), only continuously developed longitudinal reinforcement is included in the minimum reinforcement ratio, so longitudinal restraining bars and other discontinuous longitudinal reinforcement is excluded. Note that this limitation is used in lieu of the minimum ratio provided in ACI 318 as discussed in *Specification* Commentary Section I1.1.

(3) Maximum longitudinal reinforcement ratio: $A_{sr}/A_g \leq 0.08$

$$\frac{A_{sr}}{A_g} = \frac{6.32\text{ in.}^2}{576\text{ in.}^2} \leq 0.08$$
$$= 0.0110 < 0.08 \quad \textbf{o.k.}$$

This longitudinal reinforcement limitation is provided in ACI 318, Section 10.6.1.1. It is recommended that all longitudinal reinforcement, including discontinuous reinforcement not used in strength calculations, be included in this ratio as it is considered a practical limitation to mitigate congestion of reinforcement. If longitudinal reinforcement is lap spliced as opposed to mechanically coupled, this limit is effectively reduced to 4% in areas away from the splice location.

(4) Minimum number of longitudinal bars:

ACI 318, Section 10.7.3.1, requires a minimum of four longitudinal bars within rectangular or circular members with ties and six bars for columns utilizing spiral ties. The intent for rectangular sections is to provide a minimum of one bar in each corner, so irregular geometries with multiple corners require additional longitudinal bars.

8 bars provided. **o.k.**

(5) Clear spacing between longitudinal bars:

ACI 318 Section 25.2.3 requires a clear distance between bars of 1.5d_b or 1½ in.

$$s_{min} = \max\begin{Bmatrix} 1.5d_b = 1\tfrac{1}{2} \text{ in.} \\ 1\tfrac{1}{2} \text{ in.} \end{Bmatrix}$$
$$= 1\tfrac{1}{2} \text{ in. clear}$$

$$s = 9.50 \text{ in.} - 1.00 \text{ in.}$$
$$= 8.50 \text{ in.} > 1\tfrac{1}{2} \text{ in.} \quad \textbf{o.k.}$$

(6) Clear spacing between longitudinal bars and the steel core:

AISC *Specification* Section I2.1e requires a minimum clear spacing between the steel core and longitudinal reinforcement of 1.5 reinforcing bar diameters, but not less than 1½ in.

$$s_{min} = \max\begin{Bmatrix} 1.5d_b = 1\tfrac{1}{2} \text{ in.} \\ 1\tfrac{1}{2} \text{ in.} \end{Bmatrix}$$
$$= 1\tfrac{1}{2} \text{ in. clear}$$

Closest reinforcing bars to the encased section are the center bars adjacent to each flange:

$$s = \frac{h_2}{2} - \frac{d}{2} - 2.50 \text{ in.} - \frac{d_b}{2}$$
$$= \frac{24.0 \text{ in.}}{2} - \frac{10.1 \text{ in.}}{2} - 2.50 \text{ in.} - \frac{1.00 \text{ in.}}{2}$$
$$= 3.95 \text{ in.} > s_{min} = 1\tfrac{1}{2} \text{ in.} \quad \textbf{o.k.}$$

(7) Concrete cover for longitudinal reinforcement:

ACI 318, Section 20.6.1.3, provides concrete cover requirements for reinforcement. The cover requirements for column ties and primary reinforcement are the same, and the tie cover was previously determined to be acceptable, thus the longitudinal reinforcement cover is acceptable by inspection.

From ASCE/SEI, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_r = P_u$ $= 1.2(260 \text{ kips}) + 1.6(780 \text{ kips})$ $= 1{,}560 \text{ kips}$	$P_r = P_a$ $= 260 \text{ kips} + 780 \text{ kips}$ $= 1{,}040 \text{ kips}$

Available Compressive Strength

The nominal axial compressive strength without consideration of length effects, P_{no}, is determined from AISC *Specification* Section I2.1b as:

$$P_{no} = F_y A_s + F_{ysr} A_{sr} + 0.85 f_c' A_c$$ (*Spec*. Eq. I2-4)

$$= (50 \text{ ksi})(13.3 \text{ in.}^2) + (60 \text{ ksi})(6.32 \text{ in.}^2) + 0.85(5 \text{ ksi})(556 \text{ in.}^2)$$

$$= 3{,}410 \text{ kips}$$

Because the unbraced length is the same in both the *x-x* and *y-y* directions, the column will buckle about the axis having the smaller effective composite section stiffness, EI_{eff}. Noting the moment of inertia values determined previously for the concrete and reinforcing steel are similar about each axis, the column will buckle about the weak axis of the steel shape by inspection. I_{cy}, I_{sy} and I_{sry} are therefore used for calculation of length effects in accordance with AISC *Specification* Section I2.1b as follows:

$$C_1 = 0.25 + 3\left(\frac{A_s + A_{sr}}{A_g}\right) \leq 0.7$$ (*Spec*. Eq. I2-7)

$$= 0.25 + 3\left(\frac{13.3 \text{ in.}^2 + 6.32 \text{ in.}^2}{576 \text{ in.}^2}\right) \leq 0.7$$

$$= 0.352 < 0.7; \text{ therefore } C_1 = 0.352$$

$$EI_{eff} = E_s I_{sy} + E_s I_{sry} + C_1 E_c I_{cy}$$ (from *Spec*. Eq. I2-6)

$$= (29{,}000 \text{ ksi})(53.4 \text{ in.}^4) + (29{,}000 \text{ ksi})(428 \text{ in.}^4)$$

$$+ 0.352(3{,}900 \text{ ksi})(27{,}200 \text{ in.}^4)$$

$$= 51{,}300{,}000 \text{ kip-in.}^2$$

$$P_e = \pi^2 (EI_{eff}) / (L_c)^2 \text{, where } L_c = KL \text{ and } K = 1.0 \text{ for a pin-ended member}$$ (*Spec*. Eq. I2-5)

$$= \frac{\pi^2 (51{,}300{,}000 \text{ kip-in.}^2)}{[(1.0)(14 \text{ ft})(12 \text{ in./ft})]^2}$$

$$= 17{,}900 \text{ kips}$$

$$\frac{P_{no}}{P_e} = \frac{3{,}410 \text{ kips}}{17{,}900 \text{ kips}}$$

$$= 0.191 < 2.25$$

Therefore, use AISC *Specification* Equation I2-2.

$$P_n = P_{no}\left(0.658^{\frac{P_{no}}{P_e}}\right)$$ (*Spec*. Eq. I2-2)

$$= (3{,}410 \text{ kips})(0.658)^{0.191}$$

$$= 3{,}150 \text{ kips}$$

Check adequacy of the composite column for the required axial compressive strength:

LRFD	ASD
$\phi_c = 0.75$ $\phi_c P_n = 0.75(3{,}150 \text{ kips})$ $= 2{,}360 \text{ kips} > 1{,}560 \text{ kips}$ **o.k.**	$\Omega_c = 2.00$ $\dfrac{P_n}{\Omega_c} = \dfrac{3{,}150 \text{ kips}}{2.00}$ $= 1{,}580 \text{ kips} > 1{,}040 \text{ kips}$ **o.k.**

Available Compressive Strength of Composite Section Versus Bare Steel Section

Due to the differences in resistance and safety factors between composite and noncomposite column provisions, it is possible in rare instances to calculate a lower available compressive strength for an encased composite column than one would calculate for the corresponding bare steel section. However, in accordance with AISC *Specification* Section I2.1b, the available compressive strength need not be less than that calculated for the bare steel member in accordance with Chapter E.

From AISC *Manual* Table 4-1a:

LRFD	ASD
$\phi_c P_n = 359 \text{ kips} < 2{,}360 \text{ kips}$	$\dfrac{P_n}{\Omega_c} = 239 \text{ kips} < 1{,}580 \text{ kips}$

Thus, the composite section strength controls and is adequate for the required axial compressive strength as previously demonstrated.

Force Allocation and Load Transfer

Load transfer calculations for external axial forces should be performed in accordance with AISC *Specification* Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions for encased composite members is provided in Design Example I.8.

Typical Detailing Convention

Designers are directed to AISC Design Guide 6 (Griffis, 1992) for additional discussion and typical details of encased composite columns not explicitly covered in this example.

EXAMPLE I.10 ENCASED COMPOSITE MEMBER IN AXIAL TENSION

Given:

Determine if the encased composite member illustrated in Figure I.10-1 is adequate for the indicated dead load compression and wind load tension. The entire load is applied to the encased steel section.

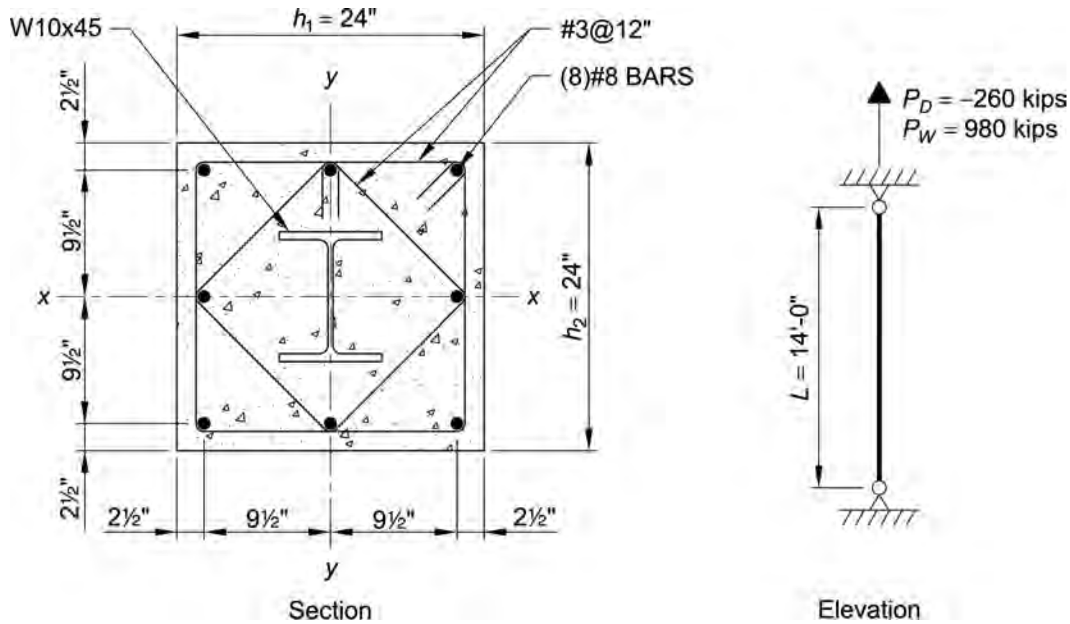


Fig. I.10-1. Encased composite member section and applied loading.

The composite member consists of an ASTM A992 W-shape encased by normal weight (145 lb/ft^3) reinforced concrete having a specified concrete compressive strength, $f_c' = 5$ ksi.

Deformed reinforcing bars conform to ASTM A615 with a minimum yield stress, F_{yr}, of 60 ksi.

Solution:

From AISC *Manual* Table 2-4, the steel material properties are:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1 and Figure I.10-1, the relevant properties of the composite section are:

$A_s = 13.3$ in.^{2}
$A_{sr} = 6.32$ in.^{2} (area of eight No. 8 bars)

Material and Detailing Limitations

Refer to Design Example I.9 for a check of material and detailing limitations specified in AISC *Specification* Chapter I for encased composite members.

Taking compression as negative and tension as positive, from ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
Governing uplift load combination = 0.9D + 1.0W $P_r = P_u$ $= 0.9(-260\text{ kips}) + 1.0(980\text{ kips})$ $= 746\text{ kips}$	Governing uplift load combination = 0.6D + 0.6W $P_r = P_a$ $= 0.6(-260\text{ kips}) + 0.6(980\text{ kips})$ $= 432\text{ kips}$

Available Tensile Strength

Available tensile strength for an encased composite member is determined in accordance with AISC *Specification* Section I2.1c.

$$P_n = F_y A_s + F_{ysr} A_{sr} \qquad \text{(\textit{Spec}. Eq. I2-8)}$$
$$= (50\text{ ksi})(13.3\text{ in.}^2) + (60\text{ ksi})(6.32\text{ in.}^2)$$
$$= 1{,}040\text{ kips}$$

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n = 0.90(1{,}040\text{ kips})$ $= 936\text{ kips} > 746\text{ kips}$ **o.k.**	$\Omega_t = 1.67$ $\dfrac{P_n}{\Omega_t} = \dfrac{1{,}040\text{ kips}}{1.67}$ $= 623\text{ kips} > 432\text{ kips}$ **o.k.**

Force Allocation and Load Transfer

In cases where all of the tension is applied to either the reinforcing steel or the encased steel shape, and the available strength of the reinforcing steel or encased steel shape by itself is adequate, no additional load transfer calculations are required.

In cases, such as the one under consideration, where the available strength of both the reinforcing steel and the encased steel shape are needed to provide adequate tension resistance, AISC *Specification* Section I6 can be modified for tensile load transfer requirements by replacing the P_{no} term in Equations I6-1 and I6-2 with the nominal tensile strength, P_n, determined from Equation I2-8.

For external tensile force applied to the encased steel section:

$$V_r' = P_r\left(1 - \frac{F_y A_s}{P_n}\right) \qquad \text{(\textit{Spec}. Eq. C-I6-1)}$$

For external tensile force applied to the longitudinal reinforcement of the concrete encasement:

$$V_r' = P_r\left(\frac{F_y A_s}{P_n}\right) \qquad \text{(\textit{Spec}. Eq. C-I6-2)}$$

where

P_n = nominal tensile strength of encased composite member from Equation I2-8, kips
P_r = required external tensile force applied to the composite member, kips

Per the problem statement, the entire external force is applied to the encased steel section, thus, AISC *Specification* Equation C-I6-1 is used as follows:

$$V_r' = P_r\left[1-\frac{(50\text{ ksi})(13.3\text{ in.}^2)}{1{,}040\text{ kips}}\right]$$
$$= 0.361P_r$$

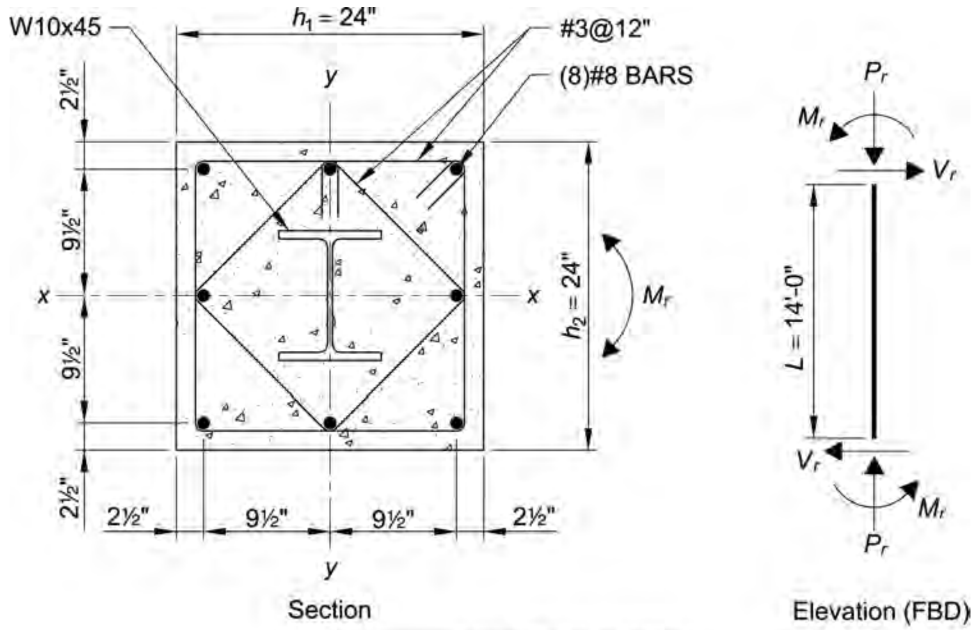
LRFD	ASD
$V_r' = 0.361(746\text{ kips})$ $= 269\text{ kips}$	$V_r' = 0.361(432\text{ kips})$ $= 156\text{ kips}$

The longitudinal shear force must be transferred between the encased steel shape and longitudinal reinforcing using the force transfer mechanisms of direct bearing or shear connection in accordance with AISC *Specification* Section I6.3 as illustrated in Example I.8.

EXAMPLE I.11 ENCASED COMPOSITE MEMBER IN COMBINED AXIAL COMPRESSION, FLEXURE AND SHEAR

Given:

Determine if the encased composite member illustrated in Figure I.11-1 is adequate for the indicated axial forces, shears and moments that have been determined in accordance with the direct analysis method of AISC *Specification* Chapter C for the controlling ASCE/SEI 7 load combinations.



	LRFD	ASD
P_r (kips)	1,170	879
M_r (kip-ft)	670	302
V_r (kips)	95.7	57.4

Fig. I.11-1. Encased composite member section and member forces.

The composite member consists of an ASTM A992 W-shape encased by normal weight (145 lb/ft^3) reinforced concrete having a specified concrete compressive strength, $f_c' = 5$ ksi.

Deformed reinforcing bars conform to ASTM A615 with a minimum yield stress, F_{yr}, of 60 ksi.

Solution:

From AISC *Manual* Table 2-4, the steel material properties are:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, Figure I.11-1, and Examples I.8 and I.9, the geometric and material properties of the composite section are:

A_s = 13.3 in.^{2}	d = 10.1 in.	h_1 = 24 in.	I_{sy} = 53.4 in.^{4}
A_g = 576 in.^{2}	b_f = 8.02 in.	h_2 = 24 in.	I_{cx} = 27,000 in.^{4}
A_c = 556 in.^{2}	t_f = 0.620 in.	E_c = 3,900 ksi	I_{cy} = 27,200 in.^{4}
A_{sr} = 6.32 in.^{2}	t_w = 0.350 in.	Z_{sx} = 54.9 in.^{3}	I_{sr} = 428 in.^{4}
c = 2½ in.	S_{sx} = 49.1 in.^{3}		

The area of continuous reinforcing located at the centerline of the composite section, A_{srs}, is determined from Figure I.11-1 as follows:

$$A_{srs} = 2\left(A_{srsi}\right)$$
$$= 2\left(0.79 \text{ in.}^2\right)$$
$$= 1.58 \text{ in.}^2$$

where

A_{srsi} = area of reinforcing bar i at centerline of composite section
$= 0.79 \text{ in.}^2$ for a No. 8 bar

For the section under consideration, A_{srs} is equal about both the x-x and y-y axis.

Classify Section for Local Buckling

In accordance with AISC *Specification* Section I1.2, local buckling effects need not be considered for encased composite members, thus all encased sections are treated as compact sections for strength calculations.

Material and Detailing Limitations

Refer to Design Example I.9 for a check of material and detailing limitations.

Interaction of Axial Force and Flexure

Interaction between flexure and axial forces in composite members is governed by AISC *Specification* Section I5, which permits the use of the methods outlined in Section I1.2.

The strain compatibility method is a generalized approach that allows for the construction of an interaction diagram based upon the same concepts used for reinforced concrete design. Application of the strain compatibility method is required for irregular/nonsymmetrical sections, and its general implementation may be found in reinforced concrete design texts and will not be discussed further here.

Plastic stress distribution methods are discussed in AISC *Specification* Commentary Section I5, which provides four procedures applicable to encased composite members. The first procedure, Method 1, invokes the interaction equations of Section H1. The second procedure, Method 2, involves the construction of a piecewise-linear interaction curve using the plastic strength equations provided in AISC *Manual* Table 6-3a. The third procedure, Method 2—Simplified, is a reduction of the piecewise-linear interaction curve that allows for the use of less conservative interaction equations than those presented in Chapter H. The fourth and final procedure, Method 3, utilizes AISC *Design Guide 6* (Griffis, 1992).

For this design example, three of the available plastic stress distribution procedures are reviewed and compared. Method 3 is not demonstrated as it is not applicable to the section under consideration due to the area of the encased steel section being smaller than the minimum limit of 4% of the gross area of the composite section provided in the earlier *Specification* upon which Design Guide 6 is based.

Method 1—Interaction Equations of Section H1

The most direct and conservative method of assessing interaction effects is through the use of the interaction equations of AISC *Specification* Section H1. Unlike concrete filled HSS shapes, the available compressive and flexural strengths of encased members are not tabulated in the AISC *Manual* due to the large variety of possible combinations. Calculations must therefore be performed explicitly using the provisions of Chapter I.

Available Compressive Strength

The available compressive strength is calculated as illustrated in Example I.9.

LRFD	ASD
$\phi_c P_n = 2{,}360 \text{ kips}$	$\dfrac{P_n}{\Omega_c} = 1{,}580 \text{ kips}$

Nominal Flexural Strength

The applied moment illustrated in Figure I.11-1 is resisted by the flexural strength of the composite section about its strong (*x-x*) axis. The strength of the section in pure flexure is calculated using the equations of AISC *Manual* Table 6-3a for Point B. Note that the calculation of the flexural strength at Point B first requires calculation of the flexural strength at Point D as follows:

$$\begin{aligned}
Z_r &= \left(A_{sr} - A_{srs}\right)\left(\frac{h_2}{2} - c\right) \\
&= \left(6.32 \text{ in.}^2 - 1.58 \text{ in.}^2\right)\left(\frac{24 \text{ in.}}{2} - 2\tfrac{1}{2} \text{ in.}\right) \\
&= 45.0 \text{ in.}^3
\end{aligned}$$

$$\begin{aligned}
Z_c &= \frac{h_1 h_2^2}{4} - Z_s - Z_r \\
&= \frac{(24 \text{ in.})(24 \text{ in.})^2}{4} - 54.9 \text{ in.}^3 - 45.0 \text{ in.}^3 \\
&= 3{,}360 \text{ in.}^3
\end{aligned}$$

$$\begin{aligned}
M_D &= F_y Z_s + F_{yr} Z_r + 0.85 f_c'\left(\frac{Z_c}{2}\right) \\
&= \left[(50 \text{ ksi})\left(54.9 \text{ in.}^3\right) + (60 \text{ ksi})\left(45.0 \text{ in.}^3\right) + 0.85(5 \text{ ksi})\left(\frac{3{,}360 \text{ in.}^3}{2}\right)\right]\left(\frac{1}{12 \text{ in./ft}}\right) \\
&= 1{,}050 \text{ kip-ft}
\end{aligned}$$

Assuming h_n is within the flange $\left(\dfrac{d}{2} - t_f < h_n \le \dfrac{d}{2}\right)$:

$$h_n = \frac{0.85 f_c'\left(A_c + A_s - db_f + A_{srs}\right) - 2F_y\left(A_s - db_f\right) - 2F_{yr}A_{srs}}{2\left[0.85 f_c'\left(h_1 - b_f\right) + 2F_y b_f\right]}$$

$$= \frac{\left\{\begin{array}{l} 0.85(5\text{ ksi})\left[556\text{ in.}^2 + 13.3\text{ in.}^2 - (10.1\text{ in.})(8.02\text{ in.}) + 1.58\text{ in.}^2\right] \\ \quad -2(50\text{ ksi})\left[13.3\text{ in.}^2 - (10.1\text{ in.})(8.02\text{ in.})\right] - 2(60\text{ ksi})\left(1.58\text{ in.}^2\right) \end{array}\right\}}{2\left[0.85(5\text{ ksi})(24\text{ in.} - 8.02\text{ in.}) + 2(50\text{ ksi})(8.02\text{ in.})\right]}$$

$$= 4.98\text{ in.}$$

Check assumption:

$$\left(\frac{10.1\text{ in.}}{2} - 0.620\text{ in.}\right) \le h_n \le \frac{10.1\text{ in.}}{2}$$

$4.43\text{ in.} < h_n = 4.98\text{ in.} < 5.05\text{ in.}$ assumption **o.k.**

$$Z_{sn} = Z_s - b_f\left(\frac{d}{2} - h_n\right)\left(\frac{d}{2} + h_n\right)$$

$$= 54.9\text{ in.}^3 - (8.02\text{ in.})\left(\frac{10.1\text{ in.}}{2} - 4.98\text{ in.}\right)\left(\frac{10.1\text{ in.}}{2} + 4.98\text{ in.}\right)$$

$$= 49.3\text{ in.}^3$$

$$Z_{cn} = h_1 h_n^2 - Z_{sn}$$

$$= (24\text{ in.})(4.98\text{ in.})^2 - 49.3\text{ in.}^3$$

$$= 546\text{ in.}^3$$

$$M_B = M_D - F_y Z_{sn} - 0.85 f_c'\left(\frac{Z_{cn}}{2}\right)$$

$$= \left[12{,}600\text{ kip-in.} - (50\text{ ksi})\left(49.3\text{ in.}^3\right) - 0.85(5\text{ ksi})\left(\frac{546\text{ in.}^3}{2}\right)\right]\left(\frac{1}{12\text{ in./ft}}\right)$$

$$= 748\text{ kip-ft}$$

Available Flexural Strength

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(748\text{ kip-ft})$ $= 673\text{ kip-ft}$	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{748\text{ kip-ft}}{1.67}$ $= 448\text{ kip-ft}$

Interaction of Axial Compression and Flexure

LRFD	ASD
$\phi_c P_n = 2{,}360$ kips $\phi_b M_n = 673$ kip-ft $\dfrac{P_r}{P_c} = \dfrac{P_u}{\phi_c P_n}$ $= \dfrac{1{,}170 \text{ kips}}{2{,}360 \text{ kips}}$ $= 0.496 > 0.2$ Therefore, use AISC *Specification* Equation H1-1a. $\dfrac{P_u}{\phi_c P_n} + \dfrac{8}{9}\left(\dfrac{M_u}{\phi_b M_n}\right) \le 1.0$ (from *Spec*. Eq. H1-1a) $0.496 + \dfrac{8}{9}\left(\dfrac{670 \text{ kip-ft}}{673 \text{ kip-ft}}\right) \le 1.0$ $1.38 > 1.0$ **n.g.**	$P_n / \Omega_c = 1{,}580$ kips $M_n / \Omega_c = 448$ kip-ft $\dfrac{P_r}{P_c} = \dfrac{P_a}{P_n / \Omega_c}$ $= \dfrac{879 \text{ kips}}{1{,}580 \text{ kips}}$ $= 0.556 > 0.2$ Therefore, use AISC *Specification* Equation H1-1a. $\dfrac{P_a}{P_n / \Omega_c} + \dfrac{8}{9}\left(\dfrac{M_a}{M_n / \Omega_b}\right) \le 1.0$ (from *Spec*. Eq. H1-1a) $0.556 + \dfrac{8}{9}\left(\dfrac{302 \text{ kip-ft}}{448 \text{ kip-ft}}\right) \le 1.0$ $1.16 > 1.0$ **n.g.**

Method 1 indicates that the section is inadequate for the applied loads. The designer can elect to choose a new section that passes the interaction check or re-analyze the current section using a less conservative design method such as Method 2. The use of Method 2 is illustrated in the following section.

Method 2—Interaction Curves from the Plastic Stress Distribution Model

The procedure for creating an interaction curve using the plastic stress distribution model is illustrated graphically in AISC *Specification* Commentary Figure C-I5.2, and repeated here.

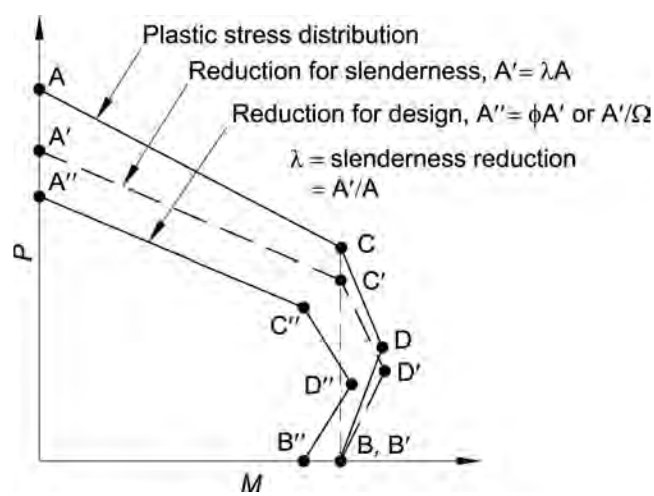


Fig. C-I5.2. Interaction diagram for composite beam-columns—Method 2.

Referencing Figure C.I5.2, the nominal strength interaction surface A, B, C, D is first determined using the equations of AISC *Manual* Table 6-3a. This curve is representative of the short column member strength without consideration of length effects. A slenderness reduction factor, λ, is then calculated and applied to each point to create surface A′, B′, C′, D′. The appropriate resistance or safety factors are then applied to create the design surface A″, B″, C″, D″. Finally, the required axial and flexural strengths from the applicable load combinations of ASCE/SEI 7 are plotted on the design surface. The member is then deemed acceptable for the applied loading if all points fall within the design surface. These steps are illustrated in detail by the following calculations.

Step 1: Construct nominal strength interaction surface A, B, C, D without length effects

Using the equations provided in Figure I-1a for bending about the *x-x* axis yields:

Point A (pure axial compression):

$$\begin{aligned} P_A &= F_y A_s + F_{yr} A_{sr} + 0.85 f_c' A_c \\ &= (50 \text{ ksi})(13.3 \text{ in.}^2) + (60 \text{ ksi})(6.32 \text{ in.}^2) + 0.85(5 \text{ ksi})(556 \text{ in.}^2) \\ &= 3{,}410 \text{ kips} \end{aligned}$$

$$M_A = 0 \text{ kip-ft}$$

Point D (maximum nominal moment strength):

$$\begin{aligned} P_D &= \frac{0.85 f_c' A_c}{2} \\ &= \frac{0.85(5 \text{ ksi})(556 \text{ in.}^2)}{2} \\ &= 1{,}180 \text{ kips} \end{aligned}$$

Calculation of M_D was demonstrated previously in Method 1.

$$M_D = 1{,}050 \text{ kip-ft}$$

Point B (pure flexure):

$$P_B = 0 \text{ kips}$$

Calculation of M_B was demonstrated previously in Method 1.

$$M_B = 748 \text{ kip-ft}$$

Point C (intermediate point):

$$\begin{aligned} P_C &= 0.85 f_c' A_c \\ &= 0.85(5 \text{ ksi})(556 \text{ in.}^2) \\ &= 2{,}360 \text{ kips} \end{aligned}$$

$$\begin{aligned} M_C &= M_B \\ &= 748 \text{ kip-ft} \end{aligned}$$

The calculated points are plotted to construct the nominal strength interaction surface without length effects as depicted in Figure I.11-2.

Step 2: Construct nominal strength interaction surface A′, B′, C′, D′ with length effects

The slenderness reduction factor, λ, is calculated for Point A using AISC *Specification* Section I2.1 in accordance with AISC *Specification* Commentary Section I5.

Because the unbraced length is the same in both the *x-x* and *y-y* directions, the column will buckle about the axis having the smaller effective composite section stiffness, EI_{eff}. Noting the moment of inertia values for the concrete and reinforcing steel are similar about each axis, the column will buckle about the weak axis of the steel shape by inspection. I_{cy}, I_{sy} and I_{sry} are therefore used for calculation of length effects in accordance with AISC *Specification* Section I2.1b.

$$P_{no} = P_A$$
$$= 3{,}410 \text{ kips}$$

$$C_1 = 0.25 + 3\left(\frac{A_s + A_{sr}}{A_g}\right) \le 0.7 \qquad \text{(}\textit{Spec}\text{. Eq. I2-7)}$$
$$= 0.25 + 3\left(\frac{13.3 \text{ in.}^2 + 6.32 \text{ in.}^2}{576 \text{ in.}^2}\right) \le 0.7$$
$$= 0.352 < 0.7; \text{ therefore } C_1 = 0.352.$$

$$EI_{eff} = E_s I_{sy} + E_s I_{sry} + C_1 E_c I_{cy} \qquad \text{(from }\textit{Spec}\text{. Eq. I2-6)}$$
$$= (29{,}000 \text{ ksi})(53.4 \text{ in.}^4) + (29{,}000 \text{ ksi})(428 \text{ in.}^4) + 0.352(3{,}900 \text{ ksi})(27{,}200 \text{ in.}^4)$$
$$= 51{,}300{,}000 \text{ kip-in.}^2$$

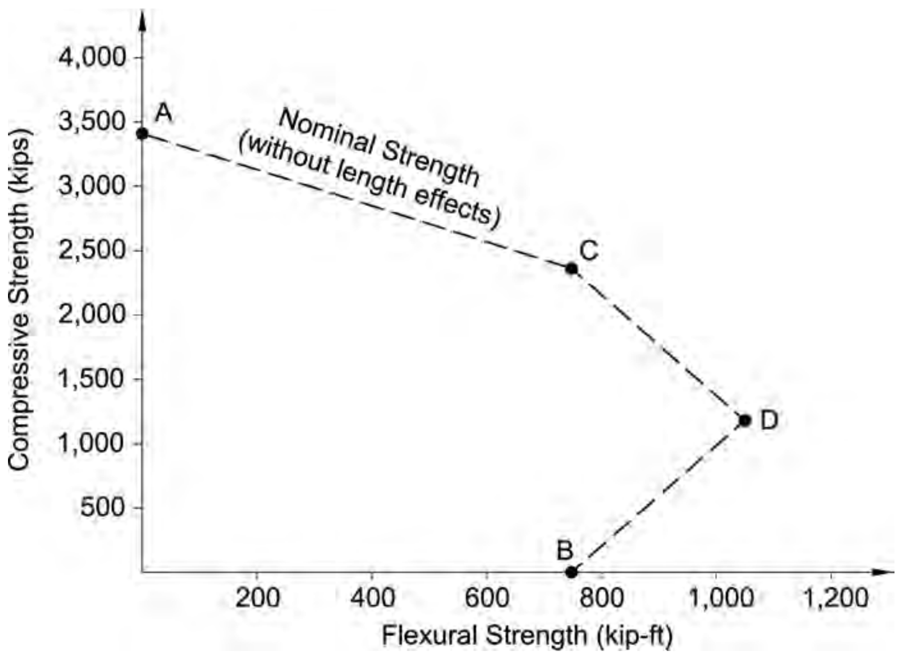


Fig. I.11-2. Nominal strength interaction surface without length effects.

$$P_e = \pi^2 (EI_{eff})/(L_c)^2 \text{, where } L_c = KL \text{ and } K = 1.0$$ (*Spec*. Eq. I2-5)

in accordance with the direct analysis method

$$= \frac{\pi^2 (51{,}300{,}000 \text{ kip-in.}^2)}{[(1.0)(14 \text{ ft})(12 \text{ in./ft})]^2}$$
$$= 17{,}900 \text{ kips}$$

$$\frac{P_{no}}{P_e} = \frac{3{,}410 \text{ kips}}{17{,}900 \text{ kips}}$$
$$= 0.191 < 2.25$$

Therefore, use AISC *Specification* Equation I2-2.

$$P_n = P_{no}\left(0.658^{\frac{P_{no}}{P_e}}\right)$$ (*Spec*. Eq. I2-2)

$$= (3{,}410 \text{ kips})(0.658)^{0.191}$$
$$= 3{,}150 \text{ kips}$$

$$\lambda = \frac{P_n}{P_{no}}$$
$$= \frac{3{,}150 \text{ kips}}{3{,}410 \text{ kips}}$$
$$= 0.924$$

In accordance with AISC *Specification* Commentary Section I5, the same slenderness reduction is applied to each of the remaining points on the interaction surface as follows:

$$P_{A'} = \lambda P_A$$
$$= 0.924(3{,}410 \text{ kips})$$
$$= 3{,}150 \text{ kips}$$

$$P_{B'} = \lambda P_B$$
$$= 0.924(0 \text{ kip})$$
$$= 0 \text{ kip}$$

$$P_{C'} = \lambda P_C$$
$$= 0.924(2{,}360 \text{ kips})$$
$$= 2{,}180 \text{ kips}$$

$$P_{D'} = \lambda P_D$$
$$= 0.924(1{,}180 \text{ kips})$$
$$= 1{,}090 \text{ kips}$$

The modified axial strength values are plotted with the flexural strength values previously calculated to construct the nominal strength interaction surface including length effects. These values are superimposed on the nominal strength surface not including length effects for comparison purposes in Figure I.11-3.

The consideration of length effects results in a vertical reduction of the nominal strength curve as illustrated by Figure I.11-3. This vertical movement creates an unsafe zone within the shaded area of the figure where flexural capacities of the nominal strength (with length effects) curve exceed the section capacity. Application of resistance or safety factors reduces this unsafe zone as illustrated in the following step; however, designers should be cognizant of the potential for unsafe designs with loads approaching the predicted flexural capacity of the section. Alternately, the use of Method 2—Simplified eliminates this possibility altogether.

Step 3: Construct design interaction surface A″, B″, C″, D″ and verify member adequacy

The final step in the Method 2 procedure is to reduce the interaction surface for design using the appropriate resistance or safety factors.

The available compressive and flexural strengths are determined as follows:

LRFD	ASD
$\phi_c = 0.75$	$\Omega_c = 2.00$
$P_{X''} = \phi_c P_{X'}$, where X = A, B, C or D	$P_{X''} = \dfrac{P_{X'}}{\Omega_c}$, where X = A, B, C or D
$P_{A''} = 0.75(3{,}150 \text{ kips})$ $= 2{,}360 \text{ kips}$	$P_{A''} = 3{,}150 \text{ kips} / 2.00$ $= 1{,}580 \text{ kips}$
$P_{B''} = 0.75(0 \text{ kip})$ $= 0 \text{ kip}$	$P_{B''} = 0 \text{ kip} / 2.00$ $= 0 \text{ kip}$
$P_{C''} = 0.75(2{,}180 \text{ kips})$ $= 1{,}640 \text{ kips}$	$P_{C''} = 2{,}180 \text{ kips} / 2.00$ $= 1{,}090 \text{ kips}$
$P_{D''} = 0.75(1{,}090 \text{ kips})$ $= 818 \text{ kips}$	$P_{D''} = 1{,}090 \text{ kips} / 2.00$ $= 545 \text{ kips}$

LRFD	ASD
$\phi_b = 0.90$	$\Omega_b = 1.67$
$M_{X''} = \phi_b M_X$, where X = A, B, C or D	$M_{X''} = \dfrac{M_X}{\Omega_b}$, where X = A, B, C or D
$M_{A''} = 0.90(0 \text{ kip-ft})$ $= 0 \text{ kip-ft}$	$M_{A''} = 0 \text{ kip-ft} / 1.67$ $= 0 \text{ kip-ft}$
$M_{B''} = 0.90(748 \text{ kip-ft})$ $= 673 \text{ kip-ft}$	$M_{B''} = 748 \text{ kip-ft} / 1.67$ $= 448 \text{ kip-ft}$
$M_{C''} = 0.90(748 \text{ kip-ft})$ $= 673 \text{ kip-ft}$	$M_{C''} = 748 \text{ kip-ft} / 1.67$ $= 448 \text{ kip-ft}$
$M_{D''} = 0.90(1{,}050 \text{ kip-ft})$ $= 945 \text{ kip-ft}$	$M_{D''} = 1{,}050 \text{ kip-ft} / 1.67$ $= 629 \text{ kip-ft}$

The available strength values for each design method can now be plotted. These values are superimposed on the nominal strength surfaces (with and without length effects) previously calculated for comparison purposes in Figure I.11-4.

By plotting the required axial and flexural strength values on the available strength surfaces indicated in Figure I.11-4, it can be seen that both ASD (M_a,P_a) and LRFD (M_u,P_u) points lie within their respective design surfaces. The member in question is therefore adequate for the applied loads.

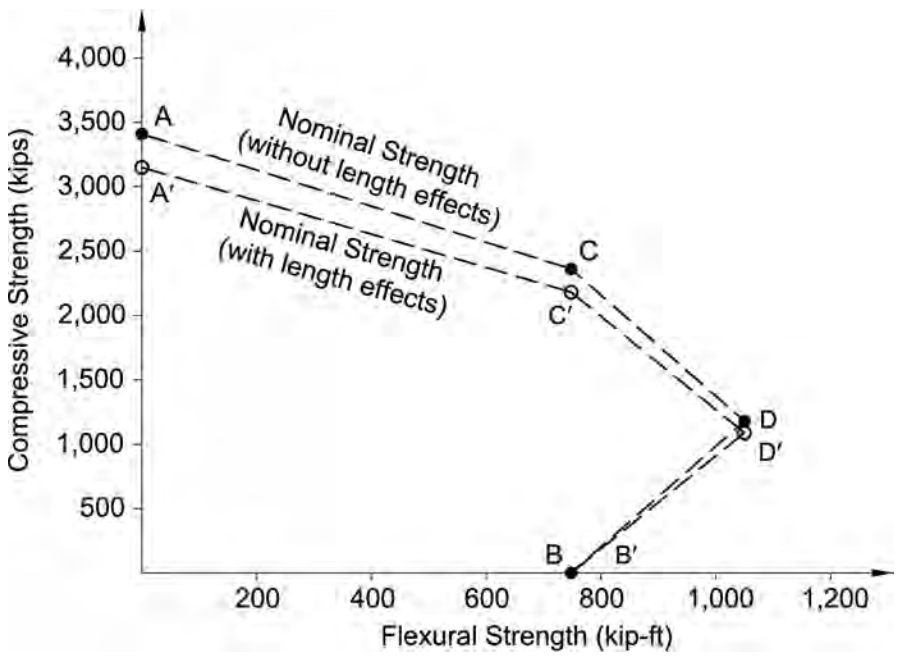


Fig. I.11-3. Nominal strength interaction surfaces (with and without length effects).

As discussed previously in Step 2 as well as in AISC *Specification* Commentary Section I5, when reducing the flexural strength of Point D for length effects and resistance or safety factors, an unsafe situation could result whereby additional flexural strength is permitted at a lower axial compressive strength than predicted by the cross section strength of the member. This effect is highlighted by the magnified portion of Figure I.11-4, where LRFD design point D′′ closely approaches the nominal strength curve. Designs falling outside the nominal strength curve are unsafe and not permitted.

Method 2—Simplified

The unsafe zone discussed in the previous section for Method 2 is avoided in the Method 2—Simplified procedure by the removal of Point D′′ from the Method 2 interaction surface leaving only points A′′, B′′ and C′′ as illustrated in Figure I.11-5. Reducing the number of interaction points also allows for a bilinear interaction check defined by AISC *Specification* Commentary Equations C-I5-1a and C-I5-1b to be performed.

Using the available strength values previously calculated in conjunction with the Commentary equations, interaction ratios are determined as follows:

LRFD	ASD
$P_r = P_u$ $= 1{,}170 \text{ kips} < P_{C''} = 1{,}640 \text{ kips}$ Therefore, use AISC *Specification* Commentary Equation C-I5-1a. $\dfrac{M_r}{M_C} = \dfrac{M_u}{M_{C''}} \le 1.0$ (from *Spec.* Comm. Eq. C-I5-1a) $\dfrac{670 \text{ kip-ft}}{673 \text{ kip-ft}} \le 1.0$ $1.0 = 1.0$ **o.k.**	$P_r = P_a$ $= 879 \text{ kips} < P_{C''} = 1{,}090 \text{ kips}$ Therefore, use AISC *Specification* Commentary Equation C-I5-1a. $\dfrac{M_r}{M_C} = \dfrac{M_a}{M_{C''}} \le 1.0$ (from *Spec.* Comm. Eq. C-I5-1a) $\dfrac{302 \text{ kip-ft}}{448 \text{ kip-ft}} \le 1.0$ $0.67 < 1.0$ **o.k.**

Thus, the member is adequate for the applied loads.

Comparison of Methods

The composite member was found to be inadequate using Method 1—Chapter H interaction equations, but was found to be adequate using both Method 2 and Method 2—Simplified procedures. A comparison between the methods is most easily made by overlaying the design curves from each method as illustrated in Figure I.11-6 for LRFD design.

From Figure I.11-6, the conservative nature of the Chapter H interaction equations can be seen. Method 2 provides the highest available strength; however, the Method 2—Simplified procedure also provides a good representation of the design curve. The procedure in Figure I-1 for calculating the flexural strength of Point C′′ first requires the calculation of the flexural strength for Point D′′. The design effort required for the Method 2—Simplified procedure, which utilizes Point C′′, is therefore not greatly reduced from Method 2.

Available Shear Strength

According to AISC *Specification* Section I4.1, there are three acceptable options for determining the available shear strength of an encased composite member:

(1) Option 1—Available shear strength of the steel section alone in accordance with AISC *Specification* Chapter G.

(2) Option 2—Available shear strength of the reinforced concrete portion alone per ACI 318.

(3) Option 3—Available shear strength of the steel section, in addition to the reinforcing steel ignoring the contribution of the concrete.

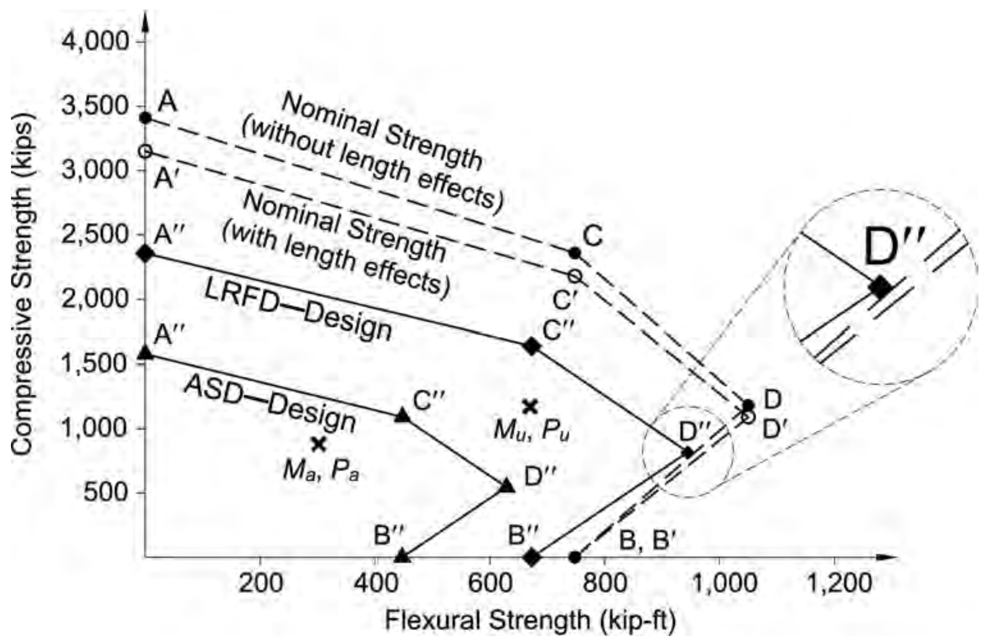


Fig. I.11-4. Available and nominal interaction surfaces.

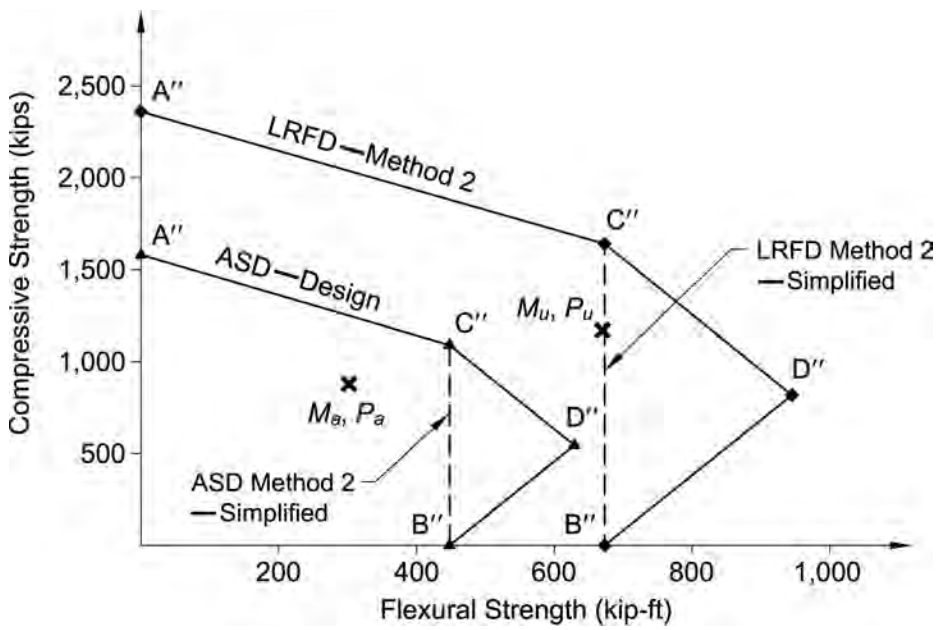


Fig. I.11-5. Comparison of Method 2 and Method 2 —Simplified.

Option 1—Available Shear Strength of Steel Section

A W10×45 member meets the criteria of AISC *Specification* Section G2.1(a) according to the User Note at the end of the section. As demonstrated in Design Example I.9, No. 3 ties at 12 in. on center as illustrated in Figure I.11-1 satisfy the minimum detailing requirements of the *Specification*. The nominal shear strength may therefore be determined as:

$C_{v1} = 1.0$ (*Spec*. Eq. G2-2)

$$\begin{aligned} A_w &= dt_w \\ &= (10.1 \text{ in.})(0.350 \text{ in.}) \\ &= 3.54 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} V_n &= 0.6F_y A_w C_{v1} \\ &= 0.6(50 \text{ ksi})(3.54 \text{ in.}^2)(1.0) \\ &= 106 \text{ kips} \end{aligned}$$ (*Spec*. Eq. G2-1)

The available shear strength of the steel section is:

LRFD	ASD
$\phi_v = 1.00$ $\phi_v V_n = 1.00(106 \text{ kips})$ $= 106 \text{ kips} > 95.7 \text{ kips}$ **o.k.**	$\Omega_v = 1.50$ $\dfrac{V_n}{\Omega_v} = \dfrac{106 \text{ kips}}{1.50}$ $= 70.7 \text{ kips} > 57.4 \text{ kips}$ **o.k.**

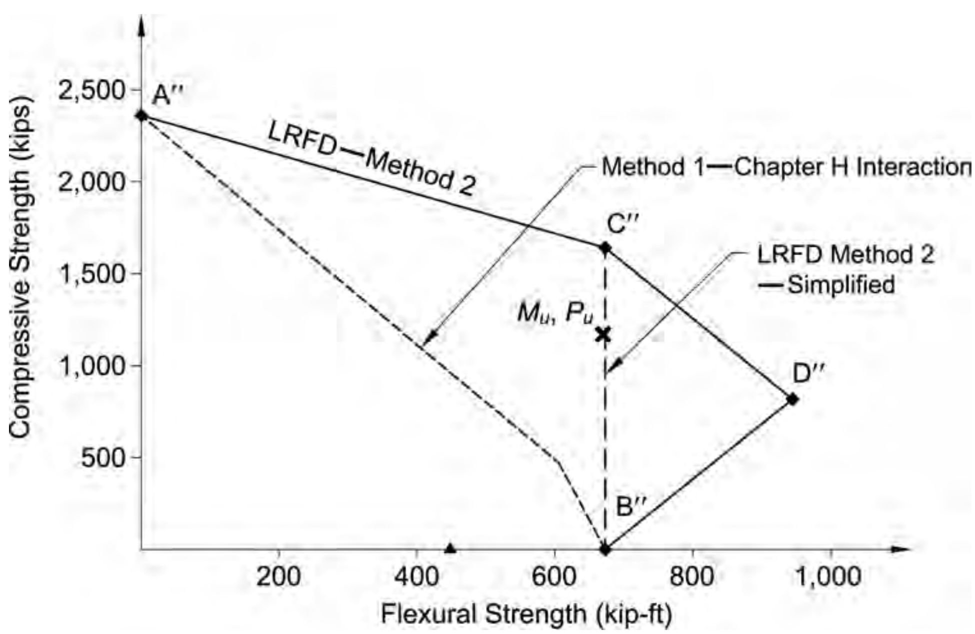


Fig. I.11-6. Comparison of interaction methods (LRFD).

Option 2—Available Shear Strength of the Reinforced Concrete (Concrete and Transverse Steel Reinforcement)

The available shear strength of the steel section alone has been shown to be sufficient; however, the amount of transverse reinforcement required for shear resistance in accordance with AISC *Specification* Section I4.1(b) will be determined for demonstration purposes.

Tie Requirements for Shear Resistance

The nominal concrete shear strength is:

$$V_c = 2\lambda\sqrt{f_c'}b_w d \qquad \text{(ACI 318, Eq. 22.5.5.1)}$$

where

λ = 1.0 for normal weight concrete from ACI 318, Table 19.2.4.2

$b_w = h_1$

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement

$= 24 \text{ in.} - 2\tfrac{1}{2} \text{ in.}$

$= 21.5 \text{ in.}$

$$V_c = 2(1.0)\sqrt{5{,}000 \text{ psi}}(24 \text{ in.})(21.5 \text{ in.})\left(\frac{1 \text{ kip}}{1{,}000 \text{ lb}}\right)$$

$$= 73.0 \text{ kips}$$

The tie requirements for shear resistance are determined from ACI 318 Chapter 22 and AISC *Specification* Section I4.1(b), as follows:

LRFD	ASD
$\phi_v = 0.75$	$\Omega_v = 2.00$
$\dfrac{A_v}{s} = \dfrac{V_u - \phi_v V_c}{\phi_v f_{yr} d}$ (from ACI 318, Eq. R22.5.10.5) $= \dfrac{95.7 \text{ kips} - 0.75(73.0 \text{ kips})}{0.75(60 \text{ ksi})(21.5 \text{ in.})}$ $= 0.0423 \text{ in.}$	$\dfrac{A_v}{s} = \dfrac{V_a - (V_c/\Omega_v)}{f_{yr} d/\Omega_v}$ (from ACI 318, Eq. R22.5.10.5) $= \dfrac{57.4 \text{ kips} - \left(\dfrac{73.0 \text{ kips}}{2.00}\right)}{\dfrac{(60 \text{ ksi})(21.5 \text{ in.})}{2.00}}$ $= 0.0324 \text{ in.}$
Using two legs of No. 3 ties with $A_v = 0.11 \text{ in.}^2$ from ACI 318, Appendix A: $\dfrac{2(0.11 \text{ in.}^2)}{s} = 0.0423 \text{ in.}$ $s = 5.20 \text{ in.}$	Using two legs of No. 3 ties with $A_v = 0.11 \text{ in.}^2$ from ACI 318, Appendix A: $\dfrac{2(0.11 \text{ in.}^2)}{s} = 0.0324 \text{ in.}$ $s = 6.79 \text{ in.}$
Using two legs of the No. 4 ties with $A_v = 0.20 \text{ in.}^2$: $\dfrac{2(0.20 \text{ in.}^2)}{s} = 0.0423 \text{ in.}$ $s = 9.46 \text{ in.}$	Using two legs of the No. 4 ties with $A_v = 0.20 \text{ in.}^2$: $\dfrac{2(0.20 \text{ in.}^2)}{s} = 0.0324 \text{ in.}$ $s = 12.3 \text{ in.}$

LRFD	ASD
From ACI 318, Section 9.7.6.2.2, the maximum spacing is: $s_{max} = \frac{d}{2} = \frac{21.5\text{ in.}}{2} = 10.8\text{ in.}$ Use No. 3 ties at 5 in. o.c. or No. 4 ties at 9 in. o.c.	From ACI 318, Section 9.7.6.2.2, the maximum spacing is: $s_{max} = \frac{d}{2} = \frac{21.5\text{ in.}}{2} = 10.8\text{ in.}$ Use No. 3 ties at 6 in. o.c. or No. 4 ties at 10 in. o.c.

Minimum Reinforcing Limits

Check that the minimum shear reinforcement is provided as required by ACI 318, Section 9.6.3.3.

$$\frac{A_{v,min}}{s} = 0.75\sqrt{f_c'}\left(\frac{b_w}{f_{yr}}\right) \geq \frac{50b_w}{f_{yr}} \quad \text{(ACI 318, Table 9.6.3.3)}$$

$$= \frac{0.75\sqrt{5{,}000\text{ psi}}\,(24\text{ in.})}{60{,}000\text{ psi}} \geq \frac{50(24\text{ in.})}{60{,}000\text{ psi}}$$

$$= 0.0212\text{ in.} > 0.0200\text{ in.}$$

LRFD	ASD
$\frac{A_v}{s} = 0.0423\text{ in.} > 0.0212\text{ in.}$ **o.k.**	$\frac{A_v}{s} = 0.0324\text{ in.} > 0.0212\text{ in.}$ **o.k.**

Maximum Reinforcing Limits

From ACI 318, Section 9.7.6.2.2, maximum stirrup spacing is reduced to $d/4$ if $V_s \geq 4\sqrt{f_c'}b_w d$. If No. 4 ties at 9 in. on center are selected:

$$V_s = \frac{A_v f_{yr} d}{s} \quad \text{(ACI 318, Eq. 22.5.10.5.3)}$$

$$= \frac{2\left(0.20\text{ in.}^2\right)(60\text{ ksi})(21.5\text{ in.})}{9\text{ in.}}$$

$$= 57.3\text{ kips}$$

$$V_{s,max} = 4\sqrt{f_c'}b_w d$$

$$= 4\sqrt{5{,}000\text{ psi}}\,(24\text{ in.})(21.5\text{ in.})\left(\frac{1\text{ kip}}{1{,}000\text{ lb}}\right)$$

$$= 146\text{ kips} > 57.3\text{ kips}$$

Therefore, the stirrup spacing is acceptable.

Option 3—Determine Available Shear Strength of the Steel Section plus Reinforcing Steel

The third procedure combines the shear strength of the reinforcing steel with that of the encased steel section, ignoring the contribution of the concrete. AISC *Specification* Section I4.1(c) provides a combined resistance and safety factor for this procedure. Note that the combined resistance and safety factor takes precedence over the factors in Chapter G used for the encased steel section alone in Option 1. The amount of transverse reinforcement required for shear resistance is determined as follows:

Tie Requirements for Shear Resistance

The nominal shear strength of the encased steel section was previously determined to be:

$$V_{n,steel} = 106 \text{ kips}$$

The tie requirements for shear resistance are determined from ACI 318, Chapter 22, and AISC *Specification* Section I4.1(c), as follows:

LRFD	ASD
$\phi_v = 0.75$ $\dfrac{A_v}{s} = \dfrac{V_u - \phi_v V_{n,steel}}{\phi_v f_{yr} d}$ $= \dfrac{95.7 \text{ kips} - 0.75(106 \text{ kips})}{0.75(60 \text{ ksi})(21.5 \text{ in.})}$ $= 0.0167 \text{ in.}$	$\Omega_v = 2.00$ $\dfrac{A_v}{s} = \dfrac{V_a - (V_{n,steel}/\Omega_v)}{f_{yr} d/\Omega_v}$ $= \dfrac{57.4 \text{ kips} - (106 \text{ kips}/2.00)}{\left[\dfrac{(60 \text{ ksi})(21.5 \text{ in.})}{2.00}\right]}$ $= 0.00682 \text{ in.}$

As determined in Option 2, the minimum value of $A_v/s = 0.0212$, and the maximum tie spacing for shear resistance is 10.8 in. Using two legs of No. 3 ties for A_v:

$$\frac{2(0.11 \text{ in.}^2)}{s} = 0.0212 \text{ in.}$$

$$s = 10.4 \text{ in.} < s_{max} = 10.8 \text{ in.}$$

Use No. 3 ties at 10 in. o.c.

Summary and Comparison of Available Shear Strength Calculations

The use of the steel section alone is the most expedient method for calculating available shear strength and allows the use of a tie spacing which may be greater than that required for shear resistance by ACI 318. Where the strength of the steel section alone is not adequate, Option 3 will generally result in reduced tie reinforcement requirements as compared to Option 2.

Force Allocation and Load Transfer

Load transfer calculations should be performed in accordance with AISC *Specification* Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions for encased composite members is provided in Design Example I.8 and AISC Design Guide 6.

EXAMPLE I.12 STEEL ANCHORS IN COMPOSITE COMPONENTS

Given:

Select an appropriate ¾-in.-diameter, Type B steel headed stud anchor to resist the dead and live loads indicated in Figure I.12-1. The anchor is part of a composite system that may be designed using the steel anchor in composite components provisions of AISC *Specification* Section I8.3.

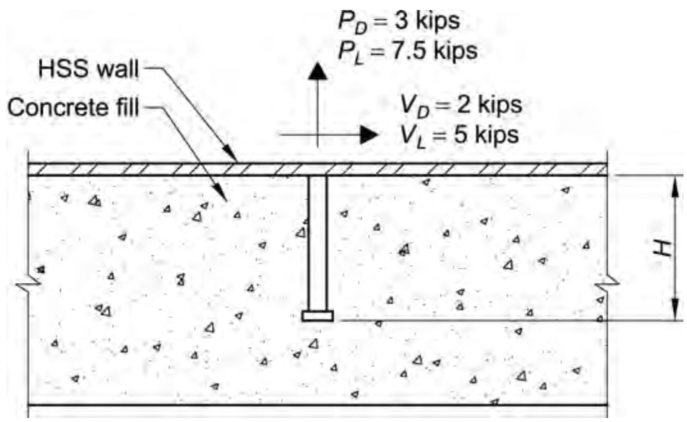


Fig. I.12-1. Steel headed stud anchor and applied loading.

The steel headed stud anchor is encased by normal weight (145 lb/ft^3) reinforced concrete having a specified concrete compressive strength, f_c' = 5 ksi. In accordance with AISC *Manual* Part 2, headed stud anchors shall be in accordance with AWS D1.1 with a specified minimum tensile stress, F_u, of 65 ksi.

The anchor is located away from edges such that concrete breakout in shear is not a viable limit state, and the nearest anchor is located 24 in. away. The concrete is considered to be uncracked.

Solution:

Minimum Anchor Length

AISC *Specification* Section I8.3 provides minimum length to shank diameter ratios for anchors subjected to shear, tension, and interaction of shear and tension in both normal weight and lightweight concrete. These ratios are also summarized in the User Note provided within Section I8.3. For normal weight concrete subject to shear and tension, $h / d_{sa} \geq 8$, thus:

$$\begin{aligned} h &\geq 8d_{sa} \\ &\geq 8(\text{¾ in.}) \\ &\geq 6.00 \text{ in.} \end{aligned}$$

This length is measured from the base of the steel headed stud anchor to the top of the head after installation. From anchor manufacturer's data, a standard stock length of 6³⁄16 in. is selected. Using a ³⁄16-in. length reduction to account for burn off during installation yields a final installed length of 6.00 in.

$$6.00 \text{ in.} = 6.00 \text{ in.} \quad \textbf{o.k.}$$

Select a ¾-in.-diameter × 6³⁄16-in.-long headed stud anchor.

Required Shear and Tensile Strength

From ASCE/SEI 7, Chapter 2, the required shear and tensile strengths are:

LRFD	ASD
Governing load combination for interaction $= 1.2D + 1.6L$ $Q_{uv} = 1.2(2 \text{ kips}) + 1.6(5 \text{ kips})$ $= 10.4 \text{ kips (shear)}$ $Q_{ut} = 1.2(3 \text{ kips}) + 1.6(7.5 \text{ kips})$ $= 15.6 \text{ kips (tension)}$	Governing load combination for interaction $= D + L$ $Q_{av} = 2 \text{ kips} + 5 \text{ kips}$ $= 7.00 \text{ kips (shear)}$ $Q_{at} = 3 \text{ kips} + 7.5 \text{ kips}$ $= 10.5 \text{ kips (tension)}$

Available Shear Strength

Per the problem statement, concrete breakout is not considered to be an applicable limit state. AISC Equation I8-3 may therefore be used to determine the available shear strength of the steel headed stud anchor as follows:

$$Q_{nv} = F_u A_{sa} \qquad \text{(}Spec\text{. Eq. I8-3)}$$

where

$$\begin{aligned} A_{sa} &= \text{cross-sectional area of steel headed stud anchor} \\ &= \frac{\pi(\text{3/4 in.})^2}{4} \\ &= 0.442 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} Q_{nv} &= (65 \text{ ksi})(0.442 \text{ in.}^2) \\ &= 28.7 \text{ kips} \end{aligned}$$

LRFD	ASD
$\phi_v = 0.65$ $\phi_v Q_{nv} = 0.65(28.7 \text{ kips})$ $= 18.7 \text{ kips}$	$\Omega_v = 2.31$ $\dfrac{Q_{nv}}{\Omega_v} = \dfrac{28.7 \text{ kips}}{2.31}$ $= 12.4 \text{ kips}$

Alternately, available shear strengths can be selected directly from Table I.12-1 located at the end of this example.

Available Tensile Strength

The nominal tensile strength of a steel headed stud anchor is determined using AISC *Specification* Equation I8-4 provided the edge and spacing limitations of AISC *Specification* Section I8.3b are met as follows:

(1) Minimum distance from centerline of anchor to free edge: $1.5h = 1.5(6.00 \text{ in.}) = 9.00 \text{ in.}$

There are no free edges, therefore this limitation does not apply.

(2) Minimum distance between centerlines of adjacent anchors: $3h = 3(6.00 \text{ in.}) = 18.0 \text{ in.}$

$18.0 \text{ in.} < 24 \text{ in.}$ **o.k.**

Equation I8-4 may therefore be used as follows:

$$\begin{aligned} Q_{nt} &= F_u A_{sa} \\ &= (65 \text{ ksi})(0.442 \text{ in.}^2) \\ &= 28.7 \text{ kips} \end{aligned}$$ (*Spec*. Eq. I8-4)

LRFD	ASD
$\phi_t = 0.75$ $\phi_t Q_{nt} = 0.75(28.7 \text{ kips})$ $= 21.5 \text{ kips}$	$\Omega_t = 2.00$ $\dfrac{Q_{nt}}{\Omega_t} = \dfrac{28.7 \text{ kips}}{2.00}$ $= 14.4 \text{ kips}$

Alternately, available tensile strengths can be selected directly from Table I.12-1 located at the end of this example.

Interaction of Shear and Tension

The detailing limits on edge distances and spacing imposed by AISC *Specification* Section I8.3c for shear and tension interaction are the same as those previously reviewed separately for tension and shear alone. Tension and shear interaction is checked using *Specification* Equation I8-5 which can be written in terms of LRFD and ASD design as follows:

LRFD	ASD
$\left(\dfrac{Q_{ut}}{\phi_t Q_{nt}}\right)^{5/3} + \left(\dfrac{Q_{uv}}{\phi_v Q_{nv}}\right)^{5/3} \le 1.0$ (from *Spec*. Eq. I8-5) $\left(\dfrac{15.6 \text{ kips}}{21.5 \text{ kips}}\right)^{5/3} + \left(\dfrac{10.4 \text{ kips}}{18.7 \text{ kips}}\right)^{5/3} = 0.96$ $0.96 < 1.0$ **o.k.**	$\left(\dfrac{Q_{at}}{Q_{nt}/\Omega_t}\right)^{5/3} + \left(\dfrac{Q_{av}}{Q_{nv}/\Omega_v}\right)^{5/3} \le 1.0$ (from *Spec*. Eq. I8-5) $\left(\dfrac{10.5 \text{ kips}}{14.4 \text{ kips}}\right)^{5/3} + \left(\dfrac{7.00 \text{ kips}}{12.4 \text{ kips}}\right)^{5/3} = 0.98$ $0.98 < 1.0$ **o.k.**

Thus, a ¾-in.-diameter × 6³⁄₁₆-in.-long headed stud anchor is adequate for the applied loads.

Limits of Application

The application of the steel anchors in composite component provisions have strict limitations as summarized in the User Note provided at the beginning of AISC *Specification* Section I8.3. These provisions do not apply to typical composite beam designs nor do they apply to hybrid construction where the steel and concrete do not resist loads together via composite action such as in embed plates. This design example is intended solely to illustrate the calculations associated with an isolated anchor that is part of an applicable composite system.

Available Strength Table

Table I.12-1 provides available shear and tension strengths for standard Type B steel headed stud anchors conforming to the requirements of AWS D1.1 for use in composite components.

Table I.12-1
Steel Headed Stud Anchor Available Strengths

Anchor Shank Diameter	A_{sa}	Q_{nv}/Ω_v	$\phi_v Q_{nv}$	Q_{nv}/Ω_v	$\phi_v Q_{nv}$
		kips	kips	kips	kips
in.	in.^{2}	ASD	LRFD	ASD	LRFD
½	0.196	5.52	8.30	6.38	9.57
⅝	0.307	8.63	13.0	9.97	15.0
¾	0.442	12.4	18.7	14.4	21.5
⅞	0.601	16.9	25.4	N/A[a]	N/A[a]
1	0.785	22.1	33.2	25.5	38.3
ASD	LRFD	[a] ⅞-in.-diameter anchors conforming to AWS D1.1, Figure 7.1, do not meet the minimum head-to-shank diameter ratio of 1.6 as required for tensile resistance per AISC *Specification* Section I8.3.			
$\Omega_v = 2.31$ $\Omega_t = 2.00$	$\phi_v = 0.65$ $\phi_t = 0.75$				

EXAMPLE I.13 COMPOSITE COLLECTOR BEAM DESIGN

Given:

Determine if the composite beam designed in Example I.1 is adequate to serve as a collector beam for the transfer of wind-induced compression forces in combination with gravity loading as indicated in Figure I.13. Applied forces were generated from an elastic analysis and stability shall be accounted for using the effective length method of design.

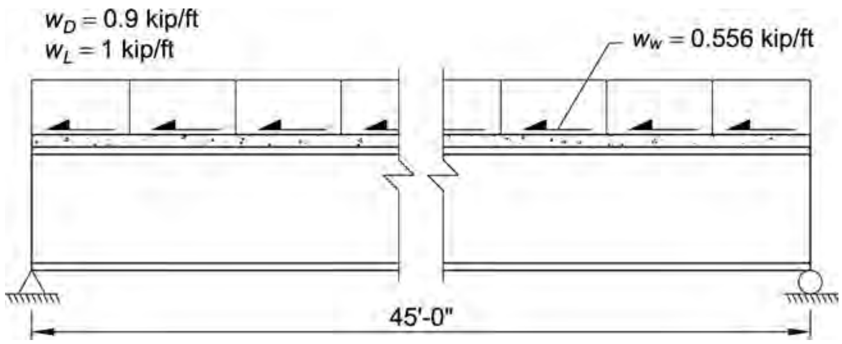


Fig. I.13. Composite collector beam and applied loading elevation.

Solution:

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W21×50

A = 14.7 in.^{2}	I_x = 984 in.^{4}	I_y = 24.9 in.^{4}	J = 1.14 in.^{4}
b_f = 6.53 in.	d = 20.8 in.	r_x = 8.18 in.	r_y = 1.30 in.
t_w = 0.380 in.	$b_f/2t_f$ = 6.10	h/t_w = 49.4	h_o = 20.3 in.

Refer to Example I.1 for additional information regarding strength and serviceability requirements associated with pre-composite and composite gravity load conditions.

Required Compressive Strength

From ASCE/SEI 7, Chapter 2, the required axial strength for the governing load combination, including wind, is:

LRFD	ASD
$P_u = 1.2D + 1.0W + L$ $= 1.2(0 \text{ kips}) + 1.0(0.556 \text{ kip/ft})(45 \text{ ft}) + 0 \text{ kips}$ $= 25.0 \text{ kips}$	$P_a = D + 0.75L + 0.75(0.6W)$ $= 0 \text{ kips} + 0.75(0 \text{ kips})$ $+ 0.75(0.6)(0.556 \text{ kip/ft})(45 \text{ ft})$ $= 11.3 \text{ kips}$

Available Compressive Strength (General)

The collector element is conservatively treated as a bare steel member for the determination of available compressive strength as discussed in AISC *Specification* Commentary Section I7. The effective length factor, K, for a pin-ended member is taken as 1.0 in accordance with Table C-A-7.1. Potential limit states are flexural buckling about both the minor and major axes, and torsional buckling.

Lateral movement is assumed to be braced by the composite slab, thus weak-axis flexural buckling will not govern by inspection as $L_{cy} = (KL)_y = 0$.

The member is slender for compression as indicated in AISC *Manual* Table 1-1, thus strong-axis flexural buckling strength is determined in accordance with AISC *Specification* Section E7 for members with slender elements for $L_{cx} = (KL)_x = 45.0$ ft.

The composite slab will prevent the member from twisting about its shear center, thus torsional buckling is not a valid limit state; however, constrained-axis torsional buckling may occur as discussed in AISC *Specification* Commentary Section E4 with $L_{cz} = (KL)_z = 1.0(45 \text{ ft}) = 45.0$ ft.

Compute the available compressive strengths for the limit states of strong-axis flexural buckling and constrained-axis torsional buckling to determine the controlling strength.

Strong-Axis Flexural Buckling

Calculate the critical stress about the strong axis, F_{crx}, in accordance with AISC *Specification* Section E3 as directed by *Specification* Section E7 for members with slender elements.

$$\frac{L_{cx}}{r_x} = \frac{(45.0 \text{ ft})(12 \text{ in./ft})}{8.18 \text{ in.}}$$
$$= 66.0$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 113 > 66.0; \text{ therefore, use AISC } \textit{Specification} \text{ Equation E3-2}$$

$$F_{ex} = \frac{\pi^2 E}{\left(\frac{L_{cx}}{r_x}\right)^2} \qquad (\textit{Spec}.\text{ Eq. E3-4})$$
$$= \frac{\pi^2(29{,}000 \text{ ksi})}{(66.0)^2}$$
$$= 65.7 \text{ ksi}$$

$$F_{crx} = \left(0.658^{\frac{F_y}{F_{ex}}}\right)F_y \qquad (\textit{Spec}.\text{ Eq. E3-2})$$
$$= \left(0.658^{\frac{50 \text{ ksi}}{65.7 \text{ ksi}}}\right)(50 \text{ ksi})$$
$$= 36.4 \text{ ksi}$$

Classify each component of the wide-flange member for local buckling.

Flange local buckling classification as determined from AISC *Specification* Table B4.1a, Case 1:

$$\lambda_r = 0.56\sqrt{\frac{E}{F_y}}$$
$$= 0.56\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 13.5$$

$$\lambda = \frac{b_f}{2t_f}$$
$$= 6.10 < 13.5; \text{ therefore, the flanges are nonslender}$$

Therefore, the flanges are fully effective.

Web local buckling classification as determined from AISC *Specification* Table B4.1a, Case 5:

$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}}$$
$$= 1.49\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 35.9$$

$$\lambda = \frac{h}{t_w}$$
$$= 49.4 > 35.9; \text{ therefore, the web is slender}$$

To evaluate the impact of web slenderness on strong-axis flexural buckling, determine if a reduced effective web width, h_e, is required in accordance with AISC *Specification* Section E7.1 as follows:

$$\lambda_r\sqrt{\frac{F_y}{F_{crx}}} = 35.9\sqrt{\frac{50 \text{ ksi}}{36.4 \text{ ksi}}}$$
$$= 42.1 < \lambda = 49.4; \text{ therefore, use AISC } \textit{Specification} \text{ Equation E7-3 to determine } h_e$$

The effective width imperfection adjustment factors, c_1 and c_2, are selected from AISC *Specification* Table E7.1, Case (a):

$$c_1 = 0.18$$
$$c_2 = 1.31$$

$$F_{el} = \left(c_2\frac{\lambda_r}{\lambda}\right)^2 F_y \qquad \text{(}\textit{Spec}\text{. Eq. E7-5)}$$
$$= \left[1.31\left(\frac{35.9}{49.4}\right)\right]^2 (50 \text{ ksi})$$
$$= 45.3 \text{ ksi}$$

$$h = \left(\frac{h}{t_w}\right)t_w$$
$$= (49.4)(0.380 \text{ in.})$$
$$= 18.8 \text{ in.}$$

$$h_e = h\left(1 - c_1\sqrt{\frac{F_{el}}{F_{cr}}}\right)\sqrt{\frac{F_{el}}{F_{cr}}}$$ (from *Spec.* Eq. E7-3)
$$= (18.8 \text{ in.})\left(1 - 0.18\sqrt{\frac{45.3 \text{ ksi}}{36.4 \text{ ksi}}}\right)\sqrt{\frac{45.3 \text{ ksi}}{36.4 \text{ ksi}}}$$
$$= 16.8 \text{ in.}$$

Calculate the effective area of the section:

$$A_e = A - (h - h_e)t_w$$
$$= 14.7 \text{ in.}^2 - (18.8 \text{ in.} - 16.8 \text{ in.})(0.380 \text{ in.})$$
$$= 13.9 \text{ in.}^2$$

Calculate the nominal compressive strength:

$$P_{nx} = F_{crx}A_e$$ (*Spec.* Eq. E7-1)
$$= (36.4 \text{ ksi})(13.9 \text{ in.}^2)$$
$$= 506 \text{ kips}$$

Calculate the available compressive strength:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(506 \text{ kips})$ $= 455 \text{ kips}$	$\Omega_c = 1.67$ $\dfrac{P_n}{\Omega_c} = \dfrac{506 \text{ kips}}{1.67}$ $= 303 \text{ kips}$

Constrained-Axis Torsional Buckling

Assuming the composite slab provides a lateral bracing point at the top flange of the beam, the constrained-axis buckling stress, F_{ez}, can be determined using AISC *Specification* Commentary Equaation C-E4-1 as follows:

The distance to bracing point from shear center along weak axis:

$$a = \frac{d}{2}$$
$$= \frac{20.8 \text{ in.}}{2}$$
$$= 10.4 \text{ in.}$$

The distance to bracing point from shear center along strong axis is:

$b = 0$

$$r_o^2 = r_x^2 + r_y^2 + a^2 + b^2 \quad \text{(Spec. Eq. C-E4-3)}$$
$$= (8.18\text{ in.})^2 + (1.30\text{ in.})^2 + (10.4\text{ in.})^2 + (0\text{ in.})^2$$
$$= 177\text{ in.}^2$$

From AISC *Specification* Commentary Section E4, the finite brace stiffness factor is:

$\omega = 0.9$

$$F_{ez} = \omega\left[\frac{\pi^2 EI_y}{(L_{cz})^2}\left(\frac{h_o{}^2}{4} + a^2\right) + GJ\right]\frac{1}{Ar_o^2} \quad \text{(Spec. Eq. C-E4-1)}$$
$$= 0.9\left\{\frac{\pi^2(29{,}000\text{ ksi})(24.9\text{ in.}^4)}{\left[(45.0\text{ ft})(12\text{ in./ft})\right]^2}\left[\frac{(20.3\text{ in.})^2}{4} + (10.4\text{ in.})^2\right] + (11{,}200\text{ ksi})(1.14\text{ in.}^4)\right\}$$
$$\times\left[\frac{1}{(14.7\text{ in.}^2)(177\text{ in.}^2)}\right]$$
$$= 6.20\text{ ksi}$$

To evaluate the impact of web slenderness on constrained-axis torsional buckling, determine if a reduced effective web width, h_e, is required in accordance with AISC *Specification* Section E7.1 as follows:

$$\lambda_r\sqrt{\frac{F_y}{F_{cr}}} = 35.9\sqrt{\frac{50\text{ ksi}}{6.20\text{ ksi}}}$$
$$= 102 > \lambda = 46.4;\text{ therefore use AISC } \textit{Specification} \text{ Equation E7-2}$$

$$h_e = h \quad \text{(from Spec. Eq. E7-2)}$$

Thus the full steel area may be used without reduction and the available compressive strength for constrained axis buckling strength is calculated as follows:

$$L_{cz} = (KL)_z$$
$$= (45.0\text{ ft})(12\text{ in./ft})$$
$$= 540\text{ in.}$$

$$\frac{F_y}{F_{ez}} = \frac{50\text{ ksi}}{6.20\text{ ksi}}$$
$$= 8.06 > 2.25,\text{ therefore, use AISC } \textit{Specification} \text{ Equation E3-3}$$

$$F_{crz} = 0.877F_{ez} \quad \text{(Spec. Eq. E3-3)}$$
$$= 0.877(6.20\text{ ksi})$$
$$= 5.44\text{ ksi}$$

The nominal compressive strength is calculated with no reduction for slenderness, $A_e = A$, as follows:

$$P_{nz} = F_{crz} A_e \quad \text{(}\textit{Spec}\text{. Eq. E7-1)}$$
$$= (5.44 \text{ ksi})(14.7 \text{ in.}^2)$$
$$= 80.0 \text{ kips}$$

The available compressive strength is determined as follows:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_{nz} = 0.90(80.0 \text{ kips})$ $= 72.0 \text{ kips}$	$\Omega_c = 1.67$ $\dfrac{P_{nz}}{\Omega_c} = \dfrac{80.0 \text{ kips}}{1.67}$ $= 47.9 \text{ kips}$

Note that it may be possible to utilize the flexural stiffness and strength of the slab as a continuous torsional restraint, resulting in increased constrained-axis torsional buckling capacity; however, that exercise is beyond the scope of this design example.

A summary of the available compressive strength for each of the viable limit states is as follows:

LRFD	ASD
Strong-axis flexural buckling: $\phi_c P_{nx} = 455 \text{ kips}$ Constrained-axis torsional buckling: $\phi_c P_{nz} = 72.0 \text{ kips}$ **controls**	Strong-axis flexural buckling: $\dfrac{P_{nx}}{\Omega_c} = 303 \text{ kips}$ Constrained-axis torsional buckling: $\dfrac{P_{nz}}{\Omega_c} = 47.9 \text{ kips}$ **controls**

Required First-Order Flexural Strength

From ASCE/SEI 7, Chapter 2, the required first-order flexural strength for the governing load combination including wind is:

LRFD	ASD
$w_u = 1.2D + 1.0W + L$ $= 1.2(0.9 \text{ kip/ft}) + 1.0(0 \text{ kip/ft}) + 1 \text{ kip/ft}$ $= 2.08 \text{ kip/ft}$ $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(2.08 \text{ kip/ft})(45 \text{ ft})^2}{8}$ $= 527 \text{ kip-ft}$	$w_a = D + 0.75L + 0.75(0.6W)$ $= 0.9 \text{ kip/ft} + 0.75(1 \text{ kip/ft}) + 0.75(0.6)(0 \text{ kip/ft})$ $= 1.65 \text{ kip/ft}$ $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(1.65 \text{ kip/ft})(45 \text{ ft})^2}{8}$ $= 418 \text{ kip-ft}$

Required Second-Order Flexural Strength

The effective length method is utilized to consider stability for this element as permitted by AISC *Specification* Section C1.2 and Appendix 7.2. The addition of axial load will magnify the required first-order flexural strength due to member slenderness (P-δ) effects. This magnification (second-order analysis) can be approximated utilizing the procedure provided in AISC *Specification* Appendix 8 as permitted by Section C2.1b.

Calculate the elastic critical buckling strength of the member in the plane of bending (in this case about the strong-axis of the beam) from AISC *Specification* Appendix 8, Section 8.2.1. For the effective length method, EI^* is taken as EI in accordance with Appendix 8.2.1, and the effective length, L_{cx} is taken as $(KL)_x$ in accordance with Appendix 7.2.3. As illustrated previously, K, is taken as 1.0 for a pin-ended member. Conservatively using the bare steel beam moment of inertia, the buckling strength is calculated as follows:

$$P_{e1} = \frac{\pi^2 EI^*}{(L_{c1})^2} \qquad \text{(}Spec\text{. Eq. A-8-5)}$$

$$= \frac{\pi^2 EI}{(KL)_x^2} \qquad \text{(for the effective length method)}$$

$$= \frac{\pi^2 (29{,}000 \text{ ksi})(984 \text{ in.}^4)}{[(45.0 \text{ ft})(12 \text{ in./ft})]^2}$$

$$= 966 \text{ kips}$$

For beam-columns subject to transverse loading between supports, the value of C_m is taken as 1.0 as permitted by AISC *Specification* Appendix 8, Section 8.2.1(b), and B_1 is calculated from *Specification* Equation A-8-3 as follows:

LRFD	ASD
$B_1 = \frac{C_m}{1-\alpha P_u/P_{e1}} \geq 1$ $= \frac{1.0}{1-1.0\left(\frac{25.0 \text{ kips}}{966 \text{ kips}}\right)} \geq 1$ $= 1.03$	$B_1 = \frac{C_m}{1-\alpha P_a/P_{e1}} \geq 1$ $= \frac{1.0}{1-1.6\left(\frac{11.3 \text{ kips}}{966 \text{ kips}}\right)} \geq 1$ $= 1.02$

Noting that the first-order moment is induced by vertical dead and live loading, it is classified as a non-translational moment, M_{nt}, in accordance with AISC *Specification* Section 8.2. The required second-order flexural strength is therefore calculated using AISC *Specification* Equation A-8-1 as:

LRFD	ASD
$M_u = B_1 M_{nt} + B_2 M_{lt}$ $= 1.03(527 \text{ kip-ft}) + 0$ $= 543 \text{ kip-ft}$	$M_a = B_1 M_{nt} + B_2 M_{lt}$ $= 1.02(418 \text{ kip-ft}) + 0$ $= 426 \text{ kip-ft}$

Available Flexural Strength

The available flexural strength of the composite beam is calculated in Example I.1 as:

LRFD	ASD
$\phi_b M_{nx} = 769 \text{ kip-ft}$	$\frac{M_{nx}}{\Omega_b} = 512 \text{ kip-ft}$

Interaction of Axial Force and Flexure

Interaction between axial forces and flexure in composite collector beams is addressed in AISC *Specification* Commentary Section I7, which states that the non-composite axial strength and the composite flexural strength may be used with the interaction equations provided in Chapter H as a reasonable simplification for design purposes. This procedure is illustrated as follows:

LRFD	ASD
$\phi_c P_n = 72.0$ kips	$\dfrac{P_n}{\Omega_c} = 47.9$ kips
$\phi_b M_{nx} = 769$ kip-ft	$\dfrac{M_{nx}}{\Omega_c} = 512$ kip-ft
$\dfrac{P_r}{P_c} = \dfrac{P_u}{\phi_c P_n}$ $= \dfrac{25.0\text{ kips}}{72.0\text{ kips}}$ $= 0.347 > 0.2$	$\dfrac{P_r}{P_c} = \dfrac{P_a}{P_n / \Omega_c}$ $= \dfrac{11.3\text{ kips}}{47.9\text{ kips}}$ $= 0.236 > 0.2$
Therefore, use AISC *Specification* Equation H1-1a.	Therefore, use AISC *Specification* Equation H1-1a.
$\dfrac{P_u}{\phi_c P_n} + \dfrac{8}{9}\left(\dfrac{M_u}{\phi_b M_{nx}}\right) \le 1.0$ $0.347 + \dfrac{8}{9}\left(\dfrac{543\text{ kip-ft}}{769\text{ kip-ft}}\right) \le 1.0$ $0.975 < 1.0$ **o.k.**	$\dfrac{P_a}{P_n / \Omega_c} + \dfrac{8}{9}\left(\dfrac{M_a}{M_{nx} / \Omega_b}\right) \le 1.0$ $0.236 + \dfrac{8}{9}\left(\dfrac{426\text{ kip-ft}}{512\text{ kip-ft}}\right) \le 1.0$ $0.976 < 1.0$ **o.k.**

The collector element is adequate to resist the imposed loads.

Load Introduction Effects

AISC *Specification* Commentary Section I7 indicates that the effect of the vertical offset between the plane of the diaphragm and the collector element should be investigated. It has been shown that the resulting eccentricity between the plane of axial load introduction in the slab and the centroid of the beam connections does not result in any additional flexural demand assuming the axial load is introduced uniformly along the length of the beam; however, this eccentricity will result in additional shear reactions (Burmeister and Jacobs, 2008). The additional shear reaction assuming an eccentricity of $d/2$ is calculated as follows:

LRFD	ASD
$V_{u\text{-}add} = \dfrac{P_u d}{2L}$ $= \dfrac{(25.0\text{ kips})(20.8\text{ in.})}{2(45\text{ ft})(12\text{ in./ft})}$ $= 0.481$ kips	$V_{a\text{-}add} = \dfrac{P_a d}{2L}$ $= \dfrac{(11.3\text{ kips})(20.8\text{ in.})}{2(45\text{ ft})(12\text{ in./ft})}$ $= 0.218$ kips

As can be seen from these results, the additional vertical shear due to the axial collector force is quite small and in most instances will be negligible versus the governing shear resulting from gravity-only load combinations.

Shear Connection

AISC *Specification* Commentary Section I7 notes that it is not required to superimpose the horizontal shear due to lateral forces with the horizontal shear due to flexure for the determination of steel anchor requirements, thus the summation of nominal strengths for all steel anchors along the beam length may be used for axial force transfer. Specific resistance and safety factors for this condition are not provided in Section I8.2 as they are implicitly accounted for within the system resistance and safety factors used for the determination of the available flexural strength of the beam. Until additional research becomes available, a conservative approach is to apply the composite component factors from *Specification* Section I8.3 to the nominal steel anchor strengths determined from *Specification* Section I8.2.

From Example I.1, the strength for ¾-in.-diameter anchors in normal weight concrete with $f_c' = 4$ ksi and deck oriented perpendicular to the beam is:

1 anchor per rib: $Q_n = 17.2$ kips/anchor
2 anchors per rib: $Q_n = 14.6$ kips/anchor

Over the entire beam length, there are 42 anchors in positions with one anchor per rib and four anchors in positions with two anchors per rib, thus the total available strength for diaphragm shear transfer is:

LRFD	ASD
$\phi_v = 0.65$ $\phi_c P_n = 0.65\left[42\left(17.2\text{ kips/anchor}\right)+4(14.6\text{ kips/anchor})\right]$ $= 508\text{ kips} > 25.0\text{ kips}$ **o.k.**	$\Omega_v = 2.31$ $\dfrac{P_n}{\Omega_c} = \dfrac{42\left(17.2\text{ kips/anchor}\right)+4\left(14.6\text{ kips/anchor}\right)}{2.31}$ $= 338\text{ kips} > 11.3\text{ kips}$ **o.k.**

Note that the longitudinal available shear strength of the diaphragm itself (consisting of the composite deck and concrete fill) will often limit the amount of force that can be introduced into the collector beam and should also be evaluated as part of the overall design.

Summary

A W21×50 collector with 46, ¾-in.-diameter by 4⅞-in.-long, steel headed stud anchors is adequate to resist the imposed loads.

CHAPTER I DESIGN EXAMPLE REFERENCES

ACI 318 (2014), *Building Code Requirements for Structural Concrete*, ACI 318-14; *and Commentary*, ACI 318R-14, American Concrete Institute, Farmington Hills, MI.

ASCE (2014), *Design Loads on Structures During Construction*, ASCE/SEI 37-14, American Society of Civil Engineers, Reston, VA.

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Chapter J
Design of Connections

AISC *Specification* Chapter J addresses the design of connections. The chapter's primary focus is the design of welded and bolted connections. Design requirements for fillers, splices, column bases, concentrated forces, anchors rods and other threaded parts are also covered. See AISC *Specification* Appendix 3 for special requirements for connections subject to fatigue.

EXAMPLE J.1 FILLET WELD IN LONGITUDINAL SHEAR

Given:

As shown in Figure J.1-1, a ¼-in.-thick × 18-in. wide plate is fillet welded to a ⅜-in.-thick plate. The plates are ASTM A572 Grade 50 and have been properly sized. Use 70-ksi electrodes. Note that the plates could be specified as ASTM A36, but F_y = 50 ksi plate has been used here to demonstrate the requirements for long welds.

Confirm that the size and length of the welds shown are adequate to resist the applied loading.

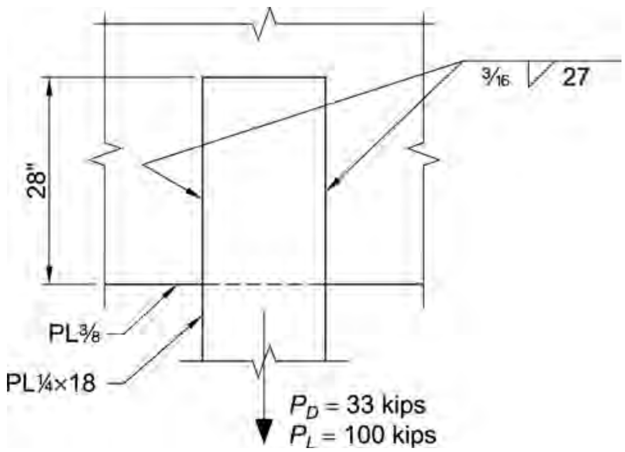


Fig. J.1-1. Geometry and loading for Example J.1.

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

ASTM A572 Grade 50
F_y = 50 ksi
F_u = 65 ksi

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_u = 1.2(33 \text{ kips}) + 1.6(100 \text{ kips})$ $= 200 \text{ kips}$	$P_a = 33 \text{ kips} + 100 \text{ kips}$ $= 133 \text{ kips}$

Maximum and Minimum Weld Size

Because the thickness of the overlapping plate is ¼ in., the maximum fillet weld size that can be used without special notation per AISC *Specification* Section J2.2b, is a 3⁄16-in. fillet weld. A 3⁄16-in. fillet weld can be deposited in the flat or horizontal position in a single pass (true up to 5⁄16-in.).

From AISC *Specification* Table J2.4, the minimum size of the fillet weld, based on a material thickness of ¼ in. is ⅛ in.

Weld Strength

The nominal weld strength per inch of 3⁄16-in. weld, determined from AISC *Specification* Section J2.4(b) is:

$$R_n = F_{nw}A_{we} \qquad \text{(Spec. Eq. J2-4)}$$
$$= (0.60F_{EXX})A_{we}$$
$$= 0.60(70\text{ ksi})\left(\frac{3/16\text{ in.}}{\sqrt{2}}\right)$$
$$= 5.57\text{ kip/in.}$$

From AISC *Specification* Section J2.2b, check the weld length to weld size ratio, because this is an end-loaded fillet weld.

$$\frac{l}{w} = \frac{27.0\text{ in.}}{3/16\text{ in.}}$$
$$= 144 > 100;\text{ therefore, AISC } \textit{Specification} \text{ Equation J2-1 must be applied}$$

$$\beta = 1.2 - 0.002(l/w) \le 1.0 \qquad \text{(Spec. Eq. J2-1)}$$
$$= 1.2 - 0.002(144) \le 1.0$$
$$= 0.912$$

The nominal weld shear rupture strength is:

$$R_n = 0.912(5.57\text{ kip/in.})(2\text{ welds})(27\text{ in.})$$
$$= 274\text{ kips}$$

From AISC *Specification* Section J2.4, the available shear rupture strength is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(274\text{ kips})$ $= 206\text{ kips} > 200\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{274\text{ kips}}{2.00}$ $= 137\text{ kips} > 133\text{ kips}$ **o.k.**

The base metal strength is determined from AISC *Specification* Section J2.4(a). The ¼-in.-thick plate controls:

$$R_n = F_{nBM}A_{BM} \qquad \text{(Spec. Eq. J2-2)}$$
$$= 0.60F_u t_p l_{weld}$$
$$= 0.60(65\text{ ksi})(1/4\text{ in.})(2\text{ welds})(27\text{ in.})$$
$$= 527\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(527\text{ kips})$ $= 395\text{ kips} > 200\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{527\text{ kips}}{2.00}$ $= 264\text{ kips} > 133\text{ kips}$ **o.k.**

EXAMPLE J.2 FILLET WELD LOADED AT AN ANGLE

Given:

Verify a fillet weld at the edge of a gusset plate is adequate to resist a force of 50 kips due to dead load and 150 kips due to live load, at an angle of 60° relative to the weld, as shown in Figure J.2-1. Assume the beam and the gusset plate thickness and length have been properly sized. Use a 70-ksi electrode.

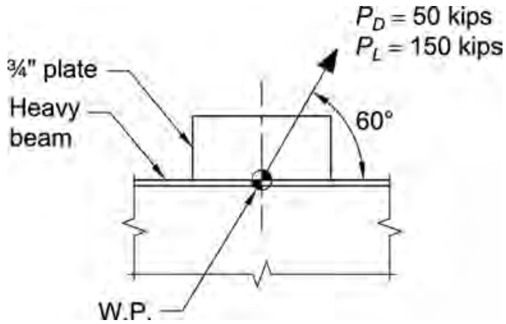


Fig. J.2-1. Geometry and loading for Example J.2.

Solution:

From ASCE/SEI 7, Chapter 2, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(50\text{ kips}) + 1.6(150\text{ kips})$ $= 300\text{ kips}$	$P_a = 50\text{ kips} + 150\text{ kips}$ $= 200\text{ kips}$

Assume a ⁵⁄₁₆-in. fillet weld is used on each side of the plate.

Note that from AISC *Specification* Table J2.4, the minimum size of fillet weld, based on a material thickness of ¾ in. is ¼ in. (assuming the beam flange thickness exceeds ¾ in.).

Available Shear Strength of the Fillet Weld Per Inch of Length

From AISC *Specification* Section J2.4(b), the nominal strength of the fillet weld is determined as follows:

$$R_n = F_{nw}A_{we} \qquad (\textit{Spec}.\text{ Eq. J2-4})$$

$$= 0.60F_{EXX}\left(1.0 + 0.50\sin^{1.5} 60°\right)A_{we}$$

$$= 0.60(70\text{ ksi})\left(1.0 + 0.50\sin^{1.5} 60°\right)\left(\frac{5/16\text{ in.}}{\sqrt{2}}\right)$$

$$= 13.0\text{ kip/in.}$$

From AISC *Specification* Section J2.4(b), the available shear strength per inch of weld for fillet welds on two sides is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(13.0 \text{ kip/in.})(2 \text{ sides})$ $= 19.5 \text{ kip/in.}$	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{13.0 \text{ kip/in.}}{2.00}(2 \text{ sides})$ $= 13.0 \text{ kip/in.}$

Required Length of Weld

LRFD	ASD
$l = \dfrac{300 \text{ kips}}{19.5 \text{ kip/in.}}$ $= 15.4 \text{ in.}$ Use 16 in. on each side of the plate.	$l = \dfrac{200 \text{ kips}}{13.0 \text{ kip/in.}}$ $= 15.4 \text{ in.}$ Use 16 in. on each side of the plate.

EXAMPLE J.3 COMBINED TENSION AND SHEAR IN BEARING-TYPE CONNECTIONS

Given:

A ¾-in.-diameter, Group A bolt with threads not excluded from the shear plane (thread condition N) is subjected to a tension force of 3.5 kips due to dead load and 12 kips due to live load, and a shear force of 1.33 kips due to dead load and 4 kips due to live load. Check the combined stresses according to AISC *Specification* Equations J3-3a and J3-3b.

Solution:

From ASCE/SEI 7, Chapter 2, the required tensile and shear strengths are:

LRFD	ASD
Tension: $T_u = 1.2(3.5\text{ kips}) + 1.6(12\text{ kips})$ $= 23.4\text{ kips}$ Shear: $V_u = 1.2(1.33\text{ kips}) + 1.6(4\text{ kips})$ $= 8.00\text{ kips}$	Tension: $T_a = 3.5\text{ kips} + 12\text{ kips}$ $= 15.5\text{ kips}$ Shear: $V_a = 1.33\text{ kips} + 4\text{ kips}$ $= 5.33\text{ kips}$

Available Tensile Strength

When a bolt is subject to combined tension and shear, the available tensile strength is determined according to the limit states of tension and shear rupture, from AISC *Specification* Section J3.7 as follows.

From AISC *Specification* Table J3.2, Group A bolts:

$F_{nt} = 90\text{ ksi}$
$F_{nv} = 54\text{ ksi}$

From AISC *Manual* Table 7-2, for a ¾-in.-diameter bolt:

$A_b = 0.442\text{ in.}^2$

The available shear stress is determined as follows and must equal or exceed the required shear stress.

LRFD	ASD
$\phi = 0.75$ $\phi F_{nv} = 0.75(54\text{ ksi})$ $= 40.5\text{ ksi}$ $f_{rv} = \dfrac{V_u}{A_b}$ $= \dfrac{8.00\text{ kips}}{0.442\text{ in.}^2}$ $= 18.1\text{ ksi} < 40.5\text{ ksi}$ **o.k.**	$\Omega = 2.00$ $\dfrac{F_{nv}}{\Omega} = \dfrac{54\text{ ksi}}{2.00}$ $= 27.0\text{ ksi}$ $f_{rv} = \dfrac{V_a}{A_b}$ $= \dfrac{5.33\text{ kips}}{0.442\text{ in.}^2}$ $= 12.1\text{ ksi} < 27.0\text{ ksi}$ **o.k.**

The available tensile strength of a bolt subject to combined tension and shear is as follows:

LRFD	ASD
$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3-3a) $= 1.3(90\text{ ksi}) - \frac{90\text{ ksi}}{40.5\text{ ksi}}(18.1\text{ ksi}) \le 90\text{ ksi}$ $= 76.8\text{ ksi}$ For combined tension and shear, ϕ = 0.75, from AISC *Specification* Section J3.7. $\phi R_n = \phi F'_{nt} A_b$ (*Spec*. Eq. J3-2) $= 0.75(76.8\text{ ksi})(0.442\text{ in.}^2)$ $= 25.5\text{ kips} > 23.4\text{ kips}$ **o.k.**	$F'_{nt} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3-3b) $= 1.3(90\text{ ksi}) - \frac{90\text{ ksi}}{27.0\text{ ksi}}(12.1\text{ ksi}) \le 90\text{ ksi}$ $= 76.7\text{ ksi}$ For combined tension and shear, Ω = 2.00, from AISC *Specification* Section J3.7. $\frac{R_n}{\Omega} = \frac{F'_{nt} A_b}{\Omega}$ (*Spec*. Eq. J3-2) $= \frac{(76.7\text{ ksi})(0.442\text{ in.}^2)}{2.00}$ $= 17.0\text{ kips} > 15.5\text{ kips}$ **o.k.**

The effects of combined shear and tensile stresses need not be investigated if either the required shear or tensile stress is less than or equal to 30% of the corresponding available stress per the User Note at the end of AISC *Specification* Section J3.7. In the example herein, both the required shear and tensile stresses exceeded the 30% threshold and evaluation of combined stresses was necessary.

AISC *Specification* Equations J3-3a and J3-3b may be rewritten so as to find a nominal shear stress, F'_{nv}, as a function of the required tensile stress as is shown in AISC *Specification* Commentary Equations C-J3-7a and C-J3-7b.

EXAMPLE J.4A SLIP-CRITICAL CONNECTION WITH SHORT-SLOTTED HOLES

Slip-critical connections shall be designed to prevent slip and for the limit states of bearing-type connections.

Given:

Refer to Figure J.4A-1 and select the number of bolts that are required to support the loads shown when the connection plates have short slots transverse to the load and no fillers are provided. Select the number of bolts required for slip resistance only.

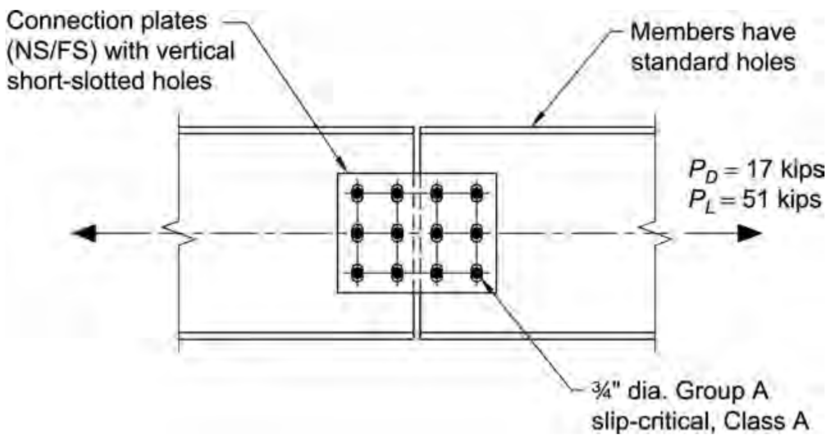


Fig. J.4A-1. Geometry and loading for Example J.4A.

Solution:

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_u = 1.2(17 \text{ kips}) + 1.6(51 \text{ kips})$ $= 102 \text{ kips}$	$P_a = 17 \text{ kips} + 51 \text{ kips}$ $= 68.0 \text{ kips}$

From AISC *Specification* Section J3.8(a), the available slip resistance for the limit state of slip for standard size and short-slotted holes perpendicular to the direction of the load is determined as follows:

ϕ = 1.00
Ω = 1.50
μ = 0.30 for Class A surface
D_u = 1.13
h_f = 1.0, no filler is provided
T_b = 28 kips, from AISC *Specification* Table J3.1, Group A
n_s = 2, number of slip planes

$$R_n = \mu D_u h_f T_b n_s \quad \text{(Spec. Eq. J3-4)}$$
$$= 0.30(1.13)(1.0)(28 \text{ kips})(2)$$
$$= 19.0 \text{ kips/bolt}$$

The available slip resistance is:

LRFD	ASD
$\phi R_n = 1.00(19.0 \text{ kips/bolt})$ $= 19.0 \text{ kips/bolt}$	$\dfrac{R_n}{\Omega} = \dfrac{19.0 \text{ kips/bolt}}{1.50}$ $= 12.7 \text{ kips/bolt}$

Required Number of Bolts

LRFD	ASD
$n_b = \dfrac{P_u}{\phi R_n}$ $= \dfrac{102 \text{ kips}}{19.0 \text{ kips/bolt}}$ $= 5.37 \text{ bolts}$ Use 6 bolts	$n_b = \dfrac{P_a}{\left(\dfrac{R_n}{\Omega}\right)}$ $= \dfrac{68.0 \text{ kips}}{12.7 \text{ kips/bolt}}$ $= 5.35 \text{ bolts}$ Use 6 bolts

Note: To complete the verification of this connection, the limit states of bolt shear, bearing, tearout, tensile yielding, tensile rupture, and block shear rupture must also be checked.

EXAMPLE J.4B SLIP-CRITICAL CONNECTION WITH LONG-SLOTTED HOLES

Given:

Repeat Example J.4A with the same loads, but assuming that the connection plates have long-slotted holes in the direction of the load, as shown in Figure J.4B-1.

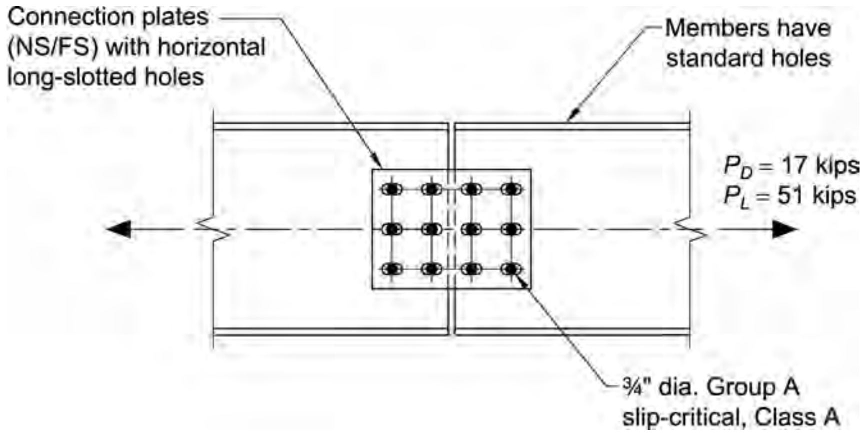


Fig. J.4B-1. Geometry and loading for Example J.4B.

Solution:

The required strength from Example J.4A is:

LRFD	ASD
$P_u = 102$ kips	$P_a = 68.0$ kips

From AISC *Specification* Section J3.8(c), the available slip resistance for the limit state of slip for long-slotted holes is determined as follows:

ϕ = 0.70
Ω = 2.14
μ = 0.30 for Class A surface
D_u = 1.13
h_f = 1.0, no filler is provided
T_b = 28 kips, from AISC *Specification* Table J3.1, Group A
n_s = 2, number of slip planes

$$R_n = \mu D_u h_f T_b n_s \qquad \text{(}\textit{Spec}\text{. Eq. J3-4)}$$
$$= 0.30(1.13)(1.0)(28 \text{ kips})(2)$$
$$= 19.0 \text{ kips/bolt}$$

The available slip resistance is:

LRFD	ASD
$\phi R_n = 0.70(19.0 \text{ kips/bolt})$ $= 13.3$ kips/bolt	$\dfrac{R_n}{\Omega} = \dfrac{19.0 \text{ kips/bolt}}{2.14}$ $= 8.88$ kips/bolt

Required Number of Bolts

LRFD	ASD
$n_b = \dfrac{P_u}{\phi R_n}$ $= \dfrac{102 \text{ kips}}{13.3 \text{ kips/bolt}}$ $= 7.67 \text{ bolts}$ Use 8 bolts	$n_b = \dfrac{P_a}{\left(\dfrac{R_n}{\Omega}\right)}$ $= \dfrac{68.0 \text{ kips}}{8.88 \text{ kips/bolt}}$ $= 7.66 \text{ bolts}$ Use 8 bolts

Note: To complete the verification of this connection, the limit states of bolt shear, bearing, tearout, tensile yielding, tensile rupture, and block shear rupture must be determined.

EXAMPLE J.5 COMBINED TENSION AND SHEAR IN A SLIP-CRITICAL CONNECTION

Because the pretension of a bolt in a slip-critical connection is used to create the clamping force that produces the shear strength of the connection, the available shear strength must be reduced for any load that produces tension in the connection.

Given:

The slip-critical bolt group shown in Figure J.5-1 is subjected to tension and shear. This example shows the design for bolt slip resistance only, and assumes that the beams and plates are adequate to transmit the loads. Determine if the bolts are adequate.

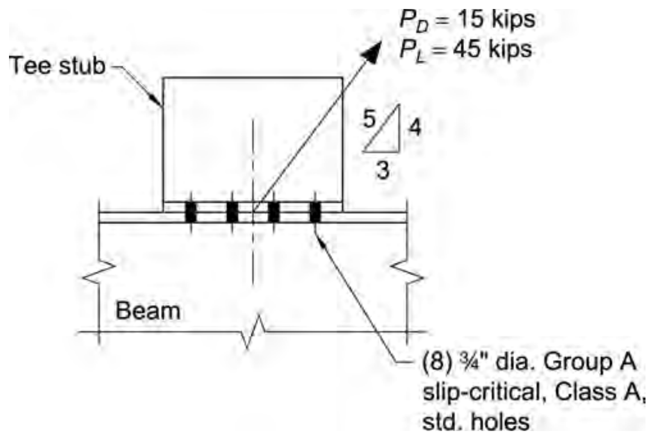


Fig. J.5-1. Geometry and loading for Example J.5.

Solution:

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_u = 1.2(15 \text{ kips}) + 1.6(45 \text{ kips})$ $= 90.0 \text{ kips}$ By geometry: $T_u = \frac{4}{5}(90.0 \text{ kips})$ $= 72.0 \text{ kips}$ $V_u = \frac{3}{5}(90.0 \text{ kips})$ $= 54.0 \text{ kips}$	$P_a = 15 \text{ kips} + 45 \text{ kips}$ $= 60.0 \text{ kips}$ By geometry: $T_a = \frac{4}{5}(60.0 \text{ kips})$ $= 48.0 \text{ kips}$ $V_a = \frac{3}{5}(60.0 \text{ kips})$ $= 36.0 \text{ kips}$

Available Bolt Tensile Strength

The available tensile strength is determined from AISC *Specification* Section J3.6.

From AISC *Specification* Table J3.2 for Group A bolts, the nominal tensile strength in ksi is, F_{nt} = 90 ksi. From AISC *Manual* Table 7-1, for a ¾-in.-diameter bolt:

$A_b = 0.442 \text{ in.}^2$

The nominal tensile strength is:

$$R_n = F_{nt} A_b \qquad \text{(from } \textit{Spec}\text{. Eq. J3-1)}$$
$$= (90 \text{ ksi})(0.442 \text{ in.}^2)$$
$$= 39.8 \text{ kips}$$

The available tensile strength is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(39.8 \text{ kips/bolt}) > \dfrac{72.0 \text{ kips}}{8 \text{ bolts}}$ $= 29.9 \text{ kips/bolt} > 9.00 \text{ kips/bolt}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{39.8 \text{ kips/bolt}}{2.00} > \dfrac{48.0 \text{ kips}}{8 \text{ bolts}}$ $= 19.9 \text{ kips/bolt} > 6.00 \text{ kips/bolt}$ **o.k.**

Note that the available tensile strength per bolt can also be taken from AISC *Manual* Table 7-2.

Available Slip Resistance per Bolt

The available slip resistance for one bolt in standard size holes is determined using AISC *Specification* Section J3.8(a):

ϕ = 1.00
Ω = 1.50
μ = 0.30 for Class A surface
D_u = 1.13
h_f = 1.0, factor for fillers, assuming no more than one filler
T_b = 28 kips, from AISC *Specification* Table J3.1, Group A
n_s = 1, number of slip planes

LRFD	ASD
Determine the available slip resistance (T_u = 0) of a bolt: $\phi R_n = \phi \mu D_u h_f T_b n_s$ (from *Spec*. Eq. J3-4) $= 1.00(0.30)(1.13)(1.0)(28 \text{ kips})(1)$ $= 9.49 \text{ kips/bolt}$	Determine the available slip resistance (T_a = 0) of a bolt: $\dfrac{R_n}{\Omega} = \dfrac{\mu D_u h_f T_b n_s}{\Omega}$ (from *Spec*. Eq. J3-4) $= \dfrac{0.30(1.13)(1.0)(28 \text{ kips})(1)}{1.50}$ $= 6.33 \text{ kips/bolt}$

Note that the available slip resistance for one bolt with a Class A faying surface can also be taken from AISC *Manual* Table 7-3.

Available Slip Resistance of the Connection

Because the slip-critical connection is subject to combined tension and shear, the available slip resistance is multiplied by a reduction factor provided in AISC *Specification* Section J3.9.

LRFD	ASD
Slip-critical combined tension and shear factor: $k_{sc} = 1 - \dfrac{T_u}{D_u T_b n_b} \geq 0$ (*Spec*. Eq. J3-5a) $= 1 - \dfrac{72.0\text{ kips}}{1.13(28\text{ kips})(8)} > 0$ $= 0.716$ $\phi R_n = \phi R_n k_{sc} n_b$ $= (9.49\text{ kips/bolt})(0.716)(8\text{ bolts})$ $= 54.4\text{ kips} > 54.0\text{ kips}$ **o.k.**	Slip-critical combined tension and shear factor: $k_{sc} = 1 - \dfrac{1.5T_a}{D_u T_b n_b} \geq 0$ (*Spec*. Eq. J3-5b) $= 1 - \dfrac{1.5(48.0\text{ kips})}{1.13(28\text{ kips})(8)} > 0$ $= 0.716$ $\dfrac{R_n}{\Omega} = \dfrac{R_n}{\Omega} k_{sc} n_b$ $= (6.33\text{ kips/bolt})(0.716)(8\text{ bolts})$ $= 36.3\text{ kips} > 36.0\text{ kips}$ **o.k.**

Note: The bolt group must still be checked for all applicable strength limit states for a bearing-type connection.

EXAMPLE J.6 BASE PLATE BEARING ON CONCRETE

Given:

As shown in Figure J.6-1, an ASTM A992 column bears on a concrete pedestal with f'_c = 3 ksi. The space between the base plate and the concrete pedestal has grout with f'_c = 4 ksi. Verify the ASTM A36 base plate will support the following loads in axial compression:

P_D = 115 kips
P_L = 345 kips

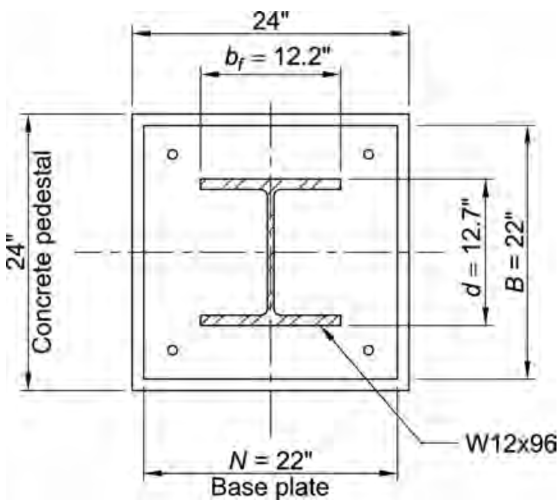


Fig. J.6-1. Geometry for Example J.6.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Base Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Column
W12×96
d = 12.7 in.
b_f = 12.2 in.
t_f = 0.900 in.
t_w = 0.550 in.

From ASCE/SEI 7, Chapter 2, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(115 \text{ kips}) + 1.6(345 \text{ kips})$ $= 690 \text{ kips}$	$P_a = 115 \text{ kips} + 345 \text{ kips}$ $= 460 \text{ kips}$

Base Plate Dimensions

Determine the required base plate area from AISC *Specification* Section J8 conservatively assuming bearing on the full area of the concrete support.

LRFD	ASD
$\phi_c = 0.65$ $A_{1(req)} = \dfrac{P_u}{\phi_c 0.85 f_c'}$ (from *Spec*. Eq. J8-1) $= \dfrac{690 \text{ kips}}{0.65(0.85)(3 \text{ ksi})}$ $= 416 \text{ in.}^2$	$\Omega_c = 2.31$ $A_{1(req)} = \dfrac{\Omega_c P_a}{0.85 f_c'}$ (from *Spec*. Eq. J8-1) $= \dfrac{2.31(460 \text{ kips})}{0.85(3 \text{ ksi})}$ $= 417 \text{ in.}^2$

Note: The strength of the grout has conservatively been neglected, as its strength is greater than that of the concrete pedestal.

Try a 22-in. × 22-in. base plate.

Verify $N \geq d + 2(3 \text{ in.})$ and $B \geq b_f + 2(3 \text{ in.})$ for anchor rod pattern shown in diagram:

$$d + 2(3 \text{ in.}) = 12.7 \text{ in.} + 2(3 \text{ in.})$$
$$= 18.7 \text{ in.} < 22 \text{ in.} \quad \textbf{o.k.}$$

$$b_f + 2(3 \text{ in.}) = 12.2 \text{ in.} + 2(3 \text{ in.})$$
$$= 18.2 \text{ in.} < 22 \text{ in.} \quad \textbf{o.k.}$$

Base plate area:

$$A_1 = NB$$
$$= (22 \text{ in.})(22 \text{ in.})$$
$$= 484 \text{ in.}^2 > 417 \text{ in.}^2$$

o.k. (conservatively compared to ASD value for $A_{1(req)}$)

Note: A square base plate with a square anchor rod pattern will be used to minimize the chance for field and shop problems.

Concrete Bearing Strength

Use AISC *Specification* Equation J8-2 because the base plate covers less than the full area of the concrete support.

Because the pedestal is square and the base plate is a concentrically located square, the full pedestal area is also the geometrically similar area. Therefore:

$$A_2 = (24 \text{ in.})(24 \text{ in.})$$
$$= 576 \text{ in.}^2$$

The available bearing strength is:

LRFD	ASD
$\phi_c = 0.65$ $\phi_c P_p = \phi_c 0.85 f'_c A_1 \sqrt{\dfrac{A_2}{A_1}} \le \phi_c 1.7 f'_c A_1$ (from *Spec*. Eq. J8-2) $= 0.65(0.85)(3 \text{ ksi})(484 \text{ in.}^2)\sqrt{\dfrac{576 \text{ in.}^2}{484 \text{ in.}^2}}$ $\le 0.65(1.7)(3 \text{ ksi})(484 \text{ in.}^2)$ $= 875 \text{ kips} < 1{,}600 \text{ kips, use } 875 \text{ kips}$ $875 \text{ kips} > 690 \text{ kips}$ **o.k.**	$\Omega_c = 2.31$ $\dfrac{P_p}{\Omega_c} = \dfrac{0.85 f'_c A_1}{\Omega_c}\sqrt{\dfrac{A_2}{A_1}} \le \dfrac{1.7 f'_c A_1}{\Omega_c}$ (from *Spec*. Eq. J8-2) $= \dfrac{0.85(3 \text{ ksi})(484 \text{ in.}^2)}{2.31}\sqrt{\dfrac{576 \text{ in.}^2}{484 \text{ in.}^2}}$ $\le \dfrac{1.7(3 \text{ ksi})(484 \text{ in.}^2)}{2.31}$ $= 583 \text{ kips} < 1{,}070 \text{ kips, use } 583 \text{ kips}$ $583 \text{ kips} > 460 \text{ kips}$ **o.k.**

Notes:

1. $A_2/A_1 \le 4$; therefore, the upper limit in AISC *Specification* Equation J8-2 does not control.
2. As the area of the base plate approaches the area of concrete, the modifying ratio, $\sqrt{A_2/A_1}$, approaches unity and AISC *Specification* Equation J8-2 converges to AISC *Specification* Equation J8-1.

Required Base Plate Thickness

The base plate thickness is determined in accordance with AISC *Manual* Part 14.

$$m = \frac{N - 0.95d}{2} \qquad \text{(\textit{Manual} Eq. 14-2)}$$
$$= \frac{22 \text{ in.} - 0.95(12.7 \text{ in.})}{2}$$
$$= 4.97 \text{ in.}$$

$$n = \frac{B - 0.8b_f}{2} \qquad \text{(\textit{Manual} Eq. 14-3)}$$
$$= \frac{22 \text{ in.} - 0.8(12.2 \text{ in.})}{2}$$
$$= 6.12 \text{ in.}$$

$$n' = \frac{\sqrt{db_f}}{4} \qquad \text{(\textit{Manual} Eq. 14-4)}$$
$$= \frac{\sqrt{(12.7 \text{ in.})(12.2 \text{ in.})}}{4}$$
$$= 3.11 \text{ in.}$$

LRFD	ASD
$X=\left[\dfrac{4db_f}{(d+b_f)^2}\right]\dfrac{P_u}{\phi_c P_p}$ (*Manual* Eq. 14-6a) $=\left[\dfrac{4(12.7\text{ in.})(12.2\text{ in.})}{(12.7\text{ in.}+12.2\text{ in.})^2}\right]\left(\dfrac{690\text{ kips}}{875\text{ kips}}\right)$ $=0.788$	$X=\left[\dfrac{4db_f}{(d+b_f)^2}\right]\dfrac{\Omega_c P_a}{P_p}$ (*Manual* Eq. 14-6b) $=\left[\dfrac{4(12.7\text{ in.})(12.2\text{ in.})}{(12.7\text{ in.}+12.2\text{ in.})^2}\right]\left(\dfrac{460\text{ kips}}{583\text{ kips}}\right)$ $=0.789$

Conservatively, use the LRFD value for *X*.

$$\lambda=\frac{2\sqrt{X}}{1+\sqrt{1-X}}\leq 1 \qquad \text{(\textit{Manual} Eq. 14-5)}$$

$$=\frac{2\sqrt{0.788}}{1+\sqrt{1-0.788}}\leq 1$$

$$=1.22>1,\ \text{use }\lambda=1$$

Note: λ can always be conservatively taken equal to 1.

$$\lambda n'=1(3.11\text{ in.})$$
$$=3.11\text{ in.}$$

$$l=\max\{m,\ n,\ \lambda n'\}$$
$$=\max\{4.97\text{ in.},\ 6.12\text{ in.},\ 3.11\text{ in.}\}$$
$$=6.12\text{ in.}$$

LRFD	ASD
$f_{pu}=\dfrac{P_u}{BN}$ $=\dfrac{690\text{ kips}}{(22\text{ in.})(22\text{ in.})}$ $=1.43\text{ ksi}$ From AISC *Manual* Equation 14-7a: $t_{min}=l\sqrt{\dfrac{2f_{pu}}{0.90F_y}}$ $=(6.12\text{ in.})\sqrt{\dfrac{2(1.43\text{ ksi})}{0.90(36\text{ ksi})}}$ $=1.82\text{ in.}$	$f_{pa}=\dfrac{P_a}{BN}$ $=\dfrac{460\text{ kips}}{(22\text{ in.})(22\text{ in.})}$ $=0.950\text{ ksi}$ From AISC *Manual* Equation 14-7b: $t_{min}=l\sqrt{\dfrac{1.67(2f_{pa})}{F_y}}$ $=(6.12\text{ in.})\sqrt{\dfrac{1.67(2)(0.950\text{ ksi})}{36\text{ ksi}}}$ $=1.82\text{ in.}$

Use PL2 in. × 22 in. × 1 ft 10 in., ASTM A36.

Chapter K
Additional Requirements for HSS and Box Section Connections

Examples K.1 through K.6 illustrate common beam-to-column shear connections that have been adapted for use with HSS columns. Example K.7 illustrates a through-plate shear connection, which is unique to HSS columns. Calculations for transverse and longitudinal forces applied to HSS are illustrated in Example K.8. Examples of HSS base plate and end plate connections are given in Examples K.9 and K.10.

EXAMPLE K.1 WELDED/BOLTED WIDE TEE CONNECTION TO AN HSS COLUMN

Given:

Verify a connection between an ASTM A992 W16×50 beam and an ASTM A500, Grade C, HSS8×8×¼ column using an ASTM A992 WT-shape, as shown in Figure K.1-1. Design, assuming a flexible support condition, for the following vertical shear loads:

P_D = 6.2 kips
P_L = 18.5 kips

Note: A tee with a flange width wider than 8 in. was selected to provide sufficient surface for flare bevel groove welds on both sides of the column, because the tee will be slightly offset from the column centerline.

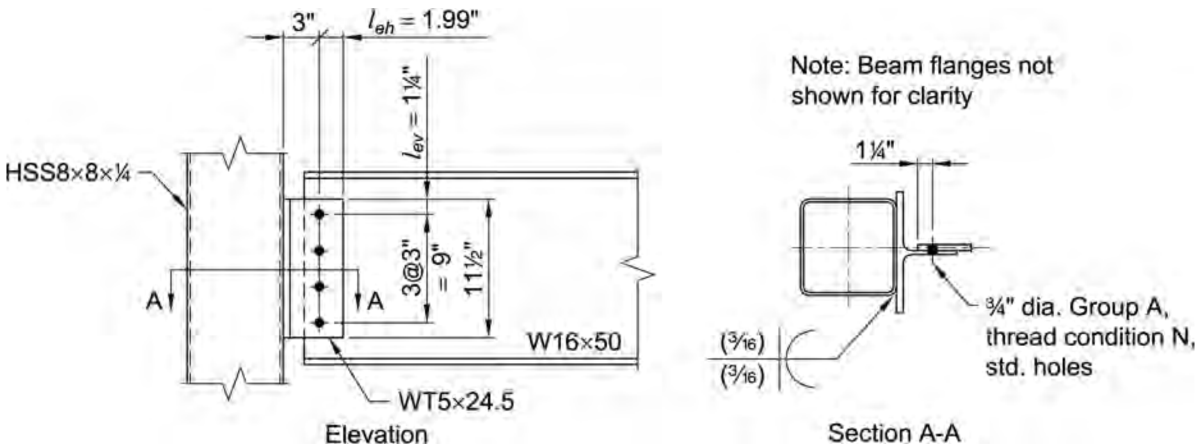


Fig K.1-1. Connection geometry for Example K.1.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Tee
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A500 Grade C
F_y = 50 ksi
F_u = 62 ksi

From AISC *Manual* Tables 1-1, 1-8 and 1-12, the geometric properties are as follows:

W16×50
$t_w = 0.380$ in.
$d = 16.3$ in.
$t_f = 0.630$ in.
$T = 13\frac{5}{8}$ in.

WT5×24.5
$t_{sw} = t_w = 0.340$ in.
$d = 4.99$ in.
$t_f = 0.560$ in.
$b_f = 10.0$ in.
$k_1 = \frac{13}{16}$ in. (see W10×49)

HSS8×8×¼
$t = 0.233$ in.
$B = 8.00$ in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_u = 1.2(6.2 \text{ kips}) + 1.6(18.5 \text{ kips})$ $= 37.0 \text{ kips}$	$P_a = 6.2 \text{ kips} + 18.5 \text{ kips}$ $= 24.7 \text{ kips}$

Calculate the available strength assuming a flexible support condition.

Required Number of Bolts

The required number of bolts will ultimately be determined using the coefficient, *C*, from AISC *Manual* Table 7-6. First, the available strength per bolt must be determined.

Determine the available shear strength of a single bolt. From AISC *Manual* Table 7-1, for ¾-in.-diameter Group A bolts:

LRFD	ASD
$\phi r_n = 17.9 \text{ kips}$	$\dfrac{r_n}{\Omega} = 11.9 \text{ kips}$

The edge distance is checked against the minimum edge distance requirement provided in AISC *Specification* Table J3.4.

$l_{ev} = 1\frac{1}{4} \text{ in.} > 1 \text{ in.}$ **o.k.**

The available bearing and tearout strength per bolt on the tee stem based on edge distance is determined from AISC *Manual* Table 7-5, for $l_{ev} = 1\frac{1}{4}$ in., as follows:

LRFD	ASD
$\phi r_n = (49.4 \text{ kip/in.})(0.340 \text{ in.})$ $= 16.8 \text{ kips}$	$\dfrac{r_n}{\Omega} = (32.9 \text{ kip/in.})(0.340 \text{ in.})$ $= 11.2 \text{ kips}$

The bolt spacing is checked against the minimum spacing requirement between centers of standard holes provided in AISC *Specification* Section J3.3.

$$2\tfrac{2}{3}d = 2\tfrac{2}{3}\left(\tfrac{3}{4}\text{ in.}\right)$$
$$= 2.00\text{ in.} > s = 3\text{ in.} \quad \textbf{o.k.}$$

The available bearing and tearout strength per bolt on the tee stem based on spacing is determined from AISC *Manual* Table 7-4, for s = 3 in., as follows:

LRFD	ASD
$\phi r_n = (87.8\text{ kip/in.})(0.340\text{ in.})$ $= 29.9\text{ kips}$	$\dfrac{r_n}{\Omega} = (58.5\text{ kip/in.})(0.340\text{ in.})$ $= 19.9\text{ kips}$

Bolt bearing and tearout strength based on edge distance controls over the available shear strength of the bolt.

Determine the coefficient for the eccentrically loaded bolt group.

LRFD	ASD
$C_{min} = \dfrac{P_u}{\phi r_n}$ $= \dfrac{37.0\text{ kips}}{16.8\text{ kips}}$ $= 2.20$ Using e = 3 in. and s = 3 in., determine C from AISC *Manual* Table 7-6, Angle = 0°. Try four rows of bolts: $C = 2.81 > 2.20$ **o.k.**	$C_{min} = \dfrac{P_a}{r_n / \Omega}$ $= \dfrac{24.7\text{ kips}}{11.2\text{ kips}}$ $= 2.21$ Using e = 3 in. and s = 3 in., determine C from AISC *Manual* Table 7-6, Angle = 0°. Try four rows of bolts: $C = 2.81 > 2.21$ **o.k.**

Tee Stem Thickness and Length

AISC *Manual* Part 9 stipulates a maximum tee stem thickness that should be provided for rotational ductility as follows:

$$t_{sw\ max} = \frac{d}{2} + \tfrac{1}{16}\text{ in.} \qquad \text{(from } \textit{Manual} \text{ Eq. 9-39)}$$
$$= \frac{\tfrac{3}{4}\text{ in.}}{2} + \tfrac{1}{16}\text{ in.}$$
$$= 0.438\text{ in.} > 0.340\text{ in.} \quad \textbf{o.k.}$$

Note: The beam web thickness is greater than the tee stem thickness. If the beam web were thinner than the tee stem, this check could be satisfied by checking the thickness of the beam web.

As discussed in AISC *Manual* Part 10, it is recommended that the minimum length of a simple shear connection is one-half the *T*-dimension of the beam to be supported. The minimum length of the tee is determined as follow:

$$l_{min} = \frac{T}{2}$$
$$= \frac{13\tfrac{5}{8}\text{ in.}}{2}$$
$$= 6.81\text{ in.}$$

As discussed in AISC *Manual* Part 10, the detailed length of connection elements must be compatible with the *T*-dimension of the beam. The tee length is checked using the number of bolts, bolt spacing, and edge distances determined previously.

$$l = 3(3\text{ in.}) + 2(1\tfrac{1}{4}\text{ in.})$$
$$= 11.5\text{ in.} < T = 13\tfrac{5}{8}\text{ in.} \quad \textbf{o.k.}$$

Try l = 11.5 in.

Tee Stem Shear Yielding Strength

Determine the available shear strength of the tee stem based on the limit state of shear yielding from AISC *Specification* Section J4.2(a).

$$A_{gv} = lt_s$$
$$= (11.5\text{ in.})(0.340\text{ in.})$$
$$= 3.91\text{ in.}^2$$

$$R_n = 0.60F_yA_{gv} \qquad \text{(\textit{Spec}. Eq. J4-3)}$$
$$= 0.60(50\text{ ksi})(3.91\text{ in.}^2)$$
$$= 117\text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(117\text{ kips})$ $= 117\text{ kips} > 37.0\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{117\text{ kips}}{1.50}$ $= 78.0\text{ kips} > 24.7\text{ kips}$ **o.k.**

Because of the geometry of the tee and because the tee flange is thicker than the stem and carries only half of the beam reaction, flexural yielding and shear yielding of the flange are not controlling limit states.

Tee Stem Shear Rupture Strength

Determine the available shear strength of the tee stem based on the limit state of shear rupture from AISC *Specification* Section J4.2(b).

$$A_{nv} = \left[l - n\left(d_n + \tfrac{1}{16}\text{ in.}\right)\right]t_s$$
$$= \left[11.5\text{ in.} - (4)\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right](0.340\text{ in.})$$
$$= 2.72\text{ in.}^2$$

$$R_n = 0.60F_uA_{nv}$$ (*Spec*. Eq. J4-4)

$$= 0.60(65 \text{ ksi})(2.72 \text{ in.}^2)$$

$$= 106 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(106 \text{ kips})$ $= 79.5 \text{ kips} > 37.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{106 \text{ kips}}{2.00}$ $= 53.0 \text{ kips} > 24.7 \text{ kips}$ **o.k.**

Tee Stem Block Shear Rupture Strength

The nominal strength for the limit state of block shear rupture is given by AISC *Specification* Section J4.3.

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.60F_yA_{gv} + U_{bs}F_uA_{nt}$$ (*Spec*. Eq. J4-5)

The available block shear rupture strength of the tee stem is determined as follows, using AISC *Manual* Tables 9-3a, 9-3b and 9-3c and AISC *Specification* Equation J4-5, with n = 4, l_{eh} = 1.99 in. (assume l_{eh} = 2.00 in. to use Table 9-3a), l_{ev} = 1¼ in. and U_{bs} = 1.0.

LRFD	ASD
Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{\phi F_uA_{nt}}{t} = 76.2 \text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{\phi 0.60F_yA_{gv}}{t} = 231 \text{ kip/in.}$ Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{\phi 0.60F_uA_{nv}}{t} = 210 \text{ kip/in.}$	Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{F_uA_{nt}}{\Omega t} = 50.8 \text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{0.60F_yA_{gv}}{\Omega t} = 154 \text{ kip/in.}$ Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{0.60F_uA_{nv}}{\Omega t} = 140 \text{ kip/in.}$

LRFD	ASD
The design block shear rupture strength is: $\phi R_n = \phi 0.60 F_u A_{vn} + \phi U_{bs} F_u A_{nt} \leq \phi 0.60 F_y A_{gv} + \phi U_{bs} F_u A_{nt}$ $= (210 \text{ kip/in.} + 76.2 \text{ kip/in.})(0.340 \text{ in.}) \leq (231 \text{ kip/in.} + 76.2 \text{ kip/in.})(0.340 \text{ in.})$ $= 97.3 \text{ kips} < 104 \text{ kips}$ $= 97.3 \text{ kips} > 37.0 \text{ kips}$ **o.k.**	The allowable block shear rupture strength is: $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega} + \dfrac{U_{bs} F_u A_{nt}}{\Omega} \leq \dfrac{0.60 F_y A_{gv}}{\Omega} + \dfrac{U_{bs} F_u A_{nt}}{\Omega}$ $= (140 \text{ kip/in.} + 50.8 \text{ kip/in.})(0.340 \text{ in.}) \leq (154 \text{ kip/in.} + 50.8 \text{ kip/in.})(0.340 \text{ in.})$ $= 64.9 \text{ kips} < 69.6 \text{ kips}$ $= 64.9 \text{ kips} > 24.7 \text{ kips}$ **o.k.**

Tee Stem Flexural Strength

The required flexural strength for the tee stem is:

LRFD	ASD
$M_u = P_u e = (37.0 \text{ kips})(3 \text{ in.}) = 111 \text{ kip-in.}$	$M_a = P_a e = (24.7 \text{ kips})(3 \text{ in.}) = 74.1 \text{ kip-in.}$

The tee stem available flexural strength due to yielding is determined as follows, from AISC *Specification* Section F11.1. The stem, in this case, is treated as a rectangular bar.

$$Z = \frac{t_s d^2}{4} = \frac{(0.340 \text{ in.})(11.5 \text{ in.})^2}{4} = 11.2 \text{ in.}^3$$

$$S_x = \frac{t_s d^2}{6} = \frac{(0.340 \text{ in.})(11.5 \text{ in.})^2}{6} = 7.49 \text{ in.}^3$$

$$M_n = M_p = F_y Z \leq 1.6 F_y S_x \qquad \text{(\textit{Spec}. Eq. F11-1)}$$

$$= (50 \text{ ksi})(11.2 \text{ in.}^3) \leq 1.6(50 \text{ ksi})(7.49 \text{ in.}^3)$$
$$= 560 \text{ kip-in.} < 599 \text{ kip-in.}$$
$$= 560 \text{ kips-in.}$$

Note: The 1.6 limit will never control for a plate because the shape factor (Z/S) for a plate is 1.5.

The tee stem available flexural yielding strength is:

LRFD	ASD
$\phi = 0.90$ $\phi M_n = 0.90(560\text{ kip-in.})$ $= 504\text{ kip-in.} > 111\text{ kip-in.}$ **o.k.**	$\Omega = 1.67$ $\dfrac{M_n}{\Omega} = \dfrac{560\text{ kip-in.}}{1.67}$ $= 335\text{ kip-in.} > 74.1\text{ kip-in.}$ **o.k.**

The tee stem available flexural strength due to lateral-torsional buckling is determined from Section F11.2.

$$\frac{L_b d}{t_s^2} = \frac{(3\text{ in.})(11.5\text{ in.})}{(0.340\text{ in.})^2}$$
$$= 298$$

$$\frac{0.08E}{F_y} = \frac{0.08(29{,}000\text{ ksi})}{50\text{ ksi}}$$
$$= 46.4$$

$$\frac{1.9E}{F_y} = \frac{1.9(29{,}000\text{ ksi})}{50\text{ ksi}}$$
$$= 1{,}102$$

Because 46.4 < 298 < 1,102, Equation F11-2 is applicable with C_b = 1.00.

$$M_n = C_b\left[1.52 - 0.274\left(\frac{L_b d}{t^2}\right)\frac{F_y}{E}\right]M_y \le M_p \qquad (\textit{Spec.}\text{ Eq. F11-2})$$
$$= 1.00\left[1.52 - 0.274(298)\left(\frac{50\text{ ksi}}{29{,}000\text{ ksi}}\right)\right](50\text{ ksi})(7.49\text{ in.}^2) \le (50\text{ ksi})(11.2\text{ in.}^3)$$
$$= 517\text{ kip-in.} < 560\text{ kip-in.}$$
$$= 517\text{ kip-in.}$$

LRFD	ASD
$\phi = 0.90$ $\phi M_n = 0.90(517\text{ kip-in.})$ $= 465\text{ kip-in.} > 111\text{ kip-in.}$ **o.k.**	$\Omega = 1.67$ $\dfrac{M_n}{\Omega} = \dfrac{517\text{ kip-in.}}{1.67}$ $= 310\text{ kip-in.} > 74.1\text{ kip-in.}$ **o.k.**

The tee stem available flexural strength is determined from AISC *Manual* Part 9 as follows:

$$Z_{net} = \frac{td^2}{4} - 2t_{sw}\left(d_h + \tfrac{1}{16}\text{ in.}\right)(1.5\text{ in.} + 4.5\text{ in.})$$
$$= \frac{(0.340\text{ in.})(11.5\text{ in.})^2}{4} - 2(0.340\text{ in.})\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)(1.5\text{ in.} + 4.5\text{ in.})$$
$$= 7.67\text{ in.}^3$$

$$M_n = F_u Z_{net}$$ (*Manual* Eq. 9-4)
$$= (65 \text{ ksi})(7.67 \text{ in.}^3)$$
$$= 499 \text{ kip-in.}$$

LRFD	ASD
$\phi_b = 0.75$ $\phi M_n = 0.75(499 \text{ kip-in.})$ $= 374 \text{ kip-in.} > 111 \text{ kip-in.}$ **o.k.**	$\Omega_b = 2.00$ $\dfrac{M_n}{\Omega} = \dfrac{499 \text{ kip-in.}}{2.00}$ $= 250 \text{ kip-in.} > 74.1 \text{ kip-in.}$ **o.k.**

Beam Web Bearing

Because t_w = 0.380 in. > t_{sw} = 0.340 in., bolt bearing does not control the strength of the beam web.

Weld Size

Because the flange width of the tee is larger than the width of the HSS, a flare bevel groove weld is required. Taking the outside radius as $R = 2t$ = 2(0.233 in.) = 0.466 in. and using AISC *Specification* Table J2.2, the effective throat thickness of the flare bevel groove weld is E = 5⁄16R = 5⁄16(0.466 in.) = 0.146 in. This effective throat thickness will be used for subsequent calculations; however, for the detail drawing, a 3⁄16-in. weld is specified.

Using AISC *Specification* Table J2.3, the minimum effective throat thickness of the flare bevel groove weld, based on the 0.233 in. thickness of the HSS column, is 1⁄8 in.

$$E = 0.146 \text{ in.} > \tfrac{1}{8} \text{ in.}$$

The equivalent fillet weld that provides the same throat dimension is:

$$\left(\frac{D}{16}\right)\left(\frac{1}{\sqrt{2}}\right) = 0.146$$
$$D = 16\sqrt{2}(0.146)$$
$$= 3.30 \text{ sixteenths of an inch}$$

The equivalent fillet weld size is used in the following calculations.

Weld Ductility

Check weld ductility using AISC *Manual* Part 9.

Let $b_f = B$ = 8.00 in.

$$b = \frac{b_f - 2k_1}{2}$$
$$= \frac{8.00 \text{ in.} - 2(\tfrac{13}{16} \text{ in.})}{2}$$
$$= 3.19 \text{ in}$$

$$w_{min} = 0.0155\frac{F_y t_f^{\,2}}{b}\left(\frac{b^2}{l^2}+2\right) \le (5/8)\,t_{sw}$$ (*Manual* Eq. 9-37)

$$= 0.0155\frac{(50\text{ ksi})(0.560\text{ in.})^2}{3.19\text{ in.}}\left[\frac{(3.19\text{ in.})^2}{(11.5\text{ in.})^2}+2\right] \le (5/8)(0.340\text{ in.})$$

$$= 0.158\text{ in.} < 0.213\text{ in.}$$

0.158 in. = 2.53 sixteenths of an inch

$D_{min} = 2.53 < 3.30$ sixteenths of an inch **o.k.**

Nominal Weld Shear Strength

The load is assumed to act concentrically with the weld group (i.e., a flexible support condition).

$a = 0$ and $k = 0$; therefore, $C = 3.71$ from AISC *Manual* Table 8-4, Angle = 0°.

$$R_n = CC_1Dl$$
$$= 3.71(1.00)(3.30\text{ sixteenths of an inch})(11.5\text{ in.})$$
$$= 141\text{ kips}$$

Shear Rupture of the HSS at the Weld

$$t_{min} = \frac{3.09D}{F_u}$$ (*Manual* Eq. 9-2)

$$= \frac{3.09(3.30\text{ sixteenths})}{62\text{ ksi}}$$
$$= 0.164\text{ in.} < 0.233\text{ in.}$$

By inspection, shear rupture of the tee flange at the welds will not control.

Therefore, the weld controls.

Available Weld Shear Strength

From AISC *Specification* Section J2.4, the available weld strength is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(141\text{ kips})$ $= 106\text{ kips} > 37.0\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\frac{R_n}{\Omega} = \frac{141\text{ kips}}{2.00}$ $= 70.5\text{ kips} > 24.7\text{ kips}$ **o.k.**

EXAMPLE K.2 WELDED/BOLTED NARROW TEE CONNECTION TO AN HSS COLUMN

Given:

Verify a connection for an ASTM A992 W16×50 beam to an ASTM A500 Grade C HSS8×8×¼ column using an ASTM A992 WT5×24.5 with fillet welds against the flat width of the HSS, as shown in Figure K.2-1. Use 70-ksi weld electrodes. Assume that, for architectural purposes, the flanges of the WT from the previous example have been stripped down to a width of 5 in. Design assuming a flexible support condition for the following vertical shear loads:

P_D = 6.2 kips
P_L = 18.5 kips

Note: This is the same problem as Example K.1 with the exception that a narrow tee will be selected which will permit fillet welds on the flat of the column. The beam will still be centered on the column centerline; therefore, the tee will be slightly offset.

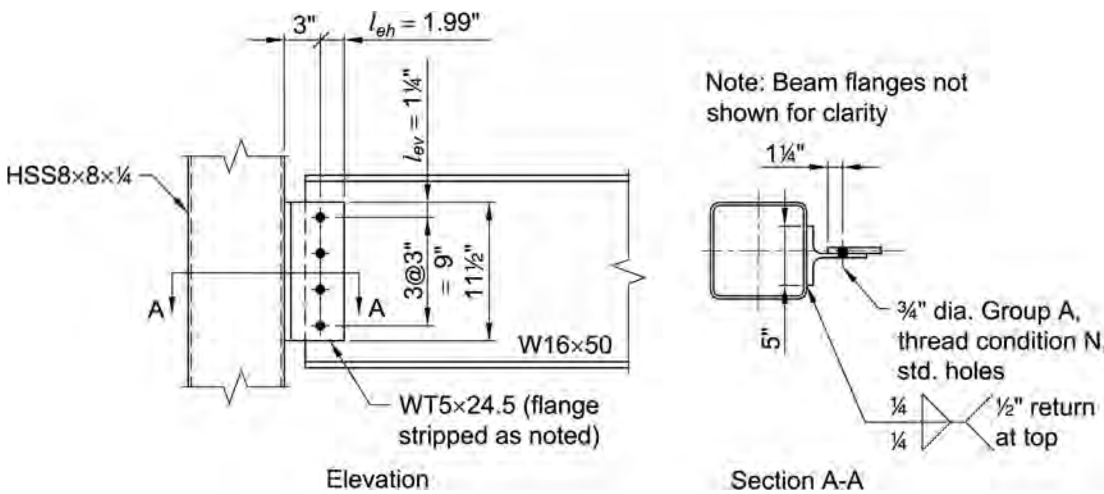


Fig K.2-1. Connection geometry for Example K.2.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Tee
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A500 Grade C
F_y = 50 ksi
F_u = 62 ksi

From AISC *Manual* Tables 1-1, 1-8 and 1-12, the geometric properties are as follows:

W16×50
t_w = 0.380 in.
d = 16.3 in.
t_f = 0.630 in.

HSS8×8×¼
t = 0.233 in.
B = 8.00 in.

WT5×24.5
t_{sw} = t_w = 0.340 in.
d = 4.99 in.
t_f = 0.560 in.
k_1 = 13⁄16 in. (see W10×49)

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_u = 1.2(6.2 \text{ kips}) + 1.6(18.5 \text{ kips})$ $= 37.0 \text{ kips}$	$P_a = 6.2 \text{ kips} + 18.5 \text{ kips}$ $= 24.7 \text{ kips}$

The tee stem thickness, tee length, tee stem strength, and beam web bearing strength are verified in Example K.1. The required number of bolts is also determined in Example K.1.

Maximum Tee Flange Width

Assume ¼-in. welds and HSS corner radius equal to 2.25 times the nominal thickness 2.25(¼ in.) = 9⁄16 in. (refer to AISC *Manual* Part 1 discussion).

The recommended minimum shelf dimension for ¼-in. fillet welds from AISC *Manual* Figure 8-13 is ½ in.

Connection offset (centerline of the column to the centerline of the tee stem):

$$\frac{0.380 \text{ in.}}{2} + \frac{0.340 \text{ in.}}{2} = 0.360 \text{ in.}$$

The stripped flange must not exceed the flat face of the tube minus the shelf dimension on each side:

$$b_f \leq 8.00 \text{ in.} - 2(\tfrac{9}{16} \text{ in.}) - 2(\tfrac{1}{2} \text{ in.}) - 2(0.360 \text{ in.})$$
$$5.00 \text{ in.} < 5.16 \text{ in.} \quad \textbf{o.k.}$$

Minimum Fillet Weld Size

From AISC *Specification* Table J2.4, the minimum fillet weld size = ⅛ in. (D = 2) for welding to 0.233-in.-thick material.

Weld Ductility

The flexible width of the connecting element, b, is defined in Figure 9-6 of AISC *Manual* Part 9:

$$b = \frac{b_f - 2k_1}{2}$$
$$= \frac{5.00 \text{ in.} - 2(^{13}\!/\!_{16} \text{ in.})}{2}$$
$$= 1.69 \text{ in.}$$

$$w_{min} = 0.0155 \frac{F_y t_f^{\,2}}{b} \left(\frac{b^2}{l^2} + 2 \right) \le (^5\!/\!_8) t_{sw} \qquad \text{(\textit{Manual} Eq. 9-37)}$$
$$= 0.0155 \frac{(50 \text{ ksi})(0.560 \text{ in.})^2}{1.69 \text{ in.}} \left[\frac{(1.69 \text{ in.})^2}{(11.5 \text{ in.})^2} + 2 \right] \le (^5\!/\!_8)(0.340 \text{ in.})$$
$$= 0.291 \text{ in.} > 0.213 \text{ in.; therefore, use } w_{min} = 0.213 \text{ in.}$$

$$D_{min} = (0.213 \text{ in.})(16)$$
$$= 3.41 \text{ sixteenths of an inch}$$

Try a ¼-in. fillet weld as a practical minimum, which is less than the maximum permitted weld size of t_f – 1⁄16 in. = 0.560 in. – 1⁄16 in. = 0.498 in., in accordance with AISC *Specification* Section J2.2b. Provide ½-in. return welds at the top of the tee to meet the criteria listed in AISC *Specification* Section J2.2b.

Minimum HSS Wall Thickness to Match Weld Strength

$$t_{min} = \frac{3.09D}{F_u} \qquad \text{(\textit{Manual} Eq. 9-2)}$$
$$= \frac{3.09(4)}{62 \text{ ksi}}$$
$$= 0.199 \text{ in.} < 0.233 \text{ in.}$$

By inspection, shear rupture of the flange of the tee at the welds will not control.

Therefore, the weld controls.

Available Weld Shear Strength

The load is assumed to act concentrically with the weld group (i.e., a flexible support condition).

a = 0 and k = 0, therefore, C = 3.71 from AISC *Manual* Table 8-4, Angle = 0°.

$$R_n = CC_1Dl$$
$$= 3.71(1.00)(4 \text{ sixteenths of an inch})(11.5 \text{ in.})$$
$$= 171 \text{ kips}$$

From AISC *Specification* Section J2.4, the available fillet weld shear strength is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(171 \text{ kips})$ $= 128 \text{ kips} > 37.0 \text{ kips}$	$\Omega = 2.00$ $\frac{R_n}{\Omega} = \frac{171 \text{ kips}}{2.00}$ $= 85.5 \text{ kips} > 24.7 \text{ kips}$

EXAMPLE K.3 DOUBLE-ANGLE CONNECTION TO AN HSS COLUMN

Given:

Use AISC *Manual* Tables 10-1 and 10-2 to design a double-angle connection for an ASTM A992 W36×231 beam to an ASTM A500 Grade C HSS14×14×½ column, as shown in Figure K.3-1. The angles are ASTM A36 material. Use 70-ksi weld electrodes. The bottom flange cope is required for erection. Use the following vertical shear loads:

P_D = 37.5 kips
P_L = 113 kips

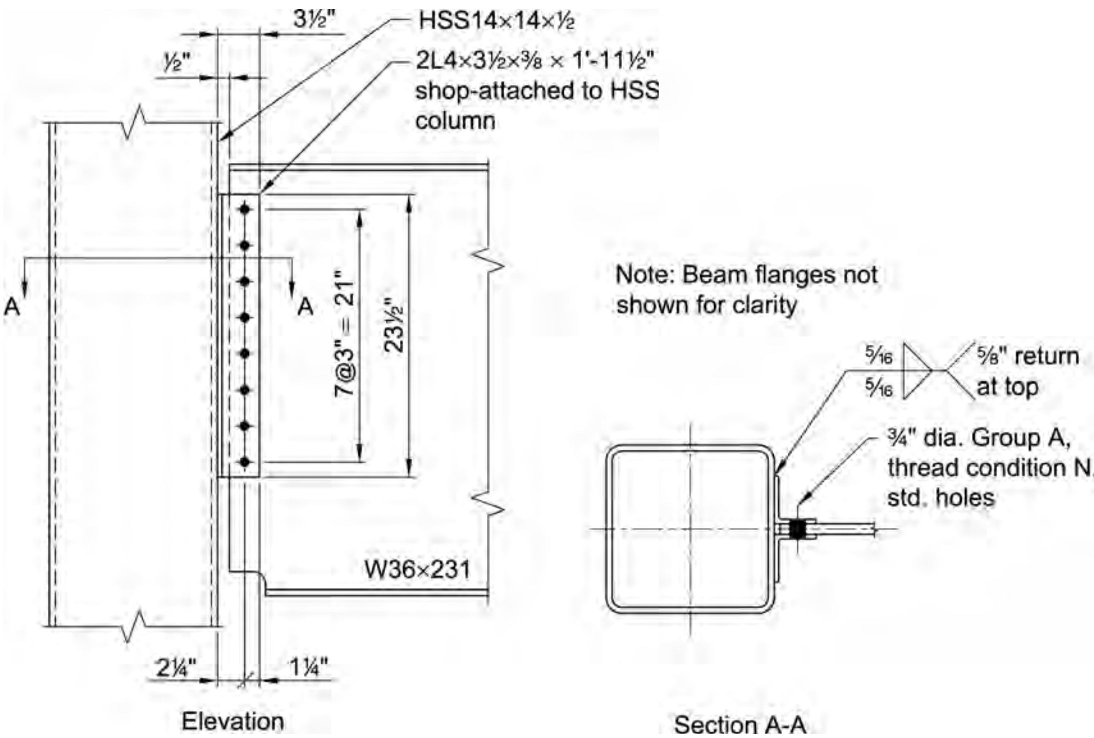


Fig K.3-1. Connection geometry for Example K.3.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A500 Grade C
F_y = 50 ksi
F_u = 62 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 1-12, the geometric properties are as follows:

W36×231
t_w = 0.760 in.
T = 31⅜ in.

HSS14×14×½
t = 0.465 in.
B = 14.0 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(37.5\text{ kips}) + 1.6(113\text{ kips})$ $= 226\text{ kips}$	$R_a = 37.5\text{ kips} + 113\text{ kips}$ $= 151\text{ kips}$

Bolt and Weld Design

Try eight rows of bolts and 5⁄16-in. welds.

Obtain the bolt group and angle available strength from AISC *Manual* Table 10-1, Group A.

LRFD	ASD
$\phi R_n = 284\text{ kips} > 226\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 189\text{ kips} > 151\text{ kips}$ **o.k.**

Obtain the available weld strength from AISC *Manual* Table 10-2 (welds B).

LRFD	ASD
$\phi R_n = 279\text{ kips} > 226\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 186\text{ kips} > 151\text{ kips}$ **o.k.**

Minimum Support Thickness

The minimum required support thickness using AISC *Manual* Table 10-2 is determined as follows for F_u = 62 ksi material.

$$0.238\text{ in.}\left(\frac{65\text{ ksi}}{62\text{ ksi}}\right) = 0.250\text{ in.} < 0.465\text{ in.} \quad \textbf{o.k.}$$

Minimum Angle Thickness

$$\begin{aligned} t_{min} &= w + \tfrac{1}{16}\text{ in., from AISC } \textit{Specification} \text{ Section J2.2b} \\ &= \tfrac{5}{16}\text{ in.} + \tfrac{1}{16}\text{ in.} \\ &= \tfrac{3}{8}\text{ in.} \end{aligned}$$

Use ⅜-in. angle thickness to accommodate the welded legs of the double-angle connection.

Use 2L4×3½×⅜×1′-11½″.

Minimum Angle Length

As discussed in AISC *Manual* Part 10, it is recommended that the minimum length of a simple shear connection is one-half the *T*-dimension of the beam to be supported. The minimum length of the connection is determined as follow:

$$\begin{aligned} l_{min} &= \frac{T}{2} \\ &= \frac{31\tfrac{3}{8}\text{ in.}}{2} \\ &= 15.7\text{ in.} < 23.5\text{ in.} \quad \textbf{o.k.} \end{aligned}$$

Minimum Column Width

The workable flat for the HSS column is 11¾ in. from AISC *Manual* Table 1-12.

The recommended minimum shelf dimension for 5⁄16-in. fillet welds from AISC *Manual* Figure 8-13 is 9⁄16 in.

The minimum acceptable width to accommodate the connection is:

$$2(4.00\text{ in.}) + 0.760\text{ in.} + 2(\tfrac{9}{16}\text{ in.}) = 9.89\text{ in.} < 11\tfrac{3}{4}\text{ in.} \quad \textbf{o.k.}$$

Available Beam Web Strength

The available beam web strength, from AISC *Manual* design table discussion for Table 10-1, is the lesser of the limit states of block shear rupture, shear yielding, shear rupture, and the sum of the effective strengths of the individual fasteners. The beam is not coped, so the only applicable limit state is the effective strength of the individual fasteners. The effective strength of an individual fastener is the lesser of the fastener shear strength, bearing strength at the bolt hole, and the tearout strength at the bolt hole.

For the limit state of fastener shear strength, with A_b = 0.442 in.^{2} from AISC *Manual* Table 7-1 for a ¾-in. bolt:

$$\begin{aligned} r_n &= F_{nv} A_b \\ &= (54\text{ ksi})(0.442\text{ in.}^2)(2\text{ shear planes}) \\ &= 47.7\text{ kips/bolt} \end{aligned}$$ (from *Spec.* Eq. J3-1)

where F_{nv} is the nominal shear strength from AISC *Specification* Table J3.2 of a Group A bolt in a bearing-type connection when threads are not excluded from the shear planes.

Assume that deformation at the bolt hole at service load is a design consideration.

For the limit state of bearing:

$$\begin{aligned} r_n &= 2.4 d t F_u \\ &= 2.4(\tfrac{3}{4}\text{ in.})(0.760\text{ in.})(65\text{ ksi}) \\ &= 88.9\text{ kips/bolt} \end{aligned}$$ (from *Spec.* Eq. J3-6a)

For the limit state of tearout:

$$r_n = 1.2 l_c t F_u \qquad \text{(from } \textit{Spec}\text{. Eq. J3-6c)}$$
$$= 1.2\left(3 \text{ in.} - \tfrac{13}{16} \text{ in.}\right)\left(0.760 \text{ in.}\right)\left(65 \text{ ksi}\right)$$
$$= 130 \text{ kips/bolt}$$

where l_c is the clear distance, in the direction of the force, between the edges of the bolt holes.

Fastener shear strength is the governing limit state for all bolts at the beam web. Fastener shear strength is one of the limit states included in the available strength given in Table 10-1 and was previously shown to be adequate.

EXAMPLE K.4 UNSTIFFENED SEATED CONNECTION TO AN HSS COLUMN

Given:

Use AISC *Manual* Table 10-6 to verify an unstiffened seated connection for an ASTM A992 W21×62 beam to an ASTM A500 Grade C HSS12×12×½ column, as shown in Figure K.4-1. The angles are ASTM A36 material. Use 70-ksi weld electrodes. Use the following vertical shear loads:

P_D = 9 kips
P_L = 27 kips

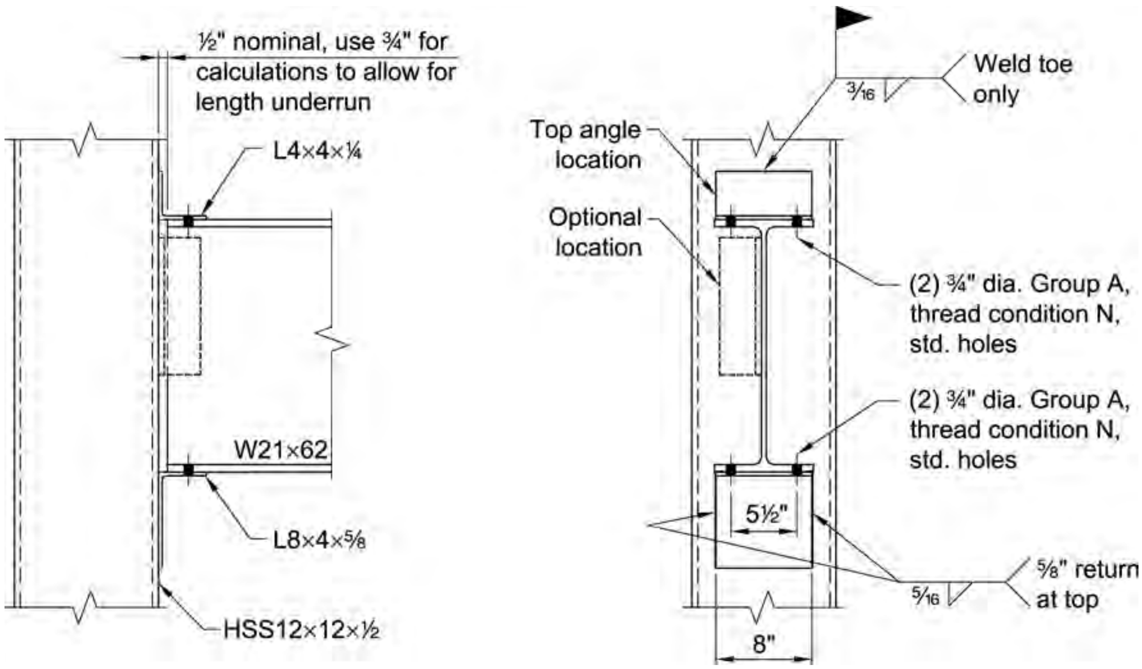


Fig K.4-1. Connection geometry for Example K.4.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A500 Grade C
F_y = 50 ksi
F_u = 62 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 1-12, the geometric properties are as follows:

W21×62
t_w = 0.400 in.
d = 21.0 in.
k_{des} = 1.12 in.

HSS12×12×½
t = 0.465 in.
B = 12.0 in.

From of ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(9 \text{ kips}) + 1.6(27 \text{ kips})$ $= 54.0 \text{ kips}$	$R_a = 9 \text{ kips} + 27 \text{ kips}$ $= 36.0 \text{ kips}$

Seat Angle and Weld Design

Check web local yielding of the W21×62 using AISC *Manual* Part 9.

LRFD	ASD
From AISC *Manual* Equation 9-46a and Table 9-4: $l_{b\ min} = \dfrac{R_u - \phi R_1}{\phi R_2} \geq k_{des}$ $= \dfrac{54.0 \text{ kips} - 56.0 \text{ kips}}{20.0 \text{ kip/in.}}$ which results in a negative quantity. Use $l_{b\ min} = k_{des}$ = 1.12 in. Check web local crippling when $l_b/d \leq 0.2$. From AISC *Manual* Equation 9-48a: $l_{b\ min} = \dfrac{R_u - \phi R_3}{\phi R_4}$ $= \dfrac{54.0 \text{ kips} - 71.7 \text{ kips}}{5.37 \text{ kip/in.}}$ which results in a negative quantity. Check web local crippling when $l_b/d > 0.2$.	From AISC *Manual* Equation 9-46b and Table 9-4: $l_{b\ min} = \dfrac{R_a - R_1/\Omega}{R_2/\Omega} \geq k_{des}$ $= \dfrac{36.0 \text{ kips} - 37.3 \text{ kips}}{13.3 \text{ kip/in.}}$ which results in a negative quantity. Use $l_{b\ min} = k_{des}$ = 1.12 in. Check web local crippling when $l_b/d \leq 0.2$. From AISC *Manual* Equation 9-48b: $l_{b\ min} = \dfrac{R_a - R_3/\Omega}{R_4/\Omega}$ $= \dfrac{36.0 \text{ kips} - 47.8 \text{ kips}}{3.58 \text{ kip/in.}}$ which results in a negative quantity. Check web local crippling when $l_b/d > 0.2$.

LRFD	ASD
From AISC *Manual* Equation 9-49a: $l_{b\,min} = \dfrac{R_u - \phi R_5}{\phi R_6} = \dfrac{54.0\text{ kips} - 64.2\text{ kips}}{7.16\text{ kip/in.}}$ which results in a negative quantity.	From AISC *Manual* Equation 9-49b: $l_{b\,min} = \dfrac{R_a - R_5/\Omega}{R_6/\Omega} = \dfrac{36.0\text{ kips} - 42.8\text{ kips}}{4.77\text{ kip/in.}}$ which results in a negative quantity.

Note: Generally, the value of l_b/d is not initially known and the larger value determined from the web local crippling equations in the preceding text can be used conservatively to determine the bearing length required for web local crippling.

For this beam and end reaction, the beam web available strength exceeds the required strength (hence the negative bearing lengths) and the lower-bound bearing length controls ($l_{b\,req} = k_{des}$ = 1.12 in.). Thus, $l_{b\,min}$ = 1.12 in.

Try an L8×4×⅝ seat with 5⁄16-in. fillet welds.

Outstanding Angle Leg Available Strength

From AISC *Manual* Table 10-6 for an 8-in. angle length and $l_{b\,req}$ = 1.12 in. ≈ 1⅛ in., the outstanding angle leg available strength is:

LRFD	ASD
$\phi R_n = 81.0\text{ kips} > 54.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 53.9\text{ kips} > 36.0\text{ kips}$ **o.k.**

Available Weld Strength

From AISC *Manual* Table 10-6, for an 8 in. x 4 in. angle and 5⁄16-in. weld size, the available weld strength is:

LRFD	ASD
$\phi R_n = 66.7\text{ kips} > 54.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 44.5\text{ kips} > 36.0\text{ kips}$ **o.k.**

Minimum HSS Wall Thickness to Match Weld Strength

$$t_{min} = \frac{3.09D}{F_u} = \frac{3.09(5)}{62\text{ ksi}} = 0.249\text{ in.} < 0.465\text{ in.}$$ (*Manual* Eq. 9-2)

Because t of the HSS is greater than t_{min} for the 5⁄16-in. weld, no reduction in the weld strength is required to account for the shear in the HSS.

Connection to Beam and Top Angle (AISC Manual *Part 10)*

Use a L4×4×¼ top angle for stability. Use a 3⁄16-in. fillet weld across the toe of the angle for attachment to the HSS. Attach both the seat and top angles to the beam flanges with two ¾-in.-diameter Group A bolts.

EXAMPLE K.5 STIFFENED SEATED CONNECTION TO AN HSS COLUMN

Given:

Use AISC *Manual* Tables 10-8 and 10-15 to verify a stiffened seated connection for an ASTM A992 W21×68 beam to an ASTM A500 Grade C HSS14×14×½ column, as shown in Figure K.5-1. Use 70-ksi electrode welds to connect the stiffener, seat plate and top angle to the HSS. The angle and plate material are ASTM A36.
Use the following vertical shear loads:

P_D = 20 kips
P_L = 60 kips

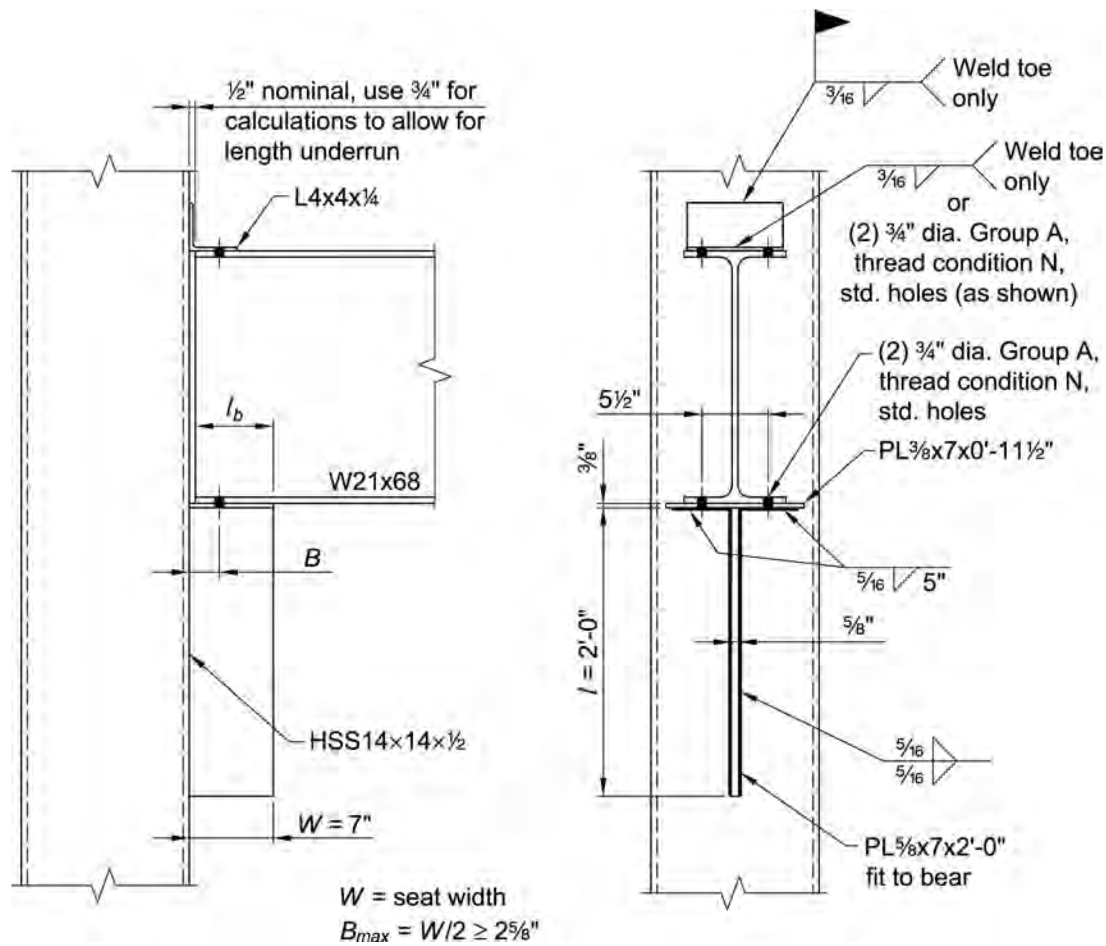


Fig K.5-1. Connection geometry for Example K.5.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A500 Grade C
F_y = 50 ksi
F_u = 62 ksi

Angles and Plates
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 1-12, the geometric properties are as follows:

W21×68
t_w = 0.430 in.
d = 21.1 in.
k_{des} = 1.19 in.

HSS14×14×½
t = 0.465 in.
B = 14.0 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_u = 1.2(20 \text{ kips}) + 1.6(60 \text{ kips})$ $= 120 \text{ kips}$	$P_a = 20 \text{ kips} + 60 \text{ kips}$ $= 80.0 \text{ kips}$

The available strength of connections to rectangular HSS with concentrated loads are determined based on the applicable limit states from Chapter J.

Stiffener Width, W, Required for Web Local Crippling and Web Local Yielding

The stiffener width is determined based on web local crippling and web local yielding of the beam, assuming a ¾-in. beam end setback in the calculations. Note that according to AISC *Specification* Section J10, the length of bearing, l_b, cannot be less than the beam k_{des}.

For web local crippling, assume $l_b/d > 0.2$ and use constants R_5 and R_6 from AISC *Manual* Table 9-4.

LRFD	ASD
From AISC *Manual* Equation 9-49a and Table 9-4: $W_{min} = \dfrac{R_u - \phi R_5}{\phi R_6} + \text{setback} \geq k_{des} + \text{setback}$ $= \dfrac{120 \text{ kips} - 75.9 \text{ kips}}{7.95 \text{ kip/in.}} + \text{¾ in.} \geq 1.19 \text{ in.} + \text{¾ in.}$ $= 6.30 \text{ in.} > 1.94 \text{ in.}$	From AISC *Manual* Equation 9-49b and Table 9-4: $W_{min} = \dfrac{R_a - R_5/\Omega}{R_6/\Omega} + \text{setback} \geq k_{des} + \text{setback}$ $= \dfrac{80.0 \text{ kips} - 50.6 \text{ kips}}{5.30 \text{ kip/in.}} + \text{¾ in.} \geq 1.19 \text{ in.} + \text{¾ in.}$ $= 6.30 \text{ in.} > 1.94 \text{ in.}$

For web local yielding, use constants R_1 and R_2 from AISC *Manual* Table 9-4.

LRFD	ASD
From AISC *Manual* Equation 9-46a and Table 9-4: $W_{min} = \dfrac{R_u - \phi R_1}{\phi R_2} + \text{setback} \geq k_{des} + \text{setback}$ $= \dfrac{120 \text{ kips} - 64.0 \text{ kips}}{21.5 \text{ kip/in.}} + \tfrac{3}{4} \text{ in.} \geq 1.19 \text{ in.} + \tfrac{3}{4} \text{ in.}$ $= 3.35 \text{ in.} > 1.94 \text{ in.}$	From AISC *Manual* Equation 9-46a and Table 9-4: $W_{min} = \dfrac{R_a - R_1 / \Omega}{R_2 / \Omega} + \text{setback} \geq k_{des} + \text{setback}$ $= \dfrac{80.0 \text{ kips} - 42.6 \text{ kips}}{14.3 \text{ kip/in.}} + \tfrac{3}{4} \text{ in.} \geq 1.19 \text{ in.} + \tfrac{3}{4} \text{ in.}$ $= 3.37 \text{ in.} > 1.94 \text{ in.}$

The minimum stiffener width, W_{min}, for web local crippling controls. The stiffener width of 7 in. is adequate.

Check the assumption that $l_b/d > 0.2$.

$$l_b = 7 \text{ in.} - \tfrac{3}{4} \text{ in.}$$
$$= 6.25 \text{ in.}$$

$$\frac{l_b}{d} = \frac{6.25 \text{ in.}}{21.1 \text{ in.}}$$
$$= 0.296 > 0.2, \text{ as assumed}$$

Weld Strength Requirements for the Seat Plate

Check the stiffener length, l = 24 in., with 5⁄16-in. fillet welds. Enter AISC *Manual* Table 10-8, using W = 7 in. as verified in the preceding text.

LRFD	ASD
$\phi R_n = 293 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 195 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

From AISC *Manual* Part 10, Figure 10-10(b), the minimum length of the seat-plate-to-HSS weld on each side of the stiffener is 0.2l = 4.80 in. This establishes the minimum weld between the seat plate and stiffener. A 5-in.-long 5⁄16-in. weld on each side of the stiffener is adequate.

Minimum HSS Wall Thickness to Match Weld Strength

The minimum HSS wall thickness required to match the shear rupture strength of the base metal to that of the weld is:

$$t_{min} = \frac{3.09D}{F_u} \quad (\textit{Manual} \text{ Eq. 9-2})$$
$$= \frac{3.09(5)}{62 \text{ ksi}}$$
$$= 0.249 \text{ in.} < 0.465 \text{ in.}$$

Because t of the HSS is greater than t_{min} for the 5⁄16-in. fillet weld, no reduction in the weld strength to account for shear in the HSS is required.

Stiffener Plate Thickness

From AISC *Manual* Part 10, Table 10-8 discussion, to develop the stiffener-to-seat-plate welds, the minimum stiffener thickness is:

$$t_{p\,min} = 2w = 2(\tfrac{5}{16}\text{ in.}) = \tfrac{5}{8}\text{ in.}$$

Also, from AISC *Manual* Part 10, Table 10-8 discussion, for a stiffener with F_y = 36 ksi and a beam with F_y = 50 ksi, the minimum stiffener thickness is:

$$t_{p\,min} = \left(\frac{F_{y\,beam}}{F_{y\,stiffener}}\right)t_w = \left(\frac{50\text{ ksi}}{36\text{ ksi}}\right)(0.430\text{ in.}) = 0.597\text{ in.}$$

The stiffener thickness of ⅝ in. is adequate.

Determine the stiffener length using AISC *Manual* Table 10-15.

The required HSS wall strength factor is:

LRFD	ASD
$\left(\frac{R_u W}{t^2}\right)_{req} = \frac{(120\text{ kips})(7\text{ in.})}{(0.465\text{ in.})^2} = 3{,}880\text{ kip/in.}$	$\left(\frac{R_a W}{t^2}\right)_{req} = \frac{(80.0\text{ kips})(7\text{ in.})}{(0.465\text{ in.})^2} = 2{,}590\text{ kip/in.}$

To satisfy the minimum, select a stiffener with l = 24 in. from AISC *Manual* Table 10-15. The HSS wall strength factor is:

LRFD	ASD
$\frac{R_u W}{t^2} = 3{,}910\text{ kip/in.} > 3{,}880\text{ kip/in.}$ **o.k.**	$\frac{R_a W}{t^2} = 2{,}600\text{ kip/in.} > 2{,}590\text{ kip/in.}$ **o.k.**

Use PL⅝ in.×7 in. × 2 ft 0 in. for the stiffener.

HSS Width Check

The minimum width is 0.4l + t_p + 2(2.25t); however, because the specified weld length of 5 in. on each side of the stiffener is greater than 0.4l, the weld length will be used. The nominal wall thickness, t_{nom}, is used, as would be used to calculate a workable flat dimension.

$$B = 14.0\text{ in.} > (2\text{ welds})(5.00\text{ in.}) + \tfrac{5}{8}\text{ in.} + 2(2.25)(\tfrac{1}{2}\text{ in.}) = 14.0\text{ in.} > 12.9\text{ in.} \quad \textbf{o.k.}$$

Seat Plate Dimensions

To accommodate two ¾-in.-diameter Group A bolts on a 5½-in. gage connecting the beam flange to the seat plate, a minimum width of 8 in. is required. To accommodate the seat-plate-to-HSS weld, the required width is:

$$2(5.00\text{ in.}) + \tfrac{5}{8}\text{ in.} = 10.6\text{ in.}$$

Note: To allow room to start and stop welds, an 11.5 in. width is used.

Use PL⅜ in.×7 in.× 0 ft-11½ in. for the seat plate.

Top Angle, Bolts and Welds (AISC Manual *Part 10)*

The minimum weld size for the HSS thickness according to AISC *Specification* Table J2.4 is 3⁄16 in. The angle thickness should be 1⁄16 in. larger.

Use L4×4×¼ with 3⁄16-in. fillet welds along the toes of the angle to the beam flange and HSS for stability. Alternatively, two ¾-in.-diameter Group A bolts may be used to connect the leg of the angle to the beam flange.

EXAMPLE K.6 SINGLE-PLATE CONNECTION TO A RECTANGULAR HSS COLUMN

Given:

Use AISC *Manual* Table 10-10a to verify the design of a single-plate connection for an ASTM A992 W18×35 beam framing into an ASTM A500 Grade C HSS6×6×⅜ column, as shown in Figure K.6-1. Use 70-ksi weld electrodes. The plate material is ASTM A36. Use the following vertical shear loads:

P_D = 6.5 kips
P_L = 19.5 kips

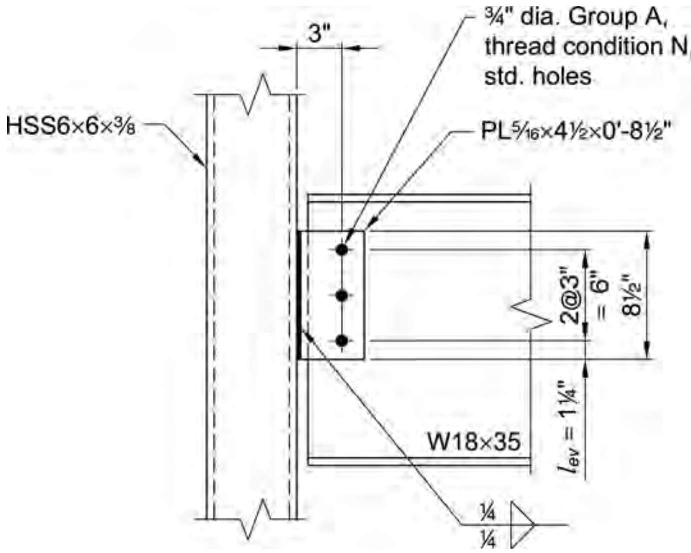


Fig K.6-1. Connection geometry for Example K.6.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A500 Grade C
F_y = 50 ksi
F_u = 62 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 1-12, the geometric properties are as follows:

W18×35
d = 17.7 in.
t_w = 0.300 in.
T = 15½ in.

HSS6×6×⅜
B = H = 6.00 in.
t = 0.349 in.
b/t = 14.2

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(6.5 \text{ kips}) + 1.6(19.5 \text{ kips})$ $= 39.0 \text{ kips}$	$R_a = 6.5 \text{ kips} + 19.5 \text{ kips}$ $= 26.0 \text{ kips}$

Single-Plate Connection

As discussed in AISC *Manual* Part 10, a single-plate connection may be used as long as the HSS wall is not classified as a slender element.

$$\frac{b}{t} \leq 1.40\sqrt{\frac{E}{F_y}}$$

$$14.2 \leq 1.40\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$

$$14.2 < 33.7$$

Therefore, the HSS wall is not slender.

The available strength of the face of the HSS for the limit state of punching shear is determined from AISC *Manual* Part 10 as follows:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$R_u e \leq \dfrac{\phi F_u t l_p^2}{5}$ (*Manual* Eq. 10-7a)	$R_a e \leq \dfrac{F_u t l_p^2}{5\Omega}$ (*Manual* Eq. 10-7b)
$(39.0 \text{ kips})(3 \text{ in.}) \leq \dfrac{0.75(62 \text{ ksi})(0.349 \text{ in.})(8.50 \text{ in.})^2}{5}$	$(26.0 \text{ kips})(3 \text{ in.}) \leq \dfrac{(62 \text{ ksi})(0.349 \text{ in.})(8.50 \text{ in.})^2}{5(2.00)}$
$117 \text{ kip-in.} < 235 \text{ kip-in.}$ **o.k.**	$78.0 \text{ kip-in.} < 156 \text{ kip-in.}$ **o.k.**

Try three rows of bolts and a 5⁄16-in. plate thickness with ¼-in. fillet welds. From AISC *Manual* Table 10-9, either the plate or the beam web must satisfy:

$$t \leq \frac{d}{2} + \tfrac{1}{16} \text{ in.}$$

$$\tfrac{5}{16} \text{ in.} \leq \frac{\tfrac{3}{4} \text{ in.}}{2} + \tfrac{1}{16} \text{ in.}$$

$$\tfrac{5}{16} \text{ in.} < 0.438 \text{ in.} \quad \textbf{o.k.}$$

Obtain the available single-plate connection strength from AISC *Manual* Table 10-10a:

LRFD	ASD
$\phi R_n = 44.2 \text{ kips} > 39.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 29.4 \text{ kips} > 26.0 \text{ kips}$ **o.k.**

Use a PL5⁄16 in.×4½ in. × 0 ft 8½ in.

HSS Shear Rupture at Welds

The minimum HSS wall thickness required to match the shear rupture strength of the HSS wall to that of the weld is:

$$\begin{aligned} t_{min} &= \frac{3.09D}{F_u} \\ &= \frac{3.09(4)}{62 \text{ ksi}} \\ &= 0.199 \text{ in.} < t = 0.349 \text{ in.} \quad \textbf{o.k.} \end{aligned}$$ (*Manual* Eq. 9-2)

Available Beam Web Strength

The available beam web strength is the lesser of the limit states of block shear rupture, shear yielding, shear rupture, and the sum of the effective strengths of the individual fasteners. The beam is not coped, so the only applicable limit state is the effective strength of the individual fasteners. The effective strength of an individual fastener is the lesser of the fastener shear strength, the bearing strength at the bolt hole and the tearout strength at the bolt hole.

For the limit state of fastener shear strength, with $A_b = 0.442 \text{ in.}^2$ from AISC *Manual* Table 7-1 for a ¾-in. bolt.:

$$\begin{aligned} r_n &= F_{nv}A_b \\ &= (54 \text{ ksi})(0.442 \text{ in.}^2) \\ &= 23.9 \text{ kips/bolt} \end{aligned}$$ (from *Spec*. Eq. J3-1)

where F_{nv} is the nominal shear strength of a Group A bolt in a bearing-type connection when threads are not excluded from the shear plane as found in AISC *Specification* Table J3.2.

Assume that deformation at the bolt hole at service load is a design consideration.

For the limit state of bearing:

$$\begin{aligned} r_n &= 2.4dtF_u \\ &= 2.4(\tfrac{3}{4} \text{ in.})(0.300 \text{ in.})(65 \text{ ksi}) \\ &= 35.1 \text{ kips/bolt} \end{aligned}$$ (from *Spec*. Eq. J3-6a)

For the limit state of tearout:

$$\begin{aligned} r_n &= 1.2l_ctF_u \\ &= 1.2(3 \text{ in.} - \tfrac{13}{16} \text{ in.})(0.300 \text{ in.})(65 \text{ ksi}) \\ &= 51.2 \text{ kips/bolt} \end{aligned}$$ (from *Spec*. Eq. J3-6c)

where l_c is the clear distance, in the direction of the force, between the edges of the bolt holes.

Fastener shear strength is the governing limit state for all bolts at the beam web. Fastener shear strength is one of the limit states included in the available strengths given in Table 10-10a and used in the preceding calculations. Thus, the effective strength of the fasteners is adequate.

EXAMPLE K.7 THROUGH-PLATE CONNECTION TO A RECTANGULAR HSS COLUMN

Given:

Use AISC *Manual* Table 10-10a to verify a through-plate connection between an ASTM A992 W18×35 beam and an ASTM A500 Grade C HSS6×4×⅛ with the connection to one of the 6 in. faces, as shown in Figure K.7-1. A thin-walled column is used to illustrate the design of a through-plate connection. Use 70-ksi weld electrodes. The plate is ASTM A36 material. Use the following vertical shear loads:

P_D = 3.3 kips
P_L = 9.9 kips

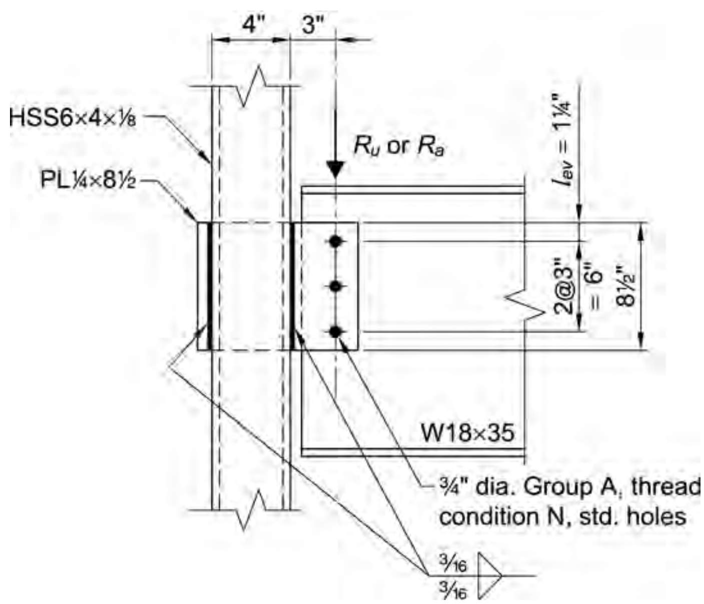


Fig K.7-1. Connection geometry for Example K.7.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A500 Grade C
F_y = 50 ksi
F_u = 62 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 1-11, the geometric properties are as follows:

W18×35
d = 17.7 in.
t_w = 0.300 in.
T = 15½ in.

HSS6×4×⅛
B = 4.00 in.
H = 6.00 in.
t = 0.116 in.
h/t = 48.7
b/t = 31.5

HSS wall slenderness

From AISC *Manual* Part 10, the limiting width-to-thickness for a nonslender HSS wall is:

$$1.40\sqrt{\frac{E}{F_y}} = 1.40\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 33.7$$

Because h/t = 48.7 > 33.7, the HSS6×4×⅛ is slender and a through-plate connection should be used instead of a single-plate connection. Through-plate connections are typically very expensive. When a single-plate connection is not adequate, another type of connection, such as a double-angle connection may be preferable to a through-plate connection.

AISC *Specification* Chapter K does not contain provisions for the design of through-plate shear connections. The following procedure treats the connection of the through-plate to the beam as a single-plate connection.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(3.3 \text{ kips}) + 1.6(9.9 \text{ kips})$ $= 19.8 \text{ kips}$	$R_a = 3.3 \text{ kips} + 9.9 \text{ kips}$ $= 13.2 \text{ kips}$

Portion of the Through-Plate Connection that Resembles a Single-Plate

Try three rows of bolts (l = 8½ in.) and a ¼-in. plate thickness with 3⁄16-in. fillet welds.

$$\frac{T}{2} = \frac{15\tfrac{1}{2} \text{ in.}}{2}$$
$$= 7.75 \text{ in.} < l = 8\tfrac{1}{2} \text{ in.} \quad \textbf{o.k.}$$

Note: From AISC *Manual* Table 10-9, the larger of the plate thickness or the beam web thickness must satisfy:

$$t \le \frac{d}{2} + \tfrac{1}{16} \text{ in.}$$
$$\tfrac{1}{4} \text{ in.} \le \frac{\tfrac{3}{4} \text{ in.}}{2} + \tfrac{1}{16} \text{ in.}$$
$$\tfrac{1}{4} \text{ in.} < 0.438 \text{ in.} \quad \textbf{o.k.}$$

Obtain the available single-plate connection strength from AISC *Manual* Table 10-10a:

LRFD	ASD
$\phi R_n = 38.3 \text{ kips} > 19.8 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 25.6 \text{ kips} > 13.2 \text{ kips}$ **o.k.**

Required Weld Strength

The available strength for the welds in this connection is checked at the location of the maximum reaction, which is along the weld line closest to the bolt line. The reaction at this weld line is determined by taking a moment about the weld line farthest from the bolt line.

$a = 3$ in. (distance from bolt line to nearest weld line)

LRFD	ASD
$V_{fu} = \dfrac{R_u(B+a)}{B}$ $= \dfrac{(19.8 \text{ kips})(4.00 \text{ in.} + 3 \text{ in.})}{4.00 \text{ in.}}$ $= 34.7 \text{ kips}$	$V_{fa} = \dfrac{R_a(B+a)}{B}$ $= \dfrac{(13.2 \text{ kips})(4.00 \text{ in.} + 3 \text{ in.})}{4.00 \text{ in.}}$ $= 23.1 \text{ kips}$

Available Weld Strength

The minimum required weld size is determined using AISC *Manual* Part 8.

LRFD	ASD
$D_{req} = \dfrac{V_{fu}}{1.392l}$ (from *Manual* Eq. 8-2a) $= \dfrac{34.7 \text{ kips}}{(1.392 \text{ kip/in.})(8.50 \text{ in.})(2)}$ $= 1.47 \text{ sixteenths} < 3 \text{ sixteenths}$ **o.k.**	$D_{req} = \dfrac{V_{fa}}{0.928l}$ (from *Manual* Eq. 8-2b) $= \dfrac{23.1 \text{ kips}}{(0.928 \text{ kip/in.})(8.50 \text{ in.})(2)}$ $= 1.46 \text{ sixteenths} < 3 \text{ sixteenths}$ **o.k.**

HSS Shear Yielding and Rupture Strength

The available shear yielding strength of the HSS is determined from AISC *Specification* Section J4.2.

LRFD	ASD
$\phi = 1.00$ $\phi R_n = \phi 0.60 F_y A_{gv}$ (from *Spec.* Eq. J4-3) $= 1.00(0.60)(50 \text{ ksi})(0.116 \text{ in.})(8.50 \text{ in.})(2)$ $= 59.2 \text{ kips} > 34.7 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ (from *Spec.* Eq. J4-3) $= \dfrac{(0.60)(50 \text{ ksi})(0.116 \text{ in.})(8.50 \text{ in.})(2)}{1.50}$ $= 39.4 \text{ kips} > 23.1 \text{ kips}$ **o.k.**

The available shear rupture strength of the HSS is determined from AISC *Specification* Section J4.2.

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi 0.60 F_u A_{nv}$ (from *Spec*. Eq. J4-4) $= 0.75(0.60)(62 \text{ ksi})(0.116 \text{ in.})(8.50 \text{ in.})(2)$ $= 55.0 \text{ kips} > 34.7 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ (from *Spec*. Eq. J4-4) $= \dfrac{(0.60)(62 \text{ ksi})(0.116 \text{ in.})(8.50 \text{ in.})(2)}{2.00}$ $= 36.7 \text{ kips} > 23.1 \text{ kips}$ **o.k.**

Available Beam Web Strength

The available beam web strength is the lesser of the limit states of block shear rupture, shear yielding, shear rupture, and the sum of the effective strengths of the individual fasteners. The beam is not coped, so the only applicable limit state is the effective strength of the individual fasteners. The effective strength of an individual fastener is the lesser of the fastener shear strength, the bearing strength at the bolt hole and the tearout strength at the bolt hole.

For the limit state of fastener shear strength, with $A_b = 0.442 \text{ in.}^2$ from AISC *Manual* Table 7-1 for a ¾-in. bolt:

$$\begin{aligned} r_n &= F_{nv} A_b \\ &= (54 \text{ ksi})(0.442 \text{ in.}^2) \\ &= 23.9 \text{ kips/bolt} \end{aligned}$$ (from *Spec*. Eq. J3-1)

where F_{nv} is the nominal shear strength of a Group A bolt in a bearing-type connection when threads are not excluded from the shear planes as found in AISC *Specification* Table J3.2.

Assume that deformation at the bolt hole at service load is a design consideration.

For the limit state of bearing:

$$\begin{aligned} r_n &= 2.4 d t F_u \\ &= 2.4(\tfrac{3}{4} \text{ in.})(0.300 \text{ in.})(65 \text{ ksi}) \\ &= 35.1 \text{ kips/bolt} \end{aligned}$$ (from *Spec*. Eq. J3-6a)

For the limit state of tearout:

$$\begin{aligned} r_n &= 1.2 l_c t F_u \\ &= 1.2(3 \text{ in.} - \tfrac{13}{16} \text{ in.})(0.300 \text{ in.})(65 \text{ ksi}) \\ &= 51.2 \text{ kips/bolt} \end{aligned}$$ (from *Spec*. Eq. J3-6c)

where l_c is the clear distance, in the direction of the force, between the edges of the bolt holes.

Fastener shear strength is the governing limit state for all bolts at the beam web. Fastener shear strength is one of the limit states included in the available strengths shown in Table 10-10a as used in the preceding calculations. Thus, the effective strength of the fasteners is adequate.

EXAMPLE K.8 LONGITUDINAL PLATE LOADED PERPENDICULAR TO THE HSS AXIS ON A ROUND HSS

Given:

Verify the local strength of the ASTM A500 Grade C HSS6.000×0.375 tension chord subject to transverse loads, P_D = 4 kips and P_L = 12 kips, applied through an ASTM A36 plate, as shown in Figure K.8-1.

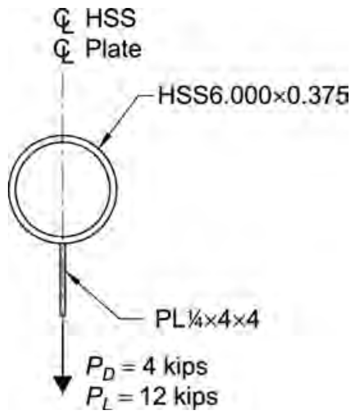


Fig K.8-1. Loading and geometry for Example K.8.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Chord
ASTM A500 Grade C
F_y = 46 ksi
F_u = 62 ksi

Plate
ASTM A36
F_{yp} = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-13, the geometric properties are as follows:

HSS6.000×0.375
D = 6.00 in.
t = 0.349 in.
D/t = 17.2

Limits of Applicability of AISC Specification *Section K2.2, Table K2.1A*

AISC *Specification* Table K2.1A provides the limits of applicability for plate-to-round connections. The applicable limits for this example are:

HSS wall slenderness:

$D/t \leq 50$ for T-connections

$17.2 < 50$ **o.k.**

Material strength:

$F_y \le 52 \text{ ksi}$

$46 \text{ ksi} < 52 \text{ ksi}$ **o.k.**

Ductility:

$$\frac{F_y}{F_u} \le 0.8$$

$$\frac{46 \text{ ksi}}{62 \text{ ksi}} \le 0.8$$

$0.741 < 0.8$ **o.k.**

End distance:

$$\begin{aligned} l_{end} &\ge D\left(1.25 - \frac{B_b/D}{2}\right) \\ &= (6.00 \text{ in.})\left[1.25 - \frac{(\frac{1}{4} \text{ in.}/6.00 \text{ in.})}{2}\right] \\ &= 7.38 \text{ in.} \end{aligned}$$

Thus, the edge of the plate must be located a minimum of 7.38 in. from the end of the HSS.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_u = 1.2(4 \text{ kips}) + 1.6(12 \text{ kips})$ $= 24.0 \text{ kips}$	$P_a = 4 \text{ kips} + 12 \text{ kips}$ $= 16.0 \text{ kips}$

HSS Plastification Limit State

The limit state of HSS plastification applies and is determined from AISC *Specification* Table K2.1.

$$R_n \sin\theta = 5.5 F_y t^2 \left(1 + 0.25 \frac{l_b}{D}\right) Q_f \qquad \text{(Spec. Eq. K2-2a)}$$

From the AISC *Specification* Table K2.1 Functions listed at the bottom of the table, for an HSS connecting surface in tension, $Q_f = 1.0$.

$$\begin{aligned} R_n &= \frac{5.5(46 \text{ ksi})(0.349 \text{ in.})^2\left[1 + 0.25\left(\frac{4 \text{ in.}}{6.00 \text{ in.}}\right)\right](1.0)}{\sin 90^\circ} \\ &= 36.0 \text{ kips} \end{aligned}$$

The available strength is:

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(36.0 \text{ kips})$ $= 32.4 \text{ kips} > 24.0 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{36.0 \text{ kips}}{1.67}$ $= 21.6 \text{ kips} > 16.0 \text{ kips}$ **o.k.**

EXAMPLE K.9 RECTANGULAR HSS COLUMN BASE PLATE

Given:

An ASTM A500 Grade C HSS6×6×½ column is supporting loads of 40 kips of dead load and 120 kips of live load. The column is supported by a 7 ft 6 in. × 7 ft 6 in. concrete spread footing with f_c' = 3,000 psi. Verify the ASTM A36 base plate size shown in Figure K.9-1 for this column.

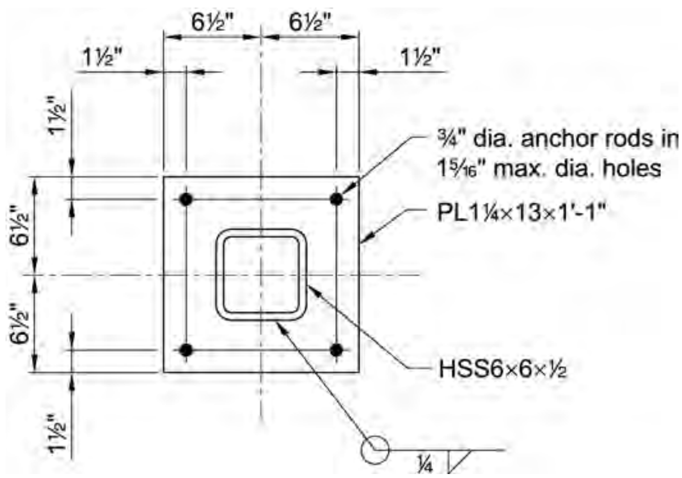


Fig K.9-1. Base plate geometry for Example K.9.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Column
ASTM A500 Grade C
F_y = 50 ksi
F_u = 62 ksi

Base Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-12, the geometric properties are as follows:

HSS6×6×½
$B = H = 6.00$ in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_u = 1.2(40\text{ kips}) + 1.6(120\text{ kips})$ $= 240\text{ kips}$	$P_a = 40\text{ kips} + 120\text{ kips}$ $= 160\text{ kips}$

Note: The procedure illustrated here is similar to that presented in AISC Design Guide 1, *Base Plate and Anchor Rod Design* (Fisher and Kloiber, 2006), and AISC *Manual* Part 14.

Try a base plate which extends 3½ in. from each face of the HSS column, or 13 in. × 13 in.

Available Strength for the Limit State of Concrete Crushing

On less than the full area of a concrete support:

$$P_p = 0.85 f_c' A_1 \sqrt{A_2 / A_1} \;\leq\; 1.7 f_c' A_1 \qquad \text{(}Spec\text{. Eq. J8-2)}$$

$$\begin{aligned} A_1 &= BN \\ &= (13 \text{ in.})(13 \text{ in.}) \\ &= 169 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_2 &= \left[(7.5 \text{ ft})(12 \text{ in./ft})\right]^2 \\ &= 8{,}100 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} P_p &= 0.85(3 \text{ ksi})\left(169 \text{ in.}^2\right)\sqrt{\frac{8{,}100 \text{ in.}^2}{169 \text{ in.}^2}} \leq 1.7(3 \text{ ksi})\left(169 \text{ in.}^2\right) \\ &= 2{,}980 \text{ kips} > 862 \text{ kips} \end{aligned}$$

Use P_p = 862 kips.

Note: The limit on the right side of AISC *Specification* Equation J8-2 will control when A_2/A_1 exceeds 4.0.

LRFD	ASD
From AISC *Specification* Section J8: $\phi_c = 0.65$ $\phi_c P_p = 0.65(862 \text{ kips})$ $= 560 \text{ kips} > 240 \text{ kips}$ **o.k.**	From AISC *Specification* Section J8: $\Omega_c = 2.31$ $\dfrac{P_p}{\Omega_c} = \dfrac{862 \text{ kips}}{2.31}$ $= 373 \text{ kips} > 160 \text{ kips}$ **o.k.**

Pressure under Bearing Plate and Required Thickness

For a rectangular HSS, the distance *m* or *n* is determined using 0.95 times the depth and width of the HSS.

$$\begin{aligned} m &= n \qquad \text{(from }Manual\text{ Eq. 14-2)} \\ &= \frac{N - 0.95(B \text{ or } H)}{2} \\ &= \frac{13 \text{ in.} - 0.95(6.00 \text{ in.})}{2} \\ &= 3.65 \text{ in.} \end{aligned}$$

Note: As discussed in AISC Design Guide 1, the $\lambda n'$ cantilever distance is not used for HSS and pipe.

The critical bending moment is the cantilever moment outside the HSS perimeter. Therefore, $m = n = l$.

LRFD	ASD
$f_{pu} = \dfrac{P_u}{A_1} = \dfrac{240\text{ kips}}{169\text{ in.}^2} = 1.42\text{ ksi}$	$f_{pa} = \dfrac{P_a}{A_1} = \dfrac{160\text{ kips}}{169\text{ in.}^2} = 0.947\text{ ksi}$
$M_u = \dfrac{f_{pu} l^2}{2}$	$M_a = \dfrac{f_{pa} l^2}{2}$
$Z = \dfrac{t_p^{\,2}}{4}$	$Z = \dfrac{t_p^{\,2}}{4}$
$\phi_b = 0.90$	$\Omega_b = 1.67$
$M_n = M_p = F_y Z$ (from *Spec*. Eq. F11-1)	$M_n = M_p = F_y Z$ (from *Spec*. Eq. F11-1)
Note: the upper limit of $1.6F_yS_x$ will not govern for a rectangular plate.	Note: the upper limit of $1.6F_yS_x$ will not govern for a rectangular plate.
Equating:	Equating:
$M_u = \phi_b M_n$ and solving for t_p gives:	$M_a = M_n/\Omega_b$ and solving for t_p gives:
$t_{p(req)} = \sqrt{\dfrac{2 f_{pu} l^2}{\phi_b F_y}} = \sqrt{\dfrac{2(1.42\text{ ksi})(3.65\text{ in.})^2}{0.90(36\text{ ksi})}} = 1.08\text{ in.}$	$t_{p(req)} = \sqrt{\dfrac{2 f_{pa} l^2}{F_y / \Omega_b}} = \sqrt{\dfrac{2(0.947\text{ ksi})(3.65\text{ in.})^2}{(36\text{ ksi})/1.67}} = 1.08\text{ in.}$
Or use AISC *Manual* Equation 14-7a:	Or use AISC *Manual* Equation 14-7b:
$t_{min} = l\sqrt{\dfrac{2P_u}{0.90F_y BN}} = (3.65\text{ in.})\sqrt{\dfrac{2(240\text{ kips})}{0.90(36\text{ ksi})(13\text{ in.})(13\text{ in})}} = 1.08\text{ in.}$	$t_{min} = l\sqrt{\dfrac{1.67(2P_a)}{F_y BN}} = (3.65\text{ in.})\sqrt{\dfrac{1.67(2)(160\text{ kips})}{(36\text{ ksi})(13\text{ in.})(13\text{ in.})}} = 1.08\text{ in.}$

Therefore, the PL1¼ in. × 13 in. × 1 ft 1 in. is adequate.

EXAMPLE K.10 RECTANGULAR HSS STRUT END PLATE

Given:

Determine the weld leg size, end-plate thickness, and the bolt size required to resist forces of 16 kips from dead load and 50 kips from live load on an ASTM A500 Grade C section, as shown in Figure K.10-1. The end plate is ASTM A36. Use 70-ksi weld electrodes.

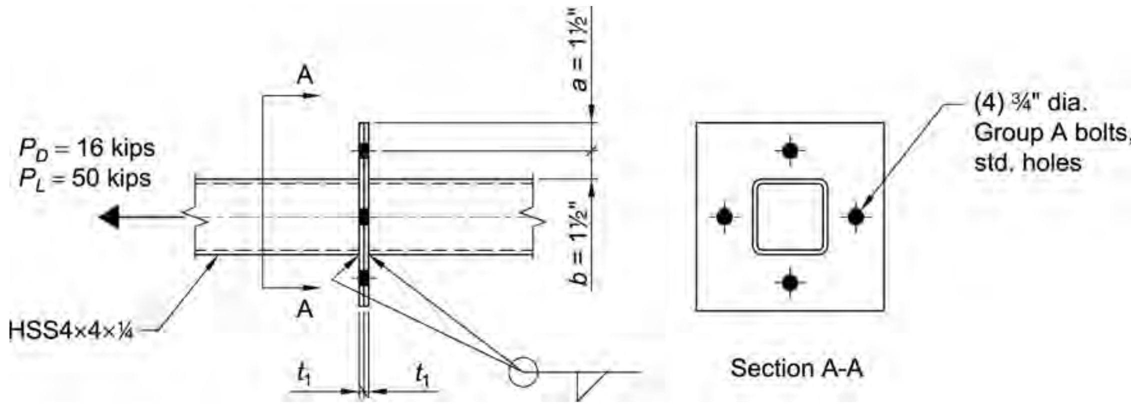


Fig K.10-1. Loading and geometry for Example K.10.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Strut
ASTM A500 Grade C
$F_y = 50$ ksi
$F_u = 62$ ksi

End Plate
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-12, the geometric properties are as follows:

HSS4×4×¼
$t = 0.233$ in.
$A = 3.37$ in.^{2}

From ASCE/SEI 7, Chapter 2, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(16\text{ kips}) + 1.6(50\text{ kips})$ $= 99.2\text{ kips}$	$P_a = 16\text{ kips} + 50\text{ kips}$ $= 66.0\text{ kips}$

Preliminary Size of the (4) Group A Bolts

LRFD	ASD
$r_{ut} = \dfrac{P_u}{n}$ $= \dfrac{99.2\text{ kips}}{4}$ $= 24.8\text{ kips}$ Using AISC *Manual* Table 7-2, try ¾-in.-diameter Group A bolts. $\phi r_n = 29.8\text{ kips}$	$r_{at} = \dfrac{P_a}{n}$ $= \dfrac{66.0\text{ kips}}{4}$ $= 16.5\text{ kips}$ Using AISC *Manual* Table 7-2, try ¾-in.-diameter Group A bolts. $\dfrac{r_n}{\Omega} = 19.9\text{ kips}$

End-Plate Thickness with Consideration of Prying Action (AISC Manual *Part 9)*

$$a' = \left(a + \frac{d_b}{2}\right) \le \left(1.25b + \frac{d_b}{2}\right) \qquad (\textit{Manual}\text{ Eq. 9-23})$$

$$= 1\tfrac{1}{2}\text{ in.} + \frac{\tfrac{3}{4}\text{ in.}}{2} \le 1.25\left(1\tfrac{1}{2}\text{ in.}\right) + \frac{\tfrac{3}{4}\text{ in.}}{2}$$

$$= 1.88\text{ in.} < 2.25\text{ in.}$$

$$= 1.88\text{ in.}$$

$$b' = b - \frac{d_b}{2} \qquad (\textit{Manual}\text{ Eq. 9-18})$$

$$= 1\tfrac{1}{2}\text{ in.} - \frac{\tfrac{3}{4}\text{ in.}}{2}$$

$$= 1.13\text{ in.}$$

$$\rho = \frac{b'}{a'} \qquad (\textit{Manual}\text{ Eq. 9-22})$$

$$= \frac{1.13}{1.88}$$

$$= 0.601$$

$$d' = \tfrac{13}{16}\text{ in.}$$

The tributary length per bolt (Packer et al., 2010),

$$p = \frac{\text{full plate width}}{\text{number of bolts per side}}$$

$$= \frac{10.0\text{ in.}}{1}$$

$$= 10.0\text{ in.}$$

$$\delta = 1 - \frac{d'}{p}$$ (*Manual* Eq. 9-20)

$$= 1 - \frac{13/16 \text{ in.}}{10.0 \text{ in.}}$$

$$= 0.919$$

LRFD	ASD
$\beta = \frac{1}{\rho}\left(\frac{\phi r_n}{r_{ut}} - 1\right)$ (from *Manual* Eq. 9-21) $= \frac{1}{0.601}\left(\frac{29.8 \text{ kips}}{24.8 \text{ kips}} - 1\right)$ $= 0.335$ Because β < 1, from AISC *Manual* Part 9: $\alpha' = \frac{1}{\delta}\left(\frac{\beta}{1-\beta}\right) \le 1.0$ $= \frac{1}{0.919}\left(\frac{0.335}{1-0.335}\right) \le 1.0$ $= 0.548$	$\beta = \frac{1}{\rho}\left(\frac{r_n / \Omega}{r_{at}} - 1\right)$ (from *Manual* Eq. 9-21) $= \frac{1}{0.601}\left(\frac{19.9 \text{ kips}}{16.5 \text{ kips}} - 1\right)$ $= 0.343$ Because β < 1, from AISC *Manual* Part 9: $\alpha' = \frac{1}{\delta}\left(\frac{\beta}{1-\beta}\right) \le 1.0$ $= \frac{1}{0.919}\left(\frac{0.343}{1-0.343}\right) \le 1.0$ $= 0.568$

Use Equation 9-19 for t_{min} in Chapter 9 of the AISC *Manual*, except that F_u is replaced by F_y per the recommendation of Willibald, Packer and Puthli (2003) and Packer et al. (2010).

LRFD	ASD
$t_{min} = \sqrt{\frac{4 r_{ut} b'}{\phi p F_y (1 + \delta\alpha')}}$ (from *Manual* Eq. 9-19a) $= \sqrt{\frac{4(24.8 \text{ kips})(1.13 \text{ in.})}{0.90(10.0 \text{ in.})(36 \text{ ksi})[1 + 0.919(0.548)]}}$ $= 0.480 \text{ in.}$ Use a ½-in.-thick end plate, t_1 > 0.480 in., further bolt check for prying not required. Use (4) ¾-in.-diameter Group A bolts.	$t_{min} = \sqrt{\frac{\Omega 4 r_{at} b'}{p F_y (1 + \delta\alpha')}}$ (from *Manual* Eq. 9-19b) $= \sqrt{\frac{1.67(4)(16.5 \text{ kips})(1.13 \text{ in.})}{(10.0 \text{ in.})(36 \text{ ksi})[1 + 0.919(0.568)]}}$ $= 0.477 \text{ in.}$ Use a ½-in.-thick end plate, t_1 > 0.477 in., further bolt check for prying not required. Use (4) ¾-in.-diameter Group A bolts.

Required Weld Size

$$R_n = F_{nw} A_{we}$$ (*Spec*. Eq. J2-4)

$$F_{nw} = 0.60 F_{EXX}\left(1.0 + 0.50 \sin^{1.5}\theta\right)$$ (*Spec*. Eq. J2-5)

$$= 0.60(70 \text{ ksi})\left(1.0 + 0.50 \sin^{1.5} 90°\right)$$

$$= 63.0 \text{ ksi}$$

$$A_{we} = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{D}{16}\right)l$$

where D is the weld size in sixteenths of an inch (i.e., D is an integer).

$$\begin{aligned} l &= 4(4.00 \text{ in.}) \\ &= 16.0 \text{ in.} \end{aligned}$$

Note: This weld length is approximate. A more accurate length could be determined by taking into account the curved corners of the HSS.

From AISC *Specification* Table J2.5:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi F_{nw} A_{we}$ $= 0.75(63.0 \text{ ksi})\left(\frac{\sqrt{2}}{2}\right)\left(\frac{D}{16}\right)(16.0 \text{ in.})$ Setting $\phi R_n = P_u$ and solving for D, $D \geq \frac{(99.2 \text{ kips})(16)}{0.75(63.0 \text{ ksi})\left(\frac{\sqrt{2}}{2}\right)(16.0 \text{ in.})}$ $= 2.97$ $D = 3$ (i.e., a 3/16 in. weld)	$\Omega = 2.00$ $\frac{R_n}{\Omega} = \left(\frac{F_{nw} A_{we}}{\Omega}\right)$ $= \frac{(63.0 \text{ ksi})\left(\frac{\sqrt{2}}{2}\right)\left(\frac{D}{16}\right)(16.0 \text{ in.})}{2.00}$ Setting $\frac{R_n}{\Omega} = P_a$ and solving for D, $D \geq \frac{2.00(66.0 \text{ kips})(16)}{(63.0 \text{ ksi})\left(\frac{\sqrt{2}}{2}\right)(16.0 \text{ in.})}$ $= 2.96$ $D = 3$ (i.e., a 3/16 in. weld)

Minimum Weld Size Requirements

For t = ¼ in., the minimum weld size = ⅛ in. from AISC *Specification* Table J2.4.

Summary:

Use a 3/16-in. weld with ½-in.-thick end plates and (4) ¾-in.-diameter Group A bolts.

CHAPTER K DESIGN EXAMPLE REFERENCES

Fisher, J.M. and Kloiber, L.A. (2006), *Base Plate and Anchor Rod Design*, Design Guide 1, 2nd Ed., AISC, Chicago, IL

Packer, J.A., Sherman, D. and Lecce, M. (2010), *Hollow Structural Section Connections*, Design Guide 24, AISC, Chicago, IL.

Willibald, S., Packer, J.A. and Puthli, R.S. (2003), "Design Recommendations for Bolted Rectangular HSS Flange Plate Connections in Axial Tension," *Engineering Journal*, AISC, Vol. 40, No. 1, pp. 15–24.

APPENDIX 6
MEMBER STABILITY BRACING

This Appendix addresses the minimum strength and stiffness necessary to provide a braced point in a column, beam or beam-column.

The governing limit states for column and beam design may include flexural, torsional and flexural-torsional buckling for columns and lateral-torsional buckling for beams. In the absence of other intermediate bracing, column unbraced lengths are defined between points of obviously adequate lateral restraint, such as floor and roof diaphragms that are part of the building's lateral force-resisting systems. Similarly, beams are often braced against lateral-torsional buckling by relatively strong and stiff bracing elements such as a continuously connected floor slab or roof diaphragm. However, at times, unbraced lengths are bounded by elements that may or may not possess adequate strength and stiffness to provide sufficient bracing. AISC *Specification* Appendix 6 provides equations for determining the required strength and stiffness of braces that have not been included in the second-order analysis of the structural system. It is not intended that the provisions of Appendix 6 apply to bracing that is part of the lateral force-resisting system. Guidance for applying these provisions to stabilize trusses is provided in AISC *Specification* Appendix 6 commentary.

Background for the provisions can be found in references cited in the Commentary including "Fundamentals of Beam Bracing" (Yura, 2001) and the *Guide to Stability Design Criteria for Metal Structures* (Ziemian, 2010). AISC *Manual* Part 2 also provides information on member stability bracing.

6.1 GENERAL PROVISIONS

Lateral column and beam bracing may be either panel or point while torsional beam bracing may be point or continuous. The User Note in AISC *Specification* Appendix 6, Section 6.1 states "A panel brace (formerly referred to as a relative brace) controls the angular deviation of a segment of the braced member between braced points (that is, the lateral displacement of one end of the segment relative to the other). A point brace (formerly referred to as a nodal brace) controls the movement at the braced point without direct interaction with adjacent braced points. A continuous bracing system consists of bracing that is attached along the entire member length." Panel and point bracing systems are discussed further in AISC *Specification* Commentary Appendix 6, Section 6.1. Examples of each bracing type are shown in AISC *Specification* Commentary Figure C-A-6.1.

In lieu of the requirements of Appendix 6, Sections 6.2, 6.3 and 6.4, alternative provisions are given in Sections 6.1(a), 6.1(b) and 6.1(c).

6.2 COLUMN BRACING

The requirements in this section apply to bracing associated with the limit state of flexural buckling. For columns that could experience torsional or flexural-torsional buckling, as addressed in AISC *Specification* Section E4, the designer must ensure that sufficient bracing to resist the torsional component of buckling is provided. See Helwig and Yura (1999).

Column braces may be panel or point. The type of bracing must be determined before the requirements for strength and stiffness can be determined. The requirements are derived for an infinite number of braces along the column and are thus conservative for most columns as explained in the Commentary. Provision is made in this section for reducing the required brace stiffness for point bracing when the column required strength is less than the available strength of the member. The Commentary also provides an approach to reduce the requirements when a finite number of point braces are provided.

6.3 BEAM BRACING

The requirements in this section apply to bracing of doubly and singly symmetric I-shaped members subject to flexure within a plane of symmetry and zero net axial force. Bracing to resist lateral-torsional buckling may be accomplished by a lateral brace, a torsional brace, or a combination of the two to prevent twist of the section. Lateral bracing should normally be connected near the compression flange. The exception is for the free ends of cantilevers and near inflection points of braced beams subject to double curvature bending. Torsional bracing may be connected anywhere on the cross section in a manner to prevent twist of the section.

According to AISC *Specification* Section F1(b), the design of members for flexure is based on the assumption that points of support are restrained against rotation about their longitudinal axis. The bracing requirements in Appendix 6 are for intermediate braces in addition to those at the support.

In members subject to double curvature, inflection points are not to be considered as braced points unless bracing is provided at that location. In addition, the bracing nearest the inflection point must be attached to prevent twist, either as a torsional brace or as lateral braces attached to both flanges as described in AISC *Specification* Appendix 6, Section 6.3.1(b).

6.3.1 Lateral Bracing

As with column bracing, beam bracing may be panel or point. In addition, it is permissible to provide torsional bracing. This section provides requirements for determining the required lateral brace strength and stiffness for panel and point braces.

For point braces, provision is made in this section to reduce the required brace stiffness when the actual unbraced length is less than the maximum unbraced length for the required flexural strength.

6.3.2 Torsional Bracing

This section provides requirements for determining the required bracing flexural strength and stiffness for point and continuous torsional bracing. Torsional bracing can be connected to the section at any cross-section location. However, if the beam has inadequate distortional (out-of-plane) bending stiffness, torsional bracing will be ineffective. Web stiffeners can be provided when necessary, to increase the web distortional stiffness for point torsional braces.

As is the case for columns and for lateral beam point braces, it is possible to reduce the required brace stiffness when the required strength of the member is less than the available strength for the provided location of bracing.

Provisions for continuous torsional bracing are also provided. A slab connected to the top flange of a beam in double curvature may provide sufficient continuous torsional bracing as discussed in the Commentary. For this condition there is no unbraced length between braces so the unbraced length used in the strength and stiffness equations is the maximum unbraced length permitted to provide the required strength in the beam. In addition, for continuous torsional bracing, stiffeners are not permitted to be used to increase web distortional stiffness.

6.4 BEAM-COLUMN BRACING

For bracing of beam-columns, the required strength and stiffness are to be determined for the column and beam independently as specified in AISC *Specification* Appendix 6, Sections 6.2 and 6.3. These values are then to be combined, depending on the type of bracing provided.

EXAMPLE A-6.1 POINT STABILITY BRACING OF A W-SHAPE COLUMN

Given:

Determine the required strength and the stiffness for intermediate point braces, such that the unbraced length for the column can be taken as 12 ft. The column is an ASTM A992 W12×72 with loading and geometry as shown in Figure A-6.1-1. The column is braced laterally and torsionally at its ends with intermediate lateral braces for the *x*- and *y*-axis provided at the one-third points as shown. Thus, the unbraced length for the limit state of flexural-torsional buckling is 36 ft and the unbraced length for flexural buckling is 12 ft. The column has sufficient strength to support the applied loads with this bracing.

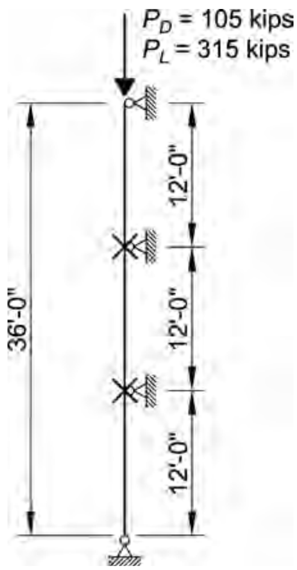


Fig. A-6.1-1. Column bracing geometry for Example A-6.1.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Required Compressive Strength of Column

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_u = 1.2(105 \text{ kips}) + 1.6(315 \text{ kips})$ $= 630 \text{ kips}$	$P_a = 105 \text{ kips} + 315 \text{ kips}$ $= 420 \text{ kips}$

Available Compressive Strength of Column

From AISC *Manual* Table 4-1a at L_{cy} = 12 ft, the available strength of the W12×72 is:

LRFD	ASD
$\phi_c P_n = 806 \text{ kips} > 630 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 536 \text{ kips} > 420 \text{ kips}$ **o.k.**

Required Point Brace Strength

From AISC *Specification* Appendix 6, Section 6.2.2, the required point brace strength is:

LRFD	ASD
$P_r = P_u$	$P_r = P_a$
$= 630 \text{ kips}$	$= 420 \text{ kips}$
$P_{br} = 0.01P_r$ (*Spec*. Eq. A-6-3)	$P_{br} = 0.01P_r$ (*Spec*. Eq. A-6-3)
$= 0.01(630 \text{ kips})$	$= 0.01(420 \text{ kips})$
$= 6.30 \text{ kips}$	$= 4.20 \text{ kips}$

Required Point Brace Stiffness

From AISC *Specification* Appendix 6, Section 6.2.2, the required point brace stiffness, with an unbraced length adjacent to the point brace L_{br} = 12 ft, is:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$P_r = P_u$	$P_r = P_a$
$= 630 \text{ kips}$	$= 420 \text{ kips}$
$\beta_{br} = \dfrac{1}{\phi}\left(\dfrac{8P_r}{L_{br}}\right)$ (*Spec*. Eq. A-6-4a)	$\beta_{br} = \Omega\left(\dfrac{8P_r}{L_{br}}\right)$ (*Spec*. Eq. A-6-4b)
$= \dfrac{1}{0.75}\left[\dfrac{8(630 \text{ kips})}{(12 \text{ ft})(12 \text{ in./ft})}\right]$	$= 2.00\left[\dfrac{8(420 \text{ kips})}{(12 \text{ ft})(12 \text{ in./ft})}\right]$
$= 46.7 \text{ kip/in.}$	$= 46.7 \text{ kip/in.}$

Determine the maximum permitted unbraced length for the required strength.

Interpolating between values, from AISC *Manual* Table 4-1a:

LRFD	ASD
$L_{cy} = 18.9$ ft for $P_u = 632$ kips	$L_{cy} = 18.9$ ft for $P_a = 421$ kips

Calculate the required point brace stiffness for this increased unbraced length

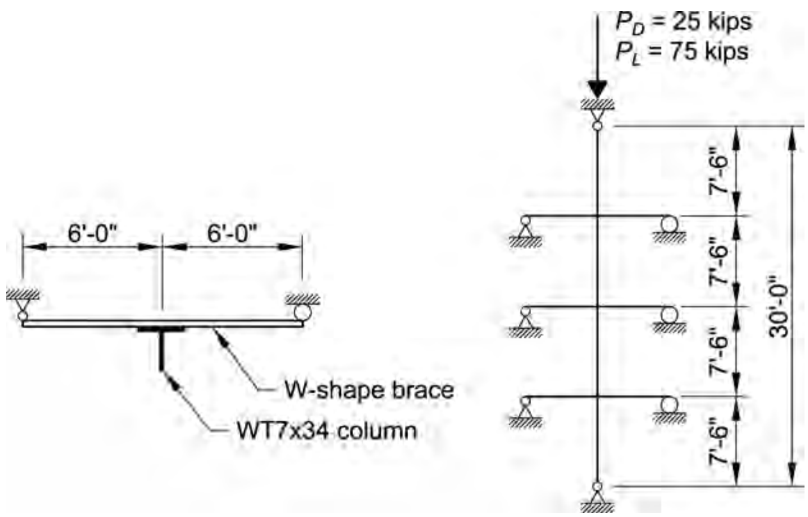
It is permissible to design the braces to provide the lower stiffness determined using the maximum unbraced length permitted to carry the required strength according to AISC *Specification* Appendix 6, Section 6.2.2.

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$P_r = P_u$ $= 630\text{ kips}$	$P_r = P_a$ $= 420\text{ kips}$
$\beta_{br} = \dfrac{1}{\phi}\left(\dfrac{8P_r}{L_{br}}\right)$ (*Spec*. Eq. A-6-4a) $= \dfrac{1}{0.75}\left[\dfrac{8(630\text{ kips})}{(18.9\text{ ft})(12\text{ in./ft})}\right]$ $= 29.6\text{ kip/in.}$	$\beta_{br} = \Omega\left(\dfrac{8P_r}{L_{br}}\right)$ (*Spec*. Eq. A-6-4b) $= 2.00\left[\dfrac{8(420\text{ kips})}{(18.9\text{ ft})(12\text{ in./ft})}\right]$ $= 29.6\text{ kip/in.}$

EXAMPLE A-6.2 POINT STABILITY BRACING OF A WT-SHAPE COLUMN

Given:

Determine the strength and stiffness requirements for the point braces and select a W-shape brace based on *x*-axis flexural buckling of the ASTM A992 WT7×34 column with loading and geometry as shown in Figure A-6.2-1. The unbraced length for this column is 7.5 ft. Bracing about the *y*-axis is provided by the axial resistance of a W-shape connected to the flange of the WT, while bracing about the *x*-axis is provided by the flexural resistance of the same W-shape loaded at the midpoint of a 12-ft-long simple span beam. Assume that the axial strength and stiffness of the W-shape are adequate to brace the *y*-axis of the WT. Also, assume the column is braced laterally and torsionally at its ends and is torsionally braced at one-quarter points by the W-shape braces.



(a) Plan *(b) Elevation*

Fig. A-6.2-1. Column bracing geometry for Example A-6.2.

Solution:

From AISC *Manual* Table 2-4, the material properties of the column and brace are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Required Compressive Strength of Column

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_u = 1.2(25\text{ kips}) + 1.6(75\text{ kips})$ $= 150\text{ kips}$	$P_a = 25\text{ kips} + 75\text{ kips}$ $= 100\text{ kips}$

Available Compressive Strength of Column

Interpolating between values, from AISC *Manual* Table 4-7, the available axial compressive strength of the WT7×34 with $L_{cx} = 7.5$ ft is:

LRFD	ASD
$\phi_c P_n = 357 \text{ kips} > 150 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 238 \text{ kips} > 100 \text{ kips}$ **o.k.**

Required Point Brace Size

From AISC *Specification* Appendix 6, Section 6.2.2, the required point brace strength is:

LRFD	ASD
$P_r = P_u$ $= 150 \text{ kips}$ $P_{br} = 0.01P_r$ (*Spec*. Eq. A-6-3) $= 0.01(150 \text{ kips})$ $= 1.50 \text{ kips}$	$P_r = P_a$ $= 100 \text{ kips}$ $P_{br} = 0.01P_r$ (*Spec*. Eq. A-6-3) $= 0.01(100 \text{ kips})$ $= 1.00 \text{ kips}$

From AISC *Specification* Appendix 6, Section 6.2.2, the required point brace stiffness is:

LRFD	ASD
$\phi = 0.75$ $P_r = P_u$ $= 150 \text{ kips}$ $\beta_{br} = \dfrac{1}{\phi}\left(\dfrac{8P_r}{L_{br}}\right)$ (*Spec*. Eq. A-6-4a) $= \dfrac{1}{0.75}\left[\dfrac{8(150 \text{ kips})}{(7.50 \text{ ft})(12 \text{ in./ft})}\right]$ $= 17.8 \text{ kip/in.}$	$\Omega = 2.00$ $P_r = P_a$ $= 100 \text{ kips}$ $\beta_{br} = \Omega\left(\dfrac{8P_r}{L_{br}}\right)$ (*Spec*. Eq. A-6-4b) $= 2.00\left[\dfrac{8(100 \text{ kips})}{(7.50 \text{ ft})(12 \text{ in./ft})}\right]$ $= 17.8 \text{ kip/in.}$

The brace is a simple-span beam loaded at its midspan. Thus, its flexural stiffness can be derived from Case 7 of AISC *Manual* Table 3-23 to be $48EI/L^3$, which must be greater than the required point brace stiffness, β_{br}. Also, the flexural strength of the beam, $\phi_b M_p$, for a compact laterally supported beam, must be greater than the moment resulting from the required brace strength over the beam's simple span, $M_{br} = P_{br}L/4$.

Based on brace stiffness, the minimum required moment of inertia of the beam is:

$$I_{br} = \frac{\beta_{br} L^3}{48E}$$

$$= \frac{(17.8 \text{ kip/in.})(12.0 \text{ ft})^3 (12 \text{ in./ft})^3}{48(29{,}000 \text{ ksi})}$$

$$= 38.2 \text{ in.}^4$$

Based on moment strength for a compact laterally supported beam, the minimum required plastic section modulus is:

LRFD	ASD
$Z_{req} = \dfrac{M_{br}}{\phi F_y}$ $= \dfrac{(1.50\text{ kips})(12.0\text{ ft})(12\text{ in./ft})}{0.90(50\text{ ksi})(4)}$ $= 1.20\text{ in.}^3$	$Z_{req} = \dfrac{\Omega M_{br}}{F_y}$ $= \dfrac{1.67(1.00\text{ kip})(12.0\text{ ft})(12\text{ in./ft})}{(50\text{ ksi})(4)}$ $= 1.20\text{ in.}^3$

From AISC *Manual* Table 3-2, select a W8×13 member with $Z_x = 11.4\text{ in.}^3$ and $I_x = 39.6\text{ in.}^4$

Note that because the live-to-dead load ratio is 3, the LRFD and ASD results are identical.

The required stiffness can be reduced if the maximum permitted unbraced length is used as described in AISC *Specification* Appendix 6, Section 6.2, and also if the actual number of braces are considered, as discussed in the Commentary. The following demonstrates how this affects the design.

Interpolating between values in AISC *Manual* Table 4-7, the maximum permitted unbraced length of the WT7×34 for the required strength is as follows:

LRFD	ASD
$L_{cx} = 18.6$ ft for $P_u = 150$ kips	$L_{cx} = 18.6$ ft for $P_a = 100$ kips

From AISC *Specification* Commentary Appendix 6, Section 6.2, determine the reduction factor for three intermediate braces:

$$\frac{2n-1}{2n} = \frac{2(3)-1}{2(3)}$$
$$= 0.833$$

Determine the required point brace stiffness for the increased unbraced length and number of braces:

LRFD	ASD
$\phi = 0.75$ $P_r = P_u$ $= 150$ kips $\beta_{br} = 0.833\left[\dfrac{1}{\phi}\left(\dfrac{8P_r}{L_{br}}\right)\right]$ (*Spec*. Eq. A-6-4a) $= 0.833\left\{\dfrac{1}{0.75}\left[\dfrac{8(150\text{ kips})}{(18.6\text{ ft})(12\text{ in./ft})}\right]\right\}$ $= 5.97$ kip/in.	$\Omega = 2.00$ $P_r = P_a$ $= 100$ kips $\beta_{br} = 0.833\left[\Omega\left(\dfrac{8P_r}{L_{br}}\right)\right]$ (*Spec*. Eq. A-6-4b) $= 0.833\left\{2.00\left[\dfrac{8(100\text{ kips})}{(18.6\text{ ft})(12\text{ in./ft})}\right]\right\}$ $= 5.97$ kip/in.

Determine the required brace size based on this new stiffness requirement.

Based on brace stiffness, the minimum required moment of inertia of the beam is:

$$I_{br} = \frac{\beta_{br} L^3}{48E}$$

$$= \frac{(5.97 \text{ kip/in.})(12.0 \text{ ft})^3 (12 \text{ in./ft})^3}{48(29{,}000 \text{ ksi})}$$

$$= 12.8 \text{ in.}^4$$

Based on the unchanged flexural strength for a compact laterally supported beam, the minimum required plastic section modulus, Z_x, was determined previously to be 1.20 in.^{3} From AISC *Manual* Table 1-1, select a W6×8.5 noncompact member with Z_x = 5.73 in.^{3} and I_x = 14.9 in.^{4}

EXAMPLE A-6.3 POINT STABILITY BRACING OF A BEAM—CASE I

Given:

A walkway in an industrial facility has a span of 28 ft as shown in Figure A-6.3.1. The walkway has a deck of grating which is not sufficient to brace the beams. The ASTM A992 W12×22 beams along walkway edges are braced against twist at the ends as required by AISC *Specification* Section F1(b) and are connected by an L3×3×¼ strut at midspan. The two diagonal ASTM A36 L5×5×5⁄16 braces are connected to the top flange of the beams at the supports and at the strut at the middle. The strut and the brace connections are welded; therefore, bolt slippage does not need to be accounted for in the stiffness calculation. The dead load on each beam is 0.05 kip/ft and the live load is 0.125 kip/ft. Determine if the diagonal braces are strong enough and stiff enough to brace this walkway.

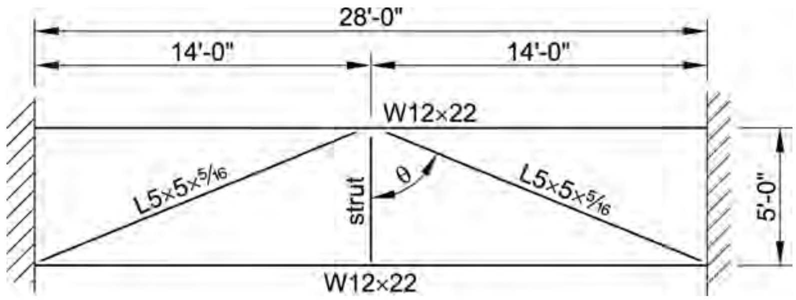


Fig. A-6.3-1. Plan view for Example A-6.3.

Solution:

Because the diagonal braces are connected directly to an unyielding support that is independent of the midspan brace point, they are designed as point braces. The strut will be assumed to be sufficiently strong and stiff to force the two beams to buckle together.

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Diagonal braces
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 1-7, the geometric properties are as follows:

Beam
W12×22
h_o = 11.9 in.

Diagonal braces
L5×5×5⁄16
A = 3.07 in.^{2}

Required Flexure Strength of Beam

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$w_u = 1.2(0.05 \text{ kip/ft}) + 1.6(0.125 \text{ kip/ft})$ $= 0.260 \text{ kip/ft}$	$w_a = 0.05 \text{ kip/ft} + 0.125 \text{ kip/ft}$ $= 0.175 \text{ kip/ft}$

Determine the required flexural strength for a uniformly loaded simply supported beam using AISC *Manual* Table 3-23, Case 1.

LRFD	ASD
$M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(0.260 \text{ kip/ft})(28 \text{ ft})^2}{8}$ $= 25.5 \text{ kip-ft}$	$M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(0.175 \text{ kip/ft})(28 \text{ ft})^2}{8}$ $= 17.2 \text{ kip-ft}$

It can be shown that the W12×22 beams are adequate with the unbraced length of 14 ft. Both beams need bracing in the same direction simultaneously.

Required Brace Strength and Stiffness

From AISC *Specification* Appendix 6, Section 6.3, determine the required point brace strength for each beam as follows, with C_d = 1.0 for bending in single curvature.

LRFD	ASD
$M_r = M_u$ $= 25.5 \text{ kip-ft}$ $P_{br} = 0.02\left(\dfrac{M_r C_d}{h_o}\right)$ (*Spec*. Eq. A-6-7) $= 0.02\left[\dfrac{(25.5 \text{ kip-ft})(12 \text{ in./ft})(1.0)}{11.9 \text{ in.}}\right]$ $= 0.514 \text{ kip}$	$M_r = M_a$ $= 17.2 \text{ kip-ft}$ $P_{br} = 0.02\left(\dfrac{M_r C_d}{h_o}\right)$ (*Spec*. Eq. A-6-7) $= 0.02\left[\dfrac{(17.2 \text{ kip-ft})(12 \text{ in./ft})(1.0)}{11.9 \text{ in.}}\right]$ $= 0.347 \text{ kip}$

Because there are two beams to be braced, the total required brace strength is:

LRFD	ASD
$P_{br} = 2(0.514 \text{ kip})$ $= 1.03 \text{ kips}$	$P_{br} = 2(0.347 \text{ kip})$ $= 0.694 \text{ kip}$

There are two beams to brace and two braces to share the load. The worst case for design of the braces will be when they are in compression.

By geometry, the diagonal bracing length is

$$L = \sqrt{(14 \text{ ft})^2 + (5 \text{ ft})^2}$$
$$= 14.9 \text{ ft}$$

The required brace strength is:

LRFD	ASD
$P_{br}\cos\theta = P_{br}\left(\dfrac{5\text{ ft}}{14.9\text{ ft}}\right) = 1.03\text{ kips}$ Because there are two braces, the required brace strength is: $P_{br} = \dfrac{1.03\text{ kips}}{2(5\text{ ft}/14.9\text{ ft})} = 1.53\text{ kips}$	$P_{br}\cos\theta = P_{br}\left(\dfrac{5\text{ ft}}{14.9\text{ ft}}\right) = 0.694\text{ kip}$ Because there are two braces, the required brace strength is: $P_{br} = \dfrac{0.694\text{ kip}}{2(5\text{ ft}/14.9\text{ ft})} = 1.03\text{ kips}$

The required point brace stiffness, with C_d = 1.0 for bending in single curvature, is determined as follows:

LRFD	ASD
$\phi = 0.75$ $M_r = M_u = 25.5\text{ kip-ft}$ $\beta_{br} = \dfrac{1}{\phi}\left(\dfrac{10M_rC_d}{L_{br}h_o}\right)$ (*Spec*. Eq. A-6-8a) $= \dfrac{1}{0.75}\left[\dfrac{10(25.5\text{ kip-ft})(12\text{ in./ft})(1.0)}{(14\text{ ft})(12\text{ in./ft})(11.9\text{ in.})}\right]$ $= 2.04\text{ kip/in.}$	$\Omega = 2.00$ $M_r = M_a = 17.2\text{ kip-ft}$ $\beta_{br} = \Omega\left(\dfrac{10M_rC_d}{L_{br}h_o}\right)$ (*Spec*. Eq. A-6-8b) $= 2.00\left[\dfrac{10(17.2\text{ kip-ft})(12\text{ in./ft})(1.0)}{(14\text{ ft})(12\text{ in./ft})(11.9\text{ in.})}\right]$ $= 2.06\text{ kip/in.}$

Because there are two beams to be braced, the total required point brace stiffness is:

LRFD	ASD
$\beta_{br} = 2(2.04\text{ kip/in.}) = 4.08\text{ kip/in.}$	$\beta_{br} = 2(2.06\text{ kip/in.}) = 4.12\text{ kip/in.}$

The beams require bracing in order to have sufficient strength to carry the given load. However, locating that brace at the midspan provides flexural strength greater than the required strength. The maximum unbraced length permitted for the required flexural strength is L_b = 18.2 ft from AISC *Manual* Table 6-2. Thus, according to AISC *Specification* Appendix 6, Section 6.3.1b, this length could be used in place of 14 ft to determine the required stiffness. However, because the required stiffness is so small, the 14 ft length will be used here.

For a single brace, the stiffness is:

$$\beta = \frac{AE\cos^2\theta}{L}$$
$$= \frac{(3.07\text{ in.}^2)(29{,}000\text{ ksi})(5\text{ ft}/14.9\text{ ft})^2}{(14.9\text{ ft})(12\text{ in./ft})}$$
$$= 56.1\text{ kip/in.}$$

Because there are two braces, the system stiffness is twice this. Thus,

$$\beta = 2(56.1 \text{ kip/in.})$$
$$= 112 \text{ kip/in.}$$

LRFD	ASD
$\beta = 112 \text{ kip/in.} > 4.08 \text{ kip/in.}$ **o.k.**	$\beta = 112 \text{ kip/in.} > 4.12 \text{ kip/in.}$ **o.k.**

Available Strength of Braces

The braces may be called upon to act in either tension or compression, depending on which transverse direction the system tries to buckle. Brace compression buckling will control over tension yielding. Therefore, determine the compressive strength of the braces assuming they are eccentrically loaded using AISC *Manual* Table 4-12.

LRFD	ASD
Interpolating for L_c = 14.9 ft: $\phi_c P_n = 17.2 \text{ kips} > 1.53 \text{ kips}$ **o.k.**	Interpolating for L_c = 14.9 ft: $\frac{P_n}{\Omega_c} = 11.2 \text{ kips} > 1.03 \text{ kips}$ **o.k.**

The L5×5×5⁄16 braces have sufficient strength and stiffness to act as the point braces for this system.

EXAMPLE A-6.4 POINT STABILITY BRACING OF A BEAM—CASE II

Given:

A walkway in an industrial facility has a span of 28 ft as shown in Figure A-6.4-1. The walkway has a deck of grating which is not sufficient to brace the beams. The ASTM A992 W12×22 beams are braced against twist at the ends, and they are connected by a strut connected at midspan. At that same point they are braced to an adjacent ASTM A500 Grade C HSS8×8×¼ column by the attachment of a 5-ft-long ASTM A36 2L3×3×¼. The brace connections are all welded; therefore, bolt slippage does not need to be accounted for in the stiffness calculation. The adjacent column is not braced at the walkway level, but is adequately braced 12 ft below and 12 ft above the walkway level. The dead load on each beam is 0.05 kip/ft and the live load is 0.125 kip/ft. Determine if the bracing system has adequate strength and stiffness to brace this walkway.

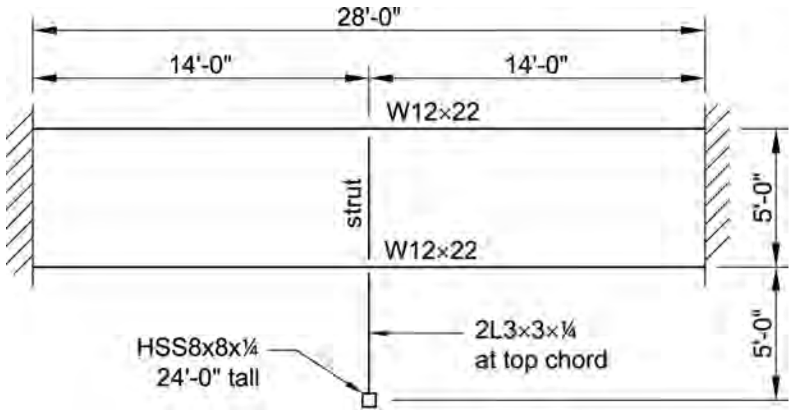


Fig. A-6.4-1. Plan view for Example A-6.4.

Solution:

Because the bracing system does not interact directly with any other braced point on the beam, the double angle and column constitute a point brace system. The strut will be assumed to be sufficiently strong and stiff to force the two beams to buckle together.

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

HSS column
ASTM A500 Grade C
F_y = 50 ksi
F_u = 62 ksi

Double-angle brace
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1, 1-12 and 1-15, the geometric properties are as follows:

Beam
W12×22
h_o = 11.9 in.

HSS column
HSS8×8×¼
I = 70.7 in.^{4}

Double-angle brace
2L3×3×¼
A = 2.88 in.^{2}

Required Flexural Strength of Beam

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$w_u = 1.2(0.05 \text{ kip/ft}) + 1.6(0.125 \text{ kip/ft})$ $= 0.260 \text{ kip/ft}$	$w_a = 0.05 \text{ kip/ft} + 0.125 \text{ kip/ft}$ $= 0.175 \text{ kip/ft}$

Determine the required flexural strength for a uniformly distributed load on the simply supported beam using AISC *Manual* Table 3-23, Case 1, as follows:

LRFD	ASD
$M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(0.260 \text{ kip/ft})(28 \text{ ft})^2}{8}$ $= 25.5 \text{ kip-ft}$	$M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(0.175 \text{ kip/ft})(28 \text{ ft})^2}{8}$ $= 17.2 \text{ kip-ft}$

It can be shown that the W12×22 beams are adequate with this unbraced length of 14 ft. Both beams need bracing in the same direction simultaneously.

Required Brace Strength and Stiffness

From AISC *Specification* Appendix 6, Section 6.3.1b, the required brace force for each beam, with C_d = 1.0 for bending in single curvature, is determined as follows:

LRFD	ASD
$M_r = M_u$ $= 25.5 \text{ kip-ft}$ $P_{br} = 0.02\left(\dfrac{M_r C_d}{h_o}\right)$ (*Spec.* Eq. A-6-7) $= 0.02\left[\dfrac{(25.5 \text{ kip-ft})(12 \text{ in./ft})(1.0)}{11.9 \text{ in.}}\right]$ $= 0.514 \text{ kip}$	$M_r = M_a$ $= 17.2 \text{ kip-ft}$ $P_{br} = 0.02\left(\dfrac{M_r C_d}{h_o}\right)$ (*Spec.* Eq. A-6-7) $= 0.02\left[\dfrac{(17.2 \text{ kip-ft})(12 \text{ in./ft})(1.0)}{11.9 \text{ in.}}\right]$ $= 0.347 \text{ kip}$

Because there are two beams, the total required brace force is:

LRFD	ASD
$P_{br} = 2(0.514 \text{ kip})$ $= 1.03 \text{ kips}$	$P_{br} = 2(0.347 \text{ kip})$ $= 0.694 \text{ kip}$

By inspection, the 2L3×3×¼ can carry the required bracing force. The HSS column can also carry the bracing force through bending on a 24-ft-long span. It will be shown that the change in length of the 2L3×3×¼ is negligible, so the available brace stiffness will come from the flexural stiffness of the column only.

From AISC *Specification* Appendix 6, Section 6.3.1b, with C_d = 1.0 for bending in single curvature, the required brace stiffness is:

LRFD	ASD
$\phi = 0.75$ $M_r = M_u$ $= 25.5 \text{ kip-ft}$ $\beta_{br} = \frac{1}{\phi}\left(\frac{10M_rC_d}{L_{br}h_o}\right)$ (*Spec*. Eq. A-6-8a) $= \frac{1}{0.75}\left[\frac{10(25.5 \text{ kip-ft})(12 \text{ in./ft})(1.0)}{(14 \text{ ft})(12 \text{ in./ft})(11.9 \text{ in.})}\right]$ $= 2.04 \text{ kip/in.}$	$\Omega = 2.00$ $M_r = M_a$ $= 17.2 \text{ kip-ft}$ $\beta_{br} = \Omega\left(\frac{10M_rC_d}{L_{br}h_o}\right)$ (*Spec*. Eq. A-6-8b) $= 2.00\left[\frac{10(17.2 \text{ kip-ft})(12 \text{ in./ft})(1.0)}{(14 \text{ ft})(12 \text{ in./ft})(11.9 \text{ in.})}\right]$ $= 2.06 \text{ kip/in.}$

The beams require one brace in order to have sufficient strength to carry the given load. However, locating that brace at midspan provides flexural strength greater than the required strength. The maximum unbraced length permitted for the required flexural strength is L_b = 18.2 ft from AISC *Manual* Table 6-2. Thus, according to AISC *Specification* Appendix 6, Section 6.3.1b, this length could be used in place of 14 ft to determine the required stiffness.

Available Stiffness of Brace

Because the brace stiffness comes from the combination of the axial stiffness of the double-angle member and the flexural stiffness of the column loaded at its midheight, the individual element stiffness will be determined and then combined.

The axial stiffness of the double angle is:

$$\beta = \frac{AE}{L}$$
$$= \frac{(2.88 \text{ in.}^2)(29{,}000 \text{ ksi})}{(5 \text{ ft})(12 \text{ in./ft})}$$
$$= 1{,}390 \text{ kip/in.}$$

The available flexural stiffness of the HSS column with a point load at midspan using AISC *Manual* Table 3-23, Case 7, is:

$$\beta = \frac{48EI}{L^3}$$

$$= \frac{48(29{,}000\text{ ksi})(70.7\text{ in.}^4)}{(24.0\text{ ft})^3(12\text{ in./ft})^3}$$

$$= 4.12\text{ kip/in.}$$

The combined stiffness is:

$$\frac{1}{\beta} = \frac{1}{\beta_{angles}} + \frac{1}{\beta_{column}}$$

$$= \frac{1}{1{,}390\text{ kip/in.}} + \frac{1}{4.12\text{ kip/in.}}$$

$$= 0.243\text{ in./kip}$$

Thus, the system stiffness is:

$$\beta = 4.12\text{ kip/in.}$$

The stiffness of the double-angle member could have reasonably been ignored.

Because the double-angle brace is ultimately bracing two beams, the required stiffness is multiplied by 2:

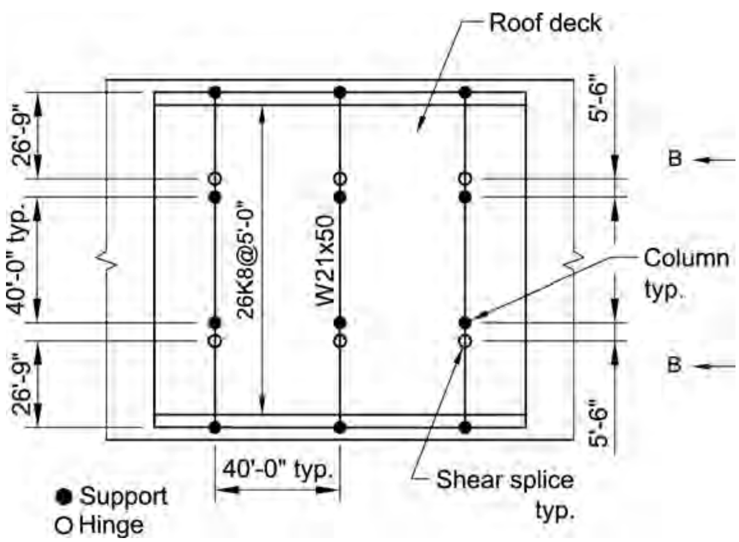
LRFD	ASD
$4.12\text{ kip/in.} \geq 2(2.04\text{ kip/in.})$ $4.12\text{ kip/in.} > 4.08\text{ kip/in.}$ **o.k.**	$4.12\text{ kip/in.} \geq 2(2.06\text{ kip/in.})$ $4.12\text{ kip/in.} = 4.12\text{ kip/in.}$ **o.k.**

The HSS8×8×¼ column is an adequate brace for the beams. However, if the column also carries an axial force, it must be checked for combined forces.

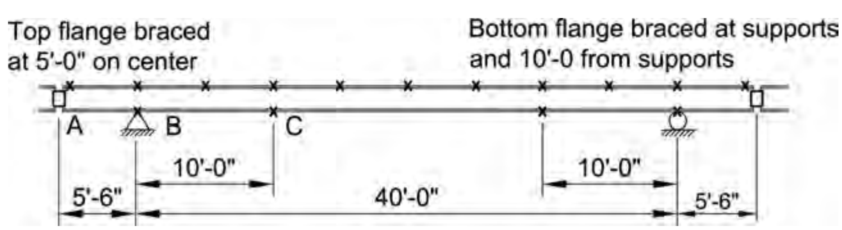
EXAMPLE A-6.5 POINT STABILITY BRACING OF A BEAM WITH REVERSE CURVATURE BENDING

Given:

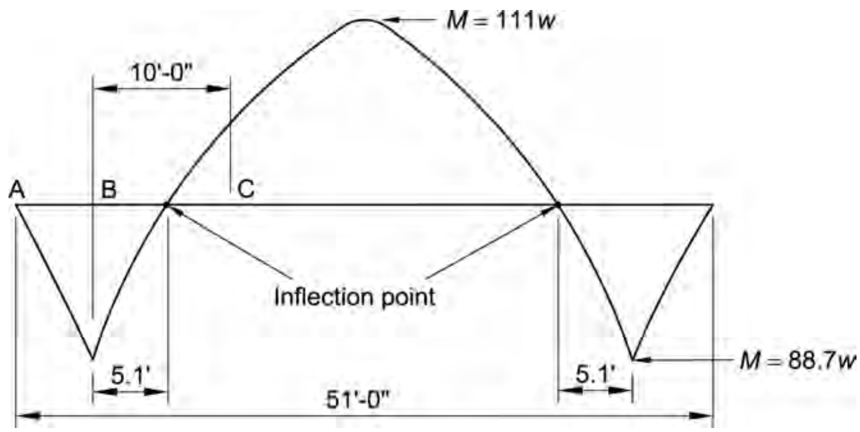
A roof system is composed of 26K8 steel joists spaced at 5-ft intervals and supported on ASTM A992 W21×50 girders as shown in Figure A-6.5-1(a). The roof dead load is 33 psf and the roof live load is 25 psf. Determine the required strength and stiffness of the braces needed to brace the girder at the support and near the inflection point. Bracing for the beam is shown in Figure A-6.5-1(b). Moment diagrams for the beam are shown in Figures A-6.5-1(c) and A-6.5-1(d). Determine the size of single-angle kickers connected to the bottom flange of the girder and the top chord of the joist, as shown in Figure A-6.5-1(e), where the brace force will be taken by a connected rigid diaphragm.



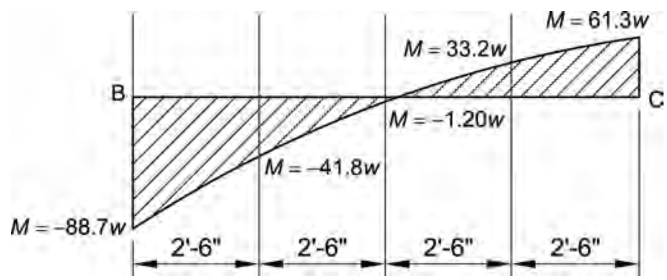
(a) Plan



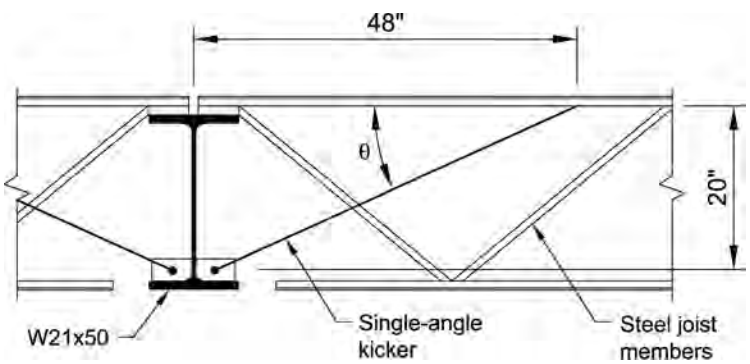
(b) Section B-B: Beam with bracing at top flanges by the steel joists and at the bottom flanges by the single-angle kickers



(c) Moment diagram of beam



(d) Moment diagram between points B and C



(e) Bracing configuration

Fig. A-6.5-1. Example A-6.5 configuration.

Solution:

Since the braces will transfer their force to a rigid roof diaphragm, they will be treated as point braces.

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Single-angle brace
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From the Steel Joist Institute:

Joist
K-Series
F_y = 50 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W21×50
h_o = 20.3 in.

Required Flexural Strength of Beam

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$w_u = 1.2(33 \text{ psf}) + 1.6(25 \text{ psf})$ $= 79.6 \text{ psf}$ $w_u = \dfrac{(79.6 \text{ psf})(40 \text{ ft})}{1{,}000 \text{ lb/kip}}$ $= 3.18 \text{ kip/ft}$ From Figure A-6.5-1(d): $M_{uB} = 88.7(3.18 \text{ kip/ft})$ $= 282 \text{ kip-ft}$	$w_a = 33 \text{ psf} + 25 \text{ psf}$ $= 58.0 \text{ psf}$ $w_a = \dfrac{(58.0 \text{ psf})(40 \text{ ft})}{1{,}000 \text{ lb/kip}}$ $= 2.32 \text{ kip/ft}$ From Figure A-6.5-1(d): $M_{aB} = 88.7(2.32 \text{ kip/ft})$ $= 206 \text{ kip-ft}$

Required Brace Strength and Stiffness

Determine the required force to brace the bottom flange of the girder with a point brace. The braces at points B and C will be determined based on the moment at B. However, because the brace at C is the closest to the inflection point, its strength and stiffness requirements are greater since they are influenced by the variable C_d which will be equal to 2.0.

From AISC *Specification* Appendix 6, Section 6.3.1b, the required brace force is determined as follows:

LRFD	ASD
$M_r = M_{uB}$ $= 282 \text{ kip-ft}$	$M_r = M_{aB}$ $= 206 \text{ kip-ft}$

LRFD	ASD
$P_{br} = 0.02\left(\dfrac{M_r C_d}{h_o}\right)$ (*Spec.* Eq. A-6-7) $= 0.02\left[\dfrac{(282\text{ kip-ft})(12\text{ in./ft})(2.0)}{20.3\text{ in.}}\right]$ $= 6.67\text{ kips}$	$P_{br} = 0.02\left(\dfrac{M_r C_d}{h_o}\right)$ (*Spec.* Eq. A-6-7) $= 0.02\left[\dfrac{(206\text{ kip-ft})(12\text{ in./ft})(2.0)}{20.3\text{ in.}}\right]$ $= 4.87\text{ kips}$

Determine the required stiffness of the point brace at point C. The required brace stiffness is a function of the unbraced length. It is permitted to use the maximum unbraced length permitted for the beam based upon the required flexural strength. Thus, determine the maximum unbraced length permitted.

Based on AISC *Specification* Section F1 and the moment diagram shown in Figure A-6.5-1(d), for the beam between points B and C, the lateral-torsional buckling modification factor, C_b, is:

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \qquad (\textit{Spec.}\text{ Eq. F1-1})$$

$$= \frac{12.5\left(\left|-88.7w\right|\right)}{2.5\left(\left|-88.7w\right|\right) + 3\left(\left|-41.8w\right|\right) + 4\left(\left|-1.2w\right|\right) + 3\left(\left|32.2w\right|\right)}$$

$$= 2.47$$

The maximum unbraced length for the required flexural strength can be determined by setting the available flexural strength based on AISC *Specification* Equation F2-3 (lateral-torsional buckling) equal to the required strength and solving for L_b (this is assuming that $L_b > L_r$).

LRFD	ASD
For a required flexural strength, M_u = 282 kip-ft, with C_b = 2.47, the unbraced length may be taken as: L_b = 22.0 ft	For a required flexural strength, M_a = 206 kip-ft, with C_b = 2.47, the unbraced length may be taken as: L_b = 20.6 ft

From AISC *Specification* Appendix 6, Section 6.3.1b, the required brace stiffness is:

LRFD	ASD
$\phi = 0.75$ $M_r = M_{uB}$ $= 282\text{ kip-ft}$ $\beta_{br} = \dfrac{1}{\phi}\left(\dfrac{10M_r C_d}{L_{br} h_o}\right)$ (*Spec.* Eq. A-6-8a) $= \dfrac{1}{0.75}\left[\dfrac{10(282\text{ kip-ft})(12\text{ in./ft})(2.0)}{(22.0\text{ ft})(12\text{ in./ft})(20.3\text{ in.})}\right]$ $= 16.8\text{ kip/in.}$	$\Omega = 2.00$ $M_r = M_{aB}$ $= 206\text{ kip-ft}$ $\beta_{br} = \Omega\left(\dfrac{10M_r C_d}{L_{br} h_o}\right)$ (*Spec.* Eq. A-6-8b) $= 2.00\left[\dfrac{10(206\text{ kip-ft})(12\text{ in./ft})(2.0)}{(20.6\text{ ft})(12\text{ in./ft})(20.3\text{ in.})}\right]$ $= 19.7\text{ kip/in.}$

Because no deformation will be considered in the connections, only the brace itself will be used to provide the required stiffness. The brace is oriented with the geometry as shown in Figure A-6.5-1(e). Thus, the force in the brace is $F_{br} = P_{br}/(\cos\theta)$ and the stiffness of the brace is $AE(\cos^2\theta)/L$. There are two braces at each brace point. One would be in tension and one in compression, depending on the direction that the girder attempts to buckle. For simplicity in design, a single brace will be selected that will be assumed to be in tension. Only the limit state of yielding will be considered.

Select a single angle to meet the requirements of strength and stiffness, with a length of:

$$L = \sqrt{(48 \text{ in.})^2 + (20 \text{ in.})^2}$$
$$= 52.0 \text{ in.}$$

Required Brace Force

LRFD	ASD
$F_{br} = \dfrac{P_{br}}{\cos\theta}$ $= \dfrac{6.67 \text{ kips}}{(48.0 \text{ in.}/52.0 \text{ in.})}$ $= 7.23 \text{ kips}$	$F_{br} = \dfrac{P_{br}}{\cos\theta}$ $= \dfrac{4.87 \text{ kips}}{(48.0 \text{ in.}/52.0 \text{ in.})}$ $= 5.28 \text{ kips}$

From AISC *Specification* Section D2(a), the required area based on available tensile strength is determined as follows:

$A_g = \dfrac{F_{br}}{\phi F_y}$ (modified *Spec.* Eq. D2-1) $= \dfrac{7.23 \text{ kips}}{0.90(36 \text{ kips})}$ $= 0.223 \text{ in.}^2$	$A_g = \dfrac{\Omega F_{br}}{F_y}$ (modified *Spec.* Eq. D2-1) $= \dfrac{1.67(5.28 \text{ kips})}{36 \text{ kips}}$ $= 0.245 \text{ in.}^2$

The required area based on stiffness is:

LRFD	ASD
$A_g = \dfrac{\beta_{br} L}{E\cos^2\theta}$ $= \dfrac{(16.8 \text{ kip/in.})(52.0 \text{ in.})}{(29{,}000 \text{ ksi})(48.0 \text{ in.}/52.0 \text{ in.})^2}$ $= 0.0354 \text{ in.}^2$	$A_g = \dfrac{\beta_{br} L}{E\cos^2\theta}$ $= \dfrac{(19.7 \text{ kip/in.})(52.0 \text{ in.})}{(29{,}000 \text{ ksi})(48.0 \text{ in.}/52.0 \text{ in.})^2}$ $= 0.0415 \text{ in.}^2$

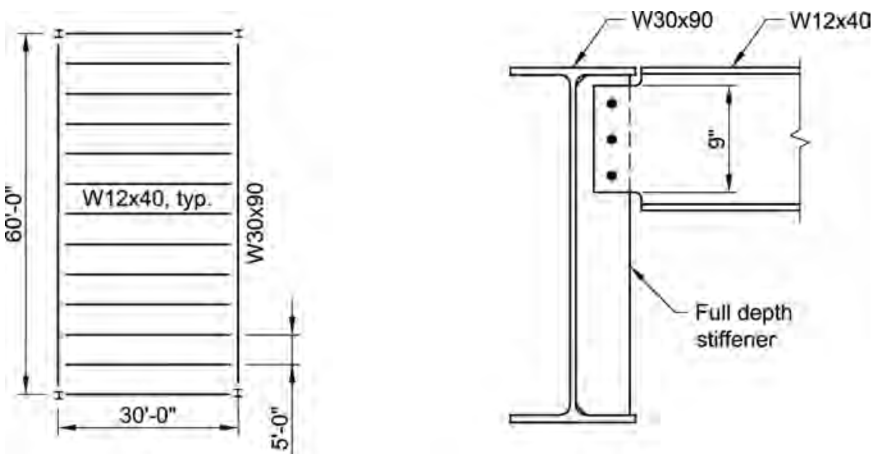
The strength requirement controls, therefore select L2×2×1/8 with $A = 0.491 \text{ in.}^2$

At the column at point B, the required strength would be one-half of that at point C, because $C_d = 1.0$ at point B instead of 2.0. However, since the smallest angle available has been selected for the brace, there is no reason to check further at the column and the same angle will be used there.

EXAMPLE A-6.6 POINT TORSIONAL STABILITY BRACING OF A BEAM

Given:

A roof system is composed of ASTM A992 W12×40 intermediate beams spaced 5 ft on center supporting a connected panel roof system that cannot be used as a diaphragm. As shown in Figure A-6.6-1, the beams span 30 ft and are supported on W30×90 girders spanning 60 ft. This is an isolated roof structure with no connections to other structures that could provide lateral support to the girder compression flanges. Thus, the flexural resistance of the attached beams must be used to provide torsional stability bracing of the girders. The roof dead load is 40 psf and the roof live load is 24 psf. Determine if the beams are sufficient to provide point torsional stability bracing.



(a) Plan *(b) Point torsional brace connection*

Fig. A-6.6-1. Roof system configuration

Solution:

Because the bracing beams are not connected in a way that would permit them to transfer an axial bracing force, they must behave as point torsional braces if they are to effectively brace the girders.

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and girder
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W12×40
t_w = 0.295 in.
I_x = 307 in.^{4}

Girder
W30×90
t_w = 0.470 in.
h_o = 28.9 in.
I_y = 115 in.^{4}

Required Flexural Strength of Girder

From ASCE/SEI 7, Chapter 2, and using AISC *Manual* Table 3-23, Case 1, the required strength of the girder is:

LRFD	ASD
$w_u = 1.2(40\text{ psf}) + 1.6(24\text{ psf})$ $= 86.4\text{ psf}$ $w_u = \dfrac{(86.4\text{ psf})(15\text{ ft})}{1{,}000\text{ lb/kip}}$ $= 1.30\text{ kip/ft}$ $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(1.30\text{ kip/ft})(60\text{ ft})^2}{8}$ $= 585\text{ kip-ft}$	$w_a = 40\text{ psf} + 24\text{ psf}$ $= 64.0\text{ psf}$ $w_a = \dfrac{(64.0\text{ psf})(15\text{ ft})}{1{,}000\text{ lb/kip}}$ $= 0.960\text{ kip/ft}$ $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(0.960\text{ kip/ft})(60\text{ ft})^2}{8}$ $= 432\text{ kip-ft}$

With C_b = 1.0, from AISC *Manual* Table 3-10, the maximum unbraced length permitted for the W30×90 based upon required flexural strength is:

LRFD	ASD
For M_{uB} = 585 kip-ft, L_b = 22.0 ft	For M_{aB} = 432 kip-ft, L_b = 20.7 ft

Point Torsional Brace Design

The required flexural strength for a point torsional brace for the girder is determined from AISC *Specification* Appendix 6, Section 6.3.2a.

LRFD	ASD
$M_r = M_{uB}$ $= 585\text{ kip-ft}$ $M_{br} = 0.02M_r$ (*Spec*. Eq. A-6-9) $= 0.02(585\text{ kip-ft})$ $= 11.7\text{ kip-ft}$	$M_r = M_{aB}$ $= 432\text{ kip-ft}$ $M_{br} = 0.02M_r$ (*Spec*. Eq. A-6-9) $= 0.02(432\text{ kip-ft})$ $= 8.64\text{ kip-ft}$

The required overall point torsional brace stiffness with braces every 5 ft, n = 11, and assuming C_b = 1.0, is determined in the following. Based on the User Note in *Specification* Section 6.3.2a:

$$I_{yeff} = I_y$$
$$= 115\text{ in.}^4$$

LRFD	ASD
$\phi = 0.75$	$\Omega = 3.00$
$\beta_T = \dfrac{1}{\phi}\dfrac{2.4L}{nEI_{yeff}}\left(\dfrac{M_r}{C_b}\right)^2$ (*Spec*. Eq. A-6-11a)	$\beta_T = \Omega\dfrac{2.4L}{nEI_{yeff}}\left(\dfrac{M_r}{C_b}\right)^2$ (*Spec*. Eq. A-6-11b)
$= \dfrac{1}{0.75}\left[\dfrac{2.4(60\text{ ft})(12\text{ in./ft})}{11(29{,}000\text{ ksi})(115\text{ in.}^4)}\right] \times \left[\dfrac{(585\text{ kip-ft})(12\text{ in./ft})}{1.0}\right]^2$	$= 3.00\left[\dfrac{2.4(60\text{ ft})(12\text{ in./ft})}{11(29{,}000\text{ ksi})(115\text{ in.}^4)}\right] \times \left[\dfrac{(432\text{ kip-ft})(12\text{ in./ft})}{1.0}\right]^2$
$= 3{,}100\text{ kip-in./rad}$	$= 3{,}800\text{ kip-in./rad}$

The distortional buckling stiffness of the girder web is a function of the web slenderness and the presence of any stiffeners. The web distortional stiffness is:

$$\beta_{sec} = \frac{3.3E}{h_o}\left(\frac{1.5h_o t_w^3}{12} + \frac{t_{st} b_s^3}{12}\right) \qquad \text{(\textit{Spec}. Eq. A-6-12)}$$

Therefore the distortional stiffness of the girder web alone is:

$$\begin{aligned}\beta_{sec} &= \frac{3.3E}{h_o}\left(\frac{1.5h_o t_w^3}{12}\right)\\ &= \frac{3.3(29{,}000\text{ ksi})}{28.9\text{ in.}}\left[\frac{1.5(28.9\text{ in.})(0.470\text{ in.})^3}{12}\right]\\ &= 1{,}240\text{ kip-in./rad}\end{aligned}$$

For AISC *Specification* Equation A-6-10 to give a nonnegative result, the web distortional stiffness given by Equation A-6-12 must be greater than the required point torsional stiffness given by Equation A-6-11. Because the web distortional stiffness of the girder is less than the required point torsional stiffness for both LRFD and ASD, web stiffeners will be required.

Determine the torsional stiffness contributed by the beams. Both girders will buckle in the same direction forcing the beams to bend in reverse curvature. Thus, the flexural stiffness of the beam using AISC *Manual* Table 3-23, Case 9, is:

$$\begin{aligned}\beta_{Tb} &= \frac{6EI}{L}\\ &= \frac{6(29{,}000\text{ ksi})(307\text{ in.}^4)}{(30\text{ ft})(12\text{ in./ft})}\\ &= 148{,}000\text{ kip-in./rad}\end{aligned}$$

Determining the required distortional stiffness of the girder will permit determination of the required stiffener size. The total stiffness is determined by summing the inverse of the distortional and flexural stiffnesses. Thus:

$$\frac{1}{\beta_T} = \frac{1}{\beta_{Tb}} + \frac{1}{\beta_{sec}}$$

Determine the minimum web distortional stiffness required to provide bracing for the girder.

LRFD	ASD
$\frac{1}{\beta_T} = \frac{1}{\beta_{Tb}} + \frac{1}{\beta_{sec}}$ $\frac{1}{3{,}100} = \frac{1}{148{,}000} + \frac{1}{\beta_{sec}}$ $\beta_{sec} = 3{,}170$ kip-in./rad	$\frac{1}{\beta_T} = \frac{1}{\beta_{Tb}} + \frac{1}{\beta_{sec}}$ $\frac{1}{3{,}800} = \frac{1}{148{,}000} + \frac{1}{\beta_{sec}}$ $\beta_{sec} = 3{,}900$ kip-in./rad

Determine the required width, b_s, of ⅜-in.-thick stiffeners.

LRFD	ASD
$\beta_{sec} = \frac{3.3E}{h_o}\left(\frac{1.5h_o t_w^3}{12} + \frac{t_{st} b_s^3}{12}\right)$ (*Spec*. Eq. A-6-12) Using the total required girder web distortional stiffness and the contribution of the girder web distortional stiffness calculated previously, solve for the required width for ⅜-in.-thick stiffeners: $3{,}170\text{ kip-in./rad} = 1{,}240\text{ kip-in./rad} + \frac{3.3(29{,}000\text{ ksi})}{28.9\text{ in.}}\left[\frac{(3/8\text{ in.})b_s^3}{12}\right]$ and $b_s = 2.65$ in.	$\beta_{sec} = \frac{3.3E}{h_o}\left(\frac{1.5h_o t_w^3}{12} + \frac{t_{st} b_s^3}{12}\right)$ (*Spec*. Eq. A-6-12) Using the total required girder web distortional stiffness and the contribution of the girder web distortional stiffness calculated previously, solve for the required width for ⅜-in.-thick stiffeners: $3{,}900\text{ kip-in./rad} = 1{,}240\text{ kip-in./rad} + \frac{3.3(29{,}000\text{ ksi})}{28.9\text{ in.}}\left[\frac{(3/8\text{ in.})b_s^3}{12}\right]$ and $b_s = 2.95$ in.

Therefore, use a 4 in. x ⅜ in. full depth one-sided stiffener at the connection of each beam.

Available Flexural Strength of Beam

Each beam is connected to a girder web stiffener. Thus, each beam will be coped at the top and bottom as shown in Figure A-6.6-1(b) with a depth at the coped section of 9 in. The available flexural strength of the coped beam is determined using the provisions of AISC *Specification* Sections J4.5 and F11.

$$M_n = M_p = F_y Z \le 1.6 F_y S_x \quad \text{(Spec. Eq. F11-1)}$$

For a rectangle, $Z < 1.6S$. Therefore, strength will be controlled by $F_y Z$ and

$$Z = \frac{(0.295\text{ in.})(9.00\text{ in.})^2}{4}$$
$$= 5.97\text{ in.}^3$$

The nominal flexural strength of the beam is:

$$M_n = F_y Z_x$$
$$= \frac{(50\text{ ksi})(5.97\text{ in.}^3)}{(12\text{ in./ft})}$$
$$= 24.9\text{ kip-ft}$$

LRFD	ASD
$\phi = 0.90$ $\phi M_n = 0.90(24.9 \text{ kip-ft})$ $= 22.4 \text{ kip-ft} > 11.7 \text{ kip-ft}$ **o.k.**	$\Omega = 1.67$ $\dfrac{M_n}{\Omega} = \dfrac{24.9 \text{ kip-ft}}{1.67}$ $= 14.9 \text{ kip-ft} > 8.64 \text{ kip-ft}$ **o.k.**

Neglecting any rotation due to the bolts moving in the holes or any influence of the end moments on the strength of the beams, this system has sufficient strength and stiffness to provide point torsional bracing to the girders.

Additional connection design limit states may also need to be checked.

APPENDIX 6 REFERENCES

Helwig, Todd A. and Yura, J.A. (1999), "Torsional Bracing of Columns," *Journal of Structural Engineering*, ASCE, Vol. 125, No. 5, pp. 547–555.

Yura, J.A. (2001), "Fundamentals of Beam Bracing," *Engineering Journal*, AISC, Vol. 38, No. 1, pp. 11–26.

Ziemian, R.D. (ed.) (2010), *Guide to Stability Design Criteria for Metal Structures,* 6th Ed., John Wiley & Sons, Inc., Hoboken, NJ.

Chapter IIA
Simple Shear Connections

The design of connecting elements are covered in Part 9 of the AISC *Manual*. The design of simple shear connections is covered in Part 10 of the AISC *Manual*.

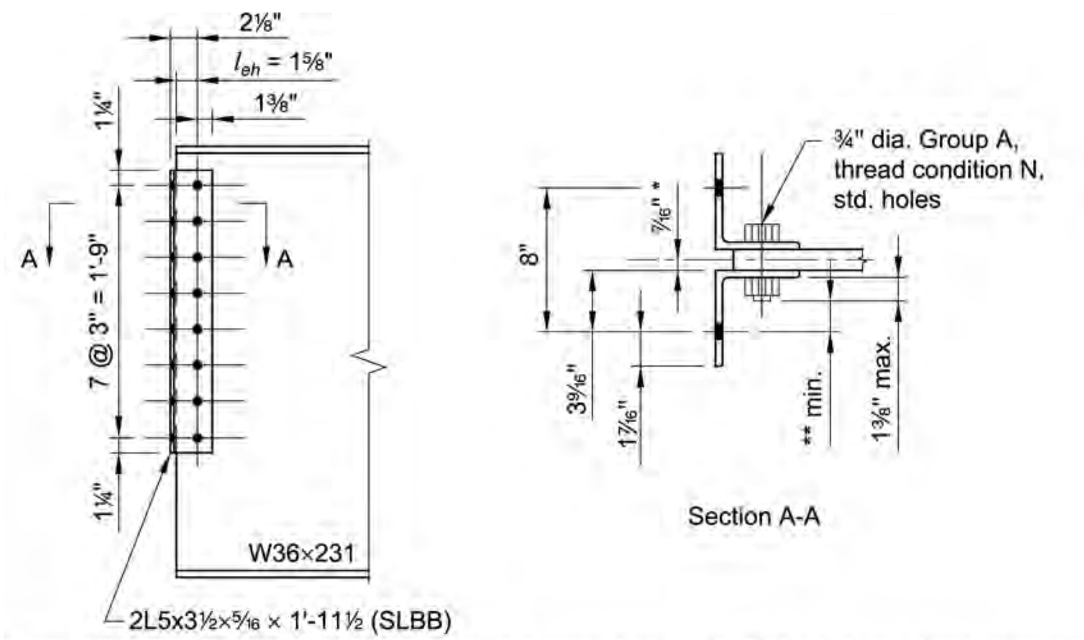
EXAMPLE II.A-1A ALL-BOLTED DOUBLE-ANGLE CONNECTION

Given:

Using the tables in AISC *Manual* Part 10, verify the available strength of an all-bolted double-angle shear connection between an ASTM A992 W36×231 beam and an ASTM A992 W14×90 column flange, as shown in Figure IIA-1A-1, supporting the following beam end reactions:

R_D = 37.5 kips
R_L = 113 kips

Use ASTM A36 angles.



* This dimension (see sketch, Section A) is determined as one-half of the decimal web thickness rounded to the next higher 1⁄16 in. Example: 0.760"/2 = 0.380"; use 7⁄16 in. This will produce spacing of holes in the supporting beam slightly larger than detailed in the angles to permit spreading of angles (angles can be spread but not closed) at time of erection to supporting member. Alternatively, consider using horizontal short slots in the support legs of the angles.
**See AISC *Manual* Tables 7-15 and 7-16 for driving clearance.

Fig. IIA-1A-1. Connection geometry for Example II.A-1A.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W36×231
t_w = 0.760 in.

Column
W14×90
t_f = 0.710 in.

From AISC *Specification* Table J3.3, the hole diameter for a ¾-in.-diameter bolt with standard holes is:

$d_h = \frac{13}{16}$ in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(37.5 \text{ kips}) + 1.6(113 \text{ kips})$ $= 226 \text{ kips}$	$R_a = 37.5 \text{ kips} + 113 \text{ kips}$ $= 151 \text{ kips}$

Connection Selection

AISC *Manual* Table 10-1 includes checks for the limit states of bolt shear, bolt bearing and tearout on the angles, shear yielding of the angles, shear rupture of the angles, and block shear rupture of the angles.

Try 8 rows of bolts and 2L5×3½×5⁄16 (SLBB). From AISC *Manual* Table 10-1:

LRFD	ASD
$\phi R_n = 248 \text{ kips} > 226 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 165 \text{ kips} > 151 \text{ kips}$ **o.k.**

Available Beam Web Strength

The available beam web strength is the lesser of the limit states of block shear rupture, shear yielding, shear rupture, and the sum of the effective strengths of the individual fasteners. Because the beam is not coped, the only applicable limit state is the effective strength of the individual fasteners, which is the lesser of the bolt shear strength per AISC *Specification* Section J3.6, and the bolt bearing and tearout strength per AISC *Specification* Section J3.10.

Bolt Shear

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear is:

LRFD	ASD
$\phi R_n = 35.8 \text{ kips/bolt}$	$\dfrac{R_n}{\Omega} = 23.9 \text{ kips/bolt}$

Bolt Bearing on Beam Web

The nominal bearing strength of the beam web per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$r_n = 2.4dtF_u \quad \text{(\textit{Spec}. Eq. J3-6a)}$$
$$= 2.4(\tfrac{3}{4}\text{ in.})(0.760\text{ in.})(65\text{ ksi})$$
$$= 88.9\text{ kips/bolt}$$

From AISC *Specification* Section J3.10, the available bearing strength of the beam web per bolt is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(88.9\text{ kips/bolt})$ $= 66.7\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{88.9\text{ kips/bolt}}{2.00}$ $= 44.5\text{ kips/bolt}$

Bolt Tearout on Beam Web

The available tearout strength of the beam web per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$l_c = 3.00\text{ in.} - \tfrac{13}{16}\text{ in.}$$
$$= 2.19\text{ in.}$$

The available tearout strength is:

$$r_n = 1.2l_c tF_u \quad \text{(\textit{Spec}. Eq. J3-6c)}$$
$$= 1.2(2.19\text{ in.})(0.760\text{ in.})(65\text{ ksi})$$
$$= 130\text{ kips/bolt}$$

From AISC *Specification* Section J3.10, the available tearout strength of the beam web per bolt is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(130\text{ kips/bolt})$ $= 97.5\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{130\text{ kips/bolt}}{2.00}$ $= 65.0\text{ kips/bolt}$

Bolt shear strength is the governing limit state for all bolts at the beam web. Bolt shear strength is one of the limit states included in the capacities shown in Table 10-1 as used above; thus, the effective strength of the fasteners is adequate.

Available Strength at the Column Flange

Since the thickness of the column flange, t_f = 0.710 in., is greater than the thickness of the angles, t = 5⁄16 in., bolt bearing will control for the angles, which was previously checked. The column flange is adequate for the required loading.

Conclusion

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-1B ALL-BOLTED DOUBLE-ANGLE CONNECTION SUBJECT TO AXIAL AND SHEAR LOADING

Given:

Verify the available strength of an all-bolted double-angle connection for an ASTM A992 W18×50 beam, as shown in Figure II.A-1B-1, to support the following beam end reactions:

LRFD	ASD
Shear, V_u = 75 kips Axial tension, N_u = 60 kips	Shear, V_a = 50 kips Axial tension, N_a = 40 kips

Use ASTM A36 double angles that will be shop-bolted to the beam.

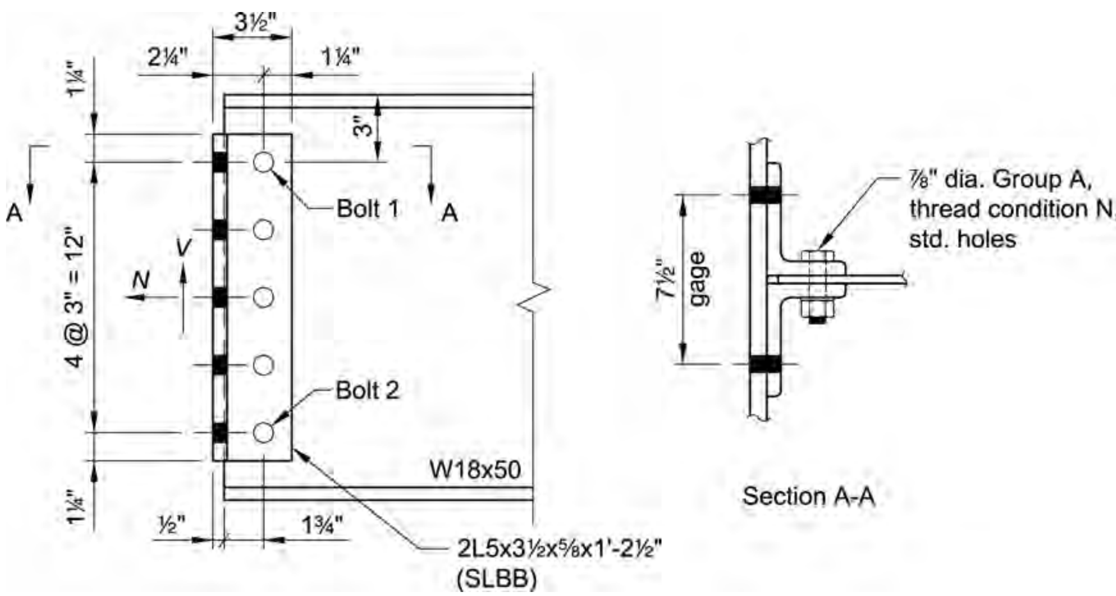


Fig. II.A-1B-1. Connection geometry for Example II.A-1B.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×50
A_g = 14.7 in.^{2}
d = 18.0 in.
t_w = 0.355 in.
t_f = 0.570 in.

From AISC *Specification* Table J3.3, the hole diameter for ⅞-in.-diameter bolts with standard holes is:

d_h = 15⁄16 in.

The resultant load is:

LRFD	ASD
$R_u = \sqrt{V_u^2 + N_u^2}$ $= \sqrt{(75 \text{ kips})^2 + (60 \text{ kips})^2}$ $= 96.0 \text{ kips}$	$R_a = \sqrt{V_a^2 + N_a^2}$ $= \sqrt{(50 \text{ kips})^2 + (40 \text{ kips})^2}$ $= 64.0 \text{ kips}$

Try 5 rows of bolts and 2L5×3½×⅝ (SLBB).

Strength of the Bolted Connection—Angles

From the Commentary to AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the invividual strengths of the individual fasteners, which may be taken as the lesser of the fastener shear strength per AISC *Specification* Section J3.6, the bearing strength at the bolt hole per AISC *Specification* Section J3.10, or the tearout strength at the bolt hole per AISC *Specification* Section J3.10.

Bolt shear

From AISC *Manual* Table 7-1, the available shear strength for ⅞-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear (or pair of bolts) is:

LRFD	ASD
$\phi r_n = 48.7$ kips/bolt (or per pair of bolts)	$\dfrac{r_n}{\Omega} = 32.5$ kips/bolt (or per pair of bolts)

Bolt bearing on angles

The available bearing strength of the angles per bolt in double shear is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$r_n = (2 \text{ angles})2.4dtF_u \qquad \text{(from } \textit{Spec.} \text{ Eq. J3-6a)}$$
$$= (2 \text{ angles})(2.4)(\tfrac{7}{8} \text{ in.})(\tfrac{5}{8} \text{ in.})(58 \text{ ksi})$$
$$= 152 \text{ kips/bolt}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(152 \text{ kips/bolt})$ $= 114 \text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{152 \text{ kips/bolt}}{2.00}$ $= 76.0 \text{ kips/bolt}$

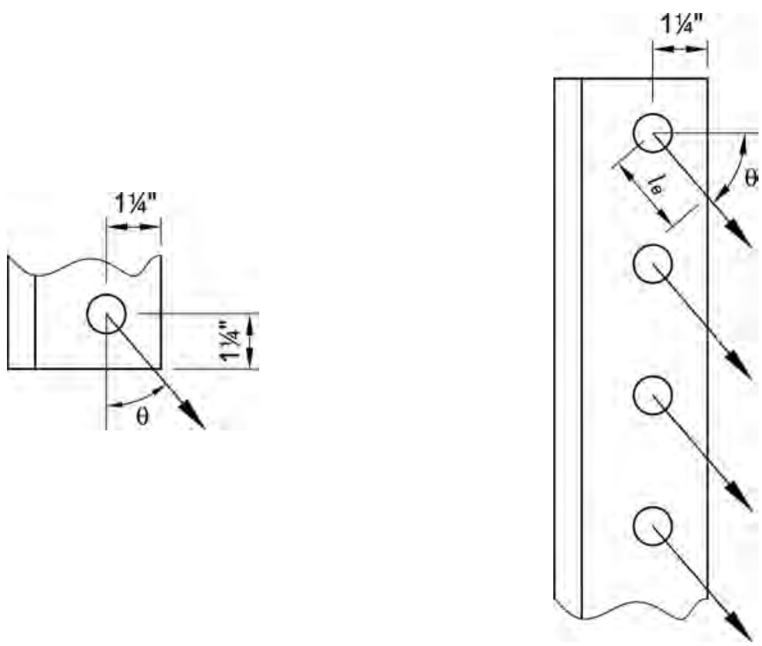
Bolt tearout on angles

From AISC *Specification* Section J3.10, the available tearout strength of the angles per bolt in double shear is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration.

As shown in Figures II.A-1B-2(a) and II.A-1B-2(b), the tearout dimensions on the angle differ between the edge bolt and the other bolts.

The angle θ, as shown in Figure II.A-1B-2(a), of the resultant force on the edge bolt is:

LRFD	ASD
$\theta = \tan^{-1}\left(\dfrac{N_u}{V_u}\right)$ $= \tan^{-1}\left(\dfrac{60 \text{ kips}}{75 \text{ kips}}\right)$ $= 38.7°$	$\theta = \tan^{-1}\left(\dfrac{N_a}{V_a}\right)$ $= \tan^{-1}\left(\dfrac{40 \text{ kips}}{50 \text{ kips}}\right)$ $= 38.7°$



(a) Edge bolt *(b) Other bolts*

Fig. II.A-1B-2. Bolt tearout on angles.

The length from the center of the bolt hole to the edge of the angle along the line of action of the force is:

$$\begin{aligned} l_e &= \frac{1\frac{1}{4}\text{ in.}}{\cos 38.7°} \\ &= 1.60\text{ in.} \end{aligned}$$

The clear distance, along the line of action of the force, between the edge of the hole and the edge of the angle is:

$$\begin{aligned} l_c &= l_e - 0.5d_h \\ &= 1.60\text{ in.} - 0.5\left(\tfrac{15}{16}\text{ in.}\right) \\ &= 1.13\text{ in.} \end{aligned}$$

The available tearout strength of the pair of angles at the edge bolt is:

$$\begin{aligned} r_n &= (2\text{ angles})1.2l_c tF_u \\ &= (2\text{ angles})(1.2)(1.13\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi}) \\ &= 98.3\text{ kips/bolt} \end{aligned}$$ (from *Spec.* Eq. J3-6c)

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(98.3\text{ kips/bolt})$ $= 73.7\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{98.3\text{ kips/bolt}}{2.00}$ $= 49.2\text{ kips/bolt}$

Therefore, bolt shear controls over bearing or tearout of the angles at the edge bolt.

The angle θ, as shown in Figure II.A-1B-2(b), of the resultant force on the other bolts is:

LRFD	ASD
$\theta = \tan^{-1}\left(\dfrac{V_u}{N_u}\right)$ $= \tan^{-1}\left(\dfrac{75\text{ kips}}{60\text{ kips}}\right)$ $= 51.3°$	$\theta = \tan^{-1}\left(\dfrac{V_a}{N_a}\right)$ $= \tan^{-1}\left(\dfrac{50\text{ kips}}{40\text{ kips}}\right)$ $= 51.3°$

The length from the center of the bolt hole to the edge of the angle along the line of action of the force is:

$$\begin{aligned} l_e &= \frac{1\frac{1}{4}\text{ in.}}{\cos 51.3°} \\ &= 2.00\text{ in.} \end{aligned}$$

The clear distance, along the line of action of the force, between the edge of the hole and the edge of the angle is:

$$\begin{aligned} l_c &= l_e - 0.5d_h \\ &= 2.00\text{ in.} - 0.5\left(\tfrac{15}{16}\text{ in.}\right) \\ &= 1.53\text{ in.} \end{aligned}$$

The available tearout strength of the pair of angles at the other bolts is:

$$r_n = (2 \text{ angles})1.2l_c tF_u$$ (from *Spec.* Eq. J3-6c)

$$= (2 \text{ angles})(1.2)(1.53 \text{ in.})(\tfrac{5}{8} \text{ in.})(58 \text{ ksi})$$

$$= 133 \text{ kips/bolt}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(133 \text{ kips/bolt})$ $= 99.8 \text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{133 \text{ kips/bolt}}{2.00}$ $= 66.5 \text{ kips/bolt}$

Therefore, bolt shear controls over bearing or tearout of the angles at the other bolt.

The effective strength for the bolted connection at the angles is determined by summing the effective strength for each bolt using the minimum available strength calculated for bolt shear, bearing on the angles, and tearout on the angles.

LRFD	ASD
$\phi R_n = n\phi r_n$ $= (5 \text{ bolts})(48.7 \text{ kips/bolt})$ $= 244 \text{ kips} > 96.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = n\dfrac{r_n}{\Omega}$ $= (5 \text{ bolts})(32.5 \text{ kips/bolt})$ $= 163 \text{ kips} > 64.0 \text{ kips}$ **o.k.**

Strength of the Bolted Connection—Beam Web

Bolt bearing on beam web

The available bearing strength of the beam web per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$r_n = 2.4dtF_u$$ (*Spec.* Eq. J3-6a)

$$= 2.4(\tfrac{7}{8} \text{ in.})(0.355 \text{ in.})(65 \text{ ksi})$$

$$= 48.5 \text{ kips/bolt}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(48.5 \text{ kips/bolt})$ $= 36.4 \text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{48.5 \text{ kips/bolt}}{2.00}$ $= 24.3 \text{ kips/bolt}$

Bolt tearout on beam web

From AISC *Specification* Section J3.10, the available tearout strength of the beam web is determined from AISC *Specification* Equation J3-6a, assuming deformation at the bolt hole is a design consideration, where the edge distance, l_c, is based on the angle of the resultant load. As shown in Figure II.A-1B-3, a horizontal edge distance of 1½ in. is used which includes a ¼ in. tolerance to account for possible mill underrun.

The angle, θ, of the resultant force is:

LRFD	ASD
$\theta = \tan^{-1}\left(\frac{V_u}{N_u}\right)$ $= \tan^{-1}\left(\frac{75\text{ kips}}{60\text{ kips}}\right)$ $= 51.3°$	$\theta = \tan^{-1}\left(\frac{V_a}{N_a}\right)$ $= \tan^{-1}\left(\frac{50\text{ kips}}{40\text{ kips}}\right)$ $= 51.3°$

The length from the center of the bolt hole to the edge of the web along the line of action of the force is:

$$\begin{aligned} l_e &= \frac{1\tfrac{1}{2}\text{ in.}}{\cos 51.3°} \\ &= 2.40\text{ in.} \end{aligned}$$

The clear distance, along the line of action of the force, between the edge of the hole and the edge of the web is:

$$\begin{aligned} l_c &= l_e - 0.5d_h \\ &= 2.40\text{ in.} - 0.5\left(\tfrac{15}{16}\text{ in.}\right) \\ &= 1.93\text{ in.} \end{aligned}$$

The available tearout strength of the beam web is determined as follows:

$$\begin{aligned} r_n &= 1.2 l_c t F_u \\ &= 1.2(1.93\text{ in.})(0.355\text{ in.})(65\text{ ksi}) \\ &= 53.4\text{ kips/bolt} \end{aligned}$$ (*Spec*. Eq. J3-6c)

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(53.4\text{ kips/bolt})$ $= 40.1\text{ kips/bolt}$	$\Omega = 2.00$ $\frac{r_n}{\Omega} = \frac{53.4\text{ kips/bolt}}{2.00}$ $= 26.7\text{ kips/bolt}$

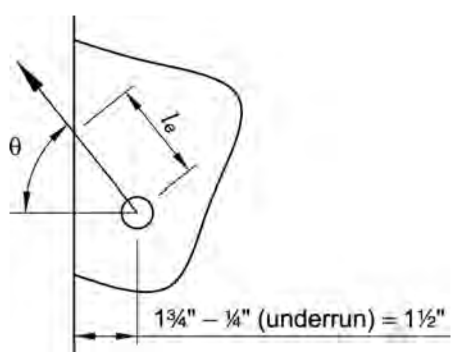


Fig. II.A-1B-3. Bolt tearout on beam web.

Therefore, bolt bearing on the beam web is the controlling limit state for all bolts.

The effective strength for the bolted connection at the beam web is determined by summing the effective strength for each bolt using the minimum available strength calculated for bolt shear, bearing on the beam web, and tearout on the beam web.

LRFD	ASD
$\phi R_n = n\phi r_n$ $= (5\text{ bolts})(36.4\text{ kips/bolt})$ $= 182\text{ kips} > 96.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = n\dfrac{r_n}{\Omega}$ $= (5\text{ bolts})(24.3\text{ kips/bolt})$ $= 122\text{ kips} > 64.0\text{ kips}$ **o.k.**

Bolt Shear and Tension Interaction—Outstanding Angle Legs

The available tensile strength of the bolts due to the effect of combined tension and shear is determined from AISC *Specification* Section J3.7.

The required shear stress is:

$$f_{rv} = \frac{V_r}{nA_b}$$

where

$A_b = 0.601\text{ in.}^2$ (from AISC *Manual* Table 7-1)

$n = 10$

LRFD	ASD
$f_{rv} = \dfrac{V_u}{nA_b}$ $= \dfrac{75\text{ kips}}{10(0.601\text{ in.}^2)}$ $= 12.5\text{ ksi}$	$f_{rv} = \dfrac{V_a}{nA_b}$ $= \dfrac{50\text{ kips}}{10(0.601\text{ in.}^2)}$ $= 8.32\text{ ksi}$

The nominal tensile strength modified to include the effects of shear stress is determined from AISC *Specification* Section J3.7 as follows. From AISC *Specification* Table J3.2:

$F_{nt} = 90\text{ ksi}$

$F_{nv} = 54\text{ ksi}$

LRFD	ASD
$\phi = 0.75$ $F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$ (*Spec.* Eq. J3-3a) $= 1.3(90\text{ ksi}) - \dfrac{90\text{ ksi}}{0.75(54\text{ ksi})}(12.5\text{ ksi}) < 90\text{ ksi}$ $= 89.2\text{ ksi} < 90\text{ ksi}$	$\Omega = 2.00$ $F'_{nt} = 1.3F_{nt} - \dfrac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$ (*Spec.* Eq. J3-3b) $= 1.3(90\text{ ksi}) - \dfrac{2.00(90\text{ ksi})}{54\text{ ksi}}(8.32\text{ ksi}) < 90\text{ ksi}$ $= 89.3\text{ ksi} < 90\text{ ksi}$

LRFD	ASD
Therefore: $F'_{nt} = 89.2$ ksi	Therefore: $F'_{nt} = 89.3$ ksi

Using the value of F'_{nt} determined for LRFD, the nominal tensile strength of one bolt is:

$$\begin{aligned} r_n &= F'_{nt} A_b \\ &= (89.2 \text{ ksi})(0.601 \text{ in.}^2) \\ &= 53.6 \text{ kips} \end{aligned}$$ (from *Spec*. Eq. J3-2)

The available tensile strength of the bolts due to combined tension and shear is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(53.6 \text{ kips/bolt})$ $= 40.2$ kips $\phi R_n = n\phi r_n$ $= (10 \text{ bolts})(40.2 \text{ kips/bolt})$ $= 402 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{53.6 \text{ kips/bolt}}{2.00}$ $= 26.8$ kips $\dfrac{R_n}{\Omega} = n\dfrac{r_n}{\Omega}$ $= (10 \text{ bolts})(26.8 \text{ kips/bolt})$ $= 268 \text{ kips} > 40 \text{ kips}$ **o.k.**

Prying Action

From AISC *Manual* Part 9, the available tensile strength of the bolts in the outstanding angle legs taking prying action into account is determined as follows:

$$\begin{aligned} a &= \frac{2(angle\ leg) + t_w - gage}{2} \\ &= \frac{2(5 \text{ in.}) + 0.355 \text{ in.} - 7\tfrac{1}{2} \text{ in.}}{2} \\ &= 1.43 \text{ in.} \end{aligned}$$

$$\begin{aligned} b &= \frac{gage - t_w - t}{2} \\ &= \frac{7\tfrac{1}{2} \text{ in.} - 0.355 \text{ in.} - \tfrac{5}{8} \text{ in.}}{2} \\ &= 3.26 \text{ in.} \end{aligned}$$

$$\begin{aligned} a' &= \left(a + \frac{d_b}{2}\right) \le \left(1.25b + \frac{d_b}{2}\right) \\ &= 1.43 \text{ in.} + \frac{\tfrac{7}{8} \text{ in.}}{2} \le 1.25(3.26 \text{ in.}) + \frac{\tfrac{7}{8} \text{ in.}}{2} \\ &= 1.87 \text{ in.} < 4.51 \text{ in.} \quad \textbf{o.k.} \end{aligned}$$ (*Manual* Eq. 9-23)

$$b' = \left(b - \frac{d_b}{2}\right) \quad \text{(\textit{Manual} Eq. 9-18)}$$
$$= 3.26 \text{ in.} - \frac{\tfrac{7}{8} \text{ in.}}{2}$$
$$= 2.82 \text{ in.}$$

$$\rho = \frac{b'}{a'} \quad \text{(\textit{Manual} Eq. 9-22)}$$
$$= \frac{2.82 \text{ in.}}{1.87 \text{ in.}}$$
$$= 1.51$$

Note that end distances of 1¼ in. are used on the angles, so p is the average pitch of the bolts:

$$p = \frac{l}{n}$$
$$= \frac{14\tfrac{1}{2} \text{ in.}}{5 \text{ rows}}$$
$$= 2.90 \text{ in.}$$

Check:

$p < s = 3.00$ in. **o.k.**

$$\delta = 1 - \frac{d'}{p} \quad \text{(\textit{Manual} Eq. 9-20)}$$
$$= 1 - \frac{\tfrac{15}{16} \text{ in.}}{2.90 \text{ in.}}$$
$$= 0.677$$

The angle thickness required to develop the available strength of the bolt with no prying action is determined as follows:

LRFD	ASD
$\phi = 0.90$ $B_c = 40.2$ kips/bolt (calculated previously) $t_c = \sqrt{\dfrac{4B_c b'}{\phi p F_u}}$ (*Manual* Eq. 9-26a) $= \sqrt{\dfrac{4(40.2 \text{ kips/bolt})(2.82 \text{ in.})}{0.90(2.90 \text{ in.})(58 \text{ ksi})}}$ $= 1.73$ in.	$\Omega = 1.67$ $B_c = 26.8$ kips/bolt (calculated previously) $t_c = \sqrt{\dfrac{\Omega 4B_c b'}{p F_u}}$ (*Manual* Eq. 9-26b) $= \sqrt{\dfrac{1.67(4)(26.8 \text{ kips/bolt})(2.82 \text{ in.})}{(2.90 \text{ in.})(58 \text{ ksi})}}$ $= 1.73$ in.

$$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right] \qquad \text{(\textit{Manual} Eq. 9-28)}$$

$$= \frac{1}{0.677(1+1.51)}\left[\left(\frac{1.73 \text{ in.}}{5/8 \text{ in.}}\right)^2 - 1\right]$$

$$= 3.92$$

Because $\alpha' > 1$, the angles have insufficient strength to develop the bolt strength, therefore:

$$Q = \left(\frac{t}{t_c}\right)^2 (1+\delta)$$

$$= \left(\frac{5/8 \text{ in.}}{1.73 \text{ in.}}\right)^2 (1+0.677)$$

$$= 0.219$$

The available tensile strength of the bolts, taking prying action into account, is determined using AISC *Manual* Equation 9-27, as follows:

LRFD	ASD
$\phi r_n = B_c Q$ $= (40.2 \text{ kips/bolt})(0.219)$ $= 8.80 \text{ kips/bolt}$	$\frac{r_n}{\Omega} = B_c Q$ $= (26.8 \text{ kips/bolt})(0.219)$ $= 5.87 \text{ kips/bolt}$
$\phi R_n = n\phi r_n$ $= (10 \text{ bolts})(8.80 \text{ kips/bolt})$ $= 88.0 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\frac{R_n}{\Omega} = n\frac{r_n}{\Omega}$ $= (10 \text{ bolts})(5.87 \text{ kips/bolt})$ $= 58.7 \text{ kips} > 40 \text{ kips}$ **o.k.**

Shear Strength of Angles

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the angles is determined as follows:

$$A_{gv} = (2 \text{ angles})lt$$

$$= (2 \text{ angles})(14\tfrac{1}{2} \text{ in.})(5/8 \text{ in.})$$

$$= 18.1 \text{ in.}^2$$

$$R_n = 0.60 F_y A_{gv} \qquad \text{(\textit{Spec.} Eq. J4-3)}$$

$$= 0.60(36 \text{ ksi})(18.1 \text{ in.}^2)$$

$$= 391 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(391 \text{ kips})$ $= 391 \text{ kips} > 96.0 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{391 \text{ kips}}{1.50}$ $= 261 \text{ kips} > 64.0 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2, the available shear rupture strength of the angle is determined using the net area determined in accordance with AISC *Specification* Section B4.3b.

$$A_{nv} = (2 \text{ angles})\left[l - n\left(d_h + \tfrac{1}{16} \text{ in.}\right)\right]t$$
$$= (2 \text{ angles})\left[14\tfrac{1}{2} \text{ in.} - 5\left(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)\right]\left(\tfrac{5}{8} \text{ in.}\right)$$
$$= 11.9 \text{ in.}^2$$

$$R_n = 0.60F_u A_{nv} \qquad \text{(}Spec. \text{ Eq. J4-4)}$$
$$= 0.60(58 \text{ ksi})\left(11.9 \text{ in.}^2\right)$$
$$= 414 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(414 \text{ kips})$ $= 311 \text{ kips} > 96.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{414 \text{ kips}}{2.00}$ $= 207 \text{ kips} > 64.0 \text{ kips}$ **o.k.**

Tensile Strength of Angles

From AISC Specification Section J4.1(a), the available tensile yielding strength of the angles is determined as follows:

$$A_g = (2 \text{ angles})lt$$
$$= (2 \text{ angles})\left(14\tfrac{1}{2} \text{ in.}\right)\left(\tfrac{5}{8} \text{ in.}\right)$$
$$= 18.1 \text{ in.}^2$$

$$R_n = F_y A_g \qquad \text{(}Spec. \text{ Eq. J4-1)}$$
$$= (36 \text{ ksi})\left(18.1 \text{ in.}^2\right)$$
$$= 652 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(652 \text{ kips})$ $= 587 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{652 \text{ kips}}{1.67}$ $= 390 \text{ kips} > 40 \text{ kips}$ **o.k.**

From AISC *Specification* Sections J4.1, the available tensile rupture strength of the angles is determined from AISC *Specification* Equation J4-2. Table D3.1, Case 1 applies in this case because the tension load is transmitted directly to the cross-sectional element by fasteners; therefore, $U = 1.00$. With $A_{nt} = A_{nv}$ (calculated previously), the effective net area is:

$$A_e = A_{nt}U \qquad \text{(Spec. Eq. D3-1)}$$
$$= (11.9 \text{ in.}^2)(1.00)$$
$$= 11.9 \text{ in.}^2$$

$$R_n = F_u A_e \qquad \text{(Spec. Eq. J4-2)}$$
$$= (58 \text{ ksi})(11.9 \text{ in.}^2)$$
$$= 690 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(690 \text{ kips})$ $= 518 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{690 \text{ kips}}{2.00}$ $= 345 \text{ kips} > 40 \text{ kips}$ **o.k.**

Block Shear Rupture of Angles—Beam Web Side

The nominal strength for the limit state of block shear rupture of the angles, assuming an L-shaped tearout due the shear load only, is determined as follows. The tearout pattern is shown in Figure II.A-1B-4.

$$R_{bsv} = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad \text{(Spec. Eq. J4-5)}$$

where

$$A_{gv} = (2 \text{ angles})(l - l_{ev})t$$
$$= (2 \text{ angles})(14\tfrac{1}{2} \text{ in.} - 1\tfrac{1}{4} \text{ in.})(\tfrac{5}{8} \text{ in.})$$
$$= 16.6 \text{ in.}^2$$

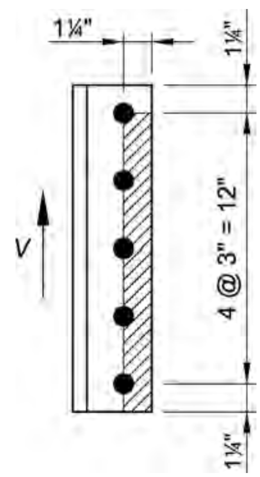


Fig. II.A-1B-4. Block shear rupture of angles for shear load only.

$$A_{nv} = A_{gv} - (2\text{ angles})(n - 0.5)(d_h + \tfrac{1}{16}\text{ in.})t$$
$$= 16.6\text{ in.}^2 - (2\text{ angles})(5 - 0.5)(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{5}{8}\text{ in.})$$
$$= 11.0\text{ in.}^2$$

$$A_{nt} = (2\text{ angles})\left[l_{eh} - 0.5(d_h + \tfrac{1}{16}\text{ in.})\right]t$$
$$= (2\text{ angles})\left[1\tfrac{1}{4}\text{ in.} - 0.5(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](\tfrac{5}{8}\text{ in.})$$
$$= 0.938\text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$R_{bsv} = 0.60(58\text{ ksi})(11.0\text{ in.}^2) + 1.0(58\text{ ksi})(0.938\text{ in.}^2) \le 0.60(36\text{ ksi})(16.6\text{ in.}^2) + 1.0(58\text{ ksi})(0.938\text{ in.}^2)$$
$$= 437\text{ kips} > 413\text{ kips}$$

Therefore:

$$R_{bsv} = 413\text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture on the angles is:

LRFD	ASD
$\phi = 0.75$ $\phi R_{bsv} = 0.75(413\text{ kips})$ $= 310\text{ kips} > 75\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_{bsv}}{\Omega} = \dfrac{413\text{ kips}}{2.00}$ $= 207\text{ kips} > 50\text{ kips}$ **o.k.**

The block shear rupture failure path due to axial load only could occur as an L- or U-shape. Assuming an L-shaped tearout relative to the axial load on the angles, the nominal block shear rupture strength in the angles is determined as follows. The tearout pattern is shown in Figure II.A-1B-5.

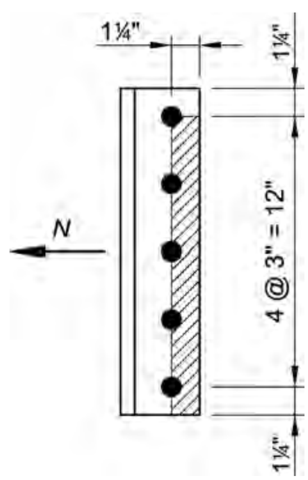


Fig. II.A-1B-5. Block shear rupture of angles for axial load only—L-shape.

$$R_{bsn} = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.60F_yA_{gv} + U_{bs}F_uA_{nt}$$ (*Spec*. Eq. J4-5)

where

$$\begin{aligned}A_{gv} &= (2 \text{ angles})l_{eh}t \\ &= (2 \text{ angles})(1\tfrac{1}{4} \text{ in.})(\tfrac{5}{8} \text{ in.}) \\ &= 1.56 \text{ in.}^2\end{aligned}$$

$$\begin{aligned}A_{nv} &= A_{gv} - (2 \text{ angles})(0.5)(d_h + \tfrac{1}{16} \text{ in.})t \\ &= 1.56 \text{ in.}^2 - (2 \text{ angles})(0.5)(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{5}{8} \text{ in.}) \\ &= 0.935 \text{ in.}^2\end{aligned}$$

$$\begin{aligned}A_{nt} &= (2 \text{ angles})\left[(l - l_{ev}) - (n - 0.5)(d_h + \tfrac{1}{16} \text{ in.})\right]t \\ &= (2 \text{ angles})\left[(14\tfrac{1}{2} \text{ in.} - 1\tfrac{1}{4} \text{ in.}) - (5 - 0.5)(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})\right](\tfrac{5}{8} \text{ in.}) \\ &= 10.9 \text{ in.}^2\end{aligned}$$

$$U_{bs} = 1.0$$

and

$$\begin{aligned}R_{bsn} &= 0.60(58 \text{ ksi})(0.935 \text{ in.}^2) + 1.0(58 \text{ ksi})(10.9 \text{ in.}^2) \le 0.60(36 \text{ ksi})(1.56 \text{ in.}^2) + 1.0(58 \text{ ksi})(10.9 \text{ in.}^2) \\ &= 665 \text{ kips} < 666 \text{ kips}\end{aligned}$$

Therefore:

$$R_{bsn} = 665 \text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture on the angles is:

LRFD	ASD
$\phi = 0.75$ $\phi R_{bsn} = 0.75(665 \text{ kips})$ $= 499 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_{bsn}}{\Omega} = \dfrac{665 \text{ kips}}{2.00}$ $= 333 \text{ kips} > 40 \text{ kips}$ **o.k.**

The nominal strength for the limit state of block shear rupture assuming an U-shaped tearout relative to the axial load on the angles is determined as follows. The tearout pattern is shown in Figure II.A-1B-6.

$$R_{bsn} = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.60F_yA_{gv} + U_{bs}F_uA_{nt}$$ (*Spec*. Eq. J4-5)

where

$$\begin{aligned}A_{gv} &= (2 \text{ angles})(2 \text{ planes})l_{eh}t \\ &= (2 \text{ angles})(2 \text{ planes})(1\tfrac{1}{4} \text{ in.})(\tfrac{5}{8} \text{ in.}) \\ &= 3.13 \text{ in.}^2\end{aligned}$$

$$A_{nv} = (2 \text{ angles})(2 \text{ planes})\left[l_{eh} - 0.5\left(d_h + \tfrac{1}{16} \text{ in.}\right)\right]t$$
$$= (2 \text{ angles})(2 \text{ planes})\left[1\tfrac{1}{4} \text{ in.} - 0.5\left(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)\right]\left(\tfrac{5}{8} \text{ in.}\right)$$
$$= 1.88 \text{ in.}^2$$

$$A_{nt} = (2 \text{ angles})\left[12.0 \text{ in.} - (n-1)\left(d_h + \tfrac{1}{16} \text{ in.}\right)\right]t$$
$$= (2 \text{ angles})\left[12.0 \text{ in.} - (5-1)\left(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)\right]\left(\tfrac{5}{8} \text{ in.}\right)$$
$$= 10.0 \text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$R_{bsn} = 0.60(58 \text{ ksi})\left(1.88 \text{ in.}^2\right) + 1.0(58 \text{ ksi})\left(10.0 \text{ in.}^2\right) \le 0.60(36 \text{ ksi})\left(3.13 \text{ in.}^2\right) + 1.0(58 \text{ ksi})\left(10.0 \text{ in.}^2\right)$$
$$= 645 \text{ kips} < 648 \text{ kips}$$

Therefore:

$$R_{bsn} = 645 \text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture on the angles is:

LRFD	ASD
$\phi = 0.75$ $\phi R_{bsn} = 0.75(645 \text{ kips})$ $= 484 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_{bsn}}{\Omega} = \dfrac{645 \text{ kips}}{2.00}$ $= 323 \text{ kips} > 40 \text{ kips}$ **o.k.**

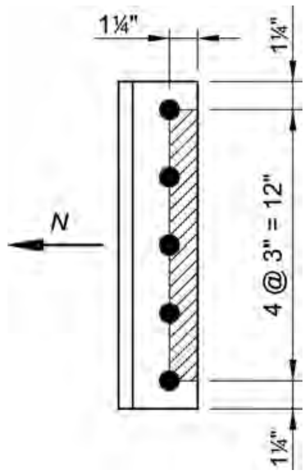


Fig. II.A-1B-6. Block shear rupture of angles for axial load only—U-shape.

Considering the interaction of shear and axial loads, apply a formulation that is similar to AISC *Manual* Equation 10-5:

LRFD	ASD
$\left(\frac{V_u}{\phi R_{bsv}}\right)^2 + \left(\frac{N_u}{\phi R_{bsn}}\right)^2 \le 1$	$\left(\frac{V_u}{R_{bsv}/\Omega}\right)^2 + \left(\frac{N_u}{R_{bsn}/\Omega}\right)^2 \le 1$
$\left(\frac{75\text{ kips}}{310\text{ kips}}\right)^2 + \left(\frac{60\text{ kips}}{484\text{ kips}}\right)^2 = 0.0739 \le 1$ **o.k.**	$\left(\frac{50\text{ kips}}{207\text{ kips}}\right)^2 + \left(\frac{40\text{ kips}}{323\text{ kips}}\right)^2 = 0.0737 \le 1$ **o.k.**

Block Shear Rupture of Angles–Outstanding Legs

The nominal strength for the limit state of block shear rupture relative to the shear load on the angles is determined as follows. The tearout pattern is shown in Figure II.A-1B-7.

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \quad \text{(\textit{Spec.} Eq. J4-5)}$$

where

$$\begin{aligned} A_{gv} &= (2\text{ angles})(l - l_{ev})t \\ &= (2\text{ angles})(14\tfrac{1}{2}\text{ in.} - 1\tfrac{1}{4}\text{ in.})(\tfrac{5}{8}\text{ in.}) \\ &= 16.6\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nv} &= A_{gv} - (2\text{ angles})(n - 0.5)(d_h + \tfrac{1}{16}\text{ in.})t \\ &= 16.6\text{ in.}^2 - (2\text{ angles})(5 - 0.5)(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{5}{8}\text{ in.}) \\ &= 11.0\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nt} &= (2\text{ angles})\left[l_{eh} - 0.5(d_h + \tfrac{1}{16}\text{ in.})\right]t \\ &= (2\text{ angles})\left[1\tfrac{7}{16}\text{ in.} - 0.5(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](\tfrac{5}{8}\text{ in.}) \\ &= 1.17\text{ in.}^2 \end{aligned}$$

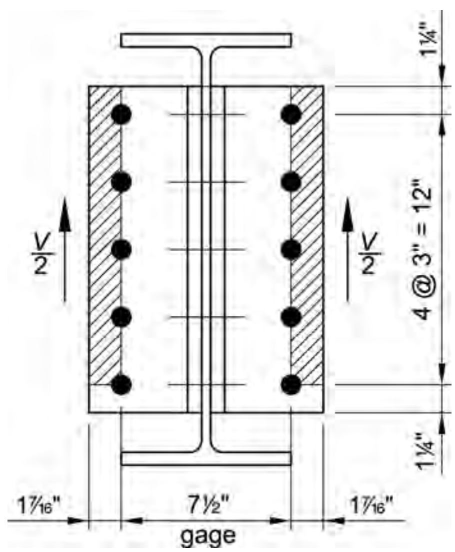


Fig. II.A-1B-7. Block shear rupture of outstanding legs of angles.

$U_{bs} = 1.0$

and

$$R_n = 0.60(58\text{ ksi})(11.0\text{ in.}^2) + 1.0(58\text{ ksi})(1.17\text{ in.}^2) \le 0.60(36\text{ ksi})(16.6\text{ in.}^2) + 1.0(58\text{ ksi})(1.17\text{ in.}^2)$$
$$= 451\text{ kips} > 426\text{ kips}$$

Therefore:

$$R_n = 426\text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture on the angles is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(426\text{ kips})$ $= 320\text{ kips} > 75\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{426\text{ kips}}{2.00}$ $= 213\text{ kips} > 50\text{ kips}$ **o.k.**

Shear Strength of Beam Web

From AISC *Specification* Section J4.2(a), the available shear yield strength of the beam web is determined as follows:

$$A_{gv} = dt_w$$
$$= (18.0\text{ in.})(0.355\text{ in.})$$
$$= 6.39\text{ in.}^2$$

$$R_n = 0.60F_yA_{gv} \qquad \text{(Spec. Eq. J4-3)}$$
$$= 0.60(50\text{ ksi})(6.39\text{ in.}^2)$$
$$= 192\text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(192\text{ kips})$ $= 192\text{ kips} > 75\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{192\text{ kips}}{1.50}$ $= 128\text{ kips} > 50\text{ kips}$ **o.k.**

The limit state of shear rupture of the beam web does not apply in this example because the beam is uncoped.

Tensile Strength of Beam

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the beam is determined as follows:

$$R_n = F_y A_g \quad \text{(Spec. Eq. J4-1)}$$
$$= (50 \text{ ksi})(14.7 \text{ in.}^2)$$
$$= 735 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(735 \text{ kips})$ $= 662 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{735 \text{ kips}}{1.67}$ $= 440 \text{ kips} > 40 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.1(b), determine the available tensile rupture strength of the beam. The effective net area is $A_e = A_nU$. No cases in AISC *Specification* Table D3.1 apply to this configuration; therefore, U is determined from AISC *Specification* Section D3.

$$A_n = A_g - n(d_h + \tfrac{1}{16} \text{ in.})(t_w)$$
$$= 14.7 \text{ in.}^2 - 5(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(0.355 \text{ in.})$$
$$= 12.9 \text{ in.}^2$$

As stated in AISC *Specification* Section D3, the value of U can be determined as the ratio of the gross area of the connected element (beam web) to the member gross area.

$$U = \frac{(d - 2t_f)(t_w)}{A_g}$$
$$= \frac{[18.0 \text{ in.} - 2(0.570 \text{ in.})](0.355 \text{ in.})}{14.7 \text{ in.}^2}$$
$$= 0.407$$

$$A_e = A_nU \quad \text{(Spec. Eq. D3-1)}$$
$$= (12.9 \text{ in.}^2)(0.407)$$
$$= 5.25 \text{ in.}^2$$

$$R_n = F_u A_e \quad \text{(Spec. Eq. J4-2)}$$
$$= (65 \text{ ksi})(5.25 \text{ in.}^2)$$
$$= 341 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(341 \text{ kips})$ $= 256 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{341 \text{ kips}}{2.00}$ $= 171 \text{ kips} > 40 \text{ kips}$ **o.k.**

Block Shear Rupture Strength of Beam Web

Block shear rupture is only applicable in the direction of the axial load, because the beam is uncoped and the limit state is not applicable for an uncoped beam subject to vertical shear. Assuming a U-shaped tearout relative to the axial load, and assuming a horizontal edge distance of l_{eh} = 1¾ in. − ¼ in. = 1½ in. to account for a possible beam underrun of ¼ in., the block shear rupture strength is:

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \qquad (Spec.\ \text{Eq. J4-5})$$

where

$$\begin{aligned}A_{gv} &= (2)l_{eh}t_w \\ &= (2)(1\tfrac{1}{2}\text{ in.})(0.355\text{ in.}) \\ &= 1.07\text{ in.}^2\end{aligned}$$

$$\begin{aligned}A_{nv} &= A_{gv} - (2)(0.5)(d_h + \tfrac{1}{16}\text{ in.})t_w \\ &= 1.07\text{ in.}^2 - (2)(0.5)(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(0.355\text{ in.}) \\ &= 0.715\text{ in.}^2\end{aligned}$$

$$\begin{aligned}A_{nt} &= \left[12.0\text{ in.} - (n-1)(d_h + \tfrac{1}{16}\text{ in.})\right]t_w \\ &= \left[12.0\text{ in.} - (5-1)(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](0.355\text{ in.}) \\ &= 2.84\text{ in.}^2\end{aligned}$$

$$U_{bs} = 1.0$$

and

$$\begin{aligned}R_n &= 0.60(65\text{ ksi})(0.710\text{ in.}^2) + 1.0(65\text{ ksi})(2.84\text{ in.}^2) \le 0.60(50\text{ ksi})(1.07\text{ in.}^2) + 1.0(65\text{ ksi})(2.84\text{ in.}^2) \\ &= 212\text{ kips} < 217\text{ kips}\end{aligned}$$

Therefore:

$$R_n = 212\text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture of the beam web is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(212\text{ kips})$ $= 159\text{ kips} > 60\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{212\text{ kips}}{2.00}$ $= 106\text{ kips} > 40\text{ kips}$ **o.k.**

Conclusion

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-1C ALL-BOLTED DOUBLE-ANGLE CONNECTION—STRUCTURAL INTEGRITY CHECK

Given:

Verify the all-bolted double-angle connection from Example II.A-1B, as shown in Figure II.A-1C-1, for the structural integrity provisions of AISC *Specification* Section B3.9. The connection is verified as a beam and girder end connection and as an end connection of a member bracing a column. Note that these checks are necessary when design for structural integrity is required by the applicable building code.

The beam is an ASTM A992 W18×50 and the angles are ASTM A36 material.

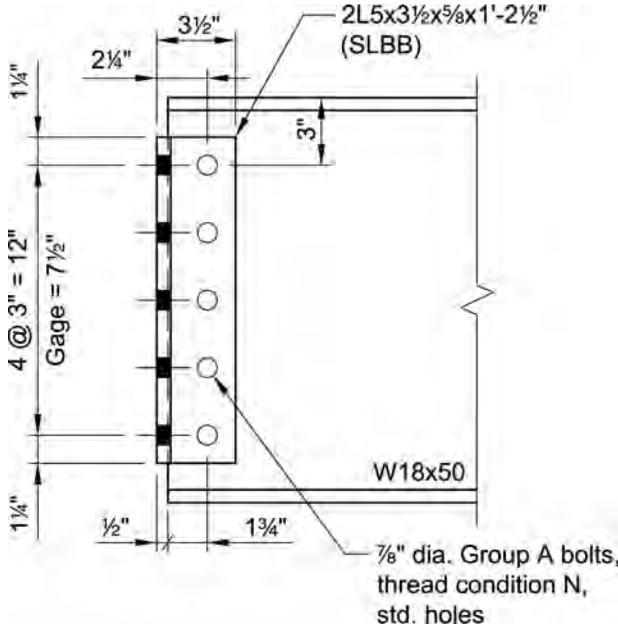


Fig. II.A-1C-1. Connection geometry for Example II.A-1C.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angle
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18x50
t_w = 0.355 in.

From AISC *Specification* Table J3.3, the hole diameter for ⅞-in.-diameter bolts with standard holes is:

d_h = 15⁄16 in.

Beam and Girder End Connection

From Example II.A-1B, the required shear strength is:

LRFD	ASD
$V_u = 75$ kips	$V_a = 50$ kips

From AISC *Specification* Section B3.9(b), the required axial tensile strength is:

LRFD	ASD
$T_u = \frac{2}{3}V_u \geq 10 \text{ kips}$ $= \frac{2}{3}(75 \text{ kips}) > 10 \text{ kips}$ $= 50 \text{ kips} > 10 \text{ kips}$ Therefore: $T_u = 50$ kips	$T_a = V_a \geq 10 \text{ kips}$ $= 50 \text{ kips} > 10 \text{ kips}$ Therefore: $T_a = 50$ kips

From AISC *Specification* Section B3.9, these strength requirements are evaluated independently from other strength requirements.

Bolt Shear

From AISC *Specification* Section J3.6, the nominal bolt shear strength is:

F_{nv} = 54 ksi, from AISC *Specification* Table J3.2

$$T_n = nF_{nv}A_b(2 \text{ shear planes}) \quad \text{(from } \textit{Spec.} \text{ Eq. J3-1)}$$
$$= (5 \text{ bolts})(54 \text{ ksi})(0.601 \text{ in.}^2)(2 \text{ shear planes})$$
$$= 325 \text{ kips}$$

Bolt Tension

From AISC *Specification* Section J3.6, the nominal bolt tensile strength is:

F_{nt} = 90 ksi, from AISC *Specification* Table J3.2

$$T_n = nF_{nt}A_b \quad \text{(from } \textit{Spec.} \text{ Eq. J3-1)}$$
$$= (10 \text{ bolts})(90 \text{ ksi})(0.601 \text{ in.}^2)$$
$$= 541 \text{ kips}$$

Bolt Bearing and Tearout

From AISC *Specification* Section B3.9, for the purpose of satisfying structural integrity requirements, inelastic deformations of the connection are permitted; therefore, AISC *Specification* Equations J3-6b and J3-6d are used to determine the nominal bearing and tearout strength. By inspection the beam web will control. For bolt bearing on the beam web:

$$T_n = (5 \text{ bolts})3.0dt_wF_u$$ (from *Spec.* Eq. J3-6b)
$$= (5 \text{ bolts})(3.0)(\tfrac{7}{8} \text{ in.})(0.355 \text{ in.})(65 \text{ ksi})$$
$$= 303 \text{ kips}$$

For bolt tearout on the beam web (including a ¼-in. tolerance to account for possible beam underrun):

$$l_c = l_{eh} - 0.5d_h$$
$$= (1\tfrac{3}{4} \text{ in.} - \tfrac{1}{4} \text{ in.}) - 0.5(\tfrac{15}{16} \text{ in.})$$
$$= 1.03 \text{ in.}$$

$$T_n = (5 \text{ bolts})1.5l_ct_wF_u$$ (from *Spec.* Eq. J3-6d)
$$= (5 \text{ bolts})(1.5)(1.03 \text{ in.})(0.355 \text{ in.})(65 \text{ ksi})$$
$$= 178 \text{ kips}$$

Angle Bending and Prying Action

From AISC *Manual* Part 9, the nominal strength of the angles accounting for prying action is determined as follows:

$$a = \frac{2(angle\ leg) + t_w - gage}{2}$$
$$= \frac{2(5 \text{ in.}) + 0.355 \text{ in.} - 7\tfrac{1}{2} \text{ in.}}{2}$$
$$= 1.43 \text{ in.}$$

$$b = \frac{gage - t_w - t}{2}$$
$$= \frac{7\tfrac{1}{2} \text{ in.} - 0.355 \text{ in.} - \tfrac{5}{8} \text{ in.}}{2}$$
$$= 3.26 \text{ in.}$$

$$a' = a + \frac{d_b}{2} \le 1.25b + \frac{d_b}{2}$$ (*Manual* Eq. 9-23)
$$= 1.43 \text{ in.} + \frac{\tfrac{7}{8} \text{ in.}}{2} \le 1.25(3.26 \text{ in.}) + \frac{\tfrac{7}{8} \text{ in.}}{2}$$
$$= 1.87 \text{ in.} < 4.51 \text{ in.}$$
$$= 1.87 \text{ in.}$$

$$b' = \left(b - \frac{d_b}{2}\right)$$ (*Manual* Eq. 9-18)
$$= 3.26 \text{ in.} - \frac{\tfrac{7}{8} \text{ in.}}{2}$$
$$= 2.82 \text{ in.}$$

$$\rho = \frac{b'}{a'}$$ (*Manual* Eq. 9-22)

$$= \frac{2.82 \text{ in.}}{1.87 \text{ in.}}$$

$$= 1.51$$

Note that end distances of 1¼ in. are used on the angles, so p is the average pitch of the bolts:

$$p = \frac{l}{n}$$

$$= \frac{14\tfrac{1}{2} \text{ in.}}{5 \text{ bolts}}$$

$$= 2.90 \text{ in.}$$

Check:

$p < s = 3.00$ in. **o.k.**

$$d' = d_h$$

$$= \tfrac{15}{16} \text{ in.}$$

$$\delta = 1 - \frac{d'}{p}$$ (*Manual* Eq. 9-20)

$$= 1 - \frac{\tfrac{15}{16} \text{ in.}}{2.90 \text{ in.}}$$

$$= 0.677$$

$$B_n = F_{nt} A_b$$

$$= (90 \text{ ksi})(0.601 \text{ in.}^2)$$

$$= 54.1 \text{ kips/bolt}$$

$$t_c = \sqrt{\frac{4B_n b'}{pF_u}}$$ (from *Manual* Eq. 9-26)

$$= \sqrt{\frac{4(54.1 \text{ kips/bolt})(2.82 \text{ in.})}{(2.90 \text{ in.})(58 \text{ ksi})}}$$

$$= 1.90 \text{ in.}$$

$$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right]$$ (*Manual* Eq. 9-28)

$$= \frac{1}{0.677(1+1.51)}\left[\left(\frac{1.90 \text{ in.}}{\tfrac{5}{8} \text{ in.}}\right)^2 - 1\right]$$

$$= 4.85$$

Because $\alpha' > 1$, the angles have insufficient strength to develop the bolt strength, therefore:

$$Q = \left(\frac{t}{t_c}\right)^2 (1+\delta)$$
$$= \left(\frac{5/8 \text{ in.}}{1.90 \text{ in.}}\right)^2 (1+0.677)$$
$$= 0.181$$

$$T_n = nB_nQ \qquad \text{(from } \textit{Manual} \text{ Eq. 9-27)}$$
$$= (10 \text{ bolts})(54.1 \text{ kips/bolt})(0.181)$$
$$= 97.9 \text{ kips}$$

Note: The 97.9 kips includes any prying forces so there is no need to calculate the prying force per bolt, q_r.

Tensile Yielding of Angles

From AISC *Specification* Section J4.1, the nominal tensile yielding strength of the angles is determined as follows:

$$A_g = (2 \text{ angles})lt$$
$$= (2 \text{ angles})(14\tfrac{1}{2} \text{ in.})(5/8 \text{ in.})$$
$$= 18.1 \text{ in.}^2$$

$$T_n = F_y A_g \qquad \text{(from } \textit{Spec.} \text{ Eq. J4-1)}$$
$$= (36 \text{ ksi})(18.1 \text{ in.}^2)$$
$$= 652 \text{ kips}$$

Tensile Rupture of Angles

From AISC *Specification* Section J4.1, the nominal tensile rupture strength of the angles is determined as follows:

$$A_n = (2 \text{ angles})\left[l - n\left(d_h + 1/16 \text{ in.}\right)\right]t$$
$$= (2 \text{ angles})\left[14\tfrac{1}{2} \text{ in.} - 5\left(15/16 \text{ in.} + 1/16 \text{ in.}\right)\right](5/8 \text{ in.})$$
$$= 11.9 \text{ in.}^2$$

AISC *Specification* Table D3.1, Case 1 applies in this case because tension load is transmitted directly to the cross-section element by fasteners; therefore, $U = 1.0$.

$$A_e = A_nU \qquad (\textit{Spec.} \text{ Eq. D3-1})$$
$$= (11.9 \text{ in.}^2)(1.0)$$
$$= 11.9 \text{ in.}^2$$

$$T_n = F_u A_e \qquad \text{(from } \textit{Spec.} \text{ Eq. J4-2)}$$
$$= (58 \text{ ksi})(11.9 \text{ in.}^2)$$
$$= 690 \text{ kips}$$

Block Shear Rupture

By inspection, block shear rupture of the beam web will control. From AISC *Specification* Section J4.3, the available block shear rupture strength of the beam web is determined as follows (account for possible ¼-in. beam underrun):

$$T_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \quad \text{(from } \textit{Spec.} \text{ Eq. J4-5)}$$

where

$$\begin{aligned} A_{gv} &= 2l_{eh}t_w \\ &= 2(1\tfrac{3}{4}\text{ in.} - \tfrac{1}{4}\text{ in.})(0.355\text{ in.}) \\ &= 1.07\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nv} &= 2\left[l_{eh} - 0.5\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]t_w \\ &= 2\left[(1\tfrac{3}{4}\text{ in.} - \tfrac{1}{4}\text{ in.}) - 0.5\left(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right](0.355\text{ in.}) \\ &= 0.710\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nt} &= \left[12.0\text{ in.} - 4\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]t_w \\ &= \left[12.0\text{ in.} - 4\left(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right](0.355\text{ in.}) \\ &= 2.84\text{ in.}^2 \end{aligned}$$

$$U_{bs} = 1.0$$

and

$$\begin{aligned} T_n &= 0.60(65\text{ ksi})(0.710\text{ in.}^2) + 1.0(65\text{ ksi})(2.84\text{ in.}^2) \le 0.60(50\text{ ksi})(1.07\text{ in.}^2) + 1.0(65\text{ ksi})(2.84\text{ in.}^2) \\ &= 212\text{ kips} < 217\text{ kips} \end{aligned}$$

Therefore:

$$T_n = 212\text{ kips}$$

Nominal Tensile Strength

The controlling nominal tensile strength, T_n, is the least of those previously calculated:

$$\begin{aligned} T_n &= \min\{325\text{ kips},\ 541\text{ kips},\ 97.9\text{ kips},\ 652\text{ kips},\ 690\text{ kips},\ 212\text{ kips}\} \\ &= 97.9\text{ kips} \end{aligned}$$

LRFD	ASD
$T_n = 97.9\text{ kips} > 50\text{ kips}$ **o.k.**	$T_n = 97.9\text{ kips} > 50\text{ kips}$ **o.k.**

Column Bracing

From AISC *Specification* Section B3.9(c), the minimum nominal tensile strength for the connection of a member bracing a column is equal to 1% of two-thirds of the required column axial strength for LRFD and equal to 1% of the required column axial for ASD. These requirements are evaluated independently from other strength requirements.

The maximum column axial force this connection is able to brace is determined as follows:

LRFD	ASD
$T_n \geq 0.01\left(\dfrac{2}{3}\right)P_u$ Solving for the column axial force: $P_u \leq 100\left(\dfrac{3}{2}\right)T_n$ $= 100\left(\dfrac{3}{2}\right)(97.9\text{ kips})$ $= 14,700\text{ kips}$	$T_n \geq 0.01P_a$ Solving for the column axial force: $P_a \leq 100T_n$ $= 100(97.9\text{ kips})$ $= 9,790\text{ kips}$

As long as the required column axial strength is less than P_u = 14,700 kips or P_a = 9,790 kips, this connection is an adequate column brace.

EXAMPLE II.A-2A BOLTED/WELDED DOUBLE-ANGLE CONNECTION

Given:

Using the tables in AISC *Manual* Part 10, verify the available strength of a double-angle shear connection with welds in the support legs (welds B) and bolts in the supported-beam-web legs, as shown in Figure II.A-2A-1. The ASTM A992 W36×231 beam is attached to an ASTM A992 W14×90 column flange supporting the following beam end reactions:

$R_D = 37.5$ kips
$R_L = 113$ kips

Use ASTM A36 angles and 70-ksi weld electrodes.

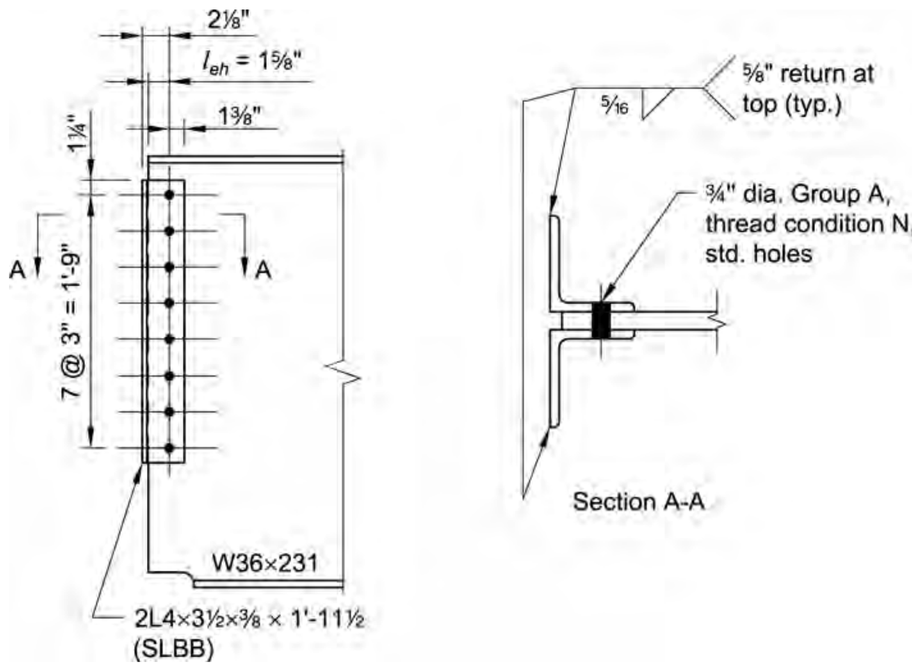


Fig. II.A-2A-1. Connection geometry for Example II.A-2A.

Note: Bottom flange coped for erection.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and column
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Angles
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W36×231
$t_w = 0.760$ in.

Column
W14×90
$t_f = 0.710$ in.

From AISC *Specification* Table J3.3, the hole diameter for ¾-in.-diameter bolts with standard holes is:

$d_h = \frac{13}{16}$ in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(37.5 \text{ kips}) + 1.6(113 \text{ kips})$ $= 226 \text{ kips}$	$R_a = 37.5 \text{ kips} + 113 \text{ kips}$ $= 151 \text{ kips}$

Weld Design

Use AISC *Manual* Table 10-2 (welds B) with n = 8. Try 5⁄16-in. weld size, l = 23½ in. From AISC *Manual* Table 10-2, the minimum support thickness is:

$t_{min} = 0.238$ in. < 0.710 in. **o.k.**

LRFD	ASD
$\phi R_n = 279 \text{ kips} > 226 \text{ kips}$ **o.k.**	$\frac{R_n}{\Omega} = 186 \text{ kips} > 151 \text{ kips}$ **o.k.**

Angle Thickness

From AISC *Specification* Section J2.2b, the minimum angle thickness for a 5⁄16-in. fillet weld is:

$$t = w + \tfrac{1}{16} \text{ in.}$$
$$= \tfrac{5}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}$$
$$= \tfrac{3}{8} \text{ in.}$$

Try 2L4×3½×⅜ (SLBB).

Angle and Bolt Design

AISC *Manual* Table 10-1 includes checks for bolt shear, bolt bearing and tearout on the angles, shear yielding of the angles, shear rupture of the angles, and block shear rupture of the angles.

Check 8 rows of bolts and ⅜-in. angle thickness.

LRFD	ASD
$\phi R_n = 284 \text{ kips} > 226 \text{ kips}$ **o.k.**	$\frac{R_n}{\Omega} = 189 \text{ kips} > 151 \text{ kips}$ **o.k.**

Beam Web Strength

The available beam web strength is the lesser of the limit states of block shear rupture, shear yielding, shear rupture, and the sum of the effective strengths of the individual fasteners. In this example, because of the relative size of the cope to the overall beam size, the coped section will not control, therefore, the strength of the bolt group will control (When this cannot be determined by inspection, see AISC *Manual* Part 9 for the design of the coped section). From the Commentary to AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the effective strengths of the individual fasteners. The effective strength of an individual fastener is the lesser of the shear strength, the bearing strength at the bolt holes, and the tearout strength at the bolt holes.

Bolt Shear

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear is:

LRFD	ASD
$\phi R_n = 35.8 \text{ kips/bolt}$	$\dfrac{R_n}{\Omega} = 23.9 \text{ kips/bolt}$

Bolt Bearing on Beam Web

The nominal bearing strength of the beam web per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$\begin{aligned} r_n &= 2.4dtF_u \qquad (\textit{Spec.}\ \text{Eq. J3-6a}) \\ &= 2.4\left(\tfrac{3}{4} \text{ in.}\right)(0.760 \text{ in.})(65 \text{ ksi}) \\ &= 88.9 \text{ kips/bolt} \end{aligned}$$

From AISC *Specification* Section J3.10, the available bearing strength of the beam web per bolt is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(88.9 \text{ kips/bolt})$ $= 66.7 \text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{88.9 \text{ kips/bolt}}{2.00}$ $= 44.5 \text{ kips/bolt}$

Bolt Tearout on Beam Web

The available tearout strength of the beam web per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$\begin{aligned} l_c &= 3.00 \text{ in.} - \tfrac{13}{16} \text{ in.} \\ &= 2.19 \text{ in.} \end{aligned}$$

$$\begin{aligned} r_n &= 1.2 l_c t F_u \qquad (\textit{Spec.}\ \text{Eq. J3-6c}) \\ &= 1.2(2.19 \text{ in.})(0.760 \text{ in.})(65 \text{ ksi}) \\ &= 130 \text{ kips/bolt} \end{aligned}$$

From AISC *Specification* Section J3.10, the available tearout strength of the beam web per bolt is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(130 \text{ kips/bolt})$ $= 97.5 \text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{130 \text{ kips/bolt}}{2.00}$ $= 65.0 \text{ kips/bolt}$

Bolt shear strength is the governing limit state for all bolts at the beam web. Bolt shear strength is one of the limit states included in the capacities shown in Table 10-1 as used above; thus, the effective strength of the fasteners is adequate.

Available strength at the column flange

Since the thickness of the column flange, t_f = 0.710 in., is greater than the thickness of the angles, t = ⅜ in., shear will control for the angles. The column flange is adequate for the required loading.

Summary

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-2B BOLTED/WELDED DOUBLE-ANGLE CONNECTION SUBJECT TO AXIAL AND SHEAR LOADING

Given:

Verify the available strength of a double-angle connection with welds in the supported-beam-web legs and bolts in the outstanding legs for an ASTM A992 W18×50 beam, as showin in Figure II.A-2B-1, to support the following beam end reactions:

LRFD	ASD
Shear, V_u = 75 kips Axial tension, N_u = 60 kips	Shear, V_a = 50 kips Axial tension, N_a = 40 kips

Use ASTM A36 angles and 70-ksi electrodes.

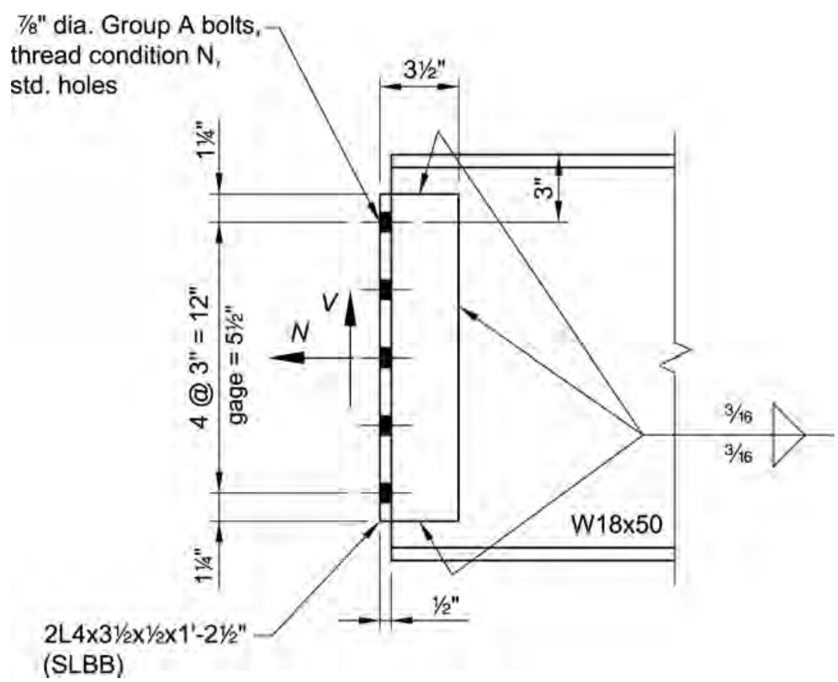


Fig. II.A-2B-1. Connection geometry for Example II.A-2B.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×50
A_g = 14.7 in.^{2}
d = 18.0 in.
t_w = 0.355 in.
b_f = 7.50 in.
t_f = 0.570 in.

From AISC *Specification* Table J3.3, the hole diameter for ⅞-in.-diameter bolts with standard holes is:

d_h = 15⁄16 in.

The resultant load is:

LRFD	ASD
$R_u = \sqrt{V_u^2 + N_u^2}$ $= \sqrt{(75 \text{ kips})^2 + (60 \text{ kips})^2}$ $= 96.0 \text{ kips}$	$R_a = \sqrt{V_a^2 + N_a^2}$ $= \sqrt{(50 \text{ kips})^2 + (40 \text{ kips})^2}$ $= 64.0 \text{ kips}$

The following bolt shear, bearing and tearout calculations are for a pair of bolts.

Bolt Shear

From AISC *Manual* Table 7-1, the available shear strength for ⅞-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear (or pair of bolts):

LRFD	ASD
$\phi r_n = 48.7 \text{ kips (for pair of bolts)}$	$\dfrac{r_n}{\Omega} = 32.5 \text{ kips (for pair of bolts)}$

Bolt Bearing on Angles

The available bearing strength of the double angle is determined from AISC *Specification* Section J3.10, assuming deformation at the bolt hole is a design consideration:

$$r_n = (2 \text{ bolts})2.4dtF_u \qquad \text{(from Spec. Eq. J3-6a)}$$
$$= (2 \text{ bolts})(2.4)(7/8 \text{ in.})(1/2 \text{ in.})(58 \text{ ksi})$$
$$= 122 \text{ kips (for pair of bolts)}$$

The available bearing strength for a pair of bolts is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(122 \text{ kips})$ $= 91.5 \text{ kips (for pair of bolts)}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{122 \text{ kips}}{2.00}$ $= 61.0 \text{ kips (for pair of bolts)}$

The bolt shear strength controls over bearing in the angles.

Bolt Tearout on Angles

The available tearout strength of the angle is determined from AISC *Specification* Section J3.10, assuming deformation at the bolt hole is a design consideration:

For the edge bolt:

$$\begin{aligned} l_c &= l_e - 0.5d_h \\ &= 1\tfrac{1}{4}\text{ in.} - 0.5\left(\tfrac{15}{16}\text{ in.}\right) \\ &= 0.781\text{ in.} \end{aligned}$$

$$\begin{aligned} r_n &= (2\text{ bolts})1.2l_c tF_u \\ &= (2\text{ bolts})(1.2)(0.781\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi}) \\ &= 54.4\text{ kips (for pair of bolts)} \end{aligned}$$

(from *Spec*. Eq. J3-6c)

The available tearout strength of the angles for a pair of edge bolts is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(54.4\text{ kips})$ $= 40.8\text{ kips}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{54.4\text{ kips}}{2.00}$ $= 27.2\text{ kips}$

The tearout strength controls over bolt shear and bearing for the edge bolts in the angles.

For the other bolts:

$$\begin{aligned} l_c &= s - d_h \\ &= 3\text{ in.} - \tfrac{15}{16}\text{ in.} \\ &= 2.06\text{ in.} \end{aligned}$$

$$\begin{aligned} r_n &= (2\text{ bolts})1.2l_c tF_u \\ &= (2\text{ bolts})(1.2)(2.06\text{ in.})(\tfrac{1}{2}\text{ in.})(58\text{ ksi}) \\ &= 143\text{ kips (for pair of bolts)} \end{aligned}$$

(*Spec*. Eq. J3-6c)

The available tearout strength for a pair of other bolts is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(143\text{ kips})$ $= 107\text{ kips (for pair of bolts)}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{143\text{ kips}}{2.00}$ $= 71.5\text{ kips (for pair of bolts)}$

Bolt shear strength controls over tearout and bearing strength for the other bolts in the angles.

Strength of Bolted Connection

The effective strength for the bolted connection at the angles is determined by summing the effective strength for each bolt using the minimum available strength calculated for bolt shear, bearing on the angles, and tearout on the angles.

LRFD	ASD
$\phi R_n = (1\text{ bolt})(40.8\text{ kips}) + (4\text{ bolts})(48.7\text{ kips})$ $= 236\text{ kips} > 75\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (1\text{ bolt})(27.2\text{ kips}) + (4\text{ bolts})(32.5\text{ kips})$ $= 157\text{ kips} > 50\text{ kips}$ **o.k.**

Shear and Tension Interaction in Bolts

The required shear stress for each bolt is determined as follows:

$$f_{rv} = \frac{V_r}{nA_b}$$

where

$A_b = 0.601\text{ in.}^2$ (from AISC *Manual* Table 7-1)

$n = 10\text{ bolts}$

LRFD	ASD
$f_{rv} = \dfrac{75\text{ kips}}{(10\text{ bolts})(0.601\text{ in.}^2)}$ $= 12.5\text{ ksi}$	$f_{rv} = \dfrac{50\text{ kips}}{(10\text{ bolts})(0.601\text{ in.}^2)}$ $= 8.32\text{ ksi}$

The nominal tensile stress modified to include the effects of shear stress is determined from AISC *Specification* Section J3.7 as follows. From AISC *Specification* Table J3.2:

$F_{nt} = 90\text{ ksi}$

$F_{nv} = 54\text{ ksi}$

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3-3a) $= 1.3(90\text{ ksi}) - \dfrac{90\text{ ksi}}{0.75(54\text{ ksi})}(12.5\text{ ksi}) \le 90\text{ ksi}$ $= 89.2\text{ ksi} < 90\text{ ksi}$ **o.k.**	$F'_{nt} = 1.3F_{nt} - \dfrac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3-3b) $= 1.3(90\text{ ksi}) - \dfrac{2.00(90\text{ ksi})}{54\text{ ksi}}(8.32\text{ ksi}) \le 90\text{ ksi}$ $= 89.3\text{ ksi} < 90\text{ ksi}$ **o.k.**

Using the value of $F'_{nt} = 89.2$ ksi determined for LRFD, the nominal tensile strength of one bolt is:

$$r_n = F'_{nt}A_b \qquad \text{(\textit{Spec}. Eq. J3-2)}$$
$$= (89.2\text{ ksi})(0.601\text{ in.}^2)$$
$$= 53.6\text{ kips}$$

The available tensile strength due to combined tension and shear is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = n\phi r_n$ $= (10 \text{ bolts})(0.75)(53.6 \text{ kips})$ $= 402 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = n\dfrac{r_n}{\Omega}$ $= (10 \text{ bolts})\left(\dfrac{53.6 \text{ kips}}{2.00}\right)$ $= 268 \text{ kips} > 40 \text{ kips}$ **o.k.**

Prying Action on Bolts

From AISC *Manual* Part 9, the available tensile strength of the bolts in the outstanding angle legs taking prying action into account is determined as follows:

$$
\begin{aligned}
a &= \frac{angle\ leg\,(2) + t_w - gage}{2} \\
&= \frac{(4.00 \text{ in.})(2) + 0.355 \text{ in.} - 5\tfrac{1}{2} \text{ in.}}{2} \\
&= 1.43 \text{ in.}
\end{aligned}
$$

Note: If the distance from the bolt centerline to the edge of the supporting element is smaller than a = 1.43 in., use the smaller a in the following calculation.

$$
\begin{aligned}
b &= \frac{gage - t_w - t}{2} \\
&= \frac{5\tfrac{1}{2} \text{ in.} - 0.355 \text{ in.} - \tfrac{1}{2} \text{ in.}}{2} \\
&= 2.32 \text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
a' &= \left(a + \frac{d_b}{2}\right) \le \left(1.25b + \frac{d_b}{2}\right) \qquad (\textit{Manual} \text{ Eq. 9-23}) \\
&= 1.43 \text{ in.} + \frac{\tfrac{7}{8} \text{ in.}}{2} \le 1.25(2.32 \text{ in.}) + \frac{\tfrac{7}{8} \text{ in.}}{2} \\
&= 1.87 \text{ in.} < 3.34 \text{ in.} \\
&= 1.87 \text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
b' &= \left(b - \frac{d_b}{2}\right) \qquad (\textit{Manual} \text{ Eq. 9-18}) \\
&= 2.32 \text{ in.} - \frac{\tfrac{7}{8} \text{ in.}}{2} \\
&= 1.88 \text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
\rho &= \frac{b'}{a'} \qquad (\textit{Manual} \text{ Eq. 9-22}) \\
&= \frac{1.88 \text{ in.}}{1.87 \text{ in.}} \\
&= 1.01
\end{aligned}
$$

Note that end distances of 1¼ in. are used on the angles, so p is the average pitch of the bolts:

$$p = \frac{l}{n}$$
$$= \frac{14\frac{1}{2}\text{ in.}}{5}$$
$$= 2.90\text{ in.}$$

Check:

$p \le s$

$2.90\text{ in.} < 3\text{ in.}$ **o.k.**

$$d' = d_h$$
$$= \frac{15}{16}\text{ in.}$$

$$\delta = 1 - \frac{d'}{p}$$ (*Manual* Eq. 9-20)
$$= 1 - \frac{\frac{15}{16}\text{ in.}}{2.90\text{ in.}}$$
$$= 0.677$$

The angle thickness required to develop the available strength of the bolt with no prying action as follows:

LRFD	ASD
$B_c = 40.2$ kips/bolt (calculated previously) $\phi = 0.90$ $t_c = \sqrt{\dfrac{4B_c b'}{\phi p F_u}}$ (*Manual* Eq. 9-26a) $= \sqrt{\dfrac{4(40.2\text{ kips/bolt})(1.88\text{ in.})}{0.90(2.90\text{ in.})(58\text{ ksi})}}$ $= 1.41\text{ in.}$	$B_c = 26.8$ kips/bolt (calculated previously) $\Omega = 1.67$ $t_c = \sqrt{\dfrac{\Omega 4B_c b'}{p F_u}}$ (*Manual* Eq. 9-26b) $= \sqrt{\dfrac{1.67(4)(26.8\text{ kips/bolt})(1.88\text{ in.})}{(2.90\text{ in.})(58\text{ ksi})}}$ $= 1.41\text{ in.}$

$$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right]$$ (*Manual* Eq. 9-28)
$$= \frac{1}{0.677(1+1.01)}\left[\left(\frac{1.41\text{ in.}}{\frac{1}{2}\text{ in.}}\right)^2 - 1\right]$$
$$= 5.11$$

Because $\alpha' > 1$, the angles have insufficient strength to develop the bolt strength, therefore:

$$Q = \left(\frac{t}{t_c}\right)^2 (1+\delta)$$
$$= \left(\frac{\frac{1}{2}\text{ in.}}{1.41\text{ in.}}\right)^2 (1+0.677)$$
$$= 0.211$$

The available tensile strength of the bolts, taking prying action into account is determined from AISC *Manual* Equation 9-27, as follows:

LRFD	ASD
$\phi r_n = B_c Q$ $= (40.2 \text{ kips/bolt})(0.211)$ $= 8.48 \text{ kips/bolt}$ $\phi R_n = n\phi r_n$ $= (10 \text{ bolts})(8.48 \text{ kips/bolt})$ $= 84.8 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\dfrac{r_n}{\Omega} = B_c Q$ $= (26.8 \text{ kips/bolt})(0.211)$ $= 5.65 \text{ kips/bolt}$ $\dfrac{R_n}{\Omega} = n\dfrac{r_n}{\Omega}$ $= (10 \text{ bolts})(5.65 \text{ kips/bolt})$ $= 56.5 \text{ kips} > 40 \text{ kips}$ **o.k.**

Weld Design

The resultant load angle on the weld is:

LRFD	ASD
$\theta = \tan^{-1}\left(\dfrac{N_u}{V_u}\right)$ $= \tan^{-1}\left(\dfrac{60 \text{ kips}}{75 \text{ kips}}\right)$ $= 38.7°$	$\theta = \tan^{-1}\left(\dfrac{N_a}{V_a}\right)$ $= \tan^{-1}\left(\dfrac{40 \text{ kips}}{50 \text{ kips}}\right)$ $= 38.7°$

From AISC *Manual* Table 8-8 for Angle = 30° (which will lead to a conservative result), using total beam setback of ½ in. + ¼ in. = ¾ in. (the ¼ in. is included to account for mill underrun):

$$l = 14\tfrac{1}{2} \text{ in.}$$

$$\begin{aligned} kl &= 3\tfrac{1}{2} \text{ in.} - \tfrac{3}{4} \text{ in.} \\ &= 2.75 \text{ in.} \end{aligned}$$

$$\begin{aligned} k &= \frac{kl}{l} \\ &= \frac{2.75 \text{ in.}}{14\tfrac{1}{2} \text{ in.}} \\ &= 0.190 \end{aligned}$$

$x = 0.027$ by interpolation

$$\begin{aligned} al &= 3\tfrac{1}{2} \text{ in.} - xl \\ &= 3\tfrac{1}{2} \text{ in.} - 0.027(14\tfrac{1}{2} \text{ in.}) \\ &= 3.11 \text{ in.} \end{aligned}$$

$$a = \frac{al}{l} = \frac{3.11 \text{ in.}}{14\tfrac{1}{2} \text{ in.}} = 0.214$$

$C = 2.69$ by interpolation

The required weld size is determined using AISC *Manual* Equation 8-21, as follows:

LRFD	ASD
$D_{min} = \frac{R_u}{\phi C C_1 l} = \frac{96.0 \text{ kips}}{0.75(2.69)(1)(14\tfrac{1}{2} \text{ in.})(2 \text{ sides})} = 1.64 \text{ sixteenths}$	$D_{min} = \frac{\Omega R_a}{C C_1 l} = \frac{2.00(64.0 \text{ kips})}{2.69(1)(14\tfrac{1}{2} \text{ in.})(2 \text{ sides})} = 1.64 \text{ sixteenths}$

Use a ³⁄₁₆-in. fillet weld (minimum size from AISC *Specification* Table J2.4).

Beam Web Strength at Fillet Weld

The minimum beam web thickness required to match the shear rupture strength of a weld both sides to that of the base metal is:

$$t_{min} = \frac{6.19 D_{min}}{F_u} \quad \text{(from } \textit{Manual} \text{ Eq. 9-3)}$$

$$= \frac{6.19(1.64)}{65 \text{ ksi}}$$

$$= 0.156 \text{ in.} < 0.355 \text{ in.} \quad \textbf{o.k.}$$

Shear Strength of Angles

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the angles is determined as follows:

$$A_{gv} = (2 \text{ angles}) lt = (2 \text{ angles})(14\tfrac{1}{2} \text{ in.})(\tfrac{1}{2} \text{ in.}) = 14.5 \text{ in.}^2$$

$$R_n = 0.60 F_y A_{gv} \quad (\textit{Spec}. \text{ Eq. J4-3})$$

$$= 0.60(36 \text{ ksi})(14.5 \text{ in.}^2)$$

$$= 313 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(313 \text{ kips})$ $= 313 \text{ kips} > 96.0 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{313 \text{ kips}}{1.50}$ $= 209 \text{ kips} > 64.0 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the angle is determined as follows. The effective net area is determined in accordance with AISC *Specification* Section B4.3b.

$$A_{nv} = (2 \text{ angles})\left[l - n\left(d_h + \tfrac{1}{16} \text{ in.}\right)\right]t$$
$$= (2 \text{ angles})\left[14\tfrac{1}{2} \text{ in.} - 5\left(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)\right](\tfrac{1}{2} \text{ in.})$$
$$= 9.50 \text{ in.}^2$$

$$R_n = 0.60 F_u A_{nv} \qquad (\textit{Spec}. \text{ Eq. J4-4})$$
$$= 0.60(58 \text{ ksi})\left(9.50 \text{ in.}^2\right)$$
$$= 331 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(331 \text{ kips})$ $= 248 \text{ kips} > 96.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{331 \text{ kips}}{2.00}$ $= 166 \text{ kips} > 64.0 \text{ kips}$ **o.k.**

Tensile Strength of Angles—Beam Web Side

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the angles is determined as follows:

$$A_g = (2 \text{ angles})lt$$
$$= (2 \text{ angles})(14\tfrac{1}{2} \text{ in.})(\tfrac{1}{2} \text{ in.})$$
$$= 14.5 \text{ in.}^2$$

$$R_n = F_y A_g \qquad (\textit{Spec}. \text{ Eq. J4-1})$$
$$= (36 \text{ ksi})\left(14.5 \text{ in.}^2\right)$$
$$= 522 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(522 \text{ kips})$ $= 470 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{522 \text{ kips}}{1.67}$ $= 313 \text{ kips} > 40 \text{ kips}$ **o.k.**

From AISC *Specification* Sections J4.1(b), the available tensile rupture strength of the angles is determined as follows:

$$R_n = F_u A_e \qquad (Spec.\ \text{Eq. J4-2})$$

Because the angle legs are welded to the beam web there is no bolt hole reduction and $A_e = A_g$; therefore, tensile rupture will not control.

Block Shear Rupture Strength of Angles–Outstanding Legs

The nominal strength for the limit state of block shear rupture of the angles assuming an L-shaped tearout relative to shear load, is determined as follows. The tearout pattern is shown in Figure II.A-2B-2.

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (Spec.\ \text{Eq. J4-5})$$

where

$$\begin{aligned} l_{eh} &= \frac{2(angle\ leg) + t_w - gage}{2} \\ &= \frac{2(4\text{ in.}) + 0.355\text{ in.} - 5\tfrac{1}{2}\text{ in.}}{2} \\ &= 1.43\text{ in.} \end{aligned}$$

$$\begin{aligned} A_{nt} &= (2\text{ angles})\left[l_{eh} - 0.5\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right](t) \\ &= (2\text{ angles})\left[1.43\text{ in.} - 0.5\left(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]\left(\tfrac{1}{2}\text{ in.}\right) \\ &= 0.930\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{gv} &= (2\text{ angles})\left[l_{ev} + (n-1)s\right](t) \\ &= (2\text{ angles})\left[1\tfrac{1}{4}\text{ in.} + (5-1)(3\text{ in.})\right]\left(\tfrac{1}{2}\text{ in.}\right) \\ &= 13.3\text{ in.}^2 \end{aligned}$$

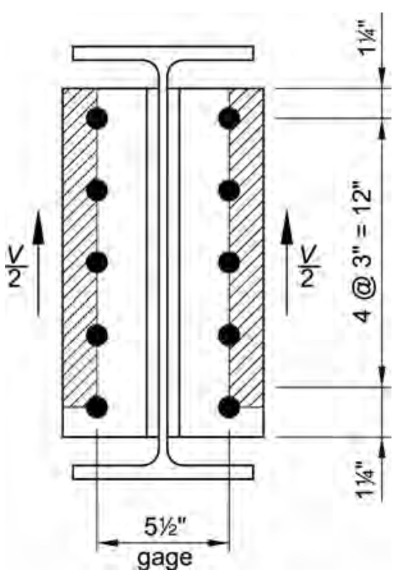


Fig. II.A-2B-2. Block shear rupture of outstanding legs of angles.

$$A_{nv} = A_{gv} - (2 \text{ angles})(n - 0.5)(d_h + \tfrac{1}{16} \text{ in.})(t)$$
$$= 13.3 \text{ in.}^2 - (2 \text{ angles})(5 - 0.5)(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{1}{2} \text{ in.})$$
$$= 8.80 \text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$R_n = 0.60(58 \text{ ksi})(8.80 \text{ in.}^2) + 1.0(58 \text{ ksi})(0.930 \text{ in.}^2) \leq 0.60(36 \text{ ksi})(13.3 \text{ in.}^2) + 1.0(58 \text{ ksi})(0.930 \text{ in.}^2)$$
$$= 360 \text{ kips} > 341 \text{ kips}$$

Therefore:

$$R_n = 341 \text{ kips}$$

The available block shear rupture strength of the angles is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(341 \text{ kips})$ $= 256 \text{ kips} > 75 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{341 \text{ kips}}{2.00}$ $= 171 \text{ kips} > 50 \text{ kips}$ **o.k.**

Shear Strength of Beam

From AISC *Specification* Section J4.2(a), the available shear yield strength of the beam web is determined as follows:

$$A_{gv} = dt_w$$
$$= (18.0 \text{ in.})(0.355 \text{ in.})$$
$$= 6.39 \text{ in.}^2$$

$$R_n = 0.60 F_y A_{gv} \quad \text{(\textit{Spec}. Eq. J4-3)}$$
$$= 0.60(50 \text{ ksi})(6.39 \text{ in.}^2)$$
$$= 192 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(192 \text{ kips})$ $= 192 \text{ kips} > 75 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{192 \text{ kips}}{1.50}$ $= 128 \text{ kips} > 50 \text{ kips}$ **o.k.**

The limit state of shear rupture of the beam web does not apply in this example because the beam is uncoped.

Block Shear Rupture Strength of Beam Web

Assuming a U-shaped tearout along the weld relative to the axial load, and a total beam setback of ¾ in. (includes ¼ in. tolerance to account for possible mill underrun), the nominal block shear rupture strength is determined as follows.

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (Spec.\ \text{Eq. J4-5})$$

where

$$\begin{aligned} A_{nt} &= lt_w \\ &= (14\tfrac{1}{2}\text{ in.})(0.355\text{ in.}) \\ &= 5.15\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{gv} &= (2)(3\tfrac{1}{2}\text{ in.} - setback)t_w \\ &= (2)(3\tfrac{1}{2}\text{ in.} - \tfrac{3}{4}\text{ in.})(0.355\text{ in.}) \\ &= 1.95\text{ in.}^2 \end{aligned}$$

Because the angles are welded and there is no reduction for bolt holes:

$$\begin{aligned} A_{nv} &= A_{gv} \\ &= 1.95\text{ in.}^2 \end{aligned}$$

$$U_{bs} = 1$$

and

$$\begin{aligned} R_n &= 0.60(65\text{ ksi})(1.95\text{ in.}^2) + 1.0(65\text{ ksi})(5.15\text{ in.}^2) \le 0.60(50\text{ ksi})(1.95\text{ in.}^2) + 1.0(65\text{ ksi})(5.15\text{ in.}^2) \\ &= 411\text{ kips} > 393\text{ kips} \end{aligned}$$

Therefore:

$$R_n = 393\text{ kips}$$

The available block shear rupture strength of the web is:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi R_n = 0.75(393\text{ kips})$ $= 295\text{ kips} > 60\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{393\text{ kips}}{2.00}$ $= 197\text{ kips} > 40\text{ kips}$ **o.k.**

Tensile Strength of Beam

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the beam is determined from AISC *Specification* Equation J4-1:

$$\begin{aligned} R_n &= F_y A_g \qquad (Spec.\ \text{Eq. J4-1}) \\ &= (50\text{ ksi})(14.7\text{ in.}^2) \\ &= 735\text{ kips} \end{aligned}$$

The available tensile yielding strength of the beam is:

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(735\text{ kips})$ $= 662\text{ kips} > 60\text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{735\text{ kips}}{1.67}$ $= 440\text{ kips} > 40\text{ kips}$ **o.k.**

From AISC *Specification* Section J4.1(b), determine the available tensile rupture strength of the beam. The effective net area is $A_e = A_nU$, where U is determined from AISC *Specification* Table D3.1, Case 2. The value of $\bar{x}$ is determined by treating the W-shape as two channels back-to-back and finding the horizontal distance to the center of gravity of one of the channels from the centerline of the beam. (Note that the fillets are ignored.)

$$\bar{x} = \frac{\Sigma(A\bar{x})}{\Sigma A}$$

$$= \frac{(0.178\text{ in.})[18.0\text{ in.} - 2(0.570\text{ in.})]\left(\dfrac{0.178\text{ in.}}{2}\right) + 2(0.570\text{ in.})\left(\dfrac{7.50\text{ in.}}{2}\right)\left(\dfrac{7.50\text{ in.}/2}{2}\right)}{\left(\dfrac{14.7\text{ in.}^2}{2}\right)}$$

$$= 1.13\text{ in.}$$

The connection length, l, used in the determination of U will be reduced by ¼ in. to account for possible mill underrun. The shear lag factor, U, is:

$$U = 1 - \frac{\bar{x}}{l}$$

$$= 1 - \frac{1.13\text{ in.}}{(3\text{ in.} - \tfrac{1}{4}\text{ in.})}$$

$$= 0.589$$

The minimum value of U can be determined from AISC *Specification* Section D3, where U is the ratio of the gross area of the connected element to the member gross area.

$$U = \frac{A_{nt}}{A_g}$$

$$= \frac{(d - 2t_f)t_w}{A_g}$$

$$= \frac{[18.0\text{ in.} - 2(0.570\text{ in.})](0.355\text{ in.})}{14.7\text{ in.}^2}$$

$$= 0.407$$

AISC *Specification* Table D3.1, Case 2 controls, use U = 0.589. Because the angles are welded and there is no reduction for bolt holes:

$$A_n = A_g$$

$$= 14.7\text{ in.}^2$$

$$A_e = A_n U \qquad \text{(Spec. Eq. D3-1)}$$
$$= (14.7\text{ in.}^2)(0.589)$$
$$= 8.66\text{ in.}^2$$

$$R_n = F_u A_e \qquad \text{(Spec. Eq. J4-2)}$$
$$= (65\text{ ksi})(8.66\text{ in.}^2)$$
$$= 563\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(563\text{ kips})$ $= 422\text{ kips} > 60\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\frac{R_n}{\Omega} = \frac{563\text{ kips}}{2.00}$ $= 282\text{ kips} > 40\text{ kips}$ **o.k.**

Conclusion

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-3 ALL-WELDED DOUBLE-ANGLE CONNECTION

Given:

Repeat Example II.A-1A using AISC *Manual* Table 10-3 and applicable provisions from the AISC *Specification* to verify the strength of an all-welded double-angle connection between an ASTM A992 W36×231 beam and an ASTM A992 W14×90 column flange, as shown in Figure II.A-3-1. Use 70-ksi electrodes and ASTM A36 angles.

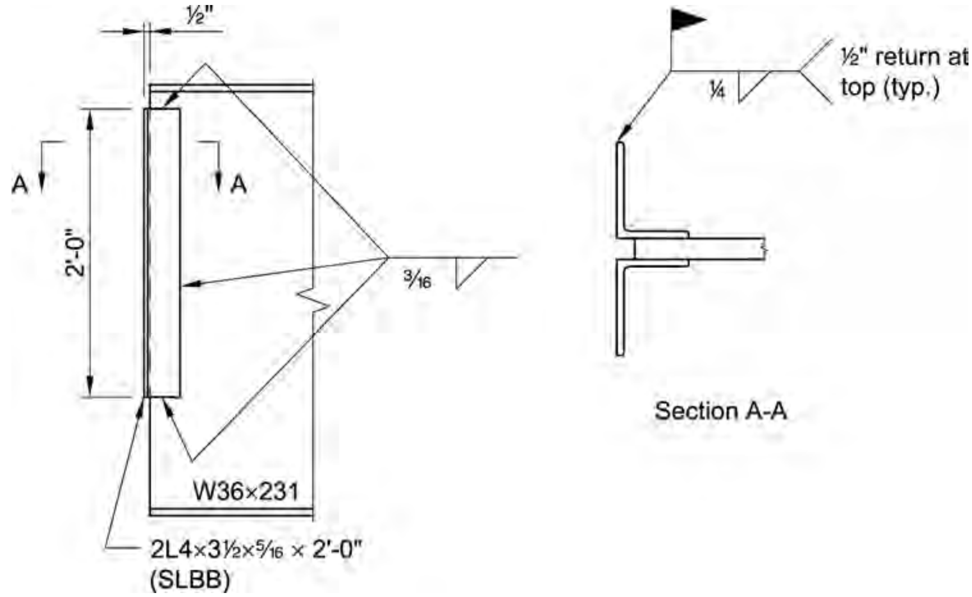


Fig. II.A-3-1. Connection geometry for Example II.A-3.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W36×231
t_w = 0.760 in.

Column
W14×90
t_f = 0.710 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(37.5 \text{ kips}) + 1.6(113 \text{ kips})$ $= 226 \text{ kips}$	$R_a = 37.5 \text{ kips} + 113 \text{ kips}$ $= 151 \text{ kips}$

Design of Weld between Beam Web and Angles

Use AISC *Manual* Table 10-3 (Welds A). Try 3⁄16-in. weld size, l = 24 in.

LRFD	ASD
$\phi R_n = 257 \text{ kips} > 226 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 171 \text{ kips} > 151 \text{ kips}$ **o.k.**

From AISC *Manual* Table 10-3, the minimum beam web thickness is:

$$t_{w\ min} = 0.286 \text{ in.} < 0.760 \text{ in.} \quad \textbf{o.k.}$$

Design of Weld between Column Flange and Angles

Use AISC *Manual* Table 10-3 (Welds B). Try ¼-in. weld size, l = 24 in.

LRFD	ASD
$\phi R_n = 229 \text{ kips} > 226 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 153 \text{ kips} > 151 \text{ kips}$ **o.k.**

From AISC *Manual* Table 10-3, the minimum column flange thickness is:

$$t_{f\ min} = 0.190 \text{ in.} < 0.710 \text{ in.} \quad \textbf{o.k.}$$

Angle Thickness

Minimum angle thickness for weld from AISC *Specification* Section J2.2b:

$$\begin{aligned} t_{min} &= w + \tfrac{1}{16} \text{ in.} \\ &= \tfrac{1}{4} \text{ in.} + \tfrac{1}{16} \text{ in.} \\ &= \tfrac{5}{16} \text{ in.} \end{aligned}$$

Try 2L4×3½×5⁄16 (SLBB).

Shear Strength of Angles

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the angles is determined as follows:

$$\begin{aligned} A_{gv} &= (2 \text{ angles}) lt \\ &= (2 \text{ angles})(24 \text{ in.})(\tfrac{5}{16} \text{ in.}) \\ &= 15.0 \text{ in.}^2 \end{aligned}$$

$$R_n = 0.60F_y A_{gv}$$ (*Spec*. Eq. J4-3)

$$= 0.60(36 \text{ ksi})(15.0 \text{ in.}^2)$$

$$= 324 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(324 \text{ kips})$ $= 324 \text{ kips} > 226 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{324 \text{ kips}}{1.50}$ $= 216 \text{ kips} > 151 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the angles is determined as follows:

$$A_{nv} = (2 \text{ angles})lt$$

$$= (2 \text{ angles})(24 \text{ in.})(\tfrac{5}{16} \text{ in.})$$

$$= 15.0 \text{ in.}^2$$

$$R_n = 0.60F_u A_{nv}$$ (*Spec*. Eq. J4-4)

$$= 0.60(58 \text{ ksi})(15.0 \text{ in.}^2)$$

$$= 522 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(522 \text{ kips})$ $= 392 \text{ kips} > 226 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{522 \text{ kips}}{2.00}$ $= 261 \text{ kips} > 151 \text{ kips}$ **o.k.**

Conclusion

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-4 ALL-BOLTED DOUBLE-ANGLE CONNECTION IN A COPED BEAM

Given:

Use AISC *Manual* Table 10-1 to verify the available strength of an all-bolted double-angle connection between an ASTM A992 W18×50 beam and an ASTM A992 W21×62 girder web, as shown in Figure II.A-4-1, to support the following beam end reactions:

R_D = 10 kips
R_L = 30 kips

The beam top flange is coped 2 in. deep by 4 in. long, l_{ev} = 1¼ in., l_{eh} = 1⅝ in. Use ASTM A36 angles.

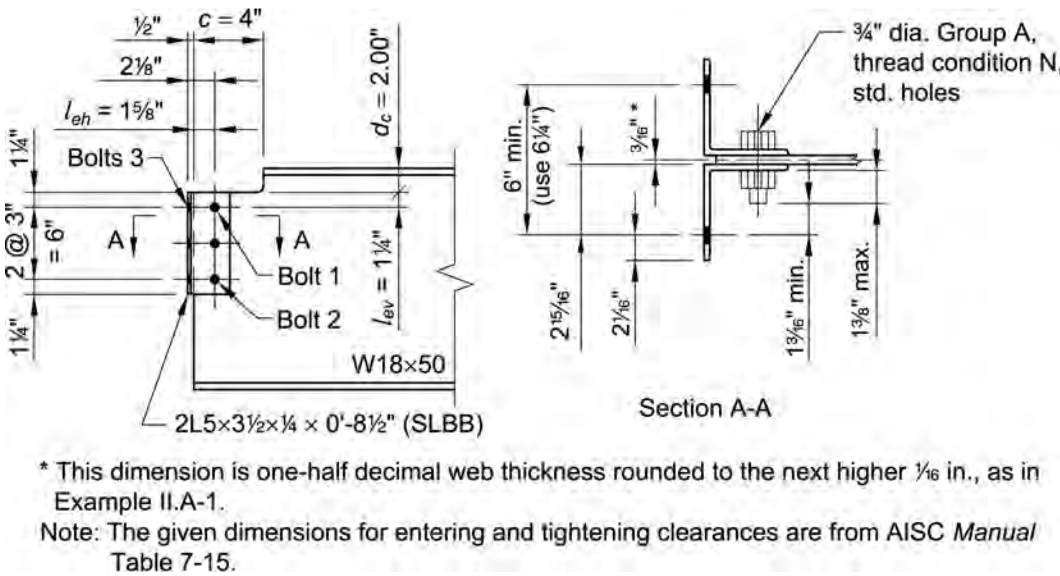


Fig. II.A-4-1. Connection geometry for Example II.A-4.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and girder
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 the geometric properties are as follows:

Beam
W18×50
d = 18.0 in.
t_w= 0.355 in.

Girder
W21×62
t_w = 0.400 in.

From AISC *Specification* Table J3.3, the hole diameter of a ¾-in.-diameter bolt in a standard hole is:

d_h = 13⁄16 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(10 \text{ kips}) + 1.6(30 \text{ kips})$ $= 60.0 \text{ kips}$	$R_a = 10 \text{ kips} + 30 \text{ kips}$ $= 40.0 \text{ kips}$

Connection Design

Tabulated values in AISC *Manual* Table 10-1 consider the limit states of bolt shear, bolt bearing and tearout on the angles, shear yielding of the angles, shear rupture of the angles, and block shear rupture of the angles.

Try 3 rows of bolts and 2L5×3½×¼ (SLBB).

LRFD	ASD
$\phi R_n = 76.7 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 51.1 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

Coped Beam Strength

From AISC *Manual* Part 9, the available coped beam web strength is the lesser of the limit states of flexural local web buckling, shear yielding, shear rupture, block shear rupture, and the sum of the effective strengths of the individual fasteners. From the Commentary to AISC *Specification* Section J3.6, the effective strength of an individual fastener is the lesser of the fastener shear strength, the bearing strength at the bolt holes and the tearout strength at the bolt holes.

Flexural local web buckling of beam web

As shown in AISC *Manual* Figure 9-2, the cope dimensions are:

c = 4 in.

d_c = 2.00 in.

$$\begin{aligned} e &= c + setback \\ &= 4 \text{ in.} + \tfrac{1}{2} \text{ in.} \\ &= 4.50 \text{ in.} \end{aligned}$$

$$\begin{aligned} h_o &= d - d_c \\ &= 18.0 \text{ in.} - 2.00 \text{ in.} \\ &= 16.0 \text{ in.} \end{aligned}$$

$$\begin{aligned} \frac{c}{d} &= \frac{4 \text{ in.}}{18.0 \text{ in.}} \\ &= 0.222 \end{aligned}$$

$$\frac{c}{h_o} = \frac{4 \text{ in.}}{16.0 \text{ in.}}$$
$$= 0.250$$

Because $\frac{c}{d} \leq 1.0$:

$$f = 2\left(\frac{c}{d}\right) \qquad \text{(\textit{Manual} Eq. 9-14a)}$$
$$= 2(0.222)$$
$$= 0.444$$

Because $\frac{c}{h_o} \leq 1.0$:

$$k = 2.2\left(\frac{h_o}{c}\right)^{1.65} \qquad \text{(\textit{Manual} Eq. 9-13a)}$$
$$= 2.2\left(\frac{16.0 \text{ in.}}{4 \text{ in.}}\right)^{1.65}$$
$$= 21.7$$

$$\lambda = \frac{h_o}{t_w} \qquad \text{(\textit{Manual} Eq. 9-11)}$$
$$= \frac{16.0 \text{ in.}}{0.355 \text{ in.}}$$
$$= 45.1$$

$$k_1 = fk \geq 1.61 \qquad \text{(\textit{Manual} Eq. 9-10)}$$
$$= (0.444)(21.7) \geq 1.61$$
$$= 9.63$$

$$\lambda_p = 0.475\sqrt{\frac{k_1 E}{F_y}} \qquad \text{(\textit{Manual} Eq. 9-12)}$$
$$= 0.475\sqrt{\frac{(9.63)(29{,}000 \text{ ksi})}{50 \text{ ksi}}}$$
$$= 35.5$$

$$2\lambda_p = 2(35.5)$$
$$= 71.0$$

Because $\lambda_p < \lambda \leq 2\lambda_p$, calculate the nominal flexural strength using AISC *Manual* Equation 9-7.

The plastic section modulus of the coped section, Z_{net}, is determined from Table IV-11 (included in Part IV of this document).

$$Z_{net} = 42.5 \text{ in.}^3$$

$$M_p = F_y Z_{net}$$
$$= (50 \text{ ksi})(42.5 \text{ in.}^3)$$
$$= 2{,}130 \text{ kip-in.}$$

From AISC *Manual* Table 9-2:

$$S_{net} = 23.4 \text{ in.}^3$$

$$M_y = F_y S_{net}$$
$$= (50 \text{ ksi})(23.4 \text{ in.}^3)$$
$$= 1{,}170 \text{ kip-in.}$$

$$M_n = M_p - (M_p - M_y)\left(\frac{\lambda}{\lambda_p} - 1\right)$$ (*Manual* Eq. 9-7)
$$= (2{,}130 \text{ kip-in.}) - (2{,}130 \text{ kip-in.} - 1{,}170 \text{ kip-in.})\left(\frac{45.1}{35.5} - 1\right)$$
$$= 1{,}870 \text{ kip-in.}$$

$$R_n = \frac{M_n}{e}$$
$$= \frac{1{,}870 \text{ kip-in.}}{4.50 \text{ in.}}$$
$$= 416 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(416 \text{ kips})$ $= 374 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\frac{R_n}{\Omega} = \frac{416 \text{ kips}}{1.67}$ $= 249 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

Shear Strength of Beam Web

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the beam web is determined as follows:

$$A_{gv} = h_o t_w$$
$$= (16.0 \text{ in.})(0.355 \text{ in.})$$
$$= 5.68 \text{ in.}^2$$

$$R_n = 0.60 F_y A_{gv}$$ (*Spec.* Eq. J4-3)
$$= 0.60(50 \text{ ksi})(5.68 \text{ in.}^2)$$
$$= 170 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(170 \text{ kips})$ $= 170 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{170 \text{ kips}}{1.50}$ $= 113 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the beam web is determined as follows:

$$A_{nv} = \left[h_o - 3\left(d_h + \tfrac{1}{16} \text{ in.}\right)\right]t_w$$
$$= \left[16.0 \text{ in.} - 3\left(\tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)\right](0.355 \text{ in.})$$
$$= 4.75 \text{ in.}^2$$

$$R_n = 0.60F_u A_{nv}$$ (*Spec*. Eq. J4-4)
$$= 0.60(65 \text{ ksi})\left(4.75 \text{ in.}^2\right)$$
$$= 185 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(185 \text{ kips})$ $= 139 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{185 \text{ kips}}{2.00}$ $= 92.5 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

Block Shear Rupture of Beam Web

From AISC *Specification* Section J4.3, the block shear rupture strength of the beam web, assuming a total beam setback of ¾ in. (includes ¼ in. tolerance to account for possible mill underrun), is determined as follows.

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs} F_u A_{nt}$$ (*Spec*. Eq. J4-5)

where

$$A_{gv} = (l_{ev} + 2s)t_w$$
$$= \left[1\tfrac{1}{4} \text{ in.} + 2(3.00 \text{ in.})\right](0.355 \text{ in.})$$
$$= 2.57 \text{ in.}^2$$

$$A_{nv} = A_{gv} - 2.5\left(d_h + \tfrac{1}{16} \text{ in.}\right)t_w$$
$$= 2.57 \text{ in.}^2 - 2.5\left(\tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)(0.355 \text{ in.})$$
$$= 1.79 \text{ in.}^2$$

$$A_{nt} = \left[l_{eh} - \tfrac{1}{4} \text{ in.(underrun)} - 0.5\left(d_h + \tfrac{1}{16} \text{ in.}\right)\right]t_w$$
$$= \left[1\tfrac{5}{8} \text{ in.} - \tfrac{1}{4} \text{ in.(underrun)} - 0.5\left(\tfrac{13}{16} + \tfrac{1}{16} \text{ in.}\right)\right](0.355 \text{ in.})$$
$$= 0.333 \text{ in.}^2$$

The block shear reduction coefficient, U_{bs}, is 1.0 for a single row beam end connection as illustrated in AISC *Specification* Commentary Figure C-J4.2.

$$R_n = 0.60(65\text{ ksi})(1.79\text{ in.}^2) + 1.0(65\text{ ksi})(0.333\text{ in.}^2) < 0.60(50\text{ ksi})(2.57\text{ in.}^2) + 1.0(65\text{ ksi})(0.333\text{ in.}^2)$$
$$= 91.5\text{ kips} \leq 98.7\text{ kips}$$

Therefore:

$$R_n = 91.5\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(91.5\text{ kips})$ $= 68.6\text{ kips} > 60.0\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{91.5\text{ kips}}{2.00}$ $= 45.8\text{ kips} > 40.0\text{ kips}$ **o.k.**

Strength of the Bolted Connection—Beam Web Side

From the Commentary to AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the invividual strengths of the individual fasteners, which may be taken as the lesser of the fastener shear strength per AISC *Specification* Section J3.6, the bearing strength at the bolt hole per AISC *Specification* Section J3.10, or the tearout strength at the bolt hole per AISC *Specification* Section J3.10.

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear (or pair of bolts) is:

LRFD	ASD
$\phi r_n = 35.8\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = 23.9\text{ kips/bolt}$

The available bearing and tearout strength of the beam web at Bolt 1, as shown in Figure II.A-4-1, is determine using AISC *Manual* Table 7-5 with l_e = 1¼ in.

LRFD	ASD
$\phi r_n = (49.4\text{ kip/in.})(0.355\text{ in.})$ $= 17.5\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (32.9\text{ kip/in.})(0.355\text{ in.})$ $= 11.7\text{ kips/bolt}$

Therefore, bearing or tearout of the beam web controls over bolt shear for Bolt 1.

The available bearing and tearout strength of the beam web at the other bolts is determine using AISC *Manual* Table 7-4 with s = 3 in.

LRFD	ASD
$\phi r_n = (87.8\text{ kip/in.})(0.355\text{ in.})$ $= 31.2\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (58.5\text{ kip/in.})(0.355\text{ in.})$ $= 20.8\text{ kips/bolt}$

Therefore, bearing or tearout of the beam web controls over bolt shear for the other bolts.

The strength of the bolt group in the beam web is determined by summing the strength of the individual fasteners as follows:

LRFD	ASD
$\phi R_n = (1 \text{ bolt})(17.5 \text{ kips/bolt}) + (2 \text{ bolts})(31.2 \text{ kips/bolt}) = 79.9 \text{ kips/bolt} > 60.0 \text{ kips}$ **o.k.**	$\frac{R_n}{\Omega} = (1 \text{ bolt})(11.7 \text{ kips/bolt}) + (2 \text{ bolts})(20.8 \text{ kips/bolt}) = 53.3 \text{ kips/bolt} > 40.0 \text{ kips}$ **o.k.**

Strength of the Bolted Connection—Support Side

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in single shear is:

LRFD	ASD
$\phi r_n = 17.9 \text{ kips/bolt}$	$\frac{r_n}{\Omega} = 11.9 \text{ kips/bolt}$

Because the girder is not coped, the available bearing and tearout strength of the girder web at all bolts is determined using AISC *Manual* Table 7-4 with s = 3 in.

LRFD	ASD
$\phi r_n = (87.8 \text{ kip/in.})(0.400 \text{ in.}) = 35.1 \text{ kips/bolt}$	$\frac{r_n}{\Omega} = (58.5 \text{ kip/in.})(0.400 \text{ in.}) = 23.4 \text{ kips/bolt}$

Therefore, bolt shear shear controls over bearing and tearout. Bolt shear strength is one of the limit states checked in previous calculations; thus, the effective strength of the fasteners is adequate.

Conclusion

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-5 WELDED/BOLTED DOUBLE-ANGLE CONNECTION IN A COPED BEAM

Given:

Use AISC *Manual* Table 10-2 to verify the available strength of a double angle shear connection welded to an ASTM A992 W18×50 beam and bolted to an ASTM A992 W21×62 girder web, as shown in Figure II.A-5-1. Use 70-ksi electrodes and ASTM A36 angles.

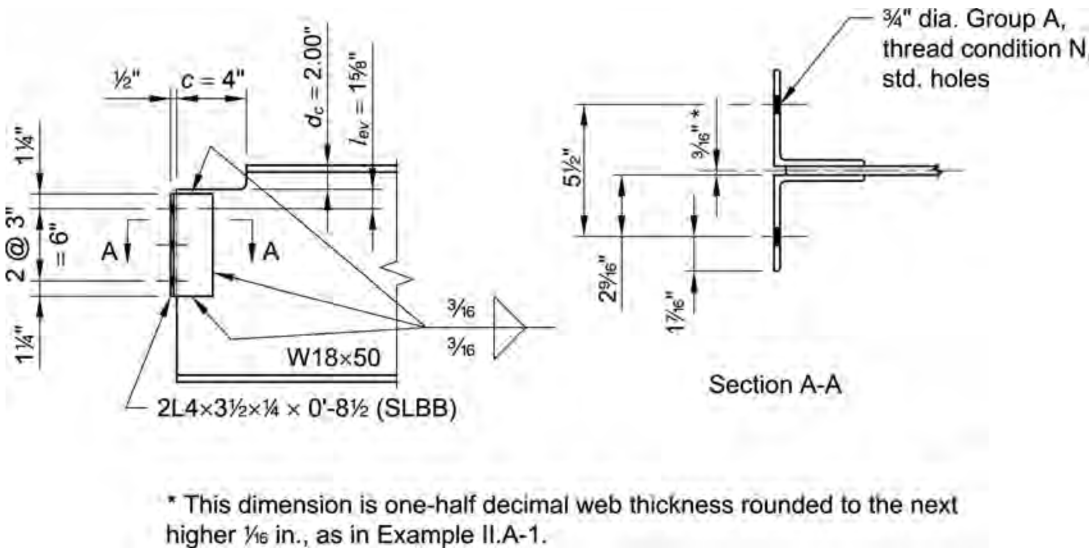


Fig. II.A-5-1. Connection geometry for Example II.A-5.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and girder
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 the geometric properties are as follows:

Beam
W18×50
d = 18.0 in.
t_w = 0.355 in.

Girder
W21×62
t_w = 0.400 in.

From AISC *Specification* Table J3.3, the hole diameter of a ¾-in.-diameter bolt in a standard hole is:

d_h = 13⁄16 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(10 \text{ kips}) + 1.6(30 \text{ kips})$ $= 60.0 \text{ kips}$	$R_a = 10 \text{ kips} + 30 \text{ kips}$ $= 40.0 \text{ kips}$

Weld Design

Use AISC *Manual* Table 10-2 (Welds A). Try 3⁄16-in. weld size, l = 8½ in.

LRFD	ASD
$\phi R_n = 110 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 73.5 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

From AISC *Manual* Table 10-2, the minimum beam web thickness is:

$$t_{w\,min} = 0.286 \text{ in.} < 0.355 \text{ in.} \quad \textbf{o.k.}$$

Minimum Angle Thickness for Weld

From AISC *Specification* Section J2.2b, the minimum angle thickness is:

$$\begin{aligned} t_{min} &= w + 1/16 \text{ in.} \\ &= 3/16 \text{ in.} + 1/16 \text{ in.} \\ &= 1/4 \text{ in.} \end{aligned}$$

Angle and Bolt Design

Tabulated values in AISC *Manual* Table 10-1 consider the limit states of bolt shear, bolt bearing and tearout on the angles, shear yielding of the angles, shear rupture of the angles, and block shear rupture of the angles.

Try 3 rows of bolts and 2L4×3½×¼ (SLBB).

LRFD	ASD
$\phi R_n = 76.7 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 51.1 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

Coped Beam Strength

The available flexural local web buckling strength of the coped beam is verified in Example II.A-4.

Block Shear Rupture of Beam Web

From AISC *Specification* Section J4.3, the block shear rupture strength of the beam web, assuming a total beam setback of ¾ in. (includes ¼ in. tolerance to account for possible mill underrun), is determined as follows.

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \leq 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \qquad \text{(Spec. Eq. J4-5)}$$

where

$$A_{gv} = (l + 3/8 \text{ in.})t_w$$
$$= (8\tfrac{1}{2} \text{ in.} + 3/8 \text{ in.})(0.355 \text{ in.})$$
$$= 3.15 \text{ in.}^2$$

$$A_{nv} = A_{gv}$$
$$= 3.15 \text{ in.}^2$$

$$A_{nt} = (3\tfrac{1}{2} \text{ in.} - 3/4 \text{ in.})t_w$$
$$= (3\tfrac{1}{2} \text{ in.} - 3/4 \text{ in.})(0.355 \text{ in.})$$
$$= 0.976 \text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$R_n = 0.60(65 \text{ ksi})(3.15 \text{ in.}^2) + 1.0(65 \text{ ksi})(0.976 \text{ in.}^2) < 0.60(50 \text{ ksi})(3.15 \text{ in.}^2) + 1.0(65 \text{ ksi})(0.976 \text{ in.}^2)$$
$$= 186 \text{ kips} > 158 \text{ kips}$$

Therefore:

$$R_n = 158 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(158 \text{ kips})$ $= 119 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{158 \text{ kips}}{2.00}$ $= 79.0 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

Shear Strength of Beam Web

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the beam web is determined as follows:

$$A_{gv} = (d - d_c)t_w$$
$$= (18.0 \text{ in.} - 2.00 \text{ in.})(0.355 \text{ in.})$$
$$= 5.68 \text{ in.}^2$$

$$R_n = 0.60F_y A_{gv} \qquad \text{(\textit{Spec.} Eq. J4-3)}$$
$$= 0.60(50 \text{ ksi})(5.68 \text{ in.}^2)$$
$$= 170 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(170 \text{ kips})$ $= 170 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{170 \text{ kips}}{1.50}$ $= 113 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the beam web is determined as follows. Because the angle is welded to the beam web, there is no reduction for bolt holes, therefore:

$$A_{nv} = A_{gv}$$
$$= 5.68 \text{ in.}^2$$

$$R_n = 0.60F_u A_{nv} \qquad \text{(Spec. Eq. J4-4)}$$
$$= 0.60(65 \text{ ksi})(5.68 \text{ in.}^2)$$
$$= 222 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(222 \text{ kips})$ $= 167 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{222 \text{ kips}}{2.00}$ $= 111 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

Effective Strength of the Fasteners to the Girder Web

The effective strength of the fasteners to the girder web is verified in Example II.A-4.

Summary

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-6 BEAM END COPED AT THE TOP FLANGE ONLY

Given:

For an ASTM A992 W21×62 beam coped 8 in. deep by 9 in. long at the top flange only, assuming a ½ in. setback (e = 9½ in.) and using an ASTM A572 Grade 50 plate for the stiffeners and doubler:

A. Calculate the available strength of the beam end, as shown in Figure II.A-6-1(a), considering the limit states of flexural yielding, flexural local buckling, shear yielding and shear rupture.

B. Choose an alternate ASTM A992 W21 shape to eliminate the need for stiffening for the following end reactions:

R_D = 23 kips
R_L = 67 kips

C. Determine the size of doubler plate needed to reinforce the W21×62, as shown in Figure II.A-6-1(c), for the given end reaction in Solution B.

D. Determine the size of longitudinal stiffeners needed to stiffen the W21, as shown in Figure II.A-6-1(d), for the given end reaction in Solution B.

Assume the shear connection is welded to the beam web.

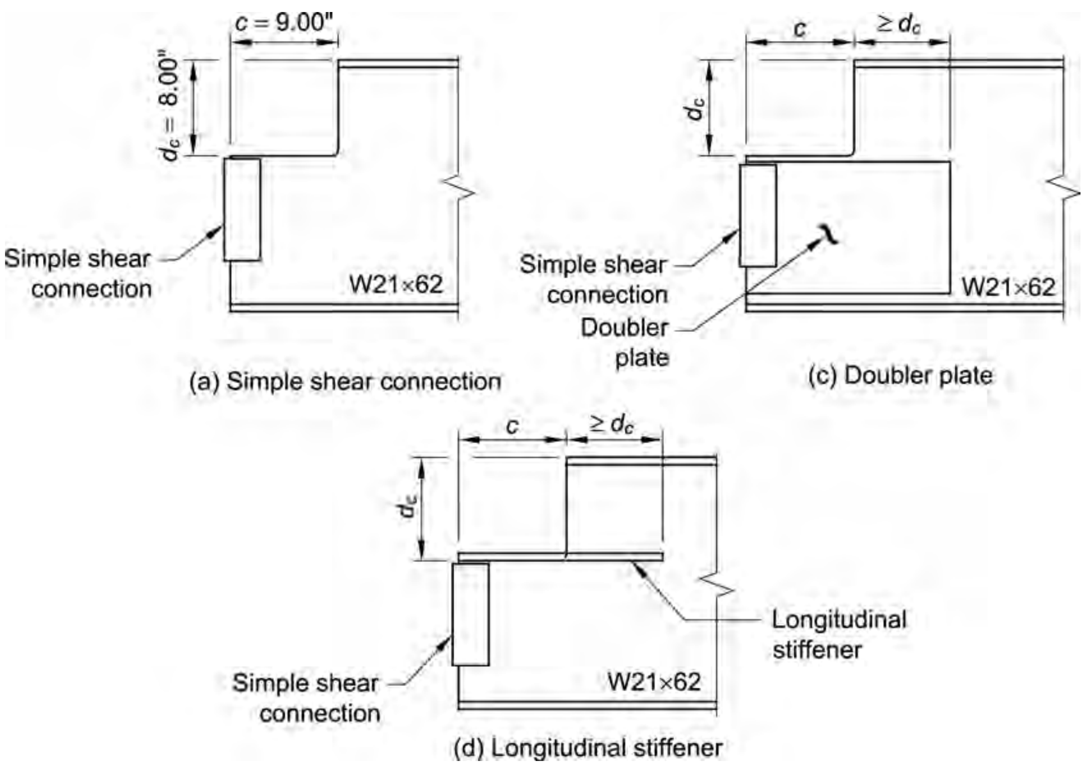


Fig. II.A-6-1. Connection geometry for Example II.A-6.

Solution A:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
W21×62
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A572 Grade 50
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1 the geometric properties are as follows:

Beam
W21×62
d = 21.0 in.
t_w = 0.400 in.
b_f = 8.24 in.
t_f = 0.615 in.

Coped Beam Strength

The beam is assumed to be braced at the end of the uncoped section. Such bracing can be provided by a bracing member or by a slab or other suitable means.

Flexural Local Buckling of Beam Web

The limit state of flexural yielding and local web buckling of the coped beam web are checked using AISC *Manual* Part 9 as follows.

$$\begin{aligned} h_o &= d - d_c \text{ (from AISC } \textit{Manual} \text{ Figure 9-2)} \\ &= 21.0 \text{ in.} - 8.00 \text{ in.} \\ &= 13.0 \text{ in.} \end{aligned}$$

$$\begin{aligned} \frac{c}{d} &= \frac{9.00 \text{ in.}}{21.0 \text{ in.}} \\ &= 0.429 \end{aligned}$$

$$\begin{aligned} \frac{c}{h_o} &= \frac{9.00 \text{ in.}}{13.0 \text{ in.}} \\ &= 0.692 \end{aligned}$$

Because $\frac{c}{d} \le 1.0$, the buckling adjustment factor, f, is calculated as:

$$\begin{aligned} f &= 2\left(\frac{c}{d}\right) \\ &= 2(0.429) \\ &= 0.858 \end{aligned}$$ (*Manual* Eq. 9-14a)

Because $\frac{c}{h_o} \le 1.0$, the plate buckling coefficient, k, is calculated as:

$$k = 2.2\left(\frac{h_o}{c}\right)^{1.65}$$ (*Manual* Eq. 9-13a)

$$= 2.2\left(\frac{13.0\text{ in.}}{9.00\text{ in.}}\right)^{1.65}$$
$$= 4.04$$

The modified plate buckling coefficient, k_1, is calculated as:

$$k_1 = fk \geq 1.61$$ (*Manual* Eq. 9-10)

$$= (0.858)(4.04) > 1.61$$
$$= 3.47$$

The plastic section modulus, Z_{net}, is determined from Table IV-11 (included in Part IV of this document):

$$Z_{net} = 32.2\text{ in.}^3$$

The plastic moment capacity, M_p, is:

$$M_p = F_y Z_{net}$$
$$= (50\text{ ksi})(32.2\text{ in.}^3)$$
$$= 1{,}610\text{ kip-in.}$$

The elastic section modulus, S_{net}, is determined from AISC *Manual* Table 9-2:

$$S_{net} = 17.8\text{ in.}^3$$

The flexural yield moment, M_y, is:

$$M_y = F_y S_{net}$$
$$= (50\text{ ksi})(17.8\text{ in.}^3)$$
$$= 890\text{ kip-in.}$$

$$\lambda = \frac{h_o}{t_w}$$ (*Manual* Eq. 9-11)

$$= \frac{13.0\text{ in.}}{0.400\text{ in.}}$$
$$= 32.5$$

$$\lambda_p = 0.475\sqrt{\frac{k_1 E}{F_y}}$$ (*Manual* Eq. 9-12)

$$= 0.475\sqrt{\frac{(3.47)(29{,}000\text{ ksi})}{50\text{ ksi}}}$$
$$= 21.3$$

$$2\lambda_p = 2(21.3)$$
$$= 42.6$$

Because $\lambda_p < \lambda \le 2\lambda_p$, the nominal flexural strength is:

$$M_n = M_p - (M_p - M_y)\left(\frac{\lambda}{\lambda_p} - 1\right) \quad \text{(Manual Eq. 9-7)}$$
$$= 1{,}610 \text{ kip-in.} - (1{,}610 \text{ kip-in.} - 890 \text{ kip-in.})\left(\frac{32.5}{21.3} - 1\right)$$
$$= 1{,}230 \text{ kip-in.}$$

The nominal strength of the coped section is:

$$R_n = \frac{M_n}{e}$$
$$= \frac{1{,}230 \text{ kip-in.}}{9.50 \text{ in.}}$$
$$= 129 \text{ kips}$$

The available strength of the coped section is:

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(129 \text{ kips})$ $= 116 \text{ kips}$	$\Omega = 1.67$ $\frac{R_n}{\Omega} = \frac{129 \text{ kips}}{1.67}$ $= 77.2 \text{ kips}$

Shear Strength of Beam Web

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the beam web is determined as follows:

$$A_{gv} = (d - d_c)t_w$$
$$= (21.0 \text{ in.} - 8.00 \text{ in.})(0.400 \text{ in.})$$
$$= 5.20 \text{ in.}^2$$

$$R_n = 0.60F_y A_{gv} \quad \text{(Spec. Eq. J4-3)}$$
$$= 0.60(50 \text{ ksi})(5.20 \text{ in.}^2)$$
$$= 156 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(156 \text{ kips})$ $= 156 \text{ kips}$	$\Omega = 1.50$ $\frac{R_n}{\Omega} = \frac{156 \text{ kips}}{1.50}$ $= 104 \text{ kips}$

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the beam web is determined as follows. Because the connection is welded to the beam web there is no reduction for bolt holes, therefore:

$$A_{nv} = A_{gv} = 5.20 \text{ in.}^2$$

$$R_n = 0.60F_u A_{nv} = 0.60(65 \text{ ksi})(5.20 \text{ in.}^2) = 203 \text{ kips}$$ (*Spec*. Eq. J4-4)

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(203 \text{ kips}) = 152 \text{ kips}$	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{203 \text{ kips}}{2.00} = 102 \text{ kips}$

Thus, the available strength of the beam is controlled by the coped section.

LRFD	ASD
$\phi R_n = 116 \text{ kips}$	$\dfrac{R_n}{\Omega} = 77.2 \text{ kips}$

Solution B:

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(23 \text{ kips}) + 1.6(67 \text{ kips}) = 135 \text{ kips}$	$R_a = 23 \text{ kips} + 67 \text{ kips} = 90.0 \text{ kips}$

Try a W21×73.

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
W21×73
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1 the geometric properties are as follows:

Beam
W21×73
d = 21.2 in.
t_w = 0.455 in.
b_f = 8.30 in.
t_f = 0.740 in.

Flexural Local Buckling of Beam Web

The limit state of flexural yielding and local web buckling of the coped beam web are checked using AISC *Manual* Part 9 as follows.

$$\begin{aligned} h_o &= d - d_c \text{ (from AISC } \textit{Manual} \text{ Figure 9-2)} \\ &= 21.2 \text{ in.} - 8.00 \text{ in.} \\ &= 13.2 \text{ in.} \end{aligned}$$

$$\begin{aligned} \frac{c}{d} &= \frac{9.00 \text{ in.}}{21.2 \text{ in.}} \\ &= 0.425 \end{aligned}$$

$$\begin{aligned} \frac{c}{h_o} &= \frac{9.00 \text{ in.}}{13.2 \text{ in.}} \\ &= 0.682 \end{aligned}$$

Because $\frac{c}{d} \le 1.0$, the buckling adjustment factor, f, is calculated as:

$$\begin{aligned} f &= 2\left(\frac{c}{d}\right) \\ &= 2(0.425) \\ &= 0.850 \end{aligned}$$ (*Manual* Eq. 9-14a)

Because $\frac{c}{h_o} \le 1.0$, the plate buckling coefficient, k, is calculated as:

$$\begin{aligned} k &= 2.2\left(\frac{h_o}{c}\right)^{1.65} \\ &= 2.2\left(\frac{13.2 \text{ in.}}{9.00 \text{ in.}}\right)^{1.65} \\ &= 4.14 \end{aligned}$$ (*Manual* Eq. 9-13a)

The modified plate buckling coefficient, k_1, is calculated as:

$$\begin{aligned} k_1 &= fk \ge 1.61 \\ &= (0.850)(4.14) > 1.61 \\ &= 3.52 \end{aligned}$$ (*Manual* Eq. 9-10)

The plastic section modulus, Z_{net}, is determined from Table IV-11 (included in Part IV of this document):

$$Z_{net} = 37.6 \text{ in.}^3$$

The plastic moment capacity, M_p, is:

$$\begin{aligned} M_p &= F_y Z_{net} \\ &= (50 \text{ ksi})(37.6 \text{ in.}^3) \\ &= 1{,}880 \text{ kip-in.} \end{aligned}$$

The elastic section modulus, S_{net}, is determined from AISC *Manual* Table 9-2:

$$S_{net} = 21.0 \text{ in.}^3$$

The flexural yield moment, M_y, is:

$$\begin{aligned} M_y &= F_y S_{net} \\ &= (50 \text{ ksi})(21.0 \text{ in.}^3) \\ &= 1{,}050 \text{ kip-in.} \end{aligned}$$

$$\begin{aligned} \lambda &= \frac{h_o}{t_w} \\ &= \frac{13.2 \text{ in.}}{0.455 \text{ in.}} \\ &= 29.0 \end{aligned}$$ (*Manual* Eq. 9-11)

$$\begin{aligned} \lambda_p &= 0.475\sqrt{\frac{k_1 E}{F_y}} \\ &= 0.475\sqrt{\frac{(3.52)(29{,}000 \text{ ksi})}{50 \text{ ksi}}} \\ &= 21.5 \end{aligned}$$ (*Manual* Eq. 9-11)

$$\begin{aligned} 2\lambda_p &= 2(21.5) \\ &= 43.0 \end{aligned}$$

Since $\lambda_p < \lambda \le 2\lambda_p$, the nominal flexural strength is:

$$\begin{aligned} M_n &= M_p - (M_p - M_y)\left(\frac{\lambda}{\lambda_p} - 1\right) \\ &= 1{,}880 \text{ kip-in.} - (1{,}880 \text{ kip-in.} - 1{,}050 \text{ kip-in.})\left(\frac{29.0}{21.5} - 1\right) \\ &= 1{,}590 \text{ kip-in.} \end{aligned}$$ (*Manual* Eq. 9-7)

The nominal strength of the coped section is:

$$\begin{aligned} R_n &= \frac{M_n}{e} \\ &= \frac{1{,}590 \text{ kip-in.}}{9.50 \text{ in.}} \\ &= 167 \text{ kips} \end{aligned}$$

The available strength of the coped section is:

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(167\text{ kips})$ $= 150\text{ kips}$	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{167\text{ kips}}{1.67}$ $= 100\text{ kips}$

Shear Strength of Beam Web

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the beam web is determined as follows:

$$\begin{aligned} A_{gv} &= (d - d_c)t_w \\ &= (21.2\text{ in.} - 8.00\text{ in.})(0.455\text{ in.}) \\ &= 6.01\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_n &= 0.60F_y A_{gv} \\ &= 0.60(50\text{ ksi})(6.01\text{ in.}^2) \\ &= 180\text{ kips} \end{aligned}$$ (*Spec*. Eq. J4-3)

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(180\text{ kips})$ $= 180\text{ kips}$	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{180\text{ kips}}{1.50}$ $= 120\text{ kips}$

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the beam web is determined as follows. Because the connection is welded to the beam web, there is no reduction for bolt holes, therefore:

$$\begin{aligned} A_{nv} &= A_{gv} \\ &= 6.01\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_n &= 0.60F_u A_{nv} \\ &= 0.60(65\text{ ksi})(6.01\text{ in.}^2) \\ &= 234\text{ kips} \end{aligned}$$ (*Spec*. Eq. J4-4)

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(234\text{ kips})$ $= 176\text{ kips}$	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{234\text{ kips}}{2.00}$ $= 117\text{ kips}$

Thus, the available strength is controlled by the coped section, therefore the available strength of the beam is:

LRFD	ASD
$\phi R_n = 150 \text{ kips} > 135 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 100 \text{ kips} > 90.0 \text{ kips}$ **o.k.**

Solution C:

Doubler Plate Design

The doubler plate is designed using AISC *Manual* Part 9. An ASTM A572 Grade 50 plate is recommended in order to match the beam yield strength. A ¼-in. minimum plate thickness will be used in order to allow the use of a 3⁄16-in. fillet weld. The depth of the plate will be set so that a compact *b/t* ratio from AISC *Specification* Table B4.1b will be satisfied. This is a conservative criterion that will allow local buckling of the doubler to be neglected.

$$\frac{d_p}{t_p} \le 1.12\sqrt{\frac{E}{F_y}}$$

Solving for d_p:

$$\begin{aligned} d_p &\le 1.12 t_p \sqrt{\frac{E}{F_y}} \\ &\le 1.12(0.250 \text{ in.})\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}} \\ &\le 6.74 \text{ in.} \end{aligned}$$

A 6.50 in. doubler plate will be used.

Using principles of mechanics, the elastic section modulus, S_{net}, and plastic section modulus, Z_{net}, are calculated neglecting the fillets and assuming the doubler plate is placed ½-in. down from the top of the cope.

$$S_{net} = 25.5 \text{ in.}^3$$
$$Z_{net} = 44.8 \text{ in.}^3$$

The plastic bending moment, M_p, of the reinforced section is:

$$\begin{aligned} M_p &= F_y Z_{net} \\ &= (50 \text{ ksi})(44.8 \text{ in.}^3) \\ &= 2{,}240 \text{ kip-in.} \end{aligned}$$

The flexural yield moment, M_y, of the reinforced section is:

$$\begin{aligned} M_y &= F_y S_{net} \\ &= (50 \text{ ksi})(25.5 \text{ in.}^3) \\ &= 1{,}280 \text{ kip-in.} \end{aligned}$$

Because $\lambda_p < \lambda \le 2\lambda_p$ for the unreinforced section, the nominal flexural strength is:

$$M_n = M_p - \left(M_p - M_y\right)\left(\frac{\lambda}{\lambda_p} - 1\right) \qquad \text{(}\textit{Manual}\text{ Eq. 9-7)}$$
$$= 2{,}240 \text{ kip-in.} - \left(2{,}240 \text{ kip-in.} - 1{,}280 \text{ kip-in.}\right)\left(\frac{32.5}{21.3} - 1\right)$$
$$= 1{,}740 \text{ kip-in.}$$

The available strength of the coped section is determined as follows:

$$R_n = \frac{M_n}{e}$$
$$= \frac{1{,}740 \text{ kip-in.}}{9.50 \text{ in.}}$$
$$= 183 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90\left(183 \text{ kips}\right)$ $= 165 \text{ kips}$	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{183 \text{ kips}}{1.67}$ $= 110 \text{ kips}$

Shear Strength of Beam Web

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the beam web reinforced with the doubler plate is determined as follows:

$$A_{gv-web} = \left(d - d_c\right)t_w$$
$$= \left(21.0 \text{ in.} - 8.00 \text{ in.}\right)\left(0.400 \text{ in.}\right)$$
$$= 5.20 \text{ in.}^2$$

$$A_{gv-plate} = d_p t_p$$
$$= \left(6.50 \text{ in.}\right)\left(\tfrac{1}{4} \text{ in.}\right)$$
$$= 1.63 \text{ in.}^2$$

$$R_n = 0.60 F_y A_{gv-web} + 0.60 F_y A_{gv-plate} \qquad \text{(from }\textit{Spec}\text{. Eq. J4-3)}$$
$$= 0.60\left(50 \text{ ksi}\right)\left(5.20 \text{ in.}^2\right) + 0.60\left(50 \text{ ksi}\right)\left(1.63 \text{ in.}^2\right)$$
$$= 205 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00\left(205 \text{ kips}\right)$ $= 205 \text{ kips}$	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{205 \text{ kips}}{1.50}$ $= 137 \text{ kips}$

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the beam web reinforced with the doubler plate is determined as follows. Because the connection is welded, there is no reduction for bolt holes, therefore:

$$A_{nv-web} = A_{gv-web}$$
$$= 5.20 \text{ in.}^2$$

$$A_{nv-plate} = A_{gv-plate}$$
$$= 1.63 \text{ in.}^2$$

$$R_n = 0.60F_u A_{nv-web} + 0.60F_u A_{nv-plate}$$ (from *Spec*. Eq. J4-4)
$$= 0.60(65 \text{ ksi})(5.20 \text{ in.}^2) + 0.60(65 \text{ ksi})(1.63 \text{ in.}^2)$$
$$= 266 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(266 \text{ kips})$ $= 200 \text{ kips}$	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{266 \text{ kips}}{2.00}$ $= 133 \text{ kips}$

Thus, the available strength of the beam is controlled by the coped section.

LRFD	ASD
$\phi R_n = 165 \text{ kips} > 135 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 110 \text{ kips} > 90.0 \text{ kips}$ **o.k.**

Weld Design

Determine the length of weld required to transfer the force into and out of the doubler plate. From Solution A, the available strength of the beam web is:

LRFD	ASD
$\phi R_n = 116 \text{ kips}$	$\dfrac{R_n}{\Omega} = 77.2 \text{ kips}$

The available strength of the beam web reinforced with the doubler plate is:

LRFD	ASD
$\phi R_n = 165 \text{ kips}$	$\dfrac{R_n}{\Omega} = 110 \text{ kips}$

The force in the doubler plate is determined as follows:

LRFD	ASD
$\phi = 0.90$ $F_d = 0.90(50 \text{ ksi})(¼ \text{ in.})(6.50 \text{ in.})\left(\dfrac{116 \text{ kips}}{165 \text{ kips}}\right)$ $= 51.4 \text{ kips}$	$\Omega = 1.67$ $F_d = \dfrac{(50 \text{ ksi})(¼ \text{ in.})(6.50 \text{ in.})\left(\dfrac{77.2 \text{ kips}}{110 \text{ kips}}\right)}{1.67}$ $= 34.1 \text{ kips}$

From AISC *Specification* Section J2.4, the doubler plate weld is designed as follows:

$R_n = 0.85R_{nwl} + 1.5R_{nwt}$ (*Spec*. Eq. J2-6b)

LRFD	ASD
From AISC *Manual* Equation 8-2a: $R_{nw} = 1.392Dl$ From AISC *Specification* Equation J2-6b: $51.4 \text{ kips} = \left[\begin{array}{l}(2 \text{ welds})(0.85)(1.392 \text{ kips/in.}) \\ \times(3 \text{ sixteenths})l_w\end{array}\right] + \left[\begin{array}{l}(1.5)(1.392 \text{ kips/in.})(3 \text{ sixteenths}) \\ \times(6.50 \text{ in.})\end{array}\right]$ Solving for l_w: $l_w = 1.50$ in.	From AISC *Manual* Equation 8-2b: $R_{nw} = 0.928Dl$ From AISC *Specification* Equation J2-6b: $34.1 \text{ kips} = \left[\begin{array}{l}(2 \text{ welds})(0.85)(0.928 \text{ kips/in.}) \\ \times(3 \text{ sixteenths})l_w\end{array}\right] + \left[\begin{array}{l}1.5(0.928 \text{ kips/in.})(3 \text{ sixteenths}) \\ \times(6.50 \text{ in.})\end{array}\right]$ Solving for l_w: $l_w = 1.47$ in..

Use 1.50 in. of 3⁄16-in. fillet weld, minimum.

The doubler plate must extend at least d_c beyond the cope. Use a PL¼ in. × 6½ in. × 1ft 5 in. with 3⁄16-in. welds all around.

Solution D:

Longitudinal Stiffener Design

Try PL¼ in.×4 in. slotted to fit over the beam web.

Determine Z_x for the stiffened section:

$$\begin{aligned}A_w &= (d - d_c - t_f)t_w \\ &= (21.0 \text{ in.} - 8.00 \text{ in.} - 0.615 \text{ in.})(0.400 \text{ in.}) \\ &= 4.95 \text{ in.}^2\end{aligned}$$

$$\begin{aligned}A_f &= b_f t_f \\ &= (8.24 \text{ in.})(0.615 \text{ in.}) \\ &= 5.07 \text{ in.}^2\end{aligned}$$

$$\begin{aligned}A_{rp} &= b_p t_p \\ &= (4.00 \text{ in.})(\tfrac{1}{4} \text{ in.}) \\ &= 1.00 \text{ in.}^2\end{aligned}$$

$$\begin{aligned}A_t &= A_w + A_f + A_{rp} \\ &= 4.95 \text{ in.}^2 + 5.07 \text{ in.}^2 + 1.00 \text{ in.}^2 \\ &= 11.0 \text{ in.}^2\end{aligned}$$

The location of the plastic neutral axis (neglecting fillets) from the inside of the flange is:

$$(0.615 \text{ in.})(8.24 \text{ in.}) + y_p(0.400 \text{ in.}) = (\tfrac{1}{4} \text{ in.})(4.00 \text{ in.}) + (12.4 \text{ in.} - y_p)(0.400 \text{ in.})$$

$$y_p = 1.12 \text{ in.}$$

From elementary mechanics, the section properties are as follows:

$$Z_x = 44.3 \text{ in.}^3$$
$$I_x = 253 \text{ in.}^4$$
$$S_{xc} = 28.6 \text{ in.}^3$$
$$S_{xt} = 57.7 \text{ in.}^3$$

$$\begin{aligned} h_c &= 2(13.0 \text{ in.} - 4.39 \text{ in.}) \\ &= 17.2 \text{ in.} \end{aligned}$$

$$\begin{aligned} h_p &= 2(13.0 \text{ in.} - 1.12 \text{ in.} - 0.615 \text{ in.}) \\ &= 22.5 \text{ in.} \end{aligned}$$

Compact section properties for the longitudinal stiffener and the web are determined from AISC *Specification* Table B4.1b, Cases 11 and 16.

$$\begin{aligned} \lambda_p &= 0.38\sqrt{\frac{E}{F_y}} \\ &= 0.38\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}} \\ &= 9.15 \end{aligned}$$ (*Spec.* Table B4.1b, Case 11)

$$\begin{aligned} \lambda &= \frac{b}{t} \\ &= \frac{(4.00 \text{ in.}/2)}{\tfrac{1}{4} \text{ in.}} \\ &= 8.00 \end{aligned}$$

Because $\lambda < \lambda_p$, the stiffener is compact in flexure.

$$\begin{aligned} \lambda_r &= 5.70\sqrt{\frac{E}{F_y}} \\ &= 5.70\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}} \\ &= 137 \end{aligned}$$ (*Spec.* Table B4.1b, Case 16)

$$\begin{aligned} \lambda &= \frac{h_c}{t_w} \\ &= \frac{17.2 \text{ in.}}{0.400 \text{ in.}} \\ &= 43.0 \end{aligned}$$

Because $\lambda < \lambda_r$, the web is not slender, therefore AISC *Specification* Section F4 applies.

Determine if lateral-torsional buckling is a design consideration.

$$a_w = \frac{h_c t_w}{b_{fc} t_{fc}}$$ (*Spec*. Eq. F4-12)

$$= \frac{(17.2 \text{ in.})(0.400 \text{ in.})}{(4.00 \text{ in.})(¼ \text{ in.})}$$

$$= 6.88$$

$$r_t = \frac{b_{fc}}{\sqrt{12\left(1+\frac{1}{6}a_w\right)}}$$ (*Spec*. Eq. F4-11)

$$= \frac{4.00 \text{ in.}}{\sqrt{12\left[1+\frac{1}{6}(6.88)\right]}}$$

$$= 0.788 \text{ in.}$$

$$L_p = 1.1 r_t \sqrt{\frac{E}{F_y}}$$ (*Spec*. Eq. F4-7)

$$= 1.1(0.788 \text{ in.})\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$

$$= 20.9 \text{ in.}$$

The stiffener will not reach a length of 20.9 in. Lateral-torsional buckling is not a design consideration.

Determine if the web of the singly-symmetric shape is compact. AISC *Specification* Table B4.1b, Case 16, applies.

$$\lambda_p = \frac{\frac{h_c}{h_p}\sqrt{\frac{E}{F_y}}}{\left(0.54\frac{M_p}{M_y} - 0.09\right)^2} \le 5.70\sqrt{\frac{E}{F_y}}$$

$$= \frac{\frac{17.2 \text{ in.}}{22.5 \text{ in.}}\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}}{\left[0.54\left(\frac{2{,}220 \text{ kip-in.}}{1{,}430 \text{ kip-in.}}\right) - 0.09\right]^2} \le 5.70\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$

$$= 32.9 < 137$$

$$= 32.9$$

$$\lambda = \frac{h_c}{t_w}$$

$$= \frac{17.2 \text{ in.}}{0.400 \text{ in.}}$$

$$= 43.0$$

Because $\lambda < \lambda_p$, the web is non-compact, therefore AISC *Specification* Section F4 applies.

Since $S_{xt} > S_{xc}$, tension flange yielding does not govern. Determine flexural strength based on compression flange yielding.

$$M_{yc} = S_{xc}F_y$$
$$= \left(28.6\text{ in.}^3\right)\left(50\text{ ksi}\right)$$
$$= 1{,}430\text{ kip-in.}$$

$$I_{yc} = \frac{\left(\tfrac{1}{4}\text{ in.}\right)\left(4.00\text{ in.}\right)^3}{12}$$
$$= 1.33\text{ in.}^4$$

$$I_y = 1.33\text{ in.}^4 + \frac{\left(0.615\text{ in.}\right)\left(8.24\text{ in.}\right)^3}{12} + \frac{\left(12.4\text{ in.}\right)\left(0.400\text{ in.}\right)^3}{12}$$
$$= 30.1\text{ in.}^4$$

$$\frac{I_{yc}}{I_y} = \frac{1.33\text{ in.}^4}{30.1\text{ in.}^4}$$
$$= 0.0442$$

Since $\frac{I_{yc}}{I_y} < 0.23$, $R_{pc} = 1.0$. Thus:

$$M_n = R_{pc}M_{yc}$$
$$= 1.0\left(1{,}430\text{ kip-in.}\right)$$
$$= 1{,}430\text{ kip-in.}$$

The nominal strength of the reinforced section is:

$$R_n = \frac{M_n}{e}$$
$$= \frac{1{,}430\text{ kip-in.}}{9.50\text{ in.}}$$
$$= 151\text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90\left(151\text{ kips}\right)$ $= 136\text{ kips} > 135\text{ kips}$ **o.k.**	$\Omega = 1.67$ $\frac{R_n}{\Omega} = \frac{151\text{ kips}}{1.67}$ $= 90.4\text{ kips} > 90.0\text{ kips}$ **o.k.**

Plate Dimensions

Since the longitudinal stiffening must extend at least d_c beyond the cope, use PL¼ in.×4 in.×1 ft 5 in. with ¼-in. welds.

Weld Strength

By calculations not shown, the moment of inertia of the reinforced section and distance from the centroid to the bottom of the reinforcement plate are:

$$I_{net} = 253 \text{ in.}^4$$

$$\bar{y} = 8.61 \text{ in.}$$

The first moment of the reinforcement plate is:

$$\begin{aligned} Q &= A_p y \\ &= (\text{¼ in.})(4.00 \text{ in.})\left[8.61 \text{ in.} + 0.5(\text{¼ in.})\right] \\ &= 8.74 \text{ in.}^3 \end{aligned}$$

where A_p is the area of the reinforcement plate and y is the distance from the centroid of the reinforced section to the centroid of the reinforcement plate.

From mechanics of materials and shear flow, the force per length that the weld must resist in the area of the cope is:

LRFD	ASD
$r_u = \dfrac{V_u Q}{I_{net}(2 \text{ welds})} = \dfrac{(135 \text{ kips})(8.74 \text{ in.}^3)}{(253 \text{ in.}^4)(2 \text{ welds})} = 2.33 \text{ kip/in.}$	$r_a = \dfrac{V_a Q}{I_{net}(2 \text{ welds})} = \dfrac{(90.0 \text{ kips})(8.74 \text{ in.}^3)}{(253 \text{ in.}^4)(2 \text{ welds})} = 1.55 \text{ kip/in.}$

From mechanics of materials, the force per length that the weld must resist to transfer the force in the reinforcement plate to the beam web is:

LRFD	ASD
$r_u = \dfrac{V_u e Q}{I_{net}(2 \text{ welds})(l-c)} = \dfrac{(135 \text{ kips})(9.50 \text{ in.})(8.74 \text{ in.}^3)}{(253 \text{ in.}^4)(2 \text{ welds})(17.0 \text{ in.} - 9.00 \text{ in.})} = 2.77 \text{ kip/in.}$ **controls**	$r_a = \dfrac{V_a e Q}{I_{net}(2 \text{ welds})(l-c)} = \dfrac{(90.0 \text{ kips})(9.50 \text{ in.})(8.74 \text{ in.}^3)}{(253 \text{ in.}^4)(2 \text{ welds})(17.0 \text{ in.} - 9.00 \text{ in.})} = 1.85 \text{ kip/in.}$ **controls**

The weld capacity from AISC *Manual* Part 8:

LRFD	ASD
$\phi r_n = (1.392 \text{ kip/in.})D$ (from *Manual* Eq. 8-2a) $= (1.392 \text{ kip/in.})(4 \text{ sixteenths}) = 5.57 \text{ kip/in.} > 2.77 \text{ kip/in.}$ **o.k.**	$\dfrac{r_n}{\Omega} = (0.928 \text{ kip/in.})D$ (from *Manual* Eq. 8-2b) $= (0.928 \text{ kip/in.})(4 \text{ sixteenths}) = 3.71 \text{ kip/in.} > 1.85 \text{ kip/in.}$ **o.k.**

Determine if the web has adequate shear rupture capacity:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = \phi 0.60 F_u A_{nv}$ (from *Spec*. Eq. J4-4) $= \dfrac{0.75(0.60)(65\text{ ksi})(0.400\text{ in.})}{2\text{ welds}}$ $= 5.85\text{ kip/in.} > 2.77\text{ kip/in.}$ **o.k.**	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ (from *Spec*. Eq. J4-4) $= \dfrac{0.60(65\text{ ksi})(0.400\text{ in.})}{2.00(2\text{ welds})}$ $= 5.85\text{ kip/in.} > 1.85\text{ kip/in.}$ **o.k.**

EXAMPLE II.A-7 BEAM END COPED AT THE TOP AND BOTTOM FLANGES

Given:

Determine the available strength for an ASTM A992 W16×40 coped 3½ in. deep by 9½ in. wide at the top flange and 2 in. deep by 9½ in. wide at the bottom flange, as shown in Figure II.A-7-1, considering the limit states of flexural yielding and local buckling. Assume a ½-in. setback from the face of the support to the end of the beam.

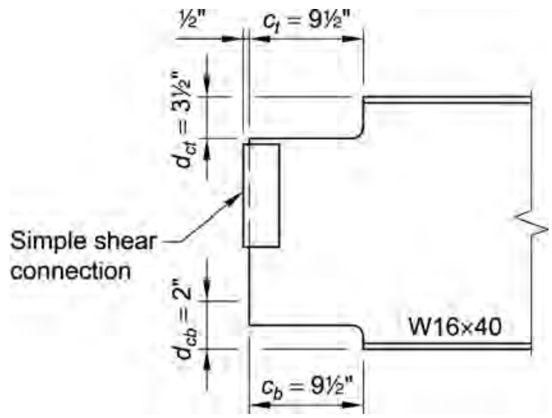


Fig. II.A-7-1. Connection geometry for Example II.A-7.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
W16×40
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1 and AISC *Manual* Figure 9-3, the geometric properties are as follows:

Beam
W16×40
$d = 16.0$ in.
$t_w = 0.305$ in.
$t_f = 0.505$ in.
$b_f = 7.00$ in.
$c_t = 9\frac{1}{2}$in.
$d_{ct} = 3\frac{1}{2}$ in.
$c_b = 9\frac{1}{2}$ in.
$d_{cb} = 2$ in.

$$\begin{aligned} e &= 9\tfrac{1}{2}\text{ in.} + \tfrac{1}{2}\text{ in.} \\ &= 10.0\text{ in.} \end{aligned}$$

$$\begin{aligned} h_o &= d - d_{ct} - d_{cb} \\ &= 16.0\text{ in.} - 3\tfrac{1}{2}\text{ in.} - 2\text{ in.} \\ &= 10.5\text{ in.} \end{aligned}$$

For a beam that is coped at both flanges, the local flexural strength is determined in accordance with AISC *Specification* Section F11.

Available Strength at Coped Section

The cope at the tension side of the beam is equal to the cope length at the compression side. From AISC *Manual* Part 9, $L_b = c_t$ and d_{ct} is the depth of the cope at the top flange.

$$C_b = \left[3 + \ln\left(\frac{L_b}{d}\right)\right]\left(1 - \frac{d_{ct}}{d}\right) \leq 1.84 \quad \text{(\textit{Manual} Eq. 9-15)}$$

$$= \left[3 + \ln\left(\frac{9\tfrac{1}{2}\text{ in.}}{16.0\text{ in.}}\right)\right]\left(1 - \frac{3\tfrac{1}{2}\text{ in.}}{16.0\text{ in.}}\right) \leq 1.84$$

$$= 1.94 > 1.84$$

Use $C_b = 1.84$.

The available strength of the coped section is determined using AISC *Specification* Section F11, with $d = h_o = 10.5$ in. and unbraced length $L_b = c_t = 9\tfrac{1}{2}$ in.

$$\frac{L_b d}{t^2} = \frac{(9\tfrac{1}{2}\text{ in.})(10.5\text{ in.})}{(0.305\text{ in.})^2}$$

$$= 1{,}070$$

$$\frac{0.08E}{F_y} = \frac{0.08(29{,}000\text{ ksi})}{50\text{ ksi}}$$

$$= 46.4$$

$$\frac{1.9E}{F_y} = \frac{1.9(29{,}000\text{ ksi})}{50\text{ ksi}}$$

$$= 1{,}100$$

Since $\frac{0.08E}{F_y} < \frac{L_b d}{t^2} \leq \frac{1.9E}{F_y}$, the limit state of lateral-torsional buckling applies. The nominal flexural strength of the coped portion of the web is determined using AISC *Specification* Section F11.2(b).

Determine the net elastic and plastic section moduli:

$$S_{net} = \frac{t_w h_o^2}{6}$$

$$= \frac{(0.305\text{ in.})(10.5\text{ in.})^2}{6}$$

$$= 5.60\text{ in.}^3$$

$$Z_{net} = \frac{t_w h_o^2}{4}$$
$$= \frac{(0.305 \text{ in.})(10.5 \text{ in.})^2}{4}$$
$$= 8.41 \text{ in.}^3$$

$$M_y = F_y S_{net}$$
$$= (50 \text{ ksi})(5.60 \text{ in.}^3)$$
$$= 280 \text{ kip-in.}$$

$$M_p = F_y Z_{net}$$
$$= (50 \text{ ksi})(8.41 \text{ in.}^3)$$
$$= 421 \text{ kip-in.}$$

$$M_n = C_b \left[1.52 - 0.274\left(\frac{L_b d}{t^2}\right)\frac{F_y}{E}\right] M_y \leq M_p$$ (*Spec*. Eq. F11-2)

$$= 1.84\left[1.52 - 0.274(1{,}070)\left(\frac{50 \text{ ksi}}{29{,}000 \text{ ksi}}\right)\right](280 \text{ kip-in.}) \leq 421 \text{ kip-in.}$$
$$= 523 \text{ kip-in.} > 421 \text{ kip-in.}$$

The nominal moment capacity of the reduced section is 421 kip-in. The nominal strength of the coped section is:

$$R_n = \frac{M_n}{e}$$
$$= \frac{421 \text{ kip-in.}}{10.0 \text{ in.}}$$
$$= 42.1 \text{ kips}$$

The available strength at the coped end is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b R_n = 0.90(42.1 \text{ kips})$ $= 37.9 \text{ kips}$	$\Omega_b = 1.67$ $\frac{R_n}{\Omega_b} = \frac{42.1 \text{ kips}}{1.67}$ $= 25.2 \text{ kips}$

EXAMPLE II.A-8 ALL-BOLTED DOUBLE-ANGLE CONNECTIONS (BEAMS-TO-GIRDER WEB)

Given:

Verify the all-bolted double-angle connections for back-to-back ASTM A992 W12×40 and W21×50 beams to an ASTM A992 W30×99 girder-web to support the end reactions shown in Figure II.A-8-1. Use ASTM A36 angles.

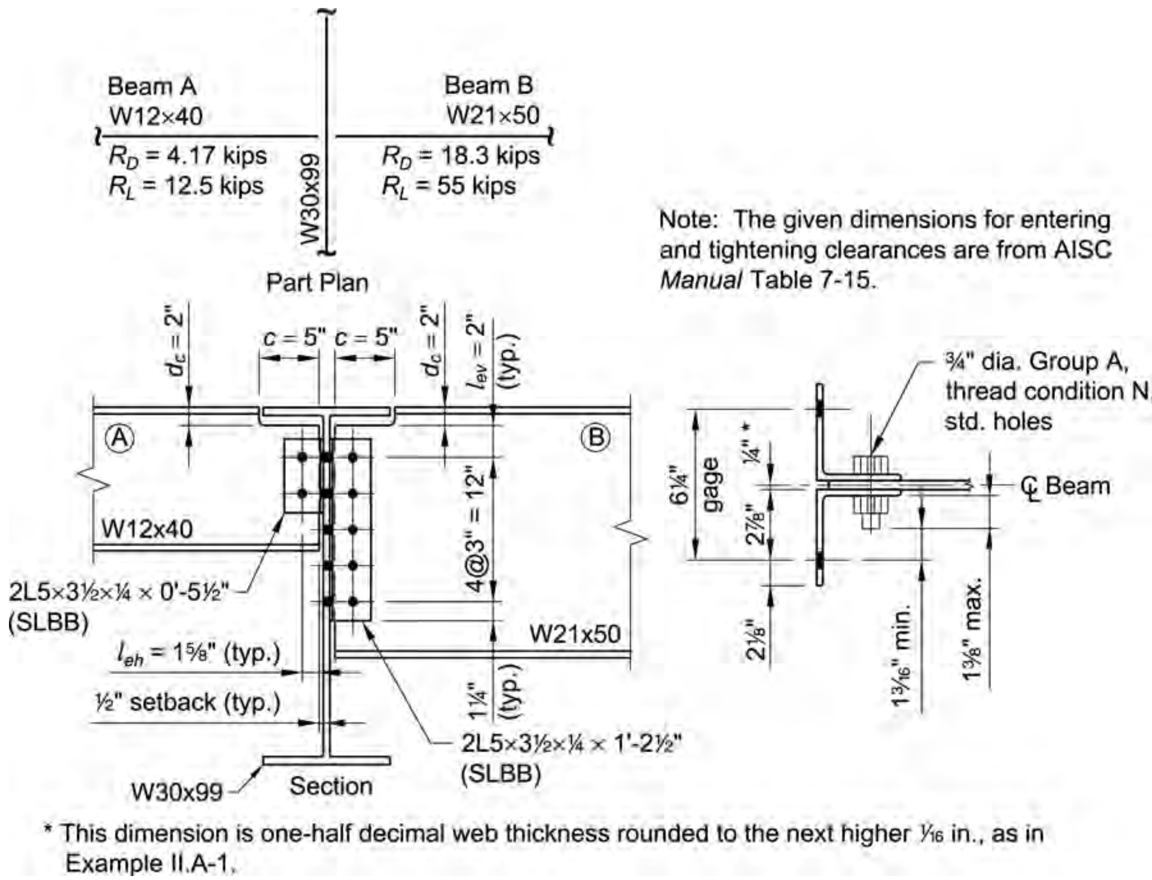


Fig. II.A-8-1. Connection geometry for Example II.A-8.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beams and girder
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1 the geometric properties are as follows:

Beam
W12×40
t_w = 0.295 in.
d = 11.9 in.

Beam
W21×50
t_w = 0.380 in.
d = 20.8 in.

Girder
W30×99
t_w = 0.520 in.
d = 29.7 in.

From AISC *Specification* Table J3.3, for ¾-in.-diameter bolts with standard holes:

d_h = 13⁄16 in.

Beam A Connection:

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(4.17\text{ kips}) + 1.6(12.5\text{ kips})$ $= 25.0\text{ kips}$	$R_a = 4.17\text{ kips} + 12.5\text{ kips}$ $= 16.7\text{ kips}$

Strength of Bolted Connection—Angles

AISC *Manual* Table 10-1 includes checks for the limit states of bolt shear, bolt bearing on the angles, tearout on the angles, shear yielding of the angles, shear rupture of the angles, and block shear rupture of the angles. For two rows of bolts and ¼-in. angle thickness:

LRFD	ASD
$\phi R_n = 48.9\text{ kips} > 25.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 32.6\text{ kips} > 16.7\text{ kips}$ **o.k.**

Strength of the Bolted Connection—Beam Web

From the Commentary to AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the invividual strengths of the individual fasteners, taken as the lesser of the fastener shear strength per AISC *Specification* Section J3.6, the bearing strength at the bolt hole per AISC *Specification* Section J3.10, or the tearout strength at the bolt hole per AISC *Specification* Section J3.10.

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear is:

LRFD	ASD
$\phi r_n = 35.8\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = 23.9\text{ kips/bolt}$

The available bearing and tearout strength of the beam web at the top bolt is determined using AISC *Manual* Table 7-5, with l_e = 2 in., as follows:

LRFD	ASD
$\phi r_n = (87.8 \text{ kip/in.})(0.295 \text{ in.})$ $= 25.9 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (58.5 \text{ kip/in.})(0.295 \text{ in.})$ $= 17.3 \text{ kips/bolt}$

The available bearing and tearout strength of the beam web at the bottom bolt (not adjacent to the edge) is determined using AISC *Manual* Table 7-4 with s = 3 in.

LRFD	ASD
$\phi r_n = (87.8 \text{ kip/in.})(0.295 \text{ in.})$ $= 25.9 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (58.5 \text{ kip/in.})(0.295 \text{ in.})$ $= 17.3 \text{ kips/bolt}$

The bearing or tearout strength controls over bolt shear for both bolts in the beam web.

The strength of the bolt group in the beam web is determined by summing the strength of the individual fasteners as follows:

LRFD	ASD
$\phi R_n = (1 \text{ bolt})(25.9 \text{ kips/bolt})$ $+ (1 \text{ bolt})(25.9 \text{ kips/bolt})$ $= 51.8 \text{ kips} > 25.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (1 \text{ bolt})(17.3 \text{ kips/bolt})$ $+ (1 \text{ bolt})(17.3 \text{ kips/bolt})$ $= 34.6 \text{ kips} > 16.7 \text{ kips}$ **o.k.**

Coped Beam Strength

From AISC *Manual* Part 9, the available coped beam web strength is the lesser of the limit states of flexural local web buckling, shear yielding, shear rupture, and block shear rupture.

Flexural local web buckling of beam web

The limit state of flexural yielding and local web buckling of the coped beam web are checked using AISC *Manual* Part 9 as follows:

$$\begin{aligned} e &= c + setback \\ &= 5 \text{ in.} + \tfrac{1}{2} \text{ in.} \\ &= 5.50 \text{ in.} \end{aligned}$$

$$\begin{aligned} h_o &= d - d_c \quad \text{(from AISC } \textit{Manual} \text{ Figure 9-2)} \\ &= 11.9 \text{ in.} - 2 \text{ in.} \\ &= 9.90 \text{ in.} \end{aligned}$$

$$\begin{aligned} \frac{c}{d} &= \frac{5 \text{ in.}}{11.9 \text{ in.}} \\ &= 0.420 \end{aligned}$$

$$\begin{aligned} \frac{c}{h_o} &= \frac{5 \text{ in.}}{9.90 \text{ in.}} \\ &= 0.505 \end{aligned}$$

Because $\dfrac{c}{d} \le 1.0,$ the buckling adjustment factor, *f*, is calculated as follows:

$$f = 2\left(\frac{c}{d}\right)$$ (*Manual* Eq. 9-14a)

$$= 2(0.420)$$
$$= 0.840$$

Because $\dfrac{c}{h_o} \le 1.0,$ the plate buckling coefficient, *k*, is calculated as follows:

$$k = 2.2\left(\frac{h_o}{c}\right)^{1.65}$$ (*Manual* Eq. 9-13a)

$$= 2.2\left(\frac{9.90 \text{ in.}}{5 \text{ in.}}\right)^{1.65}$$
$$= 6.79$$

$$\lambda = \frac{h_o}{t_w}$$ (*Manual* Eq. 9-11)

$$= \frac{9.90 \text{ in.}}{0.295 \text{ in.}}$$
$$= 33.6$$

$$k_1 = fk \ge 1.61$$ (*Manual* Eq. 9-10)

$$= (0.840)(6.79) \ge 1.61$$
$$= 5.70 > 1.61$$

$$\lambda_p = 0.475\sqrt{\frac{k_1 E}{F_y}}$$ (*Manual* Eq. 9-12)

$$= 0.475\sqrt{\frac{(5.70)(29{,}000 \text{ ksi})}{50 \text{ ksi}}}$$
$$= 27.3$$

$$2\lambda_p = 2(27.3)$$
$$= 54.6$$

Because $\lambda_p < \lambda \le 2\lambda_p$, calculate the nominal moment strength using AISC *Manual* Equation 9-7.

The plastic section modulus of the coped section, Z_{net}, is determined from Table IV-11 (included in Part IV of this document).

$$Z_{net} = 14.0 \text{ in.}^3$$

$$M_p = F_y Z_{net}$$
$$= (50\text{ ksi})(14.0\text{ in.}^3)$$
$$= 700\text{ kip-in.}$$

From AISC *Manual* Table 9-2:

$$S_{net} = 8.03\text{ in.}^3$$

$$M_y = F_y S_{net}$$
$$= (50\text{ ksi})(8.03\text{ in.}^3)$$
$$= 402\text{ kip-in.}$$

$$M_n = M_p - (M_p - M_y)\left(\frac{\lambda}{\lambda_p} - 1\right) \qquad (\textit{Manual}\text{ Eq. 9-7})$$
$$= 700\text{ kip-in.} - (700\text{ kip-in.} - 402\text{ kip-in.})\left[\left(\frac{33.6}{27.3}\right) - 1\right]$$
$$= 631\text{ kip-in.}$$

$$R_n = \frac{M_n}{e}$$
$$= \frac{631\text{ kip-in.}}{5.50\text{ in.}}$$
$$= 115\text{ kips}$$

The available strength of the coped section is:

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(115\text{ kips})$ $= 104\text{ kips} > 25.0\text{ kips}$ **o.k.**	$\Omega = 1.67$ $\frac{R_n}{\Omega} = \frac{115\text{ kips}}{1.67}$ $= 68.9\text{ kips} > 16.7\text{ kips}$ **o.k.**

Shear strength of beam web

From AISC *Specification* Section J4.2, the available shear yielding strength of the beam web is determined as follows:

$$A_{gv} = h_o t_w$$
$$= (9.90\text{ in.})(0.295\text{ in.})$$
$$= 2.92\text{ in.}^2$$

$$R_n = 0.60 F_y A_{gv} \qquad (\textit{Spec}.\text{ Eq. J4-3})$$
$$= 0.60(50\text{ ksi})(2.92\text{ in.}^2)$$
$$= 87.6\text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(87.6 \text{ kips})$ $= 87.6 \text{ kips} > 25.0 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{87.6 \text{ kips}}{1.50}$ $= 58.4 \text{ kips} > 16.7 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2, the available shear rupture strength of the beam web is determined as follows:

$$\begin{aligned} A_{nv} &= \left[h_o - n\left(d_h + \tfrac{1}{16} \text{ in.}\right)\right]t_w \\ &= \left[9.90 \text{ in.} - 2\left(\tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)\right](0.295 \text{ in.}) \\ &= 2.40 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_n &= 0.60F_u A_{nv} \\ &= 0.60(65 \text{ ksi})\left(2.40 \text{ in.}^2\right) \\ &= 93.6 \text{ kips} \end{aligned}$$ (*Spec*. Eq. J4-4)

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(93.6 \text{ kips})$ $= 70.2 \text{ kips} > 25.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{93.6 \text{ kips}}{2.00}$ $= 46.8 \text{ kips} > 16.7 \text{ kips}$ **o.k.**

Block shear rupture of beam web

The nominal strength for the limit state of block shear rupture is given by AISC *Specification* Section J4.3.

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt}$$ (*Spec*. Eq. J4-5)

The available block shear rupture strength of the beam web is determined as follows, using AISC *Manual* Tables 9-3a, 9-3b and 9-3c and AISC *Specification* Equation J4-5, with $n = 2$, $l_{eh} = 1\tfrac{3}{8}$ in. (includes ¼-in. tolerance to account for possible beam underrun), $l_{ev} = 2$ in. and $U_{bs} = 1.0$.

LRFD	ASD
Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{\phi F_u A_{nt}}{t} = 45.7 \text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{\phi 0.60 F_y A_{gv}}{t} = 113 \text{ kip/in.}$	Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{F_u A_{nt}}{\Omega t} = 30.5 \text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{0.60 F_y A_{gv}}{\Omega t} = 75.0 \text{ kip/in.}$

LRFD	ASD
Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{\phi 0.60F_u A_{nv}}{t} = 108 \text{ kip/in.}$ The design block shear rupture strength is: $\phi R_n = \phi 0.60F_u A_{nv} + \phi U_{bs} F_u A_{nt}$ $\le \phi 0.60F_y A_{gv} + \phi U_{bs} F_u A_{nt}$ $= (108 \text{ kip/in.} + 45.7 \text{ kip/in.})(0.295 \text{ in.})$ $\le (113 \text{ kip/in.} + 45.7 \text{ kip/in.})(0.295 \text{ in.})$ $= 45.3 \text{ kips} < 46.8 \text{ kips}$ Therefore: $\phi R_n = 45.3 \text{ kips} > 25.0 \text{ kips}$ **o.k.**	Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{0.60F_u A_{nv}}{\Omega t} = 71.9 \text{ kip/in.}$ The allowable block shear rupture strength is: $\dfrac{R_n}{\Omega} = \dfrac{0.60F_u A_{nv}}{\Omega} + \dfrac{U_{bs} F_u A_{nt}}{\Omega}$ $\le \dfrac{0.60F_y A_{gv}}{\Omega} + \dfrac{U_{bs} F_u A_{nt}}{\Omega}$ $= (71.9 \text{ kip/in.} + 30.5 \text{ kip/in.})(0.295 \text{ in.})$ $\le (75.0 \text{ kip/in.} + 30.5 \text{ kip/in.})(0.295 \text{ in.})$ $= 30.2 \text{ kips} < 31.1 \text{ kips}$ Therefore: $\dfrac{R_n}{\Omega} = 30.2 \text{ kips} > 16.7 \text{ kips}$ **o.k.**

Beam B Connection:

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(18.3 \text{ kips}) + 1.6(55 \text{ kips})$ $= 110 \text{ kips}$	$R_a = 18.3 \text{ kips} + 55 \text{ kips}$ $= 73.3 \text{ kips}$

Strength of the Bolted Connection—Angles

AISC *Manual* Table 10-1 includes checks for the limit states of bolt shear, bolt bearing on the angles, tearout on the angles, shear yielding of the angles, shear rupture of the angles, and block shear rupture of the angles. For five rows of bolts and ¼-in. angle thickness:

LRFD	ASD
$\phi R_n = 126 \text{ kips} > 110 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 83.8 \text{ kips} > 73.3 \text{ kips}$ **o.k.**

Strength of the Bolted Connection—Beam Web

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear is:

LRFD	ASD
$\phi r_n = 35.8 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = 23.9 \text{ kips/bolt}$

The available bearing and tearout strength of the beam web at the top edge bolt is determined using AISC *Manual* Table 7-5 with $l_e = 2$ in.

LRFD	ASD
$\phi r_n = (87.8 \text{ kip/in.})(0.380 \text{ in.})$ $= 33.4 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (58.5 \text{ kip/in.})(0.380 \text{ in.})$ $= 22.2 \text{ kips/bolt}$

The available bearing and tearout strength of the beam web at the interior bolts (not adjacent to the edge) is determined using AISC *Manual* Table 7-4 with s = 3 in.

LRFD	ASD
$\phi r_n = (87.8 \text{ kip/in.})(0.380 \text{ in.})$ $= 33.4 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (58.5 \text{ kip/in.})(0.380 \text{ in.})$ $= 22.2 \text{ kips/bolt}$

The strength of the bolt group in the beam web is determined as follows:

LRFD	ASD
$\phi R = (1 \text{ bolt})(33.4 \text{ kips/bolt})$ $+ (4 \text{ bolts})(33.4 \text{ kips/bolt})$ $= 167 \text{ kips} > 110 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (1 \text{ bolt})(22.2 \text{ kips/bolt})$ $+ (4 \text{ bolt})(22.2 \text{ kips/bolt})$ $= 111 \text{ kips} > 73.3 \text{ kips}$ **o.k.**

Coped Beam Strength

From AISC *Manual* Part 9, the available coped beam web strength is the lesser of the limit states of flexural local web buckling, shear yielding, shear rupture, and block shear rupture.

Flexural local web buckling of beam web

The limit state of flexural yielding and local web buckling of the coped beam web are checked using AISC *Manual* Part 9 as follows:

$$\begin{aligned} e &= c + setback \\ &= 5 \text{ in.} + ½ \text{ in.} \\ &= 5.50 \text{ in.} \end{aligned}$$

$$\begin{aligned} h_o &= d - d_c \quad \text{(from AISC } \textit{Manual} \text{ Figure 9-2)} \\ &= 20.8 \text{ in.} - 2 \text{ in.} \\ &= 18.8 \text{ in.} \end{aligned}$$

$$\begin{aligned} \frac{c}{d} &= \frac{5 \text{ in.}}{20.8 \text{ in.}} \\ &= 0.240 \end{aligned}$$

$$\begin{aligned} \frac{c}{h_o} &= \frac{5 \text{ in.}}{18.8 \text{ in.}} \\ &= 0.266 \end{aligned}$$

Because $\dfrac{c}{d} \le 1.0$, the buckling adjustment factor, f, is calculated as follows:

$$f = 2\left(\frac{c}{d}\right)$$ (*Manual* Eq. 9-14a)

$$= 2(0.240)$$
$$= 0.480$$

Because $\frac{c}{h_o} \leq 1.0,$ the plate buckling coefficient, k, is calculated as follows:

$$k = 2.2\left(\frac{h_o}{c}\right)^{1.65}$$ (*Manual* Eq. 9-13a)

$$= 2.2\left(\frac{18.8\text{ in.}}{5\text{ in.}}\right)^{1.65}$$
$$= 19.6$$

$$\lambda = \frac{h_o}{t_w}$$ (*Manual* Eq. 9-11)

$$= \frac{18.8\text{ in.}}{0.380\text{ in.}}$$
$$= 49.5$$

$$k_1 = fk \geq 1.61$$ (*Manual* Eq. 9-10)

$$= (0.480)(19.6) \geq 1.61$$
$$= 9.41 > 1.61$$

$$\lambda_p = 0.475\sqrt{\frac{k_1 E}{F_y}}$$ (*Manual* Eq. 9-12)

$$= 0.475\sqrt{\frac{(9.41)(29{,}000\text{ ksi})}{50\text{ ksi}}}$$
$$= 35.1$$

$$2\lambda_p = 2(35.1)$$
$$= 70.2$$

Because $\lambda_p < \lambda \leq 2\lambda_p$, calculate the nominal moment strength using AISC *Manual* Equation 9-7.

The plastic section modulus of the coped section, Z_{net}, is determined from Table IV-11 (included in Part IV of this document).

$$Z_{net} = 56.5\text{ in.}^3$$

$$M_p = F_y Z_{net}$$
$$= (50\text{ ksi})(56.5\text{ in.}^3)$$
$$= 2{,}830\text{ kip-in.}$$

From AISC *Manual* Table 9-2:

$S_{net} = 32.5 \text{ in.}^3$

$$\begin{aligned} M_y &= F_y S_{net} \\ &= (50 \text{ ksi})(32.5 \text{ in.}^3) \\ &= 1{,}630 \text{ kip-in.} \end{aligned}$$

$$M_n = M_p - (M_p - M_y)\left(\frac{\lambda}{\lambda_p} - 1\right) \quad \text{(}\textit{Manual}\text{ Eq. 9-7)}$$

$$\begin{aligned} &= 2{,}830 \text{ kip-in.} - (2{,}830 \text{ kip-in.} - 1{,}630 \text{ kip-in.})\left[\left(\frac{49.5}{35.1}\right) - 1\right] \\ &= 2{,}340 \text{ kip-in.} \end{aligned}$$

$$\begin{aligned} R_n &= \frac{M_n}{e} \\ &= \frac{2{,}340 \text{ kip-in.}}{5.50 \text{ in.}} \\ &= 425 \text{ kips} \end{aligned}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(425 \text{ kips})$ $= 383 \text{ kips} > 110 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{425 \text{ kips}}{1.67}$ $= 254 \text{ kips} > 73.3 \text{ kips}$ **o.k.**

Shear strength of beam web

From AISC *Specification* Section J4.2, the available shear yielding strength of the beam web is determined as follows:

$$\begin{aligned} A_{gv} &= h_o t_w \\ &= (18.8 \text{ in.})(0.380 \text{ in.}) \\ &= 7.14 \text{ in.}^2 \end{aligned}$$

$$R_n = 0.60 F_y A_{gv} \quad \text{(}\textit{Spec}\text{. Eq. J4-3)}$$

$$\begin{aligned} &= 0.60(50 \text{ ksi})(7.14 \text{ in.}^2) \\ &= 214 \text{ kips} \end{aligned}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(214 \text{ kips})$ $= 214 \text{ kips} > 110 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{214 \text{ kips}}{1.50}$ $= 143 \text{ kips} > 73.3 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2, the available shear rupture strength of the beam web is determined as follows:

$$A_{nv} = \left[h_o - n\left(d_h + 1/16 \text{ in.}\right)\right]t_w$$
$$= \left[18.8 \text{ in.} - 5\left(13/16 \text{ in.} + 1/16 \text{ in.}\right)\right]\left(0.380 \text{ in.}\right)$$
$$= 5.48 \text{ in.}^2$$

$$R_n = 0.60F_u A_{nv}$$ (*Spec*. Eq. J4-4)
$$= 0.60\left(65 \text{ ksi}\right)\left(5.48 \text{ in.}^2\right)$$
$$= 214 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75\left(214 \text{ kips}\right)$ $= 161 \text{ kips} > 110 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{214 \text{ kips}}{2.00}$ $= 107 \text{ kips} > 73.3 \text{ kips}$ **o.k.**

Block shear rupture of beam web

The nominal strength for the limit state of block shear rupture is given by AISC *Specification* Section J4.3.

$$R_n = 0.60F_u A_{nv} + U_{bs}F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs}F_u A_{nt}$$ (*Spec*. Eq. J4-5)

The available block shear rupture strength of the beam web is determined as follows, using AISC *Manual* Tables 9-3a, 9-3b and 9-3c and AISC *Specification* Equation J4-5, with n = 5, l_{eh} = 1⅜ in. (includes ¼ in. tolerance to account for possible beam underrun), l_{ev} = 2 in. and U_{bs} = 1.0.

LRFD	ASD
Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{\phi F_u A_{nt}}{t} = 45.7 \text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{\phi 0.60F_y A_{gv}}{t} = 315 \text{ kip/in.}$ Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{\phi 0.60F_u A_{nv}}{t} = 294 \text{ kip/in.}$	Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{F_u A_{nt}}{\Omega t} = 30.5 \text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{0.60F_y A_{gv}}{\Omega t} = 210 \text{ kip/in.}$ Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{0.60F_u A_{nv}}{\Omega t} = 196 \text{ kip/in.}$

LRFD	ASD
$\phi R_n = \phi 0.60 F_u A_{nv} + \phi U_{bs} F_u A_{nt}$ $\leq \phi 0.60 F_y A_{gv} + \phi U_{bs} F_u A_{nt}$ $= (294 \text{ kip/in.} + 45.7 \text{ kip/in.})(0.380 \text{ in.})$ $\leq (315 \text{ kip/in.} + 45.7 \text{ kip/in.})(0.380 \text{ in.})$ $= 129 \text{ kips} < 137 \text{ kips}$ Therefore: $\phi R_n = 129 \text{ kips} > 110 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega} + \dfrac{U_{bs} F_u A_{nt}}{\Omega}$ $\leq \dfrac{0.60 F_y A_{gv}}{\Omega} + \dfrac{U_{bs} F_u A_{nt}}{\Omega}$ $= (196 \text{ kip/in.} + 30.5 \text{ kip/in.})(0.380 \text{ in.})$ $\leq (210 \text{ kip/in.} + 30.5 \text{ kip/in.})(0.380 \text{ in.})$ $= 86.1 \text{ kips} < 91.4 \text{ kips}$ Therefore: $\dfrac{R_n}{\Omega} = 86.1 \text{ kips} > 73.3 \text{ kips}$ **o.k.**

Supporting Girder Connection

Supporting Girder Web

The required effective strength per bolt is the minimum from the limit states of bolt shear, bolt bearing and tearout. The bolts that are loaded by both connections will have the largest demand.. Thus, for the design of these four critical bolts, the required strength is determined as follows:

LRFD	ASD
From the W12×40 beam, each bolt must support one-fourth of 25.0 kips or 6.25 kips/bolt. From the W21×50 beam, each bolt must support one-tenth of 110 kips or 11.0 kips/bolt.	From the W12×40 beam, each bolt must support one-fourth of 16.7 kips or 4.18 kips/bolt. From the W21×50 beam, each bolt must support one-tenth of 73.3 kips or 7.33 kips/bolt.

The required strength for each of the shared bolts is:

LRFD	ASD
$R_u = 6.25 \text{ kips/bolt} + 11.0 \text{ kips/bolt}$ $= 17.3 \text{ kips/bolt}$	$R_a = 4.18 \text{ kips/bolt} + 7.33 \text{ kips/bolt}$ $= 11.5 \text{ kips/bolt}$

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear is:

LRFD	ASD
$\phi r_n = 35.8 \text{ kips/bolt} > 17.3 \text{ kips/bolt}$ **o.k.**	$\dfrac{r_n}{\Omega} = 23.9 \text{ kips/bolt} > 11.5 \text{ kips/bolt}$ **o.k.**

The available bearing and tearout strength of the girder web is determined using AISC *Manual* Table 7-4 with s = 3 in.

LRFD	ASD
$\phi r_n = (87.8 \text{ kip/in.})(0.520 \text{ in.})$ $= 45.7 \text{ kips/bolt} > 17.3 \text{ kips/bolt}$ **o.k.**	$\dfrac{r_n}{\Omega} = (58.5 \text{ kip/in.})(0.520 \text{ in.})$ $= 30.4 \text{ kips/bolt} > 11.5 \text{ kips/bolt}$ **o.k.**

Conclusion

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-9 OFFSET ALL-BOLTED DOUBLE-ANGLE CONNECTIONS (BEAMS-TO-GIRDER WEB)

Given:

Verify the all-bolted double-angle connections for back-to-back ASTM A992 W16×45 beams to an ASTM A992 W30×99 girder-web to support the end reactions shown in Figure II.A-9-1. The beam centerlines are offset 6 in. and the beam connections share a vertical row of bolts. Use ASTM A36 angles. The strength of the W16×45 beams and angles are verified in Example II.A-4 and are not repeated here.

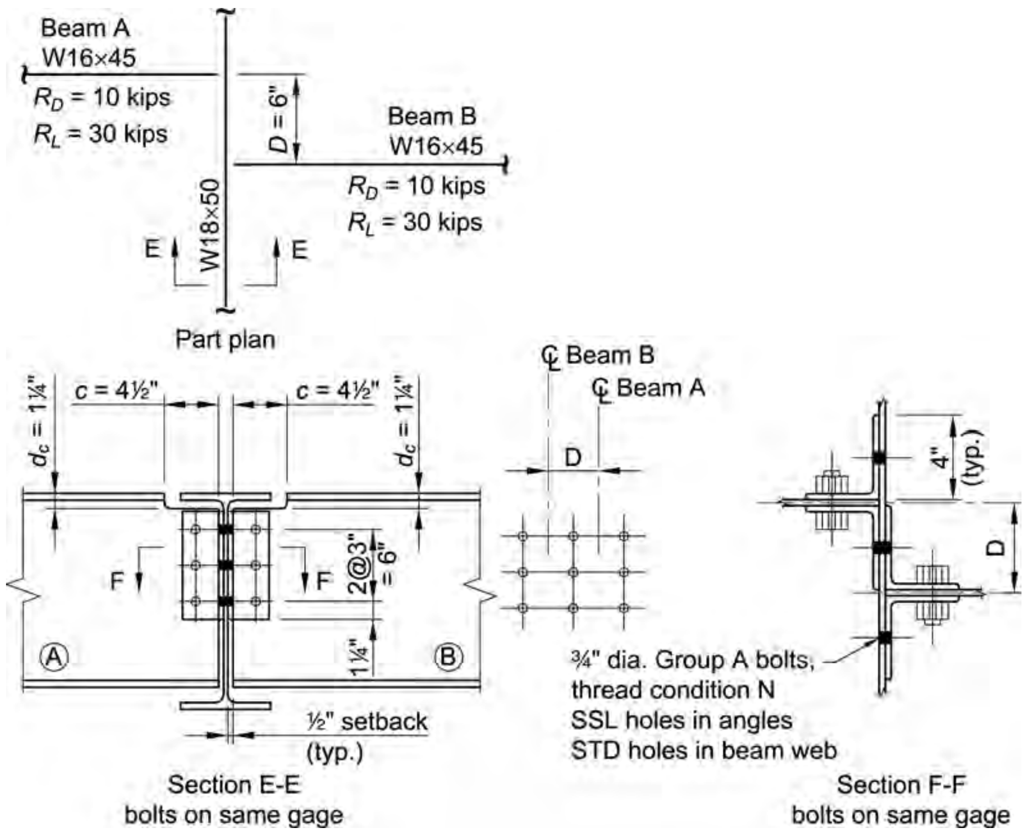


Fig. II.A-9-1. Connection geometry for Example II.A-9.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beams and girder
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Girder
W18×50
t_w = 0.355 in.
d = 18.0 in.

Beam
W16×45
t_w = 0.345 in.
d = 16.1 in.

Modify the 2L5×3½×¼ SLBB connection designed in Example II.A-4 to work in the configuration shown in Figure II.A-9-1. The offset dimension (6 in.) is approximately equal to the gage on the support from the previous example (6¼ in.) and, therefore, is not recalculated.

Thus, the available strength of the middle vertical row of bolts (through both connections) that carry a portion of the reaction for both connections must be verified for this new configuration.

From ASCE/SEI 7, Chapter 2, the required strength of the Beam A and Beam B connections to the girder web is:

LRFD	ASD
$R_u = 1.2(10 \text{ kips}) + 1.6(30 \text{ kips})$ $= 60.0 \text{ kips}$	$R_a = 10 \text{ kips} + 30 \text{ kips}$ $= 40.0 \text{ kips}$

In the girder web connection, each bolt will have the same effective strength; therefore, check the individual bolt effective strength. At the middle vertical row of bolts, the required strength for one bolt is the sum of the required shear strengths per bolt for each connection.

LRFD	ASD
$r_u = (2 \text{ sides})\left(\dfrac{60.0 \text{ kips}}{6 \text{ bolts}}\right)$ $= 20.0 \text{ kips/bolt (for middle vertical row)}$	$r_a = (2 \text{ sides})\left(\dfrac{40.0 \text{ kips}}{6 \text{ bolts}}\right)$ $= 13.3 \text{ kips/bolt (for middle vertical row)}$

Bolt Shear

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear is:

LRFD	ASD
$\phi r_n = 35.8 \text{ kips/bolt} > 20.0 \text{ kips/bolt}$ **o.k.**	$\dfrac{r_n}{\Omega} = 23.9 \text{ kips/bolt} > 13.3 \text{ kips/bolt}$ **o.k.**

Bearing on the Girder Web

The available bearing strength per bolt is determined from AISC *Manual* Table 7-4 with s = 3 in.

LRFD	ASD
$\phi r_n = (87.8 \text{ kip/in.})(0.355 \text{ in.})$ $= 31.2 \text{ kips/bolt} > 20.0 \text{ kips/bolt}$ **o.k.**	$\dfrac{r_n}{\Omega} = (58.5 \text{ kip/in.})(0.355 \text{ in.})$ $= 20.8 \text{ kips/bolt} > 13.3 \text{ kips/bolt}$ **o.k.**

Note: If the bolts are not spaced equally from the supported beam web, the force in each column of bolts should be determined by using a simple beam analogy between the bolts, and applying the laws of statics.

Conclusion

The connections are found to be adequate as given for the applied loads.

EXAMPLE II.A-10 SKEWED DOUBLE BENT-PLATE CONNECTION (BEAM-TO-GIRDER WEB)

Given:

Design the skewed double bent-plate connection between an ASTM A992 W16×77 beam and ASTM A992 W27×94 girder-web to support the following beam end reactions:

R_D = 13.3 kips
R_L = 40 kips

Use 70-ksi electrodes and ASTM A36 plates. The final design is shown in Figure II.A-10-1.

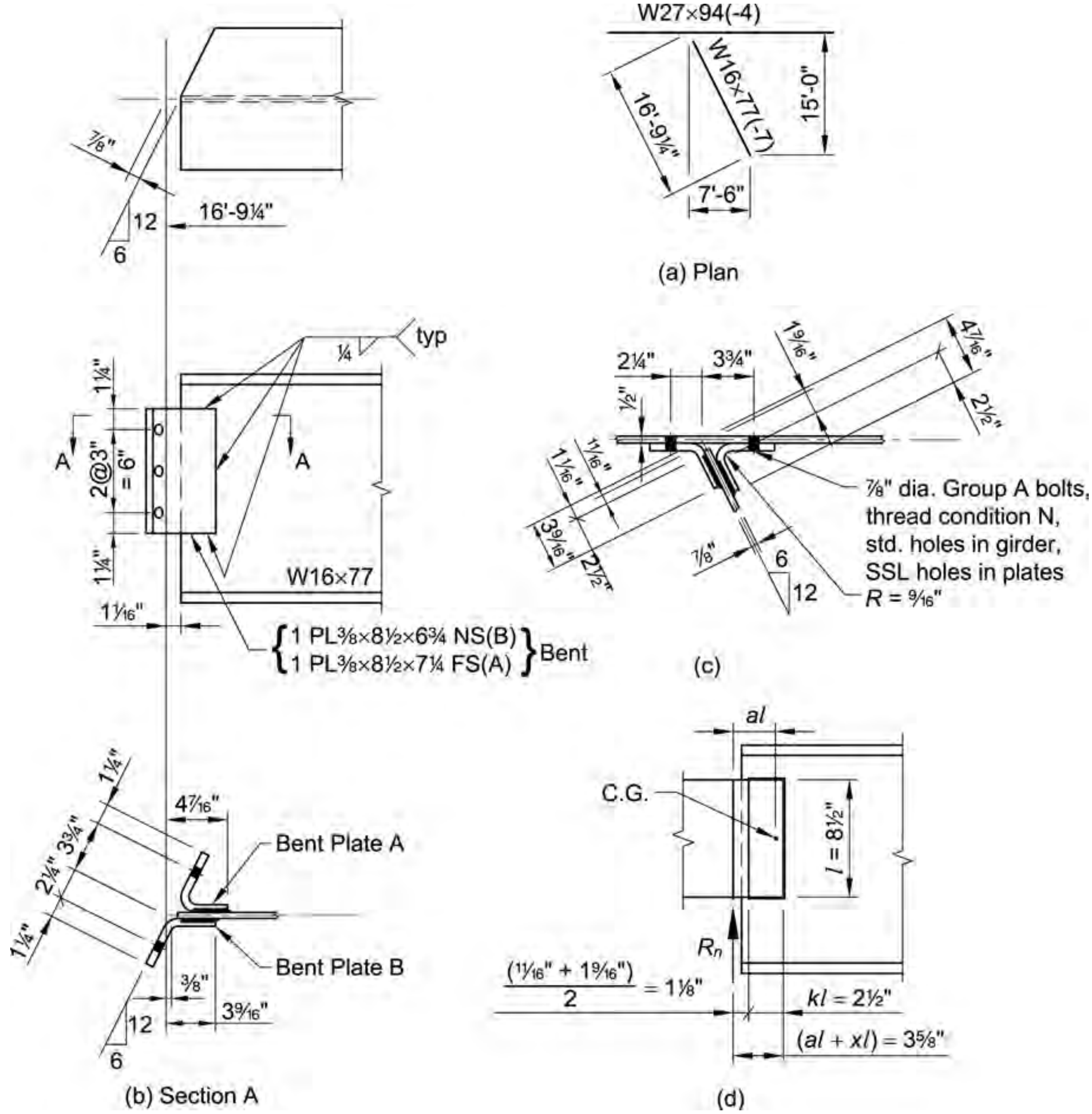


Fig. II.A-10-1. Skewed double bent-plate connection (beam-to-girder web).

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam and girder
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W16×77
t_w= 0.455 in.
d = 16.5 in.

Girder
W27×94
t_w= 0.490 in.

From AISC *Specification* Table J3.3, for ⅞-in.-diameter bolts with standard holes:

d_h = 15⁄16 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(13.3 \text{ kips}) + 1.6(40 \text{ kips})$ $= 80.0 \text{ kips}$	$R_a = 13.3 \text{ kips} + 40 \text{ kips}$ $= 53.3 \text{ kips}$

From Figure II.A-10-1(c), assign load to each vertical row of bolts by assuming a simple beam analogy between bolts and applying the principles of statics.

LRFD	ASD
Required strength for bent plate A: $R_u = \dfrac{(80.0 \text{ kips})(2\tfrac{1}{4} \text{ in.})}{6.00 \text{ in.}}$ $= 30.0 \text{ kips}$ Required strength for bent plate B: $R_u = 80.0 \text{ kips} - 30.0 \text{ kips}$ $= 50.0 \text{ kips}$	Required strength for bent plate A: $R_a = \dfrac{(53.3 \text{ kips})(2\tfrac{1}{4} \text{ in.})}{6.00 \text{ in.}}$ $= 20.0 \text{ kips}$ Required strength for bent plate B: $R_a = 53.3 \text{ kips} - 20.0 \text{ kips}$ $= 33.3 \text{ kips}$

Assume that the welds across the top and bottom of the plates will be 2½ in. long, and that the load acts at the intersection of the beam centerline and the support face.

While the welds do not coincide on opposite faces of the beam web and the weld groups are offset, the locations of the weld groups will be averaged and considered identical. See Figure II.A-10-1(d).

Weld Design

Assume a plate length of l = 8½ in.

$$k = \frac{kl}{l} = \frac{2\tfrac{1}{2}\text{ in.}}{8\tfrac{1}{2}\text{ in.}} = 0.294$$

Interpolating from AISC *Manual* Table 8-8, with angle = 0°, and k = 0.294,

$$x = 0.0544$$

$$xl = (0.0544)(8\tfrac{1}{2}\text{ in.}) = 0.462\text{ in.}$$

$$a = \frac{(al + xl) - xl}{l} = \frac{3\tfrac{5}{8}\text{ in} - 0.462\text{ in.}}{8\tfrac{1}{2}\text{ in.}} = 0.372$$

Interpolating from AISC *Manual* Table 8-8, with θ = 0°, a = 0.372, and k = 0.294,

$$C = 2.52$$

The required weld size is determined as follows:

LRFD	ASD
$\phi = 0.75$ $D_{req} = \frac{R_u}{\phi C C_1 l} = \frac{50.0\text{ kips}}{0.75(2.52)(1.0)(8\tfrac{1}{2}\text{ in.})} = 3.11\text{ sixteenths}$	$\Omega = 2.00$ $D_{req} = \frac{\Omega R_a}{C C_1 l} = \frac{2.00(33.3\text{ kips})}{2.52(1.0)(8\tfrac{1}{2}\text{ in.})} = 3.11\text{ sixteenths}$

Use ¼-in. fillet welds and at least ⁵⁄₁₆-in.-thick bent plates to allow for the welds.

Beam Web Strength at Fillet Weld

The minimum beam web thickness required to match the shear rupture strength of the weld to that of the base metal is:

$$t_{min} = \frac{6.19D_{min}}{F_u}$$ (from *Manual* Eq. 9-3)

$$= \frac{6.19(3.11)}{65 \text{ ksi}}$$

$$= 0.296 \text{ in.} < 0.455 \text{ in.} \quad \textbf{o.k.}$$

Bolt Strength

The effective strength of the individual fasteners is the lesser of the bolt shear strength per AISC *Specification* Section J3.6, and the bolt bearing and tearout strength per AISC *Specification* Section J3.10. By observation, the bent plate will govern over the girder web as it is thinner and lower strength material. Trying a 5⁄16-in. plate the available strength at the critical vertical row of bolts (bent plate B) is determined as follows.

From AISC *Manual* Table 7-1, the available shear strength per bolt for ⅞-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in single shear is:

LRFD	ASD
$\phi r_n = 24.3$ kips/bolt	$\dfrac{r_n}{\Omega} = 16.2$ kips/bolt

The available bearing and tearout strength of the bent-plate at the top edge bolt is determined using AISC *Manual* Table 7-5 with l_{ev} = 1¼ in.

LRFD	ASD
$\phi r_n = (40.8 \text{ kip/in.})(5/16 \text{ in.})$ $= 12.8$ kips/bolt	$\dfrac{r_n}{\Omega} = (27.2 \text{ kip/in.})(5/16 \text{ in.})$ $= 8.50$ kips/bolt

The available bearing and tearout strength of the bent-plate at the other bolts (not adjacent to the edge) is determined using AISC *Manual* Table 7-4 with s = 3 in.

LRFD	ASD
$\phi r_n = (91.4 \text{ kip/in.})(5/16 \text{ in.})$ $= 28.6$ kips/bolt	$\dfrac{r_n}{\Omega} = (60.9 \text{ kip/in.})(5/16 \text{ in.})$ $= 19.0$ kips/bolt

The bolt shear strength governs over bearing and tearout for the other bolts (not adjacent to the edge); therefore, the effective strength of the bolt group is determined as follows:

LRFD	ASD
$\phi R_n = (1 \text{ bolt})(12.8 \text{ kips/bolt})$ $+ (2 \text{ bolts})(24.3 \text{ kips/bolt})$ $= 61.4 \text{ kips} > 50.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (1 \text{ bolt})(8.50 \text{ kips/bolt})$ $+ (2 \text{ bolts})(16.2 \text{ kips/bolt})$ $= 40.9 \text{ kips} > 33.3 \text{ kips}$ **o.k.**

Shear Strength of Plate

From AISC *Specification* Section J4.2, the available shear yielding strength of bent plate B (see Figure II.A-10-1) is determined as follows:

$$A_{gv} = lt$$
$$= (8\tfrac{1}{2}\text{ in.})(\tfrac{5}{16}\text{ in.})$$
$$= 2.66\text{ in.}^2$$

$$R_n = 0.60F_yA_{gv} \qquad \text{(\textit{Spec}. Eq. J4-3)}$$
$$= 0.60(36\text{ ksi})(2.66\text{ in.}^2)$$
$$= 57.5\text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(57.5\text{ kips})$ $= 57.5\text{ kips} > 50.0\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{57.5\text{ kips}}{1.50}$ $= 38.3\text{ kips} > 33.3\text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2, the available shear rupture strength of bent plate B is determined as follows:

$$A_{nv} = \left[l - n\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]t$$
$$= \left[8\tfrac{1}{2}\text{ in.} - 3\left(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right](\tfrac{5}{16}\text{ in.})$$
$$= 1.72\text{ in.}^2$$

$$R_n = 0.60F_uA_{nv} \qquad \text{(\textit{Spec}. Eq. J4-4)}$$
$$= 0.60(58\text{ ksi})(1.72\text{ in.}^2)$$
$$= 59.9\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(59.9\text{ kips})$ $= 44.9\text{ kips} < 50.0\text{ kips}$ **n.g.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{59.9\text{ kips}}{2.00}$ $= 30.0\text{ kips} < 33.3\text{ kips}$ **n.g.**

Therefore, the plate thickness is increased to ⅜ in. The available shear rupture strength is:

$$A_{nv} = \left[d - n\left(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]t$$
$$= \left[8\tfrac{1}{2}\text{ in.} - 3\left(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right](\tfrac{3}{8}\text{ in.})$$
$$= 2.06\text{ in.}^2$$

$$R_n = 0.60F_uA_{nv} \qquad \text{(\textit{Spec}. Eq. J4-4)}$$
$$= 0.60(58\text{ ksi})(2.06\text{ in.}^2)$$
$$= 71.7\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(71.7 \text{ kips})$ $= 53.8 \text{ kips} > 50.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{71.7 \text{ kips}}{2.00}$ $= 35.9 \text{ kips} > 33.3 \text{ kips}$ **o.k.**

Block Shear Rupture of Plate

The nominal strength for the limit state of block shear rupture is given by AISC *Specification* Section J4.3.

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (Spec.\ \text{Eq. J4-5})$$

The available block shear rupture strength of the plate is determined as follows, using AISC *Manual* Tables 9-3a, 9-3b and 9-3c and AISC *Specification* Equation J4-5, with $n = 3$, $l_{ev} = l_{eh} = 1\frac{1}{4}$ in., and $U_{bs} = 1.0$.

LRFD	ASD
Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{\phi F_u A_{nt}}{t} = 32.6 \text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{\phi 0.6 F_y A_{gv}}{t} = 117 \text{ kip/in.}$ Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{\phi 0.6 F_u A_{nv}}{t} = 124 \text{ kip/in.}$ $\phi R_n = \phi 0.60 F_u A_{nv} + \phi U_{bs} F_u A_{nt}$ $\leq \phi 0.60 F_y A_{gv} + \phi U_{bs} F_u A_{nt}$ $= (124 \text{ kip/in.} + 32.6 \text{ kip/in.})(\frac{3}{8} \text{ in.})$ $\leq (117 \text{ kip/in.} + 32.6 \text{ kip/in.})(\frac{3}{8} \text{ in.})$ $= 58.7 \text{ kips} > 56.1 \text{ kips}$ Therefore: $\phi R_n = 56.1 \text{ kips} > 50.0 \text{ kips}$ **o.k.**	Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{F_u A_{nt}}{\Omega t} = 21.8 \text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{0.6 F_y A_{gv}}{\Omega t} = 78.3 \text{ kip/in.}$ Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{0.6 F_u A_{nv}}{\Omega t} = 82.6 \text{ kip/in.}$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega} + \dfrac{U_{bs} F_u A_{nt}}{\Omega}$ $\leq \dfrac{0.60 F_y A_{gv}}{\Omega} + \dfrac{U_{bs} F_u A_{nt}}{\Omega}$ $= (82.6 \text{ kip/in.} + 21.8 \text{ kip/in.})(\frac{3}{8} \text{ in.})$ $\leq (78.3 \text{ kip/in.} + 21.8 \text{ kip/in.})(\frac{3}{8} \text{ in.})$ $= 39.2 \text{ kips} > 37.5 \text{ kips}$ Therefore: $\dfrac{R_n}{\Omega} = 37.5 \text{ kips} > 33.3 \text{ kips}$ **o.k.**

Thus, the configuration shown in Figure II.A-10-1 can be supported using ⅜-in. bent plates, and ¼-in. fillet welds.

EXAMPLE II.A-11A SHEAR END-PLATE CONNECTION (BEAM-TO-GIRDER WEB)

Given:

Verify a shear end-plate connection to connect an ASTM A992 W18×50 beam to an ASTM A992 W21×62 girder web, as shown in Figure II.A-11A-1, to support the following beam end reactions:

R_D = 10 kips
R_L = 30 kips

Use 70-ksi electrodes and ASTM A36 plate.

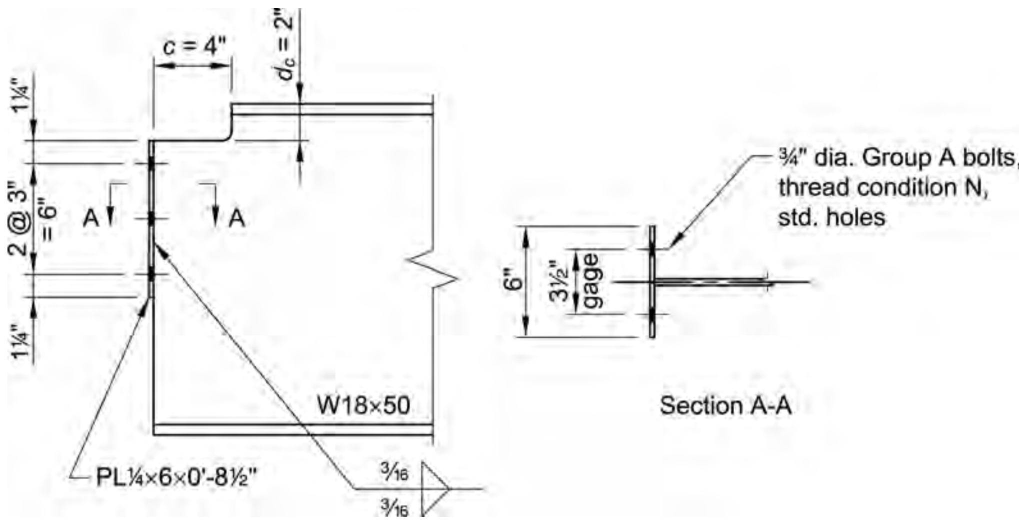


Fig. II.A-11A-1. Connection geometry for Example II.A-11A.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam and girder
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×50
t_w= 0.355 in.

Girder
W21×62
t_w = 0.400 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(10 \text{ kips}) + 1.6(30 \text{ kips})$ $= 60.0 \text{ kips}$	$R_a = 10 \text{ kips} + 30 \text{ kips}$ $= 40.0 \text{ kips}$

Bolt and End-Plate Available Strength

Tabulated values in AISC *Manual* Table 10-4 consider the limit states of bolt shear, bolt bearing on the end plate, tearout on the end plate, shear yielding of the end plate, shear rupture of the end plate, and block shear rupture of the end plate.

From AISC *Manual* Table 10-4, for three rows of ¾-in.-diameter bolts and ¼-in. plate thickness:

LRFD	ASD
$\phi R_n = 76.4 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 50.9 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

Weld and Beam Web Available Strength

Try 3⁄16-in. weld. From AISC *Manual* Table 10-4, the minimum beam web thickness is:

$$t_{w\,min} = 0.286 \text{ in.} < 0.355 \text{ in.} \quad \textbf{o.k.}$$

From AISC *Manual* Table 10-4, the weld and beam web available strength is:

LRFD	ASD
$\phi R_n = 67.9 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 45.2 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

Bolt Bearing on Girder Web

From AISC *Manual* Table 10-4:

LRFD	ASD
$\phi R_n = (527 \text{ kip/in.})(0.400 \text{ in.})$ $= 211 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (351 \text{ kip/in.})(0.400 \text{ in.})$ $= 140 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

Coped Beam Strength

As was shown in Example II.A-4, the coped section does not control the design. **o.k.**

Beam Web Shear Yielding

As was shown in Example II.A-4, beam web shear does not control the design. **o.k.**

EXAMPLE II.A-11B END-PLATE CONNECTION SUBJECT TO AXIAL AND SHEAR LOADING

Given:

Verify the available strength of an end-plate connection for an ASTM A992 W18x50 beam, as shown in Figure II.A-11B-1, to support the following beam end reactions:

LRFD	ASD
Shear, V_u = 75 kips Axial tension, N_u = 60 kips	Shear, V_a = 50 kips Axial tension, N_a = 40 kips

Use 70-ksi electrodes and ASTM A36 plate.

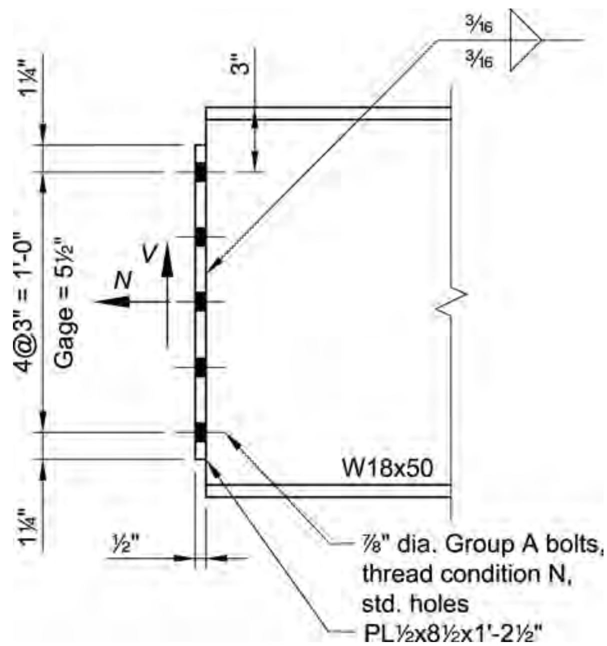


Fig. II.A-11B-1. Connection geometry for Example II.A-11B.

Solution:

From AISC *Manual* Table 2-4 and 2-5, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×50
d = 18.0 in.

t_w = 0.355 in.
A_g = 14.7 in.^{2}

From AISC *Specification* Table J3.3, for ⅞-in.-diameter bolts with standard holes:

d_h = 15⁄16 in.

The resultant load is:

LRFD	ASD
$R_u = \sqrt{V_u^2 + N_u^2}$ $= \sqrt{(75 \text{ kips})^2 + (60 \text{ kips})^2}$ $= 96.0 \text{ kips}$	$R_a = \sqrt{V_a^2 + N_a^2}$ $= \sqrt{(50 \text{ kips})^2 + (40 \text{ kips})^2}$ $= 64.0 \text{ kips}$

The connection will first be checked for the shear load. The following bolt shear, bearing and tearout calculations are for a pair of bolts.

Bolt Shear

From AISC *Manual* Table 7-1, the available shear strength for ⅞-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear, or pair of bolts in this example, is:

LRFD	ASD
$\phi r_n = 48.7 \text{ kips/pair of bolts}$	$\dfrac{r_n}{\Omega} = 32.5 \text{ kips/pair of bolts}$

Bolt Bearing on the Plate

The nominal bearing strength of the plate is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$r_n = (2 \text{ bolts/row})2.4dtF_u \quad \text{(from } \textit{Spec.} \text{ Eq. J3-6a)}$$
$$= (2 \text{ bolts/row})(2.4)(\tfrac{7}{8} \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})$$
$$= 122 \text{ kips (for a pair of bolts)}$$

From AISC *Specification* Section J3.10, the available bearing strength of the plate for a pair of bolts is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(122 \text{ kips})$ $= 91.5 \text{ kips/pair of bolts}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{122 \text{ kips}}{2.00}$ $= 61.0 \text{ kips/pair of bolts}$

Bolt Tearout on the Plate

The available tearout strength of the plate is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration. For the top edge bolts:

$$l_c = l_e - 0.5d_h$$
$$= 1\tfrac{1}{4} \text{ in.} - 0.5(\tfrac{15}{16} \text{ in.})$$
$$= 0.781 \text{ in.}$$

$$r_n = (2 \text{ bolts/row})1.2l_c tF_u$$ (from *Spec*. Eq. J3-6c)
$$= (2 \text{ bolts/row})(1.2)(0.781 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})$$
$$= 54.4 \text{ kips (for a pair of bolts)}$$

The available bolt tearout strength for the pair of top edge bolts is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(54.4 \text{ kips})$ $= 40.8 \text{ kips/pair of bolts}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{54.4 \text{ kips}}{2.00}$ $= 27.2 \text{ kips/pair of bolts}$

Tearout controls over bolt shear and bearing strength for the top edge bolts in the plate.

For interior bolts:

$$l_c = s - d_h$$
$$= 3.00 \text{ in.} - \tfrac{15}{16} \text{ in.}$$
$$= 2.06 \text{ in.}$$

$$r_n = (2 \text{ bolts/row})1.2l_c tF_u$$ (from *Spec*. Eq. J3-6c)
$$= (2 \text{ bolts/row})(1.2)(2.06 \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})$$
$$= 143 \text{ kips/pair of bolts}$$

The available bolt tearout strength for a pair of interior bolts is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(143 \text{ kips})$ $= 107 \text{ kips/pair of bolts}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{143 \text{ kips}}{2.00}$ $= 71.5 \text{ kips/pair of bolts}$

Bolt shear controls over tearout and bearing strength for the interior bolts in the plate.

Shear Strength of Bolted Connection

LRFD	ASD
$\phi R_n = (1 \text{ row})(40.8 \text{ kips/pair of bolts})$ $+ (4 \text{ rows})(48.7 \text{ kips/pair of bolts})$ $= 236 \text{ kips} > 75 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (1 \text{ row})(27.2 \text{ kips/pair of bolts})$ $+ (4 \text{ rows})(32.5 \text{ kips/pair of bolts})$ $= 157 \text{ kips} > 50 \text{ kips}$ **o.k.**

Bolt Shear and Tension Interaction

The available strength of the bolts due to the effect of combined tension and shear is determined from AISC *Specification* Section J3.7. The required shear stress is:

$$f_{rv} = \frac{V_r}{nA_b}$$

where

$A_b = 0.601 \text{ in.}^2$ (from AISC *Manual* Table 7-1)
$n = 10$ bolts

LRFD	ASD
$f_{rv} = \dfrac{75 \text{ kips}}{10\left(0.601 \text{ in.}^2\right)}$ $= 12.5 \text{ ksi}$	$f_{rv} = \dfrac{50 \text{ kips}}{10\left(0.601 \text{ in.}^2\right)}$ $= 8.32 \text{ ksi}$

The nominal tensile stress modified to include the effects of shear stress is determined from AISC *Specification* Section J3.7 as follows. From AISC *Specification* Table J3.2:

$F_{nt} = 90 \text{ ksi}$
$F_{nv} = 54 \text{ ksi}$

LRFD	ASD
$\phi = 0.75$ $F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3-3a) $= 1.3(90 \text{ ksi}) - \dfrac{90 \text{ ksi}}{0.75(54 \text{ ksi})}(12.5 \text{ ksi}) \le 90 \text{ ksi}$ $= 89.2 \text{ ksi} < 90 \text{ ksi}$ **o.k.**	$\Omega = 2.00$ $F'_{nt} = 1.3F_{nt} - \dfrac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3-3b) $= 1.3(90 \text{ ksi}) - \dfrac{2.00(90 \text{ ksi})}{54 \text{ ksi}}(8.32 \text{ ksi}) \le 90 \text{ ksi}$ $= 89.3 \text{ ksi} < 90 \text{ ksi}$ **o.k.**

Using the value of $F'_{nt} = 89.2$ ksi determined for LRFD, the nominal tensile strength of one bolt is:

$$r_n = F'_{nt} A_b \qquad \text{(\textit{Spec}. Eq. J3-2)}$$
$$= (89.2 \text{ ksi})\left(0.601 \text{ in.}^2\right)$$
$$= 53.6 \text{ kips}$$

The available tensile strength due to combined tension and shear is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(53.6 \text{ kips})$ $= 40.2 \text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{53.6 \text{ kips}}{2.00}$ $= 26.8 \text{ kips/bolt}$

LRFD	ASD
$\phi R_n = n\phi r_n$ $= (10 \text{ bolts})(40.2 \text{ kips/bolt})$ $= 402 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = n\dfrac{r_n}{\Omega}$ $= (10 \text{ bolts})(26.8 \text{ kips/bolt})$ $= 268 \text{ kips} > 40 \text{ kips}$ **o.k.**

Prying Action

From AISC *Manual* Part 9, the available tensile strength of the bolts in the end-plate taking prying action into account is determined as follows:

$$a = \frac{width\ of\ plate - gage}{2}$$
$$= \frac{8\tfrac{1}{2} \text{ in.} - 5\tfrac{1}{2} \text{ in.}}{2}$$
$$= 1.50 \text{ in.}$$

Note: If a at the supporting element is smaller than a = 1.50 in., use the smaller a in the preceding calculations.

$$b = \frac{\text{gage} - t_w}{2}$$
$$= \frac{5\tfrac{1}{2} \text{ in.} - 0.355 \text{ in.}}{2}$$
$$= 2.57 \text{ in.}$$

$$a' = \left(a + \frac{d_b}{2}\right) \le \left(1.25b + \frac{d_b}{2}\right) \qquad (\textit{Manual} \text{ Eq. 9-23})$$
$$= 1.50 \text{ in.} + \frac{\tfrac{7}{8} \text{ in.}}{2} \le 1.25(2.57 \text{ in.}) + \frac{\tfrac{7}{8} \text{ in.}}{2}$$
$$= 1.94 \text{ in.} < 3.65 \text{ in.}$$
$$= 1.94 \text{ in.}$$

$$b' = \left(b - \frac{d_b}{2}\right) \qquad (\textit{Manual} \text{ Eq. 9-18})$$
$$= 2.57 \text{ in.} - \frac{\tfrac{7}{8} \text{ in.}}{2}$$
$$= 2.13 \text{ in.}$$

$$\rho = \frac{b'}{a'} \qquad (\textit{Manual} \text{ Eq. 9-22})$$
$$= \frac{2.13 \text{ in.}}{1.94 \text{ in.}}$$
$$= 1.10$$

Note that end distances of 1¼ in. are used on the end-plate, so p is the average pitch of the bolts:

$$p = \frac{l}{n}$$
$$= \frac{14\tfrac{1}{2}\text{ in.}}{5}$$
$$= 2.90\text{ in.}$$

Check

$p \le s$

$2.90\text{ in.} < 3\text{ in.}$ **o.k.**

$$d' = d_h$$
$$= {}^{15}\!/_{16}\text{ in.}$$

$$\delta = 1 - \frac{d'}{p} \qquad \text{(\textit{Manual} Eq. 9-20)}$$
$$= 1 - \frac{{}^{15}\!/_{16}\text{ in.}}{2.90\text{ in.}}$$
$$= 0.677$$

From AISC *Manual* Equations 9-26a or 9-26b, the required end-plate thickness to develop the available strength of the bolt without prying action is:

LRFD	ASD
$\phi = 0.90$	$\Omega = 1.67$
$B_c = 40.2$ kips/bolt (calculated previously)	$B_c = 26.8$ kips/bolt (calculated previously)
$t_c = \sqrt{\dfrac{4B_c b'}{\phi p F_u}}$	$t_c = \sqrt{\dfrac{\Omega 4 B_c b'}{p F_u}}$
$= \sqrt{\dfrac{4(40.2\text{ kips/bolt})(2.13\text{ in.})}{0.90(2.90\text{ in.})(58\text{ ksi})}}$	$= \sqrt{\dfrac{1.67(4)(26.8\text{ kips/bolt})(2.13\text{ in.})}{(2.90\text{ in.})(58\text{ ksi})}}$
$= 1.50\text{ in.}$	$= 1.51\text{ in.}$

Because the end-plate thickness of ½ in. is less than t_c, using the value of $t_c = 1.51$ in. determined for ASD, calculate the effect of prying action on the bolts.

$$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right] \qquad \text{(\textit{Manual} Eq. 9-28)}$$
$$= \frac{1}{0.677(1+1.10)}\left[\left(\frac{1.51\text{ in.}}{\tfrac{1}{2}\text{ in.}}\right)^2 - 1\right]$$
$$= 5.71$$

Because $\alpha' > 1$, the end-plate has insufficient strength to develop the bolt strength, therefore:

$$Q = \left(\frac{t}{t_c}\right)^2 (1+\delta)$$
$$= \left(\frac{\text{½ in.}}{\text{1.51 in.}}\right)^2 (1+0.677)$$
$$= 0.184$$

The available tensile strength of the bolts taking prying action into account is determined from AISC *Manual* Equation 9-27 as follows:

LRFD	ASD
$T_c = B_c Q$ $= (40.2 \text{ kips/bolt})(0.186)$ $= 7.48 \text{ kips/bolt}$ $\phi R_n = T_c n$ $= (7.48 \text{ kips/bolt})(10 \text{ bolts})$ $= 74.8 \text{ kips} > 60 \text{ kips}$ **o.k.**	$T_c = B_c Q$ $= (26.8 \text{ kips/bolt})(0.184)$ $= 4.93 \text{ kips/bolt}$ $\dfrac{R_n}{\Omega} = T_c n$ $= (4.93 \text{ kips/bolt})(10 \text{ bolts})$ $= 49.3 \text{ kips} > 40 \text{ kips}$ **o.k.**

Weld Design

Assume a 3⁄16-in. fillet weld on each side of the beam web, with the weld stopping short of the end of the plate at a distance equal to the weld size.

$$l_w = 14\tfrac{1}{2} \text{ in.} - 2(\tfrac{3}{16} \text{ in.})$$
$$= 14.1 \text{ in.}$$

LRFD	ASD
$\theta = \tan^{-1}\left(\dfrac{N_u}{V_u}\right)$ $= \tan^{-1}\left(\dfrac{60 \text{ kips}}{75 \text{ kips}}\right)$ $= 38.7°$	$\theta = \tan^{-1}\left(\dfrac{N_a}{V_a}\right)$ $= \tan^{-1}\left(\dfrac{40 \text{ kips}}{50 \text{ kips}}\right)$ $= 38.7°$

From AISC *Manual* Table 8-4 for Angle = 30° (which will lead to a conservative result):

Special Case: $k = a = 0$

$C = 4.37$

The required weld size is determined from AISC *Manual* Equation 8-21 as follows:

LRFD	ASD
$\phi = 0.75$ $D_{min} = \dfrac{R_u}{\phi C C_1 l_w}$ $= \dfrac{96.0\text{ kips}}{0.75(4.37)(1.0)(14.1\text{ in.})}$ $= 2.08\text{ sixteenths}$	$\Omega = 2.00$ $D_{min} = \dfrac{\Omega R_a}{C C_1 l_w}$ $= \dfrac{2.00(64.0\text{ kips})}{(4.37)(1.0)(14.1\text{ in.})}$ $= 2.08\text{ sixteenths}$

Use a 3⁄16-in. fillet weld (minimum size from AISC *Specification* Table J2.4).

Beam Web Strength at Fillet Weld

The minimum beam web thickness required to match the shear rupture strength of the connecting element to that of the base metal is:

$$t_{min} = \frac{6.19D_{min}}{F_u} \quad \text{(from } \textit{Manual} \text{ Eq. 9-3)}$$
$$= \frac{6.19(2.08)}{65\text{ ksi}}$$
$$= 0.198\text{ in.} < 0.355\text{ in.} \quad \textbf{o.k.}$$

Shear Strength of the Plate

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the plate is determined as follows:

$$A_{gv} = 2lt$$
$$= (2)(14\tfrac{1}{2}\text{ in.})(\tfrac{1}{2}\text{ in.})$$
$$= 14.5\text{ in.}^2$$

$$R_n = 0.60F_y A_{gv} \quad (\textit{Spec}.\text{ Eq. J4-3})$$
$$= 0.60(36\text{ ksi})(14.5\text{ in.}^2)$$
$$= 313\text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(313\text{ kips})$ $= 313\text{ kips} > 96.0\text{ kips} \quad \textbf{o.k.}$	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{313\text{ kips}}{1.50}$ $= 209\text{ kips} > 64.0\text{ kips} \quad \textbf{o.k.}$

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the plate is determined as follows:

$$A_{nv} = 2\left[l - n\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]t$$
$$= 2\left[14\tfrac{1}{2}\text{ in.} - 5\left(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right](\tfrac{1}{2}\text{ in.})$$
$$= 9.50\text{ in.}^2$$

$$R_n = 0.60F_uA_{nv}$$ (*Spec*. Eq. J4-4)
$$= 0.60(58\text{ ksi})(9.50\text{ in.}^2)$$
$$= 331\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(331\text{ kips})$ $= 248\text{ kips} > 96.0\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{331\text{ kips}}{2.00}$ $= 166\text{ kips} > 64.0\text{ kips}$ **o.k.**

Block Shear Rupture Strength of the Plate

The nominal strength for the limit state of block shear rupture of the plate assuming an L-shaped tearout relative to shear load, is determined as follows. The tearout pattern is shown in Figure II.A-11B-2.

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \leq 0.60F_yA_{gv} + U_{bs}F_uA_{nt}$$ (*Spec*. Eq. J4-5)

where

$$l_{eh} = \frac{b - gage}{2}$$
$$= \frac{8\tfrac{1}{2}\text{ in.} - 5\tfrac{1}{2}\text{ in.}}{2}$$
$$= 1.50\text{ in.}$$

$$A_{gv} = (2)\left[l_{ev} + (n-1)s\right](t)$$
$$= (2)\left[1\tfrac{1}{4}\text{ in.} + (5-1)(3.00\text{ in.})\right](\tfrac{1}{2}\text{ in.})$$
$$= 13.3\text{ in.}^2$$

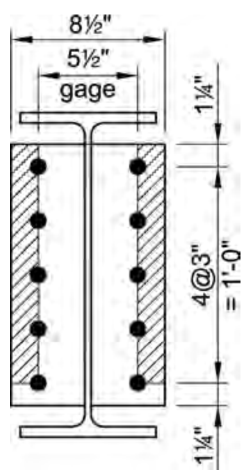


Fig. II.A-11B-2. Block shear rupture of end-plate.

$$A_{nv} = A_{gv} - (2)(n - 0.5)(d_h + 1/16\text{ in.})(t)$$
$$= 13.3\text{ in.}^2 - (2)(5 - 0.5)(15/16\text{ in.} + 1/16\text{ in.})(1/2\text{ in.})$$
$$= 8.80\text{ in.}^2$$

$$A_{nt} = (2)\left[l_{eh} - 0.5(d_h + 1/16\text{ in.})\right](t)$$
$$= (2)\left[1.50\text{ in.} - 0.5(15/16\text{ in.} + 1/16\text{ in.})\right](1/2\text{ in.})$$
$$= 1.00\text{ in.}^2$$
$$U_{bs} = 1.0$$

and

$$R_n = 0.60(58\text{ ksi})(8.80\text{ in.}^2) + 1.0(58\text{ ksi})(1.00\text{ in.}^2) \leq 0.60(36\text{ ksi})(13.3\text{ in.}^2) + 1.0(58\text{ ksi})(1.00\text{ in.}^2)$$
$$= 364\text{ kips} > 345\text{ kips}$$

Therefore:

$$R_n = 345\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(345\text{ kips})$ $= 259\text{ kips} > 75\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{345\text{ kips}}{2.00}$ $= 173\text{ kips} > 50\text{ kips}$ **o.k.**

Shear Strength of Beam

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the beam is determined as follows:

$$A_{gv} = dt_w$$
$$= (18.0\text{ in.})(0.355\text{ in.})$$
$$= 6.39\text{ in.}^2$$

$$R_n = 0.60F_yA_{gv} \quad \text{(Spec. Eq. J4-3)}$$
$$= 0.60(50\text{ ksi})(6.39\text{ in.}^2)$$
$$= 192\text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(192\text{ kips})$ $= 192\text{ kips} > 75\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{192\text{ kips}}{1.50}$ $= 128\text{ kips} > 50\text{ kips}$ **o.k.**

The limit state of shear rupture of the beam web does not apply in this example because the beam is uncoped.

Tensile Strength of Beam

From AISC *Specification* Section J4.1, the available tensile yield strength of the beam is determined as follows:

$$R_n = F_y A_g \quad \text{(Spec. Eq. J4-1)}$$
$$= (50 \text{ ksi})(14.7 \text{ in.}^2)$$
$$= 735 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(735 \text{ kips})$ $= 662 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{735 \text{ kips}}{1.67}$ $= 440 \text{ kips} > 40 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.1, determine the available tensile rupture strength of the beam. The effective net area is $A_e = A_n U$ from AISC *Specification* Section D3, where U is determined from AISC *Specification* Table D3.1, Case 3.

$$U = 1.0$$

$$A_n = \text{area of the directly connected elements}$$
$$= l_w t_w$$
$$= (14.1 \text{ in.})(0.355 \text{ in.})$$
$$= 5.01 \text{ in.}^2$$

The available tensile rupture strength is:

$$R_n = F_u A_e \quad \text{(Spec. Eq. J4-2)}$$
$$= F_u A_n U$$
$$= (65 \text{ ksi})(5.01 \text{ in.}^2)(1.0)$$
$$= 326 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(326 \text{ kips})$ $= 245 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{326 \text{ kips}}{2.00}$ $= 163 \text{ kips} > 40 \text{ kips}$ **o.k.**

Conclusion

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-11C SHEAR END-PLATE CONNECTION—STRUCTURAL INTEGRITY CHECK

Given:

Verify the shear end-plate connection from Example II.A-11B for the structural integrity provisions of AISC *Specification* Section B3.9. The ASTM A992 W18×50 beam is bracing a column and the connection geometry is shown in Figure II.A-11C-1. Note that these checks are necessary when design for structural integrity is required by the applicable building code.

Use 70-ksi electrodes and ASTM A36 plate.

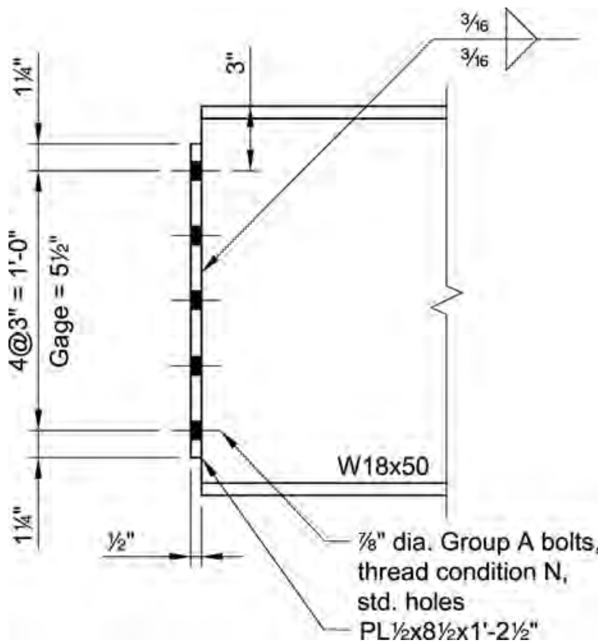


Fig. II.A-11C-1. Connection geometry for Example II.A-11C.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18x50
t_w = 0.355 in.

From Example II.A-11B, the required shear strength is:

LRFD	ASD
$V_u = 75\text{ kips}$	$V_a = 50\text{ kips}$

From AISC *Specification* Section B3.9, the required axial tensile strength is:

LRFD	ASD
$T_u = \frac{2}{3}V_u \geq 10\text{ kips}$ $= \frac{2}{3}(75\text{ kips}) > 10\text{ kips}$ $= 50\text{ kips} > 10\text{ kips}$ $= 50\text{ kips}$	$T_a = V_a \geq 10\text{ kips}$ $= 50\text{ kips} > 10\text{ kips}$ $= 50\text{ kips}$

From AISC *Specification* Section B3.9, these requirements are evaluated independently from other strength requirements.

Bolt Tension

From AISC *Specification* Section J3.6, the nominal bolt tensile strength is:

F_{nt} = 90 ksi, from AISC *Specification* Table J3.2

$$T_n = nF_{nt}A_b \qquad \text{(from } \textit{Spec.} \text{ Eq. J3-1)}$$
$$= (10\text{ bolts})(90\text{ ksi})\left(0.601\text{ in.}^2\right)$$
$$= 541\text{ kips}$$

Angle Bending and Prying Action

From AISC *Manual* Part 9, the nominal strength of the end-plate accounting for prying action is determined as follows:

$$a = \frac{width\ of\ \ plate - gage}{2}$$
$$= \frac{8\tfrac{1}{2}\text{ in.} - 5\tfrac{1}{2}\text{ in.}}{2}$$
$$= 1.50\text{ in.}$$

$$b = \frac{gage - t_w}{2}$$
$$= \frac{5\tfrac{1}{2}\text{ in.} - 0.355\text{ in.}}{2}$$
$$= 2.57\text{ in.}$$

$$a' = \left(a + \frac{d_b}{2}\right) \le \left(1.25b + \frac{d_b}{2}\right)$$ (*Manual* Eq. 9-23)

$$= 1.50 \text{ in.} + \frac{7/8 \text{ in.}}{2} \le 1.25(2.57 \text{ in.}) + \frac{7/8 \text{ in.}}{2}$$
$$= 1.94 \text{ in.} < 3.65 \text{ in.}$$
$$= 1.94 \text{ in.}$$

$$b' = b - \frac{d_b}{2}$$ (*Manual* Eq. 9-18)

$$= 2.57 \text{ in.} - \frac{7/8 \text{ in.}}{2}$$
$$= 2.13 \text{ in.}$$

$$\rho = \frac{b'}{a'}$$ (*Manual* Eq. 9-22)

$$= \frac{2.13 \text{ in.}}{1.94 \text{ in.}}$$
$$= 1.10$$

Note that end distances of 1¼ in. are used on the end-plate, so p is the average pitch of the bolts:

$$p = \frac{l}{n}$$
$$= \frac{14\tfrac{1}{2} \text{ in.}}{5}$$
$$= 2.90 \text{ in.}$$

Check

$p \le s = 3.00$ in. **o.k.**

$$d' = d_h$$
$$= 15/16 \text{ in.}$$

$$\delta = 1 - \frac{d'}{p}$$ (*Manual* Eq. 9-20)

$$= 1 - \frac{15/16 \text{ in.}}{2.90 \text{ in.}}$$
$$= 0.677$$

$$B_n = F_{nt} A_b$$
$$= (90 \text{ ksi})(0.601 \text{ in.}^2)$$
$$= 54.1 \text{ kips/bolt}$$

$$t_c = \sqrt{\frac{4B_n b'}{pF_u}}$$ (from *Manual* Eq. 9-26)

$$= \sqrt{\frac{4(54.1 \text{ kips/bolt})(2.13 \text{ in.})}{(2.90 \text{ in.})(58 \text{ ksi})}}$$

$$= 1.66 \text{ in.}$$

$$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right]$$ (*Manual* Eq. 9-28)

$$= \frac{1}{0.677(1+1.10)}\left[\left(\frac{1.66 \text{ in.}}{\text{½ in.}}\right)^2 - 1\right]$$

$$= 7.05$$

Because $\alpha' > 1$, the end-plate has insufficient strength to develop the bolt strength, therefore:

$$Q = \left(\frac{t}{t_c}\right)^2 (1+\delta)$$

$$= \left(\frac{\text{½ in.}}{1.66 \text{ in.}}\right)^2 (1+0.677)$$

$$= 0.152$$

$$T_n = B_n Q$$ (from *Manual* Eq. 9-27)

$$= (10 \text{ bolts})(54.1 \text{ kips/bolt})(0.152)$$

$$= 82.2 \text{ kips}$$

Weld Strength

From AISC *Specification* Section J2.4, the nominal tensile strength of the weld is determined as follows:

$$F_{nw} = 0.60F_{EXX}\left(1.0 + 0.50\sin^{1.5}\theta\right)$$ (*Spec.* Eq. J2-5)

$$= 0.60(70 \text{ ksi})\left(1.0 + 0.50\sin^{1.5} 90°\right)$$

$$= 63.0 \text{ ksi}$$

The weld length accounts for termination equal to the weld size.

$$l_w = l - 2w$$

$$= 14\text{½ in.} - 2(\text{3/16 in.})$$

$$= 14.1 \text{ in.}$$

The throat dimension is used to calculate the effective area of the fillet weld.

$$A_{we} = \frac{w}{\sqrt{2}} l_w (2 \text{ welds})$$
$$= \frac{\tfrac{3}{16} \text{ in.}}{\sqrt{2}} (14.1 \text{ in.})(2 \text{ welds})$$
$$= 3.74 \text{ in.}^2$$

$$T_n = F_{nw} A_{we}$$ (from *Spec.* Eq. J2-4)
$$= (63.0 \text{ ksi})(3.74 \text{ in.}^2)$$
$$= 236 \text{ kips}$$

Tensile Strength of Beam Web at the Weld

From AISC *Specification* Section J4.1, the nominal tensile strength of the beam web at the weld is:

$$A_e = l_w t_w$$
$$= (14.1 \text{ in.})(0.355 \text{ in.})$$
$$= 5.01 \text{ in.}^2$$

$$T_n = F_u A_e$$ (*Spec.* Eq. J4-2)
$$= (65 \text{ ksi})(5.01 \text{ in.}^2)$$
$$= 326 \text{ kips}$$

Nominal Tensile Strength

The controlling nominal tensile strength, T_n, is the least of those previously calculated:

$$T_n = \min\{541 \text{ kips},\ 82.2 \text{ kips},\ 236 \text{ kips},\ 326 \text{ kips}\}$$
$$= 82.2 \text{ kips}$$

LRFD	ASD
$T_n = 82.2 \text{ kips} > 50 \text{ kips}$ **o.k.**	$T_n = 82.2 \text{ kips} > 50 \text{ kips}$ **o.k.**

Column Bracing

From AISC *Specification* Section B3.9(c), the minimum axial tension strength for the connection of a member bracing a column is equal to 1% of two-thirds of the required column axial strength for LRFD and equal to 1% of the required column axial for ASD. These requirements are evaluated independently from other strength requirements.

The maximum column axial force this connection is able to brace is determined as follows:

LRFD	ASD
$T_n \geq 0.01\left(\frac{2}{3}P_u\right)$	$T_n \geq 0.01 P_a$

LRFD	ASD
Solving for the column axial force: $$P_u \le 100\left(\frac{3}{2}T_n\right)$$ $$= 100\left(\frac{3}{2}\right)(82.2\text{ kips})$$ $$= 12{,}300\text{ kips}$$	Solving for the column axial force: $$P_a \le 100T_n$$ $$= 100(82.2\text{ kips})$$ $$= 8{,}220\text{ kips}$$

As long as the required column axial strength is less than or equal to P_u = 12,300 kips or P_a = 8,220 kips, this connection is an adequate column brace.

EXAMPLE II.A-12A ALL-BOLTED UNSTIFFENED SEATED CONNECTION (BEAM-TO-COLUMN WEB)

Given:

Verify the all-bolted unstiffened seated connection between an ASTM A992 W16×50 beam and an ASTM A992 W14×90 column web, as shown in Figure II.A-12A-1, to support the following end reactions:

R_D = 9 kips
R_L = 27.5 kips

Use ASTM A36 angles.

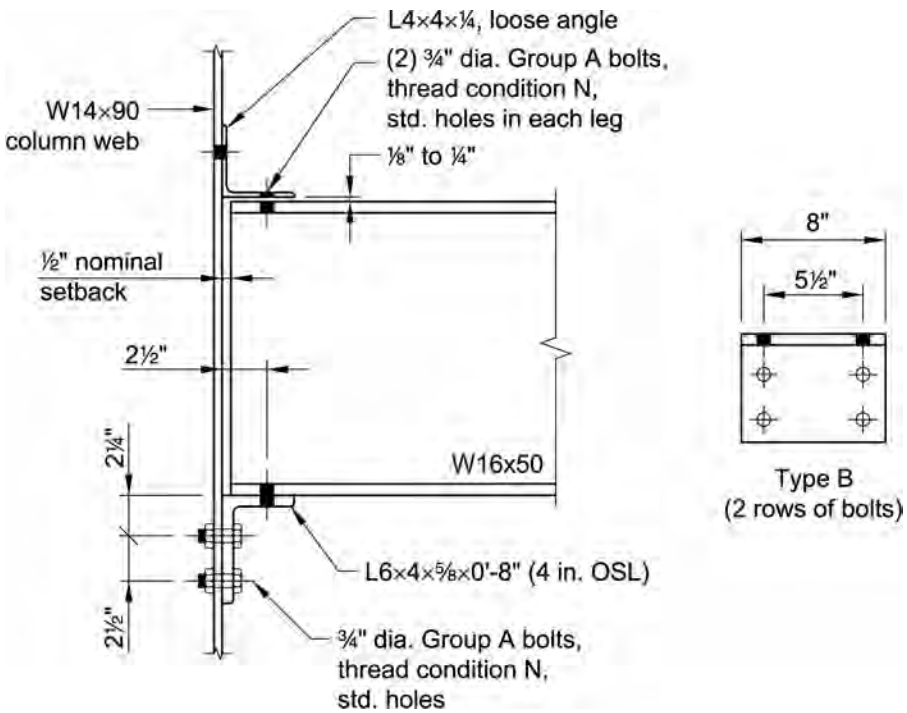


Fig. II.A-12A-1. Connection geometry for Example II.A-12A-1.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W16×50
t_w = 0.380 in.
d = 16.3 in.
b_f = 7.07 in.
t_f = 0.630 in.
k_{des} = 1.03 in.

Column
W14×90
t_w = 0.440 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(9 \text{ kips}) + 1.6(27.5 \text{ kips})$ $= 54.8 \text{ kips}$	$R_a = 9 \text{ kips} + 27.5 \text{ kips}$ $= 36.5 \text{ kips}$

Minimum Bearing Length

From AISC *Manual* Part 10, the minimum required bearing length, $l_{b\ min}$, is the length of bearing required for the limit states of web local yielding and web local crippling on the beam, but not less than k_{des}.

Using AISC *Manual* Equations 9-46a or 9-46b and AISC *Manual* Table 9-4, the minimum required bearing length for web local yielding is:

LRFD	ASD
$l_{b\ min} = \dfrac{R_u - \phi R_1}{\phi R_2} \geq k_{des}$ $= \dfrac{54.8 \text{ kips} - 48.9 \text{ kips}}{19.0 \text{ kip/in.}} \geq 1.03 \text{ in.}$ $= 0.311 \text{ in.} < 1.03 \text{ in.}$ Therefore, $l_{b\ min} = k_{des} = 1.03 \text{ in.}$	$l_{b\ min} = \dfrac{R_a - R_1/\Omega}{R_2/\Omega} \geq k_{des}$ $= \dfrac{36.5 \text{ kips} - 32.6 \text{ kips}}{12.7 \text{ kip/in.}} \geq 1.03 \text{ in.}$ $= 0.307 \text{ in.} < 1.03 \text{ in.}$ Therefore, $l_{b\ min} = k_{des} = 1.03 \text{ in.}$

For web local crippling, the maximum bearing length-to-depth ratio is determined as follows (including ¼-in. tolerance to account for possible beam underrun):

$$\left(\frac{l_b}{d}\right)_{max} = \frac{3.25 \text{ in.}}{16.3 \text{ in.}}$$
$$= 0.199 < 0.2$$

Using AISC *Manual* Equations 9-48a or 9-48b and AISC *Manual* Table 9-4, when $\dfrac{l_b}{d} \leq 0.2$:

LRFD	ASD
$l_{b\,min} = \dfrac{R_u - \phi R_3}{\phi R_4}$ $= \dfrac{54.8\text{ kips} - 67.2\text{ kips}}{5.79\text{ kip/in.}}$ This results in a negative quantity; therefore, $l_{b\,min} = k_{des} = 1.03\text{ in.}$	$l_{b\,min} = \dfrac{R_a - R_3 / \Omega}{R_4 / \Omega}$ $= \dfrac{36.5\text{ kips} - 44.8\text{ kips}}{3.86\text{ kip/in.}}$ This results in a negative quantity; therefore, $l_{b\,min} = k_{des} = 1.03\text{ in.}$

Connection Selection

AISC *Manual* Table 10-5 includes checks for the limit states of shear yielding and flexural yielding of the outstanding angle leg.

For an 8-in. angle length with a ⅝-in. thickness, a 3½-in. minimum outstanding leg, and conservatively using $l_{b,\,req} = 1\tfrac{1}{16}$ in., from AISC *Manual* Table 10-5:

LRFD	ASD
$\phi R_n = 90.0\text{ kips} > 54.8\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 59.9\text{ kips} > 36.5\text{ kips}$ **o.k.**

The L6×4×⅝ (4-in. OSL), 8-in. long with 5½-in. bolt gage, Connection Type B (four bolts), is acceptable.

From the bottom portion of AISC *Manual* Table 10-5 for L6, with ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N), the available shear strength is:

LRFD	ASD
$\phi R_n = 71.6\text{ kips} > 54.8\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 47.7\text{ kips} > 36.5\text{ kips}$ **o.k.**

Bolt Bearing and Tearout on the Angle

Due to the presence and location of the bolts in the outstanding leg of the angle, tearout does not control. The nominal bearing strength of the angles is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$R_n = (4\text{ bolts})2.4dtF_u \quad \text{(from } \textit{Spec.}\text{ Eq. J3-6a)}$$
$$= (4\text{ bolts})(2.4)(\tfrac{3}{4}\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})$$
$$= 261\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(261\text{ kips})$ $= 196\text{ kips} > 54.8\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{261\text{ kips}}{2.00}$ $= 131\text{ kips} > 36.5\text{ kips}$ **o.k.**

Note that the effective strength of the bolt group is controlled by bolt shear.

Bolt Bearing on the Column

The nominal bearing strength of the column web determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration, is:

$$R_n = (4 \text{ bolts})2.4dt_wF_u$$ (from *Spec.* Eq. J3-6a)

$$= (4 \text{ bolts})(2.4)(\tfrac{3}{4} \text{ in.})(0.440 \text{ in.})(65 \text{ ksi})$$

$$= 206 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(206 \text{ kips})$ $= 155 \text{ kips} > 54.8 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{206 \text{ kips}}{2.00}$ $= 103 \text{ kips} > 36.5 \text{ kips}$ **o.k.**

Top Angle and Bolts

As discussed in AISC *Manual* Part 10, use an L4×4×¼ with two ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) through each leg.

Conclusion

The connection design shown in Figure II.A-12A-1 is acceptable.

EXAMPLE II.A-12B ALL-BOLTED UNSTIFFENED SEATED CONNECTION—STRUCTURAL INTEGRITY CHECK

Given:

Verify the all-bolted unstiffened seated connection from Example II.A-12A, as shown in Figure II.A-12B-1, for the structural integrity provisions of AISC *Specification* Section B3.9. The connection is verified as a beam and girder end connection and as an end connection of a member bracing a column. Note that these checks are necessary when design for structural integrity is required by the applicable building code.

The beam is an ASTM A992 W16×50 and the angles are ASTM A36 material.

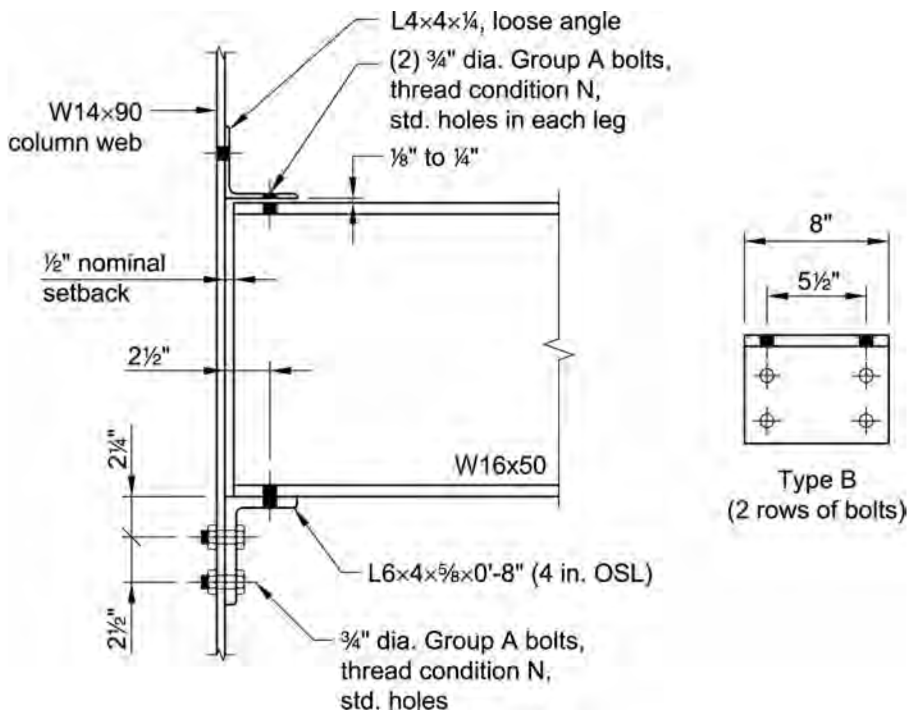


Fig. II.A-12B-1. Connection geometry for Example II.A-12B.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Angle
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W16x50
b_f = 7.07 in.
t_f = 0.630 in.

From AISC *Specification* Table J3.3, the hole diameter for ¾-in.-diameter bolts with standard holes is:

d_h = 13⁄16 in.

From Example II.A-12A, the required shear strength is:

LRFD	ASD
$V_u = 54.8$ kips	$V_a = 36.5$ kips

From AISC *Specification* Section B3.9(b), the required axial tensile strength is:

LRFD	ASD
$T_u = \frac{2}{3}V_u \geq 10 \text{ kips}$ $= \frac{2}{3}(54.8 \text{ kips}) > 10 \text{ kips}$ $= 36.5 \text{ kips}$	$T_a = V_a \geq 10 \text{ kips}$ $= 36.5 \text{ kips} > 10 \text{ kips}$ $= 36.5 \text{ kips}$

From AISC *Specification* Section B3.9, these strength requirements are evaluated independently from other strength requirements.

Bolt Shear

Bolt shear is checked for the outstanding leg of the seat angle. From AISC *Specification* Section J3.6, the nominal bolt shear strength is:

F_{nv} = 54 ksi, from AISC *Specification* Table J3.2

$$T_n = nF_{nv}A_b \quad \text{(from } \textit{Spec.} \text{ Eq. J3-1)}$$
$$= (2 \text{ bolts})(54 \text{ ksi})(0.442 \text{ in.}^2)$$
$$= 47.7 \text{ kips}$$

Bolt Tension

Bolt tension is checked for the top row of bolts on the support leg of the seat angle. From AISC *Specification* Section J3.6, the nominal bolt tensile strength is:

F_{nt} = 90 ksi, from AISC *Specification* Table J3.2

$$T_n = nF_{nt}A_b \quad \text{(from } \textit{Spec.} \text{ Eq. J3-1)}$$
$$= (2 \text{ bolts})(90 \text{ ksi})(0.442 \text{ in.}^2)$$
$$= 79.6 \text{ kips}$$

Bolt Bearing and Tearout

Bolt bearing and tearout is checked for the outstanding leg of the seat angle. From AISC *Specification* Section B3.9, for the purpose of satisfying structural integrity requirements, inelastic deformations of the connection are permitted; therefore, AISC *Specification* Equations J3-6b and J3-6d are used to determine the nominal bearing and tearout strength. By inspection, bolt bearing and tearout will control for the angle.

For bolt bearing on the angle:

$$T_n = n3.0dtF_u \qquad \text{(from } \textit{Spec.} \text{ Eq. J3-6b)}$$
$$= (2 \text{ bolts})(3.0)(\tfrac{3}{4} \text{ in.})(\tfrac{5}{8} \text{ in.})(58 \text{ ksi})$$
$$= 163 \text{ kips}$$

For bolt tearout on the angle:

$$l_c = leg - 2\tfrac{1}{2} \text{ in.} - 0.5d_h$$
$$= 4.00 \text{ in.} - 2\tfrac{1}{2} \text{ in.} - 0.5(\tfrac{13}{16} \text{ in.})$$
$$= 1.09 \text{ in.}$$

$$T_n = n1.5l_ctF_u \qquad \text{(from } \textit{Spec.} \text{ Eq. J3-6d)}$$
$$= (2 \text{ bolts})(1.5)(1.09 \text{ in.})(\tfrac{5}{8} \text{ in.})(58 \text{ ksi})$$
$$= 119 \text{ kips}$$

Angle Bending and Prying Action

From AISC *Manual* Part 9, the nominal strength of the angle accounting for prying action is determined as follows:

$$b = 2\tfrac{1}{4} \text{ in.} - \frac{\tfrac{5}{8} \text{ in.}}{2}$$
$$= 1.94 \text{ in.}$$

$$a = \min\{2.50 \text{ in.}, 1.25b\}$$
$$= \min\{2.50 \text{ in.}, 1.25(1.94 \text{ in.})\}$$
$$= 2.43 \text{ in.}$$

$$b' = \left(b - \frac{d_b}{2}\right) \qquad \text{(\textit{Manual} Eq. 9-18)}$$
$$= 1.94 \text{ in.} - \frac{\tfrac{3}{4} \text{ in.}}{2}$$
$$= 1.57 \text{ in.}$$

$$a' = a + \frac{d_b}{2} \qquad \text{(from \textit{Manual} Eq. 9-23)}$$
$$= 2.43 \text{ in.} + \frac{\tfrac{3}{4} \text{ in.}}{2}$$
$$= 2.81 \text{ in.}$$

$$\rho = \frac{b'}{a'} \qquad \text{(}\textit{Manual}\text{ Eq. 9-22)}$$

$$= \frac{1.57 \text{ in.}}{2.81 \text{ in.}}$$

$$= 0.559$$

Note that end distances of 1¼ in. are used on the angles, so p is the average pitch of the bolts:

$$p = \frac{l}{n}$$

$$= \frac{8.00 \text{ in.}}{2}$$

$$= 4.00 \text{ in.}$$

Check

$p \le s = 5\tfrac{1}{2}$ in.   **o.k.**

$$d' = d_h$$

$$= \tfrac{13}{16} \text{ in.}$$

$$\delta = 1 - \frac{d'}{p} \qquad \text{(}\textit{Manual}\text{ Eq. 9-20)}$$

$$= 1 - \frac{\tfrac{13}{16} \text{ in.}}{4.00 \text{ in.}}$$

$$= 0.797$$

$$B_n = F_{nt} A_b$$

$$= (90 \text{ ksi})(0.442 \text{ in.}^2)$$

$$= 39.8 \text{ kips/bolt}$$

$$t_c = \sqrt{\frac{4B_n b'}{pF_u}} \qquad \text{(from }\textit{Manual}\text{ Eq. 9-26)}$$

$$= \sqrt{\frac{4(39.8 \text{ kips/bolt})(1.57 \text{ in.})}{(4.00 \text{ in.})(58 \text{ ksi})}}$$

$$= 1.04 \text{ in.}$$

$$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right] \qquad \text{(}\textit{Manual}\text{ Eq. 9-28)}$$

$$= \frac{1}{0.797(1+0.559)}\left[\left(\frac{1.04 \text{ in.}}{\tfrac{5}{8} \text{ in.}}\right)^2 - 1\right]$$

$$= 1.42$$

Because $\alpha' > 1$, the angle has insufficient strength to develop the bolt strength, therefore:

$$Q = \left(\frac{t}{t_c}\right)^2 (1+\delta)$$
$$= \left(\frac{5/8 \text{ in.}}{1.04 \text{ in.}}\right)^2 (1+0.797)$$
$$= 0.649$$

$$T_n = B_n Q \qquad \text{(from } \textit{Manual} \text{ Eq. 9-27)}$$
$$= (2 \text{ bolts})(39.8 \text{ kips/bolt})(0.649)$$
$$= 51.7 \text{ kips}$$

Block Shear Rupture

By comparison of the seat angle length and flange width, block shear rupture of the beam flange will control. The block shear rupture failure path is shown in Figure II.A-12B-2. From AISC *Specification* Section J4.3, the available block shear rupture strength of the beam flange is determined as follows (account for a possible ¼-in. beam underrun):

$$T_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60 F_y A_{gv} + U_{bs} F_u A_{nt} \qquad \text{(from } \textit{Spec.} \text{ Eq. J4-5)}$$

where

$$A_{gv} = (2) l_e t_f$$
$$= (2)(1\tfrac{3}{4} \text{ in.})(0.630 \text{ in.})$$
$$= 2.21 \text{ in.}^2$$

$$A_{nv} = (2)\left[l_e - 0.5\left(d_h + \tfrac{1}{16} \text{ in.}\right)\right] t_f$$
$$= (2)\left[1\tfrac{3}{4} \text{ in.} - 0.5\left(\tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)\right](0.630 \text{ in.})$$
$$= 1.65 \text{ in.}^2$$

$$A_{nt} = (2)\left[\frac{b_f - gage}{2} - 0.5\left(d_h + \tfrac{1}{16} \text{ in.}\right)\right] t_f$$
$$= (2)\left[\frac{7.07 \text{ in.} - 5\tfrac{1}{2} \text{ in.}}{2} - 0.5\left(\tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)\right](0.630 \text{ in.})$$
$$= 0.438 \text{ in.}^2$$

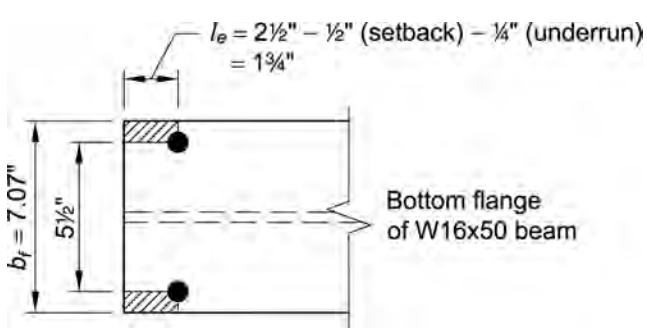


Fig. II.A-12B-2. Beam flange block shear rupture.

$U_{bs} = 1.0$

and

$$T_n = 0.60(65\text{ ksi})(1.65\text{ in.}^2) + 1.0(65\text{ ksi})(0.438\text{ in.}^2) \le 0.60(50\text{ ksi})(2.21\text{ in.}^2) + 1.0(65\text{ ksi})(0.438\text{ in.}^2)$$
$$= 92.8\text{ kips} < 94.8\text{ kips}$$
$$= 92.8\text{ kips}$$

Nominal Tensile Strength

The controlling tensile strength, T_n, is the least of those previously calculated:

$$T_n = \min\{47.7\text{ kips},\ 79.6\text{ kips},\ 163\text{ kips},\ 119\text{ kips},\ 51.7\text{ kips},\ 92.8\text{ kips}\}$$
$$= 47.7\text{ kips}$$

LRFD	ASD
$T_n = 47.7\text{ kips} > 36.5\text{ kips}$ **o.k.**	$T_n = 47.7\text{ kips} > 36.5\text{ kips}$ **o.k.**

Column Bracing

From AISC *Specification* Section B3.9(c), the minimum axial tension strength for the connection of a member bracing a column is equal to 1% of two-thirds of the required column axial strength for LRFD and equal to 1% of the required column axial for ASD. These requirements are evaluated independently from other strength requirements.

The maximum column axial force this connection is able to brace is determined as follows,

LRFD	ASD
$T_n \ge 0.01\left(\frac{2}{3}P_u\right)$ Solving for the column axial force: $P_u \le 100\left(\frac{3}{2}T_n\right)$ $= 100\left(\frac{3}{2}\right)(47.7\text{ kips})$ $= 7{,}160\text{ kips}$	$T_n \ge 0.01P_a$ Solving for the column axial force: $P_a \le 100T_n$ $= 100(47.7\text{ kips})$ $= 4{,}770\text{ kips}$

As long as the required column axial strength is less than P_u = 7,160 kips or P_a = 4,770 kips, this connection is an adequate column brace.

EXAMPLE II.A-13 BOLTED/WELDED UNSTIFFENED SEATED CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Verify the unstiffened seated connection between an ASTM A992 W21×62 beam and an ASTM A992 W14×61 column flange, as shown in Figure II.A-13-1, to support the following beam end reactions:

R_D = 9 kips
R_L = 27.5 kips

Use ASTM A36 angles and 70-ksi weld electrodes.

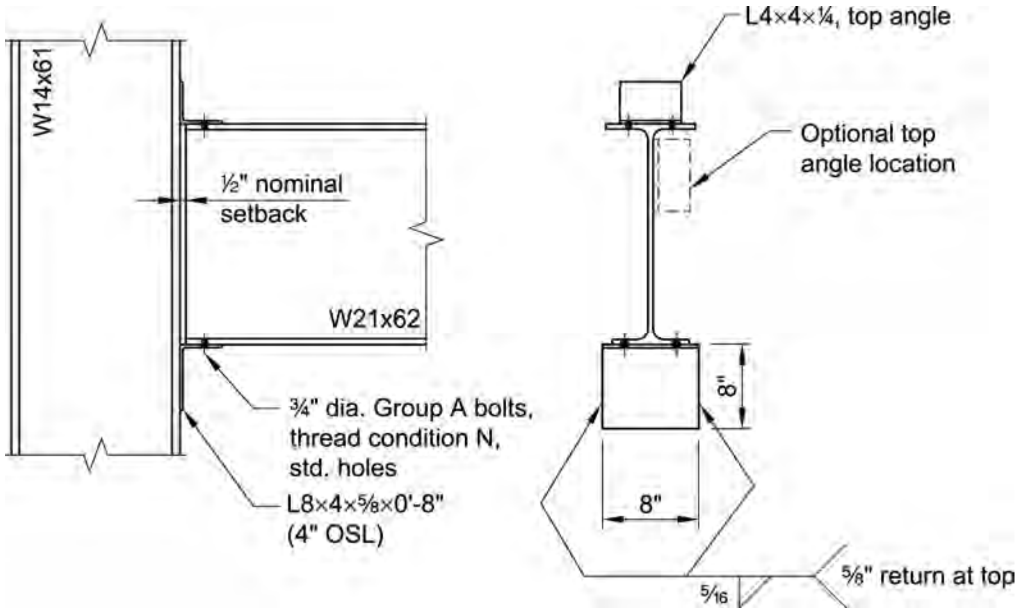


Fig. II.A-13-1. Connection geometry for Example II.A-13.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W21×62
t_w = 0.400 in.
d = 21.0 in.
b_f = 8.24 in.
t_f = 0.615 in.
k_{des}= 1.12 in.

Column
W14×61
t_f = 0.645 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(9 \text{ kips}) + 1.6(27.5 \text{ kips})$ $= 54.8 \text{ kips}$	$R_a = 9 \text{ kips} + 27.5 \text{ kips}$ $= 36.5 \text{ kips}$

Minimum Bearing Length

From AISC *Manual* Part 10, the minimum required bearing length, $l_{b\ min}$, is the length of bearing required for the limit states of web local yielding and web local crippling on the beam, but not less than k_{des}.

Using AISC *Manual* Equations 9-46a or 9-46b and AISC *Manual* Table 9-4, the minimum required bearing length for web local yielding is:

LRFD	ASD
$l_{b\ min} = \dfrac{R_u - \phi R_1}{\phi R_2} \geq k_{des}$ $= \dfrac{54.8 \text{ kips} - 56.0 \text{ kips}}{20.0 \text{ kip/in.}} \geq 1.12 \text{ in.}$ which results in a negative quantity. Therefore, $l_{b\,min} = k_{des} = 1.12 \text{ in.}$	$l_{b\ min} = \dfrac{R_a - R_1/\Omega}{R_2/\Omega} \geq k_{des}$ $= \dfrac{36.5 \text{ kips} - 37.3 \text{ kips}}{13.3 \text{ kip/in.}} \geq 1.12 \text{ in.}$ which results in a negative quantity. Therefore, $l_{b\,min} = k_{des} = 1.12 \text{ in.}$

For web local crippling, the maximum bearing length-to-depth ratio is determined as follows (including a ¼-in. tolerance to account for possible beam underrun):

$$\left(\frac{l_b}{d}\right)_{max} = \frac{3.25 \text{ in.}}{21.0 \text{ in.}}$$
$$= 0.155 < 0.2$$

From AISC *Manual* Equations 9-48a or 9-48b and AISC *Manual* Table 9-4, when $\dfrac{l_b}{d} \leq 0.2$:

LRFD	ASD
$l_{b\ min} = \dfrac{R_u - \phi R_3}{\phi R_4}$ $= \dfrac{54.8 \text{ kips} - 71.7 \text{ kips}}{5.37 \text{ kip/in.}}$	$l_{b\ min} = \dfrac{R_a - R_3/\Omega}{R_4/\Omega}$ $= \dfrac{36.5 \text{ kips} - 47.8 \text{ kips}}{3.58 \text{ kip/in.}}$

LRFD	ASD
This results in a negative quantity; therefore, $l_{b\,min} = k_{des} = 1.12$ in.	This results in a negative quantity; therefore, $l_{b\,min} = k_{des} = 1.12$ in.

Connection Selection

AISC *Manual* Table 10-6 includes checks for the limit states of shear yielding and flexural yielding of the outstanding angle leg.

For an 8-in. angle length with a ⅝-in. thickness, a 3½-in. minimum outstanding leg, and conservatively using $l_{b,\,req}$ = 1⅛ in., from AISC *Manual* Table 10-6:

LRFD	ASD
$\phi R_n = 81.0 \text{ kips} > 54.8 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 53.9 \text{ kips} > 36.5 \text{ kips}$ **o.k.**

From AISC *Manual* Table 10-6, for a L8×4×⅝ (4-in. OSL), 8-in. long, with 5⁄16-in. fillet welds, the weld available strength is:

LRFD	ASD
$\phi R_n = 66.7 \text{ kips} > 54.8 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 44.5 \text{ kips} > 36.5 \text{ kips}$ **o.k.**

Use two ¾-in.-diameter bolts with threads not excluded from the shear plane (thread condition N) to connect the beam to the seat angle.

The strength of the bolts, welds and angles must be verified if horizontal forces are added to the connection.

Top Angle, Bolts and Welds

Use an L4×4×¼ with two ¾-in.-diameter bolts with threads not excluded from the shear plane (thread condition N) through the supported beam leg of the angle. Use a 3⁄16-in. fillet weld along the toe of the angle to the column flange. See the discussion in AISC *Manual* Part 10.

Conclusion

The connection design shown in Figure II.A-13-1 is acceptable.

EXAMPLE II.A-14 BOLTED/WELDED STIFFENED SEATED CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Verify the bolted/welded stiffened seated connection between an ASTM A992 W21×68 beam and an ASTM A992 W14×90 column flange, as shown in Figure II.A-14-1, to support the following end reactions:

R_D = 21 kips
R_L = 62.5 kips

Use 70-ksi weld electrodes and ASTM A36 angles and plate.

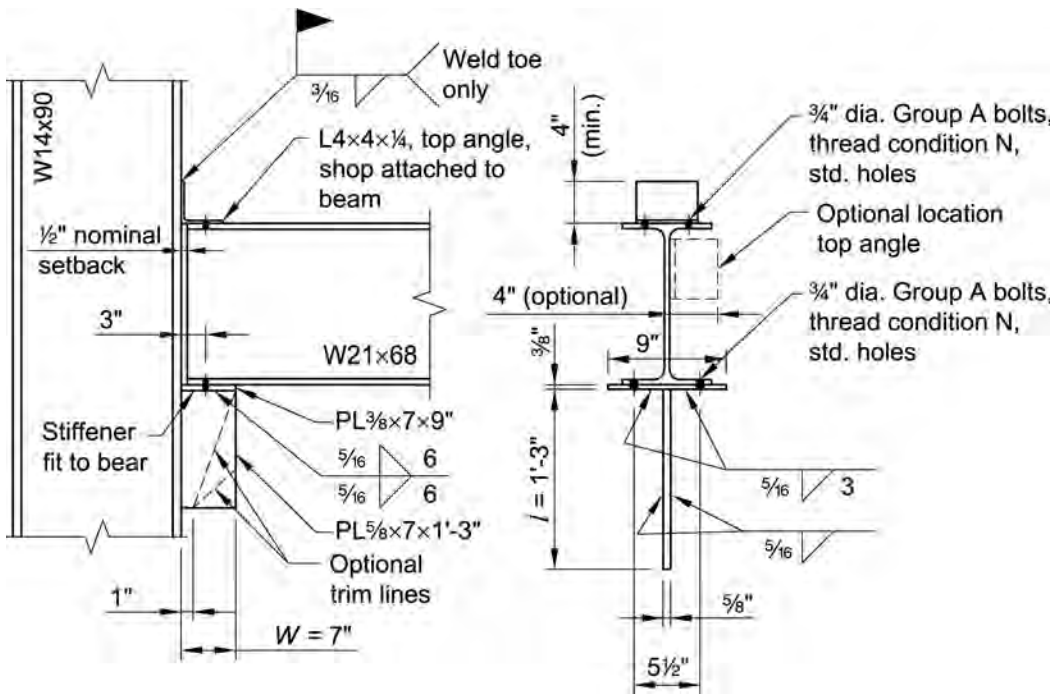


Fig. II.A-14-1. Connection geometry for Example II.A-14.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam and column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angle and plates
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W21×68
t_w = 0.430 in.
d = 21.1 in.
b_f = 8.27 in.
t_f = 0.685 in.
k_{des} = 1.19 in.

Column
W14×90
t_f = 0.710 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(21\text{ kips}) + 1.6(62.5\text{ kips})$ $= 125\text{ kips}$	$R_a = 21\text{ kips} + 62.5\text{ kips}$ $= 83.5\text{ kips}$

Required Stiffener Width

The minimum stiffener width, W_{min}, is determined based on limit states of web local yielding and web local crippling for the beam.

The minimum stiffener width for web local crippling of the beam web, for the force applied less than one-half of the depth of the beam from the end of the beam and assuming $l_b/d > 0.2$, is determined from AISC *Manual* Equations 9-49a or 9-49b and AISC *Manual* Table 9-4, as follows (including a ¼-in. tolerance to account for possible beam underrun):

LRFD	ASD
$W_{min} = \dfrac{R_u - \phi R_5}{\phi R_6} + \text{setback} + \text{underrun}$ $= \dfrac{125\text{ kips} - 75.9\text{ kips}}{7.95\text{ kip/in.}} + \tfrac{1}{2}\text{ in.} + \tfrac{1}{4}\text{ in.}$ $= 6.93\text{ in.}$	$W_{min} = \dfrac{R_a - R_5/\Omega}{R_6/\Omega} + \text{setback} + \text{underrun}$ $= \dfrac{83.5\text{ kips} - 50.6\text{ kips}}{5.30\text{ kip/in.}} + \tfrac{1}{2}\text{ in.} + \tfrac{1}{4}\text{ in.}$ $= 6.96\text{ in.}$

The minimum stiffener width for web local yielding of the beam, for the force applied less than the depth of the beam from the end of the beam, is determined from AISC *Manual* Equations 9-46a or 9-46b and AISC *Manual* Table 9-4, as follows (including a ¼-in. tolerance to account for possible beam underrun):

LRFD	ASD
$W_{min} = \dfrac{R_u - \phi R_1}{\phi R_2} + \text{setback} + \text{underrun}$ $= \dfrac{125\text{ kips} - 64.0\text{ kips}}{21.5\text{ kip/in.}} + \tfrac{1}{2}\text{ in.} + \tfrac{1}{4}\text{ in.}$ $= 3.59\text{ in.}$	$W_{min} = \dfrac{R_a - R_1/\Omega}{R_2/\Omega} + \text{setback} + \text{underrun}$ $= \dfrac{83.5\text{ kips} - 42.6\text{ kips}}{14.3\text{ kip/in.}} + \tfrac{1}{2}\text{ in.} + \tfrac{1}{4}\text{ in.}$ $= 3.61\text{ in.}$

Use W = 7 in.

Check assumption:

$$\frac{l_b}{d} = \frac{6.25\text{ in.}}{21.1\text{ in.}}$$
$$= 0.296 > 0.2 \quad \textbf{o.k.}$$

Stiffener Length and Stiffene- to-Column Flange Weld Size

Use a stiffener with l = 15 in. and 5⁄16-in. fillet welds.

From AISC *Manual* Table 10-8, with W = 7 in.:

LRFD	ASD
$\phi R_n = 139\text{ kips} > 125\text{ kips}$ **o.k.**	$\frac{R_n}{\Omega} = 93.0\text{ kips} > 83.5\text{ kips}$ **o.k.**

Seat Plate Welds

Use 5⁄16-in. fillet welds on each side of the stiffener. From AISC *Manual* Figure 10-10(b), minimum length of seat plate-to-column flange weld is 0.2(L) = 3 in. As discussed in AISC *Manual* Part 10, the weld between the seat plate and stiffener plate is required to have a strength equal to or greater than the weld between the seat plate and the column flange, use 5⁄16-in. fillet welds on each side of the stiffener to the seat plate; length of weld = 6 in. per side.

Seat Plate Dimensions

A dimension of 9 in. is adequate to accommodate the ¾-in.-diameter bolts on a 5½-in. gage connecting the beam flange to the seat plate.

Use a PL⅜ in.×7 in.×9 in. for the seat.

Stiffener Plate Thickness

As discussed in AISC *Manual* Part 10, the minimum stiffener plate thickness to develop the seat plate weld for F_y = 36 ksi plate material is:

$$t_{min} = 2w$$
$$= 2(5/16\text{ in.})$$
$$= 5/8\text{ in.}$$

As discussed in AISC *Manual* Part 10, the minimum plate thickness for a stiffener with F_y = 36 ksi and a beam with F_y = 50 ksi is:

$$t_{min} = \left(\frac{50\text{ ksi}}{36\text{ ksi}}\right)t_w$$
$$= \left(\frac{50\text{ ksi}}{36\text{ ksi}}\right)(0.430\text{ in.})$$
$$= 0.597\text{ in.} < 5/8\text{ in.}$$

Use a PL⅝ in.×7 in.×1 ft 3 in.

Top Angle, Bolts and Welds

Use an L4×4×¼ with two ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) through the supported beam leg of the angle. Use a 3⁄16-in. fillet weld along the toe of the angle to the column flange. See discussion in AISC *Manual* Part 10.

Conclusion

The connection design shown in Figure II.A-14-1 is acceptable.

EXAMPLE II.A-15 BOLTED/WELDED STIFFENED SEATED CONNECTION (BEAM-TO-COLUMN WEB)

Given:

Verify the stiffened seated connection between an ASTM A992 W21×68 beam and an ASTM A992 W14×90 column web, as shown in Figure II.A-15-1, to support the following beam end reactions:

R_D = 21 kips
R_L = 62.5 kips

Use 70-ksi weld electrodes and ASTM A36 angles and plate.

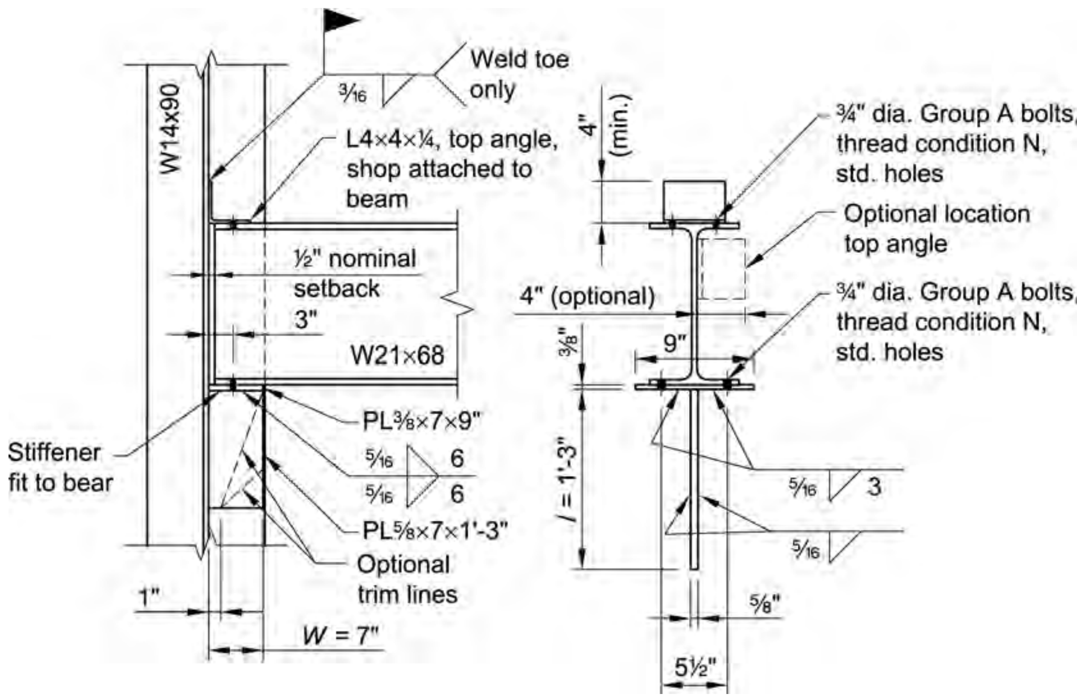


Fig. II.A-15-1. Connection geometry for Example II.A-15.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam and column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angle and Plates
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W21×68
t_w = 0.430 in.
d = 21.1 in.
b_f = 8.27 in.
t_f = 0.685 in.
k_{des} = 1.19 in.

Column
W14×90
t_w= 0.440 in.
T = 10 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(21 \text{ kips}) + 1.6(62.5 \text{ kips})$ $= 125 \text{ kips}$	$R_a = 21 \text{ kips} + 62.5 \text{ kips}$ $= 83.5 \text{ kips}$

Required Stiffener Width

The minimum stiffener width, W_{min}, is determined based on limit states of web local yielding and web local crippling for the beam.

The minimum stiffener width for web local crippling of the beam web, for the force applied less than one-half of the depth of the beam from the end of the beam and assuming l_b/d > 0.2, is determined from AISC *Manual* Equations 9-49a or 9-49b and AISC *Manual* Table 9-4, as follows (including a ¼-in. tolerance to account for possible beam underrun):

LRFD	ASD
$W_{min} = \dfrac{R_u - \phi R_5}{\phi R_6} + \text{setback} + \text{underrun}$ $= \dfrac{125 \text{ kips} - 75.9 \text{ kips}}{7.95 \text{ kip/in.}} + \tfrac{1}{2} \text{ in.} + \tfrac{1}{4} \text{ in.}$ $= 6.93 \text{ in.}$	$W_{min} = \dfrac{R_a - R_5/\Omega}{R_6/\Omega} + \text{setback} + \text{underrun}$ $= \dfrac{83.5 \text{ kips} - 50.6 \text{ kips}}{5.30 \text{ kip/in.}} + \tfrac{1}{2} \text{ in.} + \tfrac{1}{4} \text{ in.}$ $= 6.96 \text{ in.}$

The minimum stiffener width for web local yielding of the beam, for the force applied less than the depth of the beam from the end of the beam, is determined from AISC *Manual* Equations 9-46a or 9-46b and AISC *Manual* Table 9-4, as follows (including a ¼-in. tolerance to account for possible beam underrun):

LRFD	ASD
$W_{min} = \dfrac{R_u - \phi R_1}{\phi R_2} + \text{setback} + \text{underrun}$ $= \dfrac{125 \text{ kips} - 64.0 \text{ kips}}{21.5 \text{ kip/in.}} + \tfrac{1}{2} \text{ in.} + \tfrac{1}{4} \text{ in.}$ $= 3.59 \text{ in.}$	$W_{min} = \dfrac{R_a - R_1/\Omega}{R_2/\Omega} + \text{setback} + \text{underrun}$ $= \dfrac{83.5 \text{ kips} - 42.6 \text{ kips}}{14.3 \text{ kip/in.}} + \tfrac{1}{2} \text{ in.} + \tfrac{1}{4} \text{ in.}$ $= 3.61 \text{ in.}$

Use W = 7 in.

Check assumption:

$$\frac{l_b}{d} = \frac{6.25\text{ in.}}{21.1\text{ in.}}$$
$$= 0.296 > 0.2 \quad \textbf{o.k.}$$

Stiffener Length and Stiffener to Column Flange Weld Size

Use a stiffener with l = 15 in. and 5⁄16-in. fillet welds.

From AISC *Manual* Table 10-8, with W = 7 in., the weld available strength is:

LRFD	ASD
$\phi R_n = 139\text{ kips} > 125\text{ kips} \quad \textbf{o.k.}$	$\dfrac{R_n}{\Omega} = 93.0\text{ kips} > 83.5\text{ kips} \quad \textbf{o.k.}$

Seat Plate Welds

Use 5⁄16-in. fillet welds on each side of the stiffener. From AISC *Manual* Figure 10-10(b), minimum length of seat plate-to-column flange weld is 0.2(L) = 3 in. As discussed in AISC *Manual* Part 10, the weld between the seat plate and stiffener plate is required to have a strength equal to or greater than the weld between the seat plate and the column flange, use 5⁄16-in. fillet welds on each side of the stiffener to the seat plate; length of weld = 6 in. per side.

Seat Plate Dimensions

A dimension of 9 in. is adequate to accommodate the ¾-in.-diameter bolts on a 5½-in. gage connecting the beam flange to the seat plate.

Use a PL⅜ in.×7 in.×9 in. for the seat.

Stiffener Plate Thickness

As discussed in AISC *Manual* Part 10, the minimum stiffener plate thickness to develop the seat plate weld for F_y = 36 ksi plate material is:

$$t_{min} = 2w$$
$$= 2\left(\tfrac{5}{16}\text{ in.}\right)$$
$$= \tfrac{5}{8}\text{ in.}$$

As discussed in AISC *Manual* Part 10, the minimum plate thickness for a stiffener with F_y = 36 ksi and a beam with F_y = 50 ksi is:

$$t_{min} = \left(\frac{50\text{ ksi}}{36\text{ ksi}}\right)t_w$$
$$= \left(\frac{50\text{ ksi}}{36\text{ ksi}}\right)(0.430\text{ in.})$$
$$= 0.597\text{ in.} < \tfrac{5}{8}\text{ in.}$$

Use a PL⅝ in.×7 in.×1 ft 3 in.

Top Angle, Bolts and Welds

Use an L4×4×¼ with two ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) through the supported beam leg of the angle. Use a 3⁄16-in. fillet weld along the toe of the angle to the column web. See discussion in AISC *Manual* Part 10.

Column Web

If the seat is welded to a column web, the base metal strength of the column must be checked.

If only one side of the column web has a stiffened seated connection, then:

$$t_{min} = \frac{3.09D}{F_u} \quad \text{(\textit{Manual} Eq. 9-2)}$$

$$= \frac{3.09(5 \text{ sixteenths})}{65 \text{ ksi}}$$

$$= 0.238 \text{ in.} < 0.440 \text{ in.} \quad \textbf{o.k.}$$

If both sides of the column web have a stiffened seated connection, then:

$$t_{min} = \frac{6.19D}{F_u} \quad \text{(\textit{Manual} Eq. 9-3)}$$

$$= \frac{6.19(5 \text{ sixteenths})}{65 \text{ ksi}}$$

$$= 0.476 \text{ in.} > 0.440 \text{ in.} \quad \textbf{n.g.}$$

The column is sufficient for a one-sided stiffened seated connection. For a two-sided connection the weld available strength must be reduced as discussed in AISC *Manual* Part 10.

Note: Additional detailing considerations for stiffened seated connections are given in Part 10 of the AISC *Manual*.

Conclusion

The connection design shown in Figure II.A-15-1 is acceptable.

EXAMPLE II.A-16 OFFSET UNSTIFFENED SEATED CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Verify the seat angle and weld size required for the unstiffened seated connection between an ASTM A992 W14×38 beam and an ASTM A992 W12×65 column flange connection with an offset of 5½ in., as shown in Figure II.A-16-1, to support the following beam end reactions:

R_D = 5 kips
R_L = 15 kips

Use an ASTM A36 angle and 70-ksi weld electrodes.

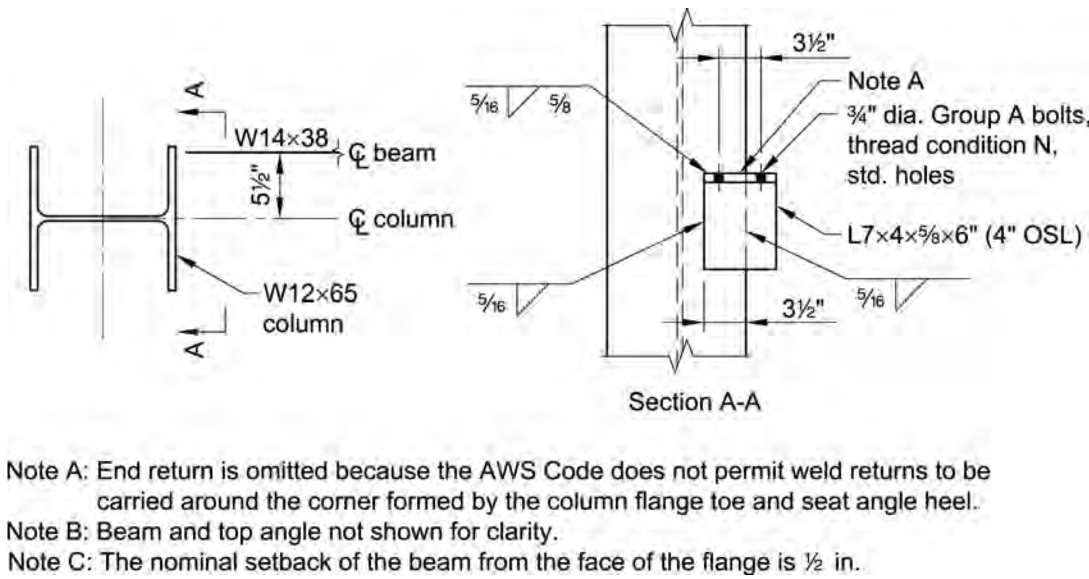


Fig. II.A-16-1. Connection geometry for Example II.A-16.

Solution:

From AISC *Manual* Tables 2-4, the material properties are as follows:

Beam and column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angle
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W14×38
d = 14.1 in.
k_{des} = 0.915 in.

Column
W12×65
t_f = 0.605 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(5\text{ kips}) + 1.6(15\text{ kips})$ $= 30.0\text{ kips}$	$R_a = 5\text{ kips} + 15\text{ kips}$ $= 20.0\text{ kips}$

Minimum Bearing Length

From AISC *Manual* Part 10, the minimum required bearing length, $l_{b\ min}$, is the length of bearing required for the limit states of web local yielding and web local crippling on the beam, but not less than k_{des}.

From AISC *Manual* Equations 9-46a or 9-46b and AISC *Manual* Table 9-4, the minimum required bearing length for web local yielding is:

LRFD	ASD
$l_{b\ min} = \dfrac{R_u - \phi R_1}{\phi R_2} \geq k_{des}$ $= \dfrac{30.0\text{ kips} - 35.5\text{ kips}}{15.5\text{ kip/in.}} \geq 0.915\text{ in.}$ This results in a negative quantity; therefore, $l_{b\ min} = k_{des} = 0.915\text{ in.}$	$l_{b\ min} = \dfrac{R_a - R_1/\Omega}{R_2/\Omega} \geq k_{des}$ $= \dfrac{20.0\text{ kips} - 23.6\text{ kips}}{10.3\text{ kip/in.}} \geq 0.915\text{ in.}$ This results in a negative quantity; therefore, $l_{b\ min} = k_{des} = 0.915\text{ in.}$

From AISC *Manual* Equations 9-48a or 9-48b and AISC *Manual* Table 9-4, the minimum required bearing length for web local crippling, assuming $l_b/d \leq 0.2$, is:

LRFD	ASD
$l_{b\ min} = \dfrac{R_u - \phi R_3}{\phi R_4} \geq k_{des}$ $= \dfrac{30.0\text{ kips} - 44.7\text{ kips}}{4.45\text{ kip/in.}} \geq 0.915\text{ in.}$ This results in a negative quantity; therefore, $l_{b\ min} = k_{des} = 0.915\text{ in.}$	$l_{b\ min} = \dfrac{R_a - R_3/\Omega}{R_4/\Omega} \geq k_{des}$ $= \dfrac{20.0\text{ kips} - 29.8\text{ kips}}{2.96\text{ kip/in.}} \geq 0.915\text{ in.}$ This results in a negative quantity; therefore, $l_{b\ min} = k_{des} = 0.915\text{ in.}$

Check assumption:

$$\frac{l_b}{d} = \frac{0.915\text{ in.}}{14.1\text{ in.}}$$
$$= 0.0649 < 0.2 \quad \textbf{o.k.}$$

Seat Angle and Welds

The required strength for the righthand weld can be determined by summing moments about the lefthand weld.

LRFD	ASD
$R_{uR} = \dfrac{(30.0\text{ kips})(3.00\text{ in.})}{3.50\text{ in.}}$ $= 25.7\text{ kips}$	$R_{aR} = \dfrac{(20.0\text{ kips})(3.00\text{ in.})}{3.50\text{ in.}}$ $= 17.1\text{ kips}$

Conservatively design the seat for twice the force in the more highly loaded weld. Therefore, design the seat for the following:

LRFD	ASD
$R_u = 2(25.7\text{ kips})$ $= 51.4\text{ kips}$	$R_a = 2(17.1\text{ kips})$ $= 34.2\text{ kips}$

Use a 6-in. angle length with a ⅝-in. thickness and a 3½-in. minimum outstanding leg and conservatively using $l_{b,\,req}$ = 15⁄16 in., from AISC *Manual* Table 10-6:

LRFD	ASD
$\phi R_n = 81.0\text{ kips} > 51.4\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 54.0\text{ kips} > 34.2\text{ kips}$ **o.k.**

Use an L7×4×⅝ (4-in. OSL), 6-in. long with 5⁄16-in. fillet welds. From AISC *Manual* Table 10-6, the weld available strength is:

LRFD	ASD
$\phi R_n = 53.4\text{ kips} > 51.4\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 35.6\text{ kips} > 34.2\text{ kips}$ **o.k.**

Use an L7×4×⅝×0 ft 6 in. for the seat angle. Use two ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) to connect the beam to the seat angle. Weld the angle to the column with 5⁄16-in. fillet welds.

Top Angle, Bolts and Welds

Use an L4×4×¼ with two ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) through the outstanding leg of the angle.

Use a 3⁄16-in. fillet weld along the toe of the angle to the column flange [maximum size permitted by AISC *Specification* Section J2.2b(b)(2)].

Conclusion

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-17A SINGLE-PLATE CONNECTION (CONVENTIONAL BEAM-TO-COLUMN FLANGE)

Given:

Verify a single-plate connection between an ASTM A992 W16×50 beam and an ASTM A992 W14×90 column flange, as shown in Figure II.A-17A-1, to support the following beam end reactions:

R_D = 8 kips
R_L = 25 kips

Use 70-ksi electrodes and an ASTM A36 plate.

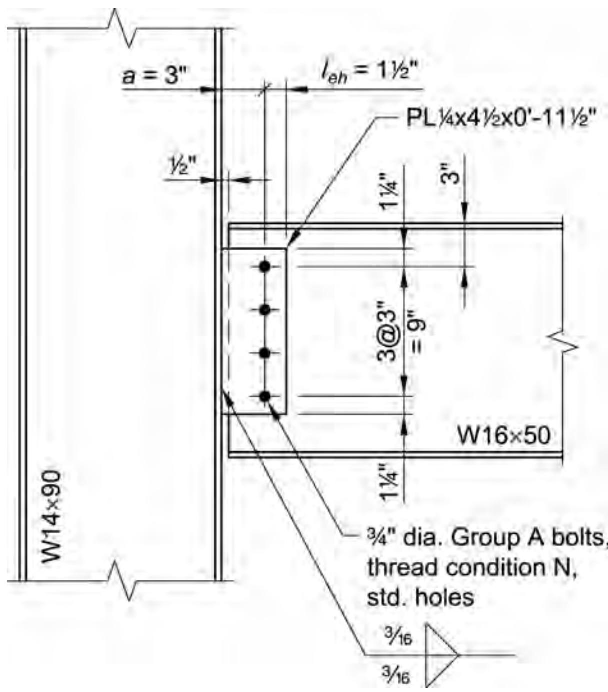


Fig. II.A-17A-1. Connection geometry for Example II.A-17A.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam and column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W16×50
t_w = 0.380 in.
d = 16.3 in.

Column
W14×90
t_f = 0.710 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(8 \text{ kips}) + 1.6(25 \text{ kips})$ $= 49.6 \text{ kips}$	$R_a = 8 \text{ kips} + 25 \text{ kips}$ $= 33.0 \text{ kips}$

Connection Selection

AISC *Manual* Table 10-10a includes checks for the limit states of bolt shear, bolt bearing on the plate, tearout on the plate, shear yielding of the plate, shear rupture of the plate, block shear rupture of the plate, and weld shear.

Use four rows of ¾-in.-diameter bolts in standard holes, ¼-in. plate thickness, and 3⁄16-in. fillet weld size. From AISC *Manual* Table 10-10a, the bolt, weld and single-plate available strength is:

LRFD	ASD
$\phi R_n = 52.2 \text{ kips} > 49.6 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 34.8 \text{ kips} > 33.0 \text{ kips}$ **o.k.**

Bolt Bearing and Tearout for Beam Web

Similar to the discussion in AISC *Manual* Part 10 for conventional, single-plate shear connections, the bearing and tearout are checked in accordance with AISC *Specification* Section J3.10, assuming the reaction is applied concentrically.

The available bearing and tearout strength of the beam web is determined using AISC *Manual* Table 7-4 with s = 3 in.

LRFD	ASD
$\phi R_n = (4 \text{ bolts})(87.8 \text{ kip/in.})(0.380 \text{ in.})$ $= 134 \text{ kips} > 49.6 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (4 \text{ bolts})(58.5 \text{ kip/in.})(0.380 \text{ in.})$ $= 88.9 \text{ kips} > 33.0 \text{ kips}$ **o.k.**

Note: To provide for stability during erection, it is recommended that the minimum plate length be one-half the T-dimension of the beam to be supported. AISC *Manual* Table 10-1 may be used as a reference to determine the recommended maximum and minimum connection lengths for a supported beam. Block shear rupture, shear yielding, and shear rupture will not control for an uncoped section.

Conclusion

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-17B SINGLE-PLATE CONNECTION SUBJECT TO AXIAL AND SHEAR LOADING (BEAM-TO-COLUMN FLANGE)

Given:

Verify the available strength of a single-plate connection for an ASTM A992 W18×50 beam connected to an ASTM A992 W14×90 column flange, as shown in Figure II.A-17B-1, to support the following beam end reactions:

LRFD	ASD
Shear, V_u = 75 kips Axial tension, N_u = 60 kips	Shear, V_a = 50 kips Axial tension, N_a = 40 kips

Use 70-ksi electrodes and an ASTM A572 Grade 50 plate.

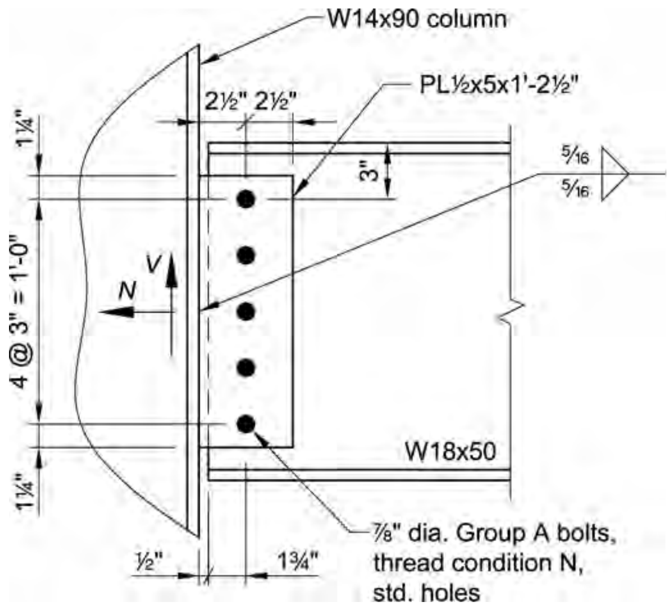


Fig. II.A-17B-1. Connection geometry for Example II.A-17B.

Solution:

From AISC *Manual* Table 2-4 and Table 2-5, the material properties are as follows:

Beam and column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A572 Grade 50
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×50
A_g = 14.7 in.^{2}
d = 18.0 in.

t_w = 0.355 in.
t_f = 0.570 in.

Column
W14×90
t_f = 0.710 in.

From AISC *Specification* Table J3.3, for ⅞-in.-diameter bolts with standard holes:

d_h = 15⁄16 in.

The resultant load is:

LRFD	ASD
$R_u = \sqrt{V_u^2 + N_u^2}$ $= \sqrt{(75\text{ kips})^2 + (60\text{ kips})^2}$ $= 96.0\text{ kips}$	$R_a = \sqrt{V_a^2 + N_a^2}$ $= \sqrt{(50\text{ kips})^2 + (40\text{ kips})^2}$ $= 64.0\text{ kips}$

The resultant load angle, measured from the vertical, is:

LRFD	ASD
$\theta = \tan^{-1}\left(\frac{60\text{ kips}}{75\text{ kips}}\right)$ $= 38.7°$	$\theta = \tan^{-1}\left(\frac{40\text{ kips}}{50\text{ kips}}\right)$ $= 38.7°$

Bolt Shear Strength

From AISC *Manual* Table 10-9, for single-plate shear connections with standard holes and n = 5:

$$e = \frac{a}{2} = \frac{2\frac{1}{2}\text{ in.}}{2} = 1.25\text{ in.}$$

The coefficient for eccentrically loaded bolts is determined by interpolating from AISC *Manual* Table 7-6 for Angle = 30°, n = 5 and e_x = 1.25 in. Note that 30° is used conservatively in order to employ AISC *Manual* Table 7-6. A direct analysis method can be performed to obtain a more precise value using the instantaneous center of rotation method.

C = 4.60

From AISC *Manual* Table 7-1, the available shear strength for a ⅞-in.-diameter Group A bolt with threads not excluded from the shear plane (thread condition N) is:

LRFD	ASD
ϕr_n = 24.3 kips/bolt	$\frac{r_n}{\Omega}$ = 16.2 kips/bolt

Bolt Bearing on the Beam Web

Note that bolt bearing and tearout of the beam web will control over bearing and tearout of the plate because the beam web is thinner and has less edge distance than the plate; therefore, those limit states will only be checked on the beam web.

The nominal bearing strength is determined using AISC *Specification* Equation J3-6b in lieu of Equation J3-6a, because plowing of the bolts in the beam web is desirable to provide some flexibility in the connection.

$$\begin{aligned} r_n &= 3.0dtF_u \\ &= 3.0(\tfrac{7}{8}\text{ in.})(0.355\text{ in.})(65\text{ ksi}) \\ &= 60.6\text{ kips/bolt} \end{aligned}$$

(*Spec*. Eq. J3-6b)

From AISC *Specification* Section J3.10, the available bearing strength of the beam per bolt is:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(60.6\text{ kips/bolt})$ $= 45.5\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{60.6\text{ kips/bolt}}{2.00}$ $= 30.3\text{ kips/bolt}$

Bolt Tearout on the Beam Web

The nominal tearout strength is determined using AISC *Specification* Equation J3-6d in lieu of Equation J3-6c, because plowing of the bolts in the beam web is desirable to provide some flexibility in the connection.

Because the direction of the load on the bolt is unknown, the minimum bolt edge distance is used to determine a worst case available tearout strength. The bolt edge distance for the web in the horizontal direction controls for this design. If a computer program is available, the true l_c can be calculated based on the instantaneous center of rotation. Therefore, for worst case edge distance in the beam web, and considering possible length underrun of ¼ in. on the beam length:

$$\begin{aligned} l_c &= l_{eh} - 0.5d_h - \text{underrun} \\ &= 1\tfrac{3}{4}\text{ in.} - 0.5(\tfrac{15}{16}\text{ in.}) - \tfrac{1}{4}\text{ in.} \\ &= 1.03\text{ in.} \end{aligned}$$

$$\begin{aligned} r_n &= 1.5l_ctF_u \\ &= 1.5(1.03\text{ in.})(0.355\text{ in.})(65\text{ ksi}) \\ &= 35.7\text{ kips/bolt} \end{aligned}$$

(*Spec*. Eq. J3-6d)

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(35.7\text{ kips})$ $= 26.8\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{35.7\text{ kips}}{2.00}$ $= 17.9\text{ kips/bolt}$

Strength of Bolted Connection

Bolt shear is the controlling limit state for all bolts at the connection to the beam web. The available strength of the connection is:

LRFD	ASD
$\phi R_n = C\phi r_n$ $= 4.60(24.3 \text{ kips/bolt})$ $= 112 \text{ kips} > 96.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{Cr_n}{\Omega}$ $= 4.60(16.2 \text{ kips/bolt})$ $= 74.5 \text{ kips} > 64.0 \text{ kips}$ **o.k.**

Strength of Weld

From AISC *Manual* Part 10, a weld size of (⅝)t_p is used to develop the strength of the shear plate, because, in general, the moment generated by this connection is indeterminate.

$$w = \tfrac{5}{8}t_p$$
$$= \tfrac{5}{8}(\tfrac{1}{2} \text{ in.})$$
$$= \tfrac{5}{16} \text{ in.}$$

Use a two-sided 5⁄16-in. fillet weld.

Shear Strength of Supporting Column Flange

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the column flange is determined as follows:

$$A_{nv} = (2 \text{ shear planes})lt_f$$
$$= (2 \text{ shear planes})(14.5 \text{ in.})(0.710 \text{ in.})$$
$$= 20.6 \text{ in.}^2$$

$$R_n = 0.60F_uA_{nv} \qquad (\textit{Spec}. \text{ Eq. J4-4})$$
$$= 0.60(65 \text{ ksi})(20.6 \text{ in.}^2)$$
$$= 803 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(803 \text{ kips})$ $= 602 \text{ kips} > 75 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{803 \text{ kips}}{2.00}$ $= 402 \text{ kips} > 50 \text{ kips}$ **o.k.**

The available shear yielding strength of the column flange need not be checked because $A_{nv} = A_{gv}$ and shear rupture will control.

Shear Yielding Strength of the Plate

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the plate is determined as follows:

$$A_{gv} = lt$$
$$= (14.5 \text{ in.})(\tfrac{1}{2} \text{ in.})$$
$$= 7.25 \text{ in.}^2$$

$R_n = 0.60F_y A_{gv}$ (*Spec*. Eq. J4-3)

$= 0.60(50 \text{ ksi})(7.25 \text{ in.}^2)$

$= 218 \text{ kips}$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(218 \text{ kips})$ $= 218 \text{ kips} > 75 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{218 \text{ kips}}{1.50}$ $= 145 \text{ kips} > 50 \text{ kips}$ **o.k.**

Tensile Yielding Strength of the Plate

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the plate is determined as follows:

$A_g = lt$

$= (14.5 \text{ in.})(\tfrac{1}{2} \text{ in.})$

$= 7.25 \text{ in.}^2$

$R_n = F_y A_g$ (*Spec*. Eq. J4-1)

$= (50 \text{ ksi})(7.25 \text{ in.}^2)$

$= 363 \text{ kips}$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(363 \text{ kips})$ $= 327 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{363 \text{ kips}}{1.67}$ $= 217 \text{ kips} > 40 \text{ kips}$ **o.k.**

Flexural Yielding of the Plate

The required flexural strength is calculated based upon the required shear strength and the eccentricity previously calculated:

LRFD	ASD
$M_u = V_u e$ $= (75 \text{ kips})(1.25 \text{ in.})$ $= 93.8 \text{ kip-in.}$	$M_a = V_a e$ $= (50 \text{ kips})(1.25 \text{ in.})$ $= 62.5 \text{ kip-in.}$

From AISC *Manual* Part 10, the plate buckling will not control for the conventional configuration. The flexural yielding strength is determined as follows:

$$Z_g = \frac{t_p l^2}{4}$$

$$= \frac{(\tfrac{1}{2} \text{ in.})(14.5 \text{ in.})^2}{4}$$

$$= 26.3 \text{ in.}^3$$

$$M_n = F_y Z_g$$
$$= (50 \text{ ksi})(26.3 \text{ in.}^3)$$
$$= 1{,}320 \text{ kip-in.}$$

LRFD	ASD
$\phi = 0.90$ $\phi M_n = 0.90(1{,}320 \text{ kip-in.})$ $= 1{,}190 \text{ kip-in.} > 93.8 \text{ kip-in.}$ **o.k.**	$\Omega = 1.67$ $\dfrac{M_n}{\Omega} = \dfrac{1{,}320 \text{ kip-in.}}{1.67}$ $= 790 \text{ kip-in.} > 62.5 \text{ kip-in.}$ **o.k.**

Interaction of Axial, Flexural and Shear Yielding in Plate

AISC *Specification* Chapter H does not address combined flexure and shear. The method employed here is derived from Chapter H in conjunction with AISC *Manual* Equation 10-5.

LRFD	ASD
$\dfrac{N_u}{\phi R_{np}} = \dfrac{60 \text{ kips}}{327 \text{ kips}}$ $= 0.183$ Because $\dfrac{N_u}{\phi R_{np}} < 0.2$: $\left(\dfrac{N_u}{2\phi R_{np}} + \dfrac{M_u}{\phi M_n}\right)^2 + \left(\dfrac{V_u}{\phi R_{nv}}\right)^2 \leq 1$ $\left[\dfrac{60 \text{ kips}}{2(327 \text{ kips})} + \dfrac{93.8 \text{ kip-in.}}{1{,}190 \text{ kip-in.}}\right]^2 + \left(\dfrac{75 \text{ kips}}{218 \text{ kips}}\right)^2 \leq 1$ $0.147 < 1$ **o.k.**	$\dfrac{N_a}{R_{np}/\Omega} = \dfrac{40 \text{ kips}}{217 \text{ kips}}$ $= 0.184$ Because $\dfrac{N_a}{R_{np}/\Omega} < 0.2$: $\left(\dfrac{\Omega N_a}{2R_{np}} + \dfrac{\Omega M_a}{M_n}\right)^2 + \left(\dfrac{\Omega V_a}{R_{nv}}\right)^2 \leq 1$ $\left[\dfrac{40 \text{ kips}}{2(217 \text{ kips})} + \dfrac{62.5 \text{ kip-in.}}{790 \text{ kip-in.}}\right]^2 + \left(\dfrac{50 \text{ kips}}{145 \text{ kips}}\right)^2 \leq 1$ $0.148 < 1$ **o.k.**

Shear Rupture Strength of the Plate

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the plate is determined as follows:

$$A_{nv} = \left[l - n(d_h + \tfrac{1}{16} \text{ in.})\right] t$$
$$= \left[14.5 \text{ in.} - 5(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})\right](\tfrac{1}{2} \text{ in.})$$
$$= 4.75 \text{ in.}^2$$

$$R_n = 0.60 F_u A_{nv} \qquad (\textit{Spec}. \text{ Eq. J4-4})$$
$$= 0.60(65 \text{ ksi})(4.75 \text{ in.}^2)$$
$$= 185 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(185 \text{ kips})$ $= 139 \text{ kips} > 75 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{185 \text{ kips}}{2.00}$ $= 92.5 \text{ kips} > 50 \text{ kips}$ **o.k.**

Tensile Rupture of the Plate

From AISC *Specification* Section J4.1(b), the available tensile rupture strength of the plate is determined as follows:

$$\begin{aligned} A_n &= \left[l - n\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right] t \\ &= \left[14.5\text{ in.} - 5\left(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]\left(\tfrac{1}{2}\text{ in.}\right) \\ &= 4.75\text{ in.}^2 \end{aligned}$$

Table D3.1, Case 1, applies in this case because the tension load is transmitted directly to the cross-sectional element by fasteners; therefore, $U = 1.0$.

$$\begin{aligned} A_e &= A_n U \\ &= \left(4.75\text{ in.}^2\right)(1.0) \\ &= 4.75\text{ in.}^2 \end{aligned}$$ (*Spec*. Eq. D3-1)

$$\begin{aligned} R_n &= F_u A_e \\ &= (65\text{ ksi})\left(4.75\text{ in.}^2\right) \\ &= 309\text{ kips} \end{aligned}$$ (*Spec*. Eq. J4-2)

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(309 \text{ kips})$ $= 232 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{309 \text{ kips}}{2.00}$ $= 155 \text{ kips} > 40 \text{ kips}$ **o.k.**

Flexural Rupture of the Plate

The available flexural rupture strength of the plate is determined as follows:

$$\begin{aligned} Z_{net} &= Z_g - \frac{t_p}{4}\left[\left(d_h + \tfrac{1}{16}\text{ in.}\right)(s)\left(n^2 - 1\right) + \left(d_h + \tfrac{1}{16}\text{ in.}\right)^2\right] \\ &= 26.3\text{ in.}^3 - \frac{\tfrac{1}{2}\text{ in.}}{4}\left[\left(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)(3.00\text{ in.})\left(5^2 - 1\right) + \left(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)^2\right] \\ &= 17.2\text{ in.}^3 \end{aligned}$$

$$\begin{aligned} M_n &= F_u Z_{net} \\ &= (65\text{ ksi})\left(17.2\text{ in.}^3\right) \\ &= 1{,}120\text{ kip-in.} \end{aligned}$$ (*Manual* Eq. 9-4)

LRFD	ASD
$\phi = 0.75$ $\phi M_n = 0.75(1{,}120 \text{ kip-in.})$ $= 840 \text{ kip-in.} > 93.8 \text{ kip-in.}$ **o.k.**	$\Omega = 2.00$ $\dfrac{M_n}{\Omega} = \dfrac{1{,}120 \text{ kip-in.}}{2.00}$ $= 560 \text{ kip-in.} > 62.5 \text{ kip-in.}$ **o.k.**

Interaction of Axial, Flexure and Shear Rupture in Plate

AISC *Specification* Chapter H does not address combined flexure and shear. The method employed here is derived from Chapter H in conjuction with AISC *Manual* Equation 10-5.

LRFD	ASD
$\dfrac{N_u}{\phi R_{np}} = \dfrac{60 \text{ kips}}{232 \text{ kips}}$ $= 0.259$ Because $\dfrac{N_u}{\phi R_{np}} > 0.2$: $\left(\dfrac{N_u}{\phi R_{np}} + \dfrac{8}{9}\dfrac{M_u}{\phi M_n}\right)^2 + \left(\dfrac{V_u}{\phi R_{nv}}\right)^2 \leq 1$ $\left[\dfrac{60 \text{ kips}}{232 \text{ kips}} + \dfrac{8}{9}\left(\dfrac{93.8 \text{ kip-in.}}{840 \text{ kip-in.}}\right)\right]^2 + \left(\dfrac{75 \text{ kips}}{139 \text{ kips}}\right)^2 \leq 1$ $0.419 < 1$ **o.k.**	$\dfrac{N_a}{R_{np}/\Omega} = \dfrac{40 \text{ kips}}{155 \text{ kips}}$ $= 0.258$ Because $\dfrac{N_a}{R_{np}/\Omega} > 0.2$: $\left(\dfrac{\Omega N_a}{R_{np}} + \dfrac{8}{9}\dfrac{\Omega M_a}{M_n}\right)^2 + \left(\dfrac{\Omega V_a}{R_{nv}}\right)^2 \leq 1$ $\left[\dfrac{40 \text{ kips}}{155 \text{ kips}} + \dfrac{8}{9}\left(\dfrac{62.5 \text{ kip-in.}}{560 \text{ kip-in.}}\right)\right]^2 + \left(\dfrac{50 \text{ kips}}{92.5 \text{ kips}}\right)^2 \leq 1$ $0.420 < 1$ **o.k.**

Block Shear Rupture Strength of the Plate—Beam Shear Direction

The nominal strength for the limit state of block shear rupture of the angles, assuming an L-shaped tearout due the shear load only, is determined as follows:

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \quad \text{(\textit{Spec}. Eq. J4-5)}$$

where

$$A_{gv} = (l - l_{ev})t$$
$$= [14.5 \text{ in.} - 1\tfrac{1}{4} \text{ in.}](\tfrac{1}{2} \text{ in.})$$
$$= 6.63 \text{ in.}^2$$

$$A_{nv} = A_{gv} - (n - 0.5)(d_h + \tfrac{1}{16} \text{ in.})t$$
$$= 6.63 \text{ in.}^2 - (5 - 0.5)(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{1}{2} \text{ in.})$$
$$= 4.38 \text{ in.}^2$$

$$A_{nt} = [l_{eh} - 0.5(d_h + \tfrac{1}{16} \text{ in.})]t$$
$$= [2\tfrac{1}{2} \text{ in.} - 0.5(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})](\tfrac{1}{2} \text{ in.})$$
$$= 1.00 \text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$R_n = 0.60(65 \text{ ksi})(4.38 \text{ in.}^2) + 1.0(65 \text{ ksi})(1.00 \text{ in.}^2) \leq 0.60(50 \text{ ksi})(6.63 \text{ in.}^2) + 1.0(65 \text{ ksi})(1.00 \text{ in.}^2)$$
$$= 236 \text{ kips} < 264 \text{ kips}$$

Therefore:

$$R_n = 236 \text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture on the plate is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(236 \text{ kips})$ $= 177 \text{ kips} > 75 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{236 \text{ kips}}{2.00}$ $= 118 \text{ kips} > 50 \text{ kips}$ **o.k.**

Block Shear Rupture Strength of the Plate—Beam Axial Direction

The plate block shear rupture failure path due to axial load only could occur as an L- or U-shape. Assuming an L-shaped failure path due to axial load only, the available block shear rupture strength of the plate is:

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \quad (Spec. \text{ Eq. J4-5})$$

where

$$A_{gv} = l_{eh} t$$
$$= (2\tfrac{1}{2} \text{ in.})(\tfrac{1}{2} \text{ in.})$$
$$= 1.25 \text{ in.}^2$$

$$A_{nv} = A_{gv} - 0.5(d_h + \tfrac{1}{16} \text{ in.})t$$
$$= 1.25 \text{ in.}^2 - 0.5(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{1}{2} \text{ in.})$$
$$= 1.00 \text{ in.}^2$$

$$A_{nt} = \left[l - l_{ev} - (n - 0.5)(d_h + \tfrac{1}{16} \text{ in.})\right]t$$
$$= \left[14.5 \text{ in.} - 1\tfrac{1}{4} \text{ in.} - (5 - 0.5)(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})\right](\tfrac{1}{2} \text{ in.})$$
$$= 4.38 \text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$R_n = 0.60(65 \text{ ksi})(1.00 \text{ in.}^2) + 1.0(65 \text{ ksi})(4.38 \text{ in.}^2) \leq 0.60(50 \text{ ksi})(1.25 \text{ in.}^2) + 1.0(65 \text{ ksi})(4.38 \text{ in.}^2)$$
$$= 324 \text{ kips} > 322 \text{ kips}$$

Therefore:

$$R_n = 322 \text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture on the plate is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(322\text{ kips})$ $= 242\text{ kips} > 60\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{322\text{ kips}}{2.00}$ $= 161\text{ kips} > 40\text{ kips}$ **o.k.**

Assuming a U-shaped failure path in the plate due to axial load, the available block shear rupture strength of the plate is:

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (Spec.\ \text{Eq. J4-5})$$

where

$$\begin{aligned} A_{gv} &= (2\text{ shear planes})l_{eh}t_p \\ &= (2\text{ shear planes})(2\tfrac{1}{2}\text{ in.})(\tfrac{1}{2}\text{ in.}) \\ &= 2.50\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nv} &= A_{gv} - (2\text{ shear planes})(0.5)(d_h + \tfrac{1}{16}\text{ in.})t \\ &= 2.50\text{ in.}^2 - (2\text{ shear planes})(0.5)(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{1}{2}\text{ in.}) \\ &= 2.00\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nt} &= \left[l - 2l_{ev} - (n-1)(d_h + \tfrac{1}{16}\text{ in.})\right]t \\ &= \left[14.5\text{ in.} - 2(1\tfrac{1}{4}\text{ in.}) - (5-1)(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](\tfrac{1}{2}\text{ in.}) \\ &= 4.00\text{ in.}^2 \end{aligned}$$

$$U_{bs} = 1.0$$

and

$$\begin{aligned} R_n &= 0.60(65\text{ ksi})(2.00\text{ in.}^2) + 1.0(65\text{ ksi})(4.00\text{ in.}^2) \le 0.60(50\text{ ksi})(2.50\text{ in.}^2) + 1.0(65\text{ ksi})(4.00\text{ in.}^2) \\ &= 338\text{ kips} > 335\text{ kips} \end{aligned}$$

Therefore:

$$R_n = 335\text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture on the plate is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(335\text{ kips})$ $= 251\text{ kips} > 60\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{335\text{ kips}}{2.00}$ $= 168\text{ kips} > 40\text{ kips}$ **o.k.**

The L-shaped failure path controls in the shear plate.

Check shear and tension interaction for plate block shear on the L-shaped failure plane:

LRFD	ASD
$\left(\dfrac{V_u}{\phi R_{nv}}\right)^2 + \left(\dfrac{N_u}{\phi R_{nt}}\right)^2 \leq 1$ $\left(\dfrac{75\text{ kips}}{177\text{ kips}}\right)^2 + \left(\dfrac{60\text{ kips}}{242\text{ kips}}\right)^2 = 0.241 < 1$ **o.k.**	$\left(\dfrac{V_a}{R_{nv}/\Omega}\right)^2 + \left(\dfrac{N_a}{R_{nt}/\Omega}\right)^2 \leq 1$ $\left(\dfrac{50\text{ kips}}{118\text{ kips}}\right)^2 + \left(\dfrac{40\text{ kips}}{161\text{ kips}}\right)^2 = 0.241 < 1$ **o.k.**

Shear Strength of the Beam Web

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the beam is determined as follows:

$$\begin{aligned} A_{gv} &= dt_w \\ &= (18.0\text{ in.})(0.355\text{ in.}) \\ &= 6.39\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_n &= 0.60F_yA_{gv} \\ &= 0.60(50\text{ ksi})(6.39\text{ in.}^2) \\ &= 192\text{ kips} \end{aligned}$$ (*Spec*. Eq. J4-3)

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(192\text{ kips})$ $= 192\text{ kips} > 75\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{192\text{ kips}}{1.50}$ $= 128\text{ kips} > 50\text{ kips}$ **o.k.**

The limit state of shear rupture of the beam web will not control in this example because the beam is uncoped.

Tensile Strength of the Beam

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the beam is determined as follows:

$$\begin{aligned} R_n &= F_yA_g \\ &= (50\text{ ksi})(14.7\text{ in.}^2) \\ &= 735\text{ kips} \end{aligned}$$ (*Spec*. Eq. J4-1)

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(735\text{ kips})$ $= 662\text{ kips} > 60\text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{735\text{ kips}}{1.67}$ $= 440\text{ kips} > 40\text{ kips}$ **o.k.**

From AISC *Specification* Sections J4.1, the available tensile rupture strength of the beam is determined from AISC *Specification* Equation J4-2. No cases in Table D3.1 apply to this configuration; therefore, U is determined in accordance with AISC *Specification* Section D3, where the minimum value of U is the ratio of the gross area of the connected element to the member gross area.

$$U = \frac{(d - 2t_f)t_w}{A_g}$$
$$= \frac{[18.0 \text{ in.} - 2(0.570 \text{ in.})](0.355 \text{ in.})}{14.7 \text{ in.}^2}$$
$$= 0.407$$

$$A_n = A_g - n(d_h + \tfrac{1}{16} \text{ in.})t_w$$
$$= 14.7 \text{ in.}^2 - 5(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(0.355 \text{ in.})$$
$$= 12.9 \text{ in.}$$

$$A_e = A_n U \qquad \text{(Spec. Eq. D3-1)}$$
$$= (12.9 \text{ in.}^2)(0.407)$$
$$= 5.25 \text{ in.}^2$$

$$R_n = F_u A_e \qquad \text{(Spec. Eq. J4-2)}$$
$$= (65 \text{ ksi})(5.25 \text{ in.}^2)$$
$$= 341 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(341 \text{ kips})$ $= 256 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{341 \text{ kips}}{2.00}$ $= 171 \text{ kips} > 40 \text{ kips}$ **o.k.**

Block Shear Rupture of the Beam Web

Block shear rupture is only applicable in the direction of the axial load, because the beam is uncoped and the limit state is not applicable for an uncoped beam subject to vertical shear. Assuming a U-shaped tearout relative to the axial load, and assuming a horizontal edge distance of l_{eh} = 1¾ in. – ¼ in. = 1½ in. to account for a possible beam underrun of ¼ in., the block shear rupture strength is:

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad \text{(Spec. Eq. J4-5)}$$

where

$$A_{gv} = (2 \text{ shear planes})l_{eh}t_w$$
$$= (2 \text{ shear planes})(1\tfrac{1}{2} \text{ in.})(0.355 \text{ in.})$$
$$= 1.07 \text{ in.}^2$$

$$A_{nv} = A_{gv} - (2 \text{ shear planes})(0.5)(d_h + \tfrac{1}{16} \text{ in.})t_w$$
$$= 1.07 \text{ in.}^2 - (2 \text{ shear planes})(0.5)(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(0.355 \text{ in.})$$
$$= 0.715 \text{ in.}^2$$

$$A_{nt} = \left[12.0\text{ in.} - (n-1)\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]t_w$$
$$= \left[12.0\text{ in.} - (5-1)\left(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right](0.355\text{ in.})$$
$$= 2.84\text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$R_n = 0.60(65\text{ ksi})\left(0.715\text{ in.}^2\right) + 1.0(65\text{ ksi})\left(2.84\text{ in.}^2\right) \le 0.60(50\text{ ksi})\left(1.07\text{ in.}^2\right) + 1.0(65\text{ ksi})\left(2.84\text{ in.}^2\right)$$
$$= 212\text{ kips} < 217\text{ kips}$$

Therefore:

$$R_n = 212\text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture of the beam web is:

LRFD	ASD
$\phi R_n = 0.75(212\text{ kips})$ $= 159\text{ kips} > 60\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{212\text{ kips}}{2.00}$ $= 106\text{ kips} > 40\text{ kips}$ **o.k.**

Conclusion

The connection is found to be adequate as given for the applied loads. Note that because the supported member was assumed to be continuously laterally braced, it is not necessary to check weak-axis moment.

EXAMPLE II.A-17C SINGLE-PLATE CONNECTION—STRUCTURAL INTEGRITY CHECK

Given:

Verify the single plate connection from Example II.A-17A, as shown in Figure II.A-17C-1, for the structural integrity provisions of AISC *Specification* Section B3.9. The connection is verified as a beam and girder end connection and as an end connection of a member bracing a column. Note that these checks are necessary when design for structural integrity is required by the applicable building code.

Use 70-ksi electrodes and an ASTM A36 plate.

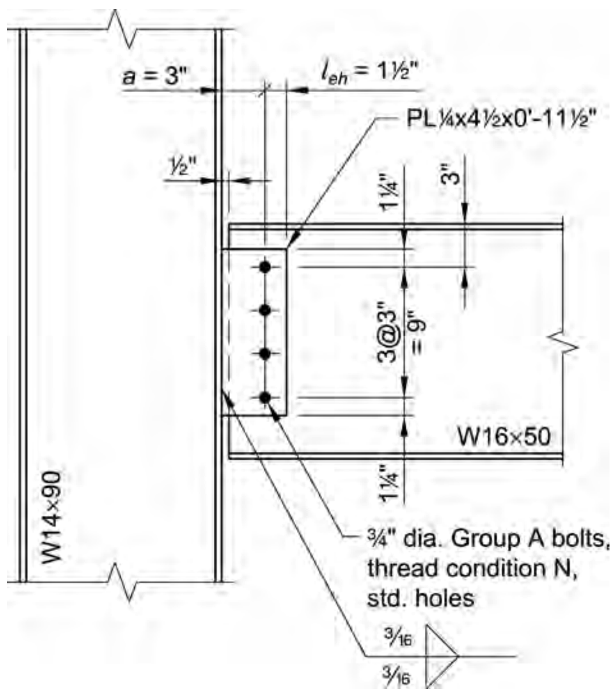


Fig. II.A-17C-1. Connection geometry for Example II.A-17C.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W16x50
$t_w = 0.380$ in.

From AISC *Specification* Table J3.3, the hole diameter for ¾-in.-diameter bolts with standard holes is:

$d_h = {}^{13}\!/_{16}$ in.

Beam and Girder End Connection

From Example II.A-17A, the required shear strength is:

LRFD	ASD
$V_u = 49.6$ kips	$V_a = 33.0$ kips

From AISC *Specification* Section B3.9(b), the required axial tensile strength is:

LRFD	ASD
$T_u = \frac{2}{3}V_u \geq 10$ kips $= \frac{2}{3}(49.6 \text{ kips}) > 10$ kips $= 33.1 \text{ kips} > 10$ kips $= 33.1$ kips	$T_a = V_a \geq 10$ kips $= 33.0 \text{ kips} > 10$ kips $= 33.0$ kips

Bolt Shear

From AISC Specification Section J3.6, the nominal bolt shear strength is:

$F_{nv} = 54$ ksi, from AISC *Specification* Table J3.2

$$T_n = nF_{nv}A_b \qquad \text{(from } Spec. \text{ Eq. J3-1)}$$
$$= (4 \text{ bolts})(54 \text{ ksi})(0.442 \text{ in.}^2)$$
$$= 95.5 \text{ kips}$$

Bolt Bearing and Tearout

From AISC *Specification* Section B3.9, for the purpose of satisfying structural integrity requirements inelastic deformations of the connection are permitted; therefore, AISC *Specification* Equations J3-6b and J3-6d are used to determine the nominal bearing and tearout strength. By inspection, bolt bearing and tearout will control for the plate. For bolt bearing on the plate:

$$T_n = (4 \text{ bolts})3.0dtF_u \qquad \text{(from } Spec. \text{ Eq. J3-6b)}$$
$$= (4 \text{ bolts})(3.0)(\tfrac{3}{4} \text{ in.})(\tfrac{1}{4} \text{ in.})(58 \text{ ksi})$$
$$= 131 \text{ kips}$$

For bolt tearout on the plate:

$$l_c = l_{eh} - 0.5\left(d_h + \tfrac{1}{16}\text{ in.}\right)$$
$$= 1\tfrac{1}{2}\text{ in.} - 0.5\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)$$
$$= 1.06\text{ in.}$$

$$T_n = (4\text{ bolts})1.5 l_c t F_u$$ (from *Spec.* Eq. J3-6d)
$$= (4\text{ bolts})(1.5)(1.06\text{ in.})(\tfrac{1}{4}\text{ in.})(58\text{ ksi})$$
$$= 92.2\text{ kips}$$

Tensile Yielding of Plate

From AISC *Specification* Section J4.1, the nominal tensile yielding strength of the shear plate is determined as follows:

$$A_g = lt$$
$$= (11.5\text{ in.})(\tfrac{1}{4}\text{ in.})$$
$$= 2.88\text{ in.}^2$$

$$T_n = F_y A_g$$ (from *Spec.* Eq. J4-1)
$$= (36\text{ ksi})\left(2.88\text{ in.}^2\right)$$
$$= 104\text{ kips}$$

Tensile Rupture of Plate

From AISC *Specification* Section J4.1, the nominal tensile rupture strength of the shear plate is determined as follows:

$$A_n = \left[l - n\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]t$$
$$= \left[11.5\text{ in.} - (4\text{ bolts})\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right](\tfrac{1}{4}\text{ in.})$$
$$= 2.00\text{ in.}^2$$

AISC *Specification* Table D3.1, Case 1 applies in this case because tension load is transmitted directly to the cross-section element by fasteners; therefore, $U = 1.0$.

$$A_e = A_n U$$ (*Spec.* Eq. D3-1)
$$= \left(2.00\text{ in.}^2\right)(1.0)$$
$$= 2.00\text{ in.}^2$$

$$T_n = F_u A_e$$ (from *Spec.* Eq. J4-2)
$$= (58\text{ ksi})\left(2.00\text{ in.}^2\right)$$
$$= 116\text{ kips}$$

Block Shear Rupture—Plate

From AISC *Specification* Section J4.3, the nominal block shear rupture strength, due to axial load, of the shear plate is determined as follows:

$$T_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs} F_u A_{nt}$$ (from *Spec.* Eq. J4-5)

where

$$A_{gv} = (2\text{ shear planes})l_{eh}t$$
$$= (2\text{ shear planes})(1\tfrac{1}{2}\text{ in.})(\tfrac{1}{4}\text{ in.})$$
$$= 0.750\text{ in.}^2$$

$$A_{nv} = (2\text{ shear planes})\left[l_{eh} - 0.5(d_h + \tfrac{1}{16}\text{ in.})\right]t_p$$
$$= (2\text{ shear planes})\left[1\tfrac{1}{2}\text{ in.} - 0.5(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](\tfrac{1}{4}\text{ in.})$$
$$= 0.531\text{ in.}^2$$

$$A_{nt} = \left[l - 2l_{ev} - (n-1)(d_h + \tfrac{1}{16}\text{ in.})\right]t$$
$$= \left[11.5\text{ in.} - 2(1\tfrac{1}{4}\text{ in.}) - (4-1)(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](\tfrac{1}{4}\text{ in.})$$
$$= 1.59\text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$T_n = 0.60(58\text{ ksi})(0.531\text{ in.}^2) + 1.0(58\text{ ksi})(1.59\text{ in.}^2) \leq 0.60(36\text{ ksi})(0.750\text{ in.}^2) + 1.0(58\text{ ksi})(1.59\text{ in.}^2)$$
$$= 111\text{ kips} > 108\text{ kips}$$
$$= 108\text{ kips}$$

Block Shear Rupture—Beam Web

From AISC *Specification* Section J4.3, the nominal block shear rupture strength, due to axial load, of the beam web is determined as follows (accounting for a possible ¼-in. beam underrun):

$$T_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs} F_u A_{nt}$$ (*Spec.* Eq. J4-5)

where

$$A_{gv} = (2\text{ shear planes})(l_{eh} - underrun)t_w$$
$$= (2\text{ shear planes})(2\tfrac{1}{2}\text{ in.} - \tfrac{1}{4}\text{ in.})(0.380\text{ in.})$$
$$= 1.71\text{ in.}^2$$

$$A_{nv} = (2\text{ shear planes})\left[l_{eh} - underrun - 0.5(d_h + \tfrac{1}{16}\text{ in.})\right]t_w$$
$$= (2\text{ shear planes})\left[2\tfrac{1}{2}\text{ in.} - \tfrac{1}{4}\text{ in.} - 0.5(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](0.380\text{ in.})$$
$$= 1.38\text{ in.}^2$$

$$A_{nt} = \left[9.00\text{ in.} - 3(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](0.380\text{ in.})$$
$$= 2.42\text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$T_n = 0.60(65 \text{ ksi})(1.38 \text{ in.}^2) + 1.0(65 \text{ ksi})(2.42 \text{ in.}^2) \leq 0.60(50 \text{ ksi})(1.71 \text{ in.}^2) + 1.0(65 \text{ ksi})(2.42 \text{ in.}^2)$$
$$= 211 \text{ kips} > 209 \text{ kips}$$
$$= 209 \text{ kips}$$

Weld Strength

From AISC *Specification* Section J2.4, the nominal tensile strength of the weld is determined as follows:

$$F_{nw} = 0.60F_{EXX}\left(1.0 + 0.50\sin^{1.5}\theta\right) \quad \text{(Spec. Eq. J2-5)}$$
$$= 0.60(70 \text{ ksi})\left(1.0 + 0.50\sin^{1.5}90°\right)$$
$$= 63.0 \text{ ksi}$$

The throat dimension is used to calculate the effective area of the fillet weld.

$$A_{we} = \frac{w}{\sqrt{2}}l(2 \text{ welds})$$
$$= \frac{3/16 \text{ in.}}{\sqrt{2}}(11.5 \text{ in.})(2 \text{ welds})$$
$$= 3.05 \text{ in.}^2$$

$$T_n = F_{nw}A_{we} \quad \text{(from Spec. Eq. J2-4)}$$
$$= (63.0 \text{ ksi})(3.05 \text{ in.}^2)$$
$$= 192 \text{ kips}$$

Nominal Tensile Strength

The controlling tensile strength, T_n, is the least of those previously calculated:

$$T_n = \min\{95.5 \text{ kips},\ 131 \text{ kips},\ 92.2 \text{ kips},\ 104 \text{ kips},\ 116 \text{ kips},\ 108 \text{ kips},\ 209 \text{ kips},\ 192 \text{ kips}\}$$
$$= 92.2 \text{ kips}$$

LRFD	ASD
$T_n = 92.2 \text{ kips} > 33.1 \text{ kips}$ **o.k.**	$T_n = 92.2 \text{ kips} > 33.0 \text{ kips}$ **o.k.**

Column Bracing

From AISC *Specification* Section B3.9(c), the minimum axial tension strength for the connection of a member bracing a column is equal to 1% of two-thirds of the required column axial strength for LRFD and equal to 1% of the required column axial for ASD. These requirements are evaluated independently from other strength requirements.

The maximum column axial force this connection is able to brace is determined as follows:

LRFD	ASD
$T_n \geq 0.01\left(\frac{2}{3}P_u\right)$	$T_n \geq 0.01P_a$

LRFD	ASD
Solving for the column axial force: $$P_u \le 100\left(\frac{3}{2}T_n\right)$$ $$= 100\left(\frac{3}{2}\right)(92.2 \text{ kips})$$ $$= 13{,}800 \text{ kips}$$	Solving for the column axial force: $$P_a \le 100T_n$$ $$= 100(92.2 \text{ kips})$$ $$= 9{,}220 \text{ kips}$$

As long as the required column axial strength is less than P_u = 13,800 kips or P_a = 9,220 kips, this connection is an adequate column brace.

EXAMPLE II.A-18 SINGLE-PLATE CONNECTION (BEAM-TO-GIRDER WEB)

Given:

Verify a single-plate connection between an ASTM A992 W18×35 beam and an ASTM A992 W21×62 girder web, as shown in Figure II.A-18-1, to support the following beam end reactions:

R_D = 6.5 kips
R_L = 20 kips

The top flange is coped 2 in. deep by 4 in. long, l_{ev} = 1½ in. Use 70-ksi electrodes and an ASTM A36 plate.

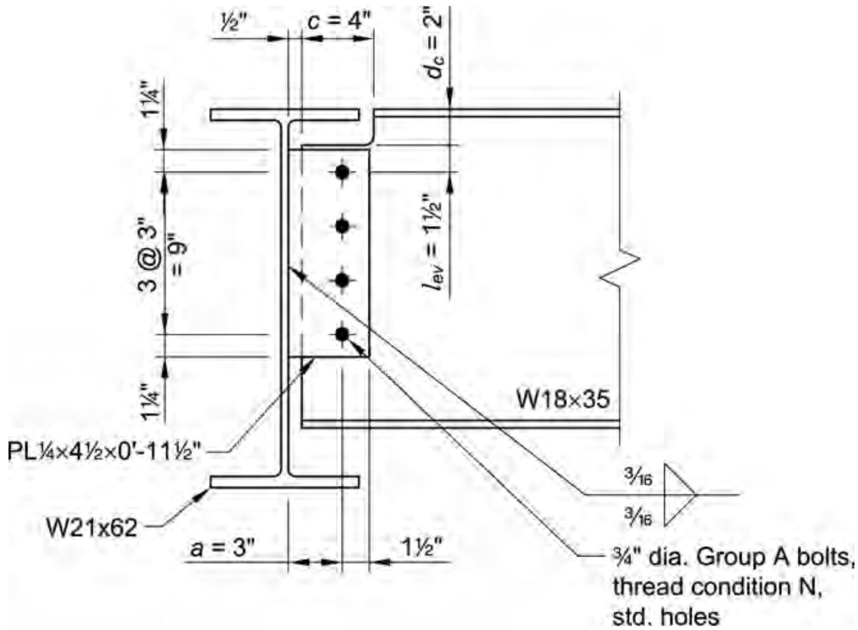


Fig. II.A-18-1. Connection geometry for Example II.A-18.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam and girder
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×35
t_w = 0.300 in.
d = 17.7 in.
t_f = 0.425 in.

Girder
W21×62
t_w = 0.400 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(6.5\text{ kips}) + 1.6(20\text{ kips})$ $= 39.8\text{ kips}$	$R_a = 6.5\text{ kips} + 20\text{ kips}$ $= 26.5\text{ kips}$

Connection Selection

AISC *Manual* Table 10-10a includes checks for the limit states of bolt shear, bolt bearing on the plate, tearout on the plate, shear yielding of the plate, shear rupture of the plate, block shear rupture of the plate and weld shear.

Use four rows of bolts, ¼-in. plate thickness, and 3⁄16-in. fillet weld size. From AISC *Manual* Table 10-10a:

LRFD	ASD
$\phi R_n = 52.2\text{ kips} > 39.8\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 34.8\text{ kips} > 26.5\text{ kips}$ **o.k.**

Block Shear Rupture of Beam Web

The nominal strength for the limit state of block shear rupture is given by AISC *Specification* Section J4.3.

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \leq 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \qquad (Spec.\text{ Eq. J4-5})$$

The available block shear rupture strength of the beam web is determined as follows, using AISC *Manual* Tables 9-3a, 9-3b and 9-3c and AISC *Specification* Equation J4-5, with n = 4, l_{eh} = 2¼ in. (reduced ¼ in. to account for beam underrun), l_{ev} = 1½ in. and U_{bs} = 1.0.

LRFD	ASD
Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{\phi F_uA_{nt}}{t} = 88.4\text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{\phi 0.60F_yA_{gv}}{t} = 236\text{ kip/in.}$	Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{F_uA_{nt}}{\Omega t} = 58.9\text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{0.60F_yA_{gv}}{\Omega t} = 158\text{ kip/in.}$

LRFD	ASD
Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{\phi 0.60F_u A_{nv}}{t} = 218\text{ kip/in.}$ The design block shear rupture strength is: $\phi R_n = \phi 0.60F_u A_{nv} + \phi U_{bs} F_u A_{nt} \le \phi 0.60F_y A_{gv} + \phi U_{bs} F_u A_{nt} = (218\text{ kip/in.} + 88.4\text{ kip/in.})(0.300\text{ in.}) \le (236\text{ kip/in.} + 88.4\text{ kip/in.})(0.300\text{ in.}) = 91.9\text{ kips} < 97.3\text{ kips}$ Therefore: $\phi R_n = 91.9\text{ kips} > 39.8\text{ kips}$ **o.k.**	Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{0.60F_u A_{nv}}{\Omega t} = 145\text{ kip/in.}$ The allowable block shear rupture strength is: $\dfrac{R_n}{\Omega} = \dfrac{0.60F_u A_{nv}}{\Omega} + \dfrac{U_{bs} F_u A_{nt}}{\Omega} \le \dfrac{0.60F_y A_{gv}}{\Omega} + \dfrac{U_{bs} F_u A_{nt}}{\Omega} = (145\text{ kip/in.} + 58.9\text{ kip/in.})(0.300\text{ in.}) \le (158\text{ kip/in.} + 58.9\text{ kip/in.})(0.300\text{ in.}) = 61.2\text{ kips} < 65.1\text{ kips}$ Therefore: $\dfrac{R_n}{\Omega} = 61.2\text{ kips} > 26.5\text{ kips}$ **o.k.**

Strength of the Bolted Connection—Beam Web

From the Commentary to AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the individual strengths of the individual fasteners, which may be taken as the lesser of the fastener shear strength per AISC *Specification* Section J3.6, the bearing strength at the bolt hole per AISC *Specification* Section J3.10, or the tearout strength at the bolt hole per AISC *Specification* Section J3.10.

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) is:

LRFD	ASD
$\phi r_n = 17.9\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = 11.9\text{ kips/bolt}$

The available bearing and tearout strength of the beam web edge bolt (top bolt shown in Figure II.A-18-1) is determined using AISC *Manual* Table 7-5, conservatively using l_e = 1¼ in.

LRFD	ASD
$\phi r_n = (49.4\text{ kip/in.})(0.300\text{ in.}) = 14.8\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (32.9\text{ kip/in.})(0.300\text{ in.}) = 9.87\text{ kips/bolt}$

The bearing or tearout strength controls over bolt shear for the edge bolt.

The available bearing and tearout strength of the beam web at the interior bolts is determined using AISC *Manual* Table 7-4 with s = 3 in.

LRFD	ASD
$\phi r_n = (87.8\text{ kip/in.})(0.300\text{ in.})$ $= 26.3\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (58.5\text{ kip/in.})(0.300\text{ in.})$ $= 17.6\text{ kips/bolt}$

Bolt shear strength controls for the interior bolts.

The strength of the bolt group in the beam web is determined by summing the strength of the individual fasteners as follows:

LRFD	ASD
$\phi R_n = (1\text{ bolt})(14.8\text{ kips/bolt})$ $+ (3\text{ bolts})(17.9\text{ kips/bolt})$ $= 68.5\text{ kips} > 39.8\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (1\text{ bolt})(9.87\text{ kips/bolt})$ $+ (3\text{ bolts})(11.9\text{ kips/bolt})$ $= 45.6\text{ kips} > 26.5\text{ kips}$ **o.k.**

Strength of the Bolted Connection—Single Plate

The available bearing and tearout strength of the plate at the edge bolt (bottom bolt shown in Figure II.A-18-1) is determined using AISC *Manual* Table 7-5 with l_e = 1¼ in.

LRFD	ASD
$\phi r_n = (44.0\text{ kip/in.})(\text{¼ in.})$ $= 11.0\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (29.4\text{ kip/in.})(\text{¼ in.})$ $= 7.35\text{ kips/bolt}$

The bearing or tearout strength controls over bolt shear for the edge bolt.

The available bearing and tearout strength of the plate at the interior bolts is determined using AISC *Manual* Table 7-4 with s = 3 in.

LRFD	ASD
$\phi r_n = (78.3\text{ kip/in.})(\text{¼ in.})$ $= 19.6\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (52.2\text{ kip/in.})(\text{¼ in.})$ $= 13.1\text{ kips/bolt}$

Bolt shear strength controls for the interior bolts.

The strength of the bolt group in the plate is determined by summing the strength of the individual fasteners as follows:

LRFD	ASD
$\phi R_n = (1\text{ bolt})(11.0\text{ kips/bolt})$ $+ (3\text{ bolts})(17.9\text{ kips/bolt})$ $= 64.7\text{ kips} > 39.8\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (1\text{ bolt})(7.35\text{ kips/bolt})$ $+ (3\text{ bolts})(11.9\text{ kips/bolt})$ $= 43.1\text{ kips} > 26.5\text{ kips}$ **o.k.**

Shear Rupture of the Girder Web at the Weld

The minimum support thickness to match the shear rupture strength of the weld is determined as follows:

$$t_{min} = \frac{3.09D}{F_u} \qquad \text{(Manual Eq. 9-2)}$$

$$= \frac{3.09(3 \text{ sixteenths})}{65 \text{ ksi}}$$

$$= 0.143 \text{ in.} < 0.400 \text{ in.} \quad \textbf{o.k.}$$

Note: For coped beam sections, the limit states of flexural yielding and local buckling should be checked independently per AISC *Manual* Part 9. The supported beam web should also be checked for shear yielding and shear rupture per AISC *Specification* Section J4.2. However, for the shallow cope in this example, these limit states do not govern. For an illustration of these checks, see Example II.A-4.

Conclusion

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-19A EXTENDED SINGLE-PLATE CONNECTION (BEAM-TO-COLUMN WEB)

Given:

Verify the connection between an ASTM A992 W16×36 beam and the web of an ASTM A992 W14×90 column, as shown in Figure II.A-19A-1, to support the following beam end reactions:

R_D = 6 kips
R_L = 18 kips

Use 70-ksi electrodes and ASTM A36 plate.

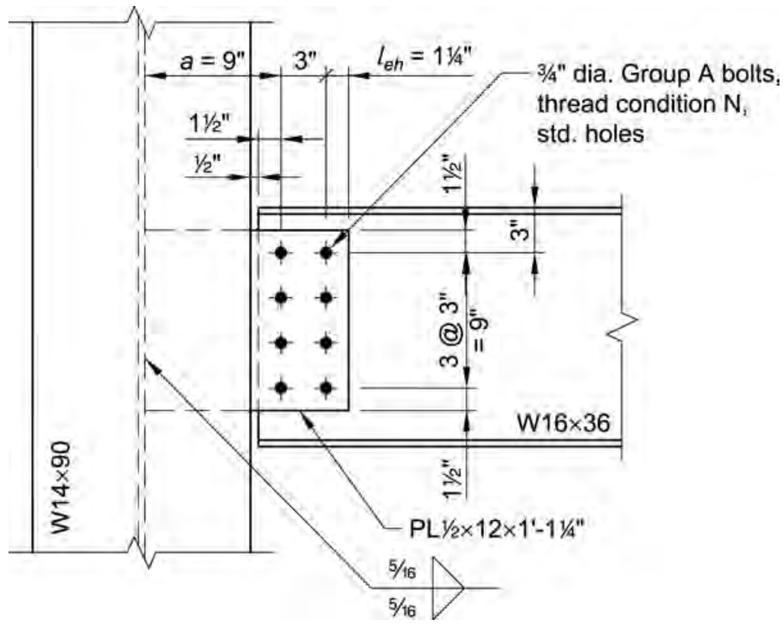


Fig. II.A-19A-1. Connection geometry for Example II.A-19A.

Note: All dimensional limitations are satisfied.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam and column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W16×36
t_w = 0.295 in.
d = 15.9 in.

Column
W14×90
t_w = 0.440 in.
b_f = 14.5 in.

From AISC *Specification* Table J3.3, the hole diameter for a ¾-in.-diameter bolt with standard holes is:

$d_h = {}^{13}\!/_{16}$ in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(6\text{ kips}) + 1.6(18\text{ kips})$ $= 36.0\text{ kips}$	$R_a = 6\text{ kips} + 18\text{ kips}$ $= 24.0\text{ kips}$

Strength of the Bolted Connection—Beam Web

From AISC *Manual* Part 10, determine the distance from the support to the first line of bolts and the distance to the center of gravity of the bolt group.

$$a = 9\text{ in.}$$

$$e = 9\text{ in.} + \frac{3\text{ in.}}{2}$$
$$= 10.5\text{ in.}$$

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) is:

LRFD	ASD
$\phi r_n = 17.9\text{ kips}$	$\dfrac{r_n}{\Omega} = 11.9\text{ kips}$

Tearout for the bolts in the beam web does not control due to the presence of the beam top flange.

The available bearing strength of the beam web per bolt is determined using AISC *Manual* Table 7-4 with s = 3 in.

LRFD	ASD
$\phi r_n = (87.8\text{ kip/in.})(0.295\text{ in.})$ $= 25.9\text{ kips}$	$\dfrac{r_n}{\Omega} = (58.5\text{ kip/in.})(0.295\text{ in.})$ $= 17.3\text{ kips}$

Therefore, bolt shear controls over bearing.

The strength of the bolt group is determined by interpolating AISC *Manual* Table 7-7, with e = 10.5 in. and Angle = 0°:

$$C = 2.33$$

LRFD	ASD
$\phi R_n = C\phi r_n$ $= 2.33(17.9 \text{ kips})$ $= 41.7 \text{ kips} > 36.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{Cr_n}{\Omega}$ $= 2.33(11.9 \text{ kips})$ $= 27.7 \text{ kips} > 24.0 \text{ kips}$ **o.k.**

Maximum Plate Thickness

From AISC *Manual* Part 10, determine the maximum plate thickness, t_{max}, that will result in the plate yielding before the bolts shear.

F_{nv} = 54 ksi from AISC *Specification* Table J3.2

C' = 26.0 in. from AISC *Manual* Table 7-7 for the moment-only case (Angle = 0°)

$$M_{max} = \frac{F_{nv}}{0.90}(A_b C') \qquad \text{(}\textit{Manual}\text{ Eq. 10-4)}$$

$$= \left(\frac{54 \text{ ksi}}{0.90}\right)(0.442 \text{ in.}^2)(26.0 \text{ in.})$$

$$= 690 \text{ kip-in.}$$

$$t_{max} = \frac{6M_{max}}{F_y l^2} \qquad \text{(}\textit{Manual}\text{ Eq. 10-3)}$$

$$= \frac{6(690 \text{ kip-in.})}{(36 \text{ ksi})(12.0 \text{ in.})^2}$$

$$= 0.799 \text{ in.}$$

Try a plate thickness of ½ in.

Strength of the Bolted Connection—Plate

The available bearing strength of the plate per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration, as follows:

$$r_n = 2.4dtF_u \qquad \text{(}\textit{Spec.}\text{ Eq. J3-6a)}$$

$$= 2.4(\tfrac{3}{4} \text{ in.})(\tfrac{1}{2} \text{ in.})(58 \text{ ksi})$$

$$= 52.2 \text{ kips/bolt}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(52.2 \text{ kips/bolt})$ $= 39.2 \text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{52.2 \text{ kips/bolt}}{2.00}$ $= 26.1 \text{ kips/bolt}$

The available tearout strength of the bottom edge bolt in the plate is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration, as follows:

$$l_c = l_{eh} - 0.5d_h$$
$$= 1\tfrac{1}{2}\text{ in.} - 0.5\left(\tfrac{13}{16}\text{ in.}\right)$$
$$= 1.09\text{ in.}$$

$$r_n = 1.2l_c tF_u$$ (*Spec*. Eq. J3-6c)
$$= 1.2\left(1.09\text{ in.}\right)\left(\tfrac{1}{2}\text{ in.}\right)\left(58\text{ ksi}\right)$$
$$= 37.9\text{ kips/bolt}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75\left(37.9\text{ kips/bolt}\right)$ $= 28.4\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{37.9\text{ kips/bolt}}{2.00}$ $= 19.0\text{ kips/bolt}$

Therefore, the bolt shear determined previously controls for the bolt group in the plate.

Shear Strength of Plate

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the plate is determined as follows:

$$A_{gv} = lt$$
$$= \left(12.0\text{ in.}\right)\left(\tfrac{1}{2}\text{ in.}\right)$$
$$= 6.00\text{ in.}^2$$

$$R_n = 0.60F_y A_{gv}$$ (*Spec*. Eq. J4-3)
$$= 0.60\left(36\text{ ksi}\right)\left(6.00\text{ in.}^2\right)$$
$$= 130\text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00\left(130\text{ kips}\right)$ $= 130\text{ kips} > 36.0\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{130\text{ kips}}{1.50}$ $= 86.7\text{ kips} > 24.0\text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the plate is determined using the net area determined in accordance with AISC *Specification* Section B4.3b.

$$A_{nv} = \left[l - n\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]t$$
$$= \left[12.0\text{ in.} - 4\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]\left(\tfrac{1}{2}\text{ in.}\right)$$
$$= 4.25\text{ in.}^2$$

$$R_n = 0.60F_u A_{nv}$$ (*Spec*. Eq. J4-4)
$$= 0.60\left(58\text{ ksi}\right)\left(4.25\text{ in.}^2\right)$$
$$= 148\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(148\text{ kips})$ $= 111\text{ kips} > 36.0\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{148\text{ kips}}{2.00}$ $= 74.0\text{ kips} > 24.0\text{ kips}$ **o.k.**

Block Shear Rupture of Plate

From AISC *Specification* Section J4.3, the block shear rupture strength of the plate is determined as follows.

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \qquad (\textit{Spec.}\text{ Eq. J4-5})$$

where

$$\begin{aligned}A_{gv} &= (l - l_{ev})t \\ &= (12.0\text{ in.} - 1\tfrac{1}{2}\text{ in.})(\tfrac{1}{2}\text{ in.}) \\ &= 5.25\text{ in.}^2\end{aligned}$$

$$\begin{aligned}A_{nv} &= A_{gv} - (n - 0.5)(d_h + \tfrac{1}{16}\text{ in.})t \\ &= 5.25\text{ in.}^2 - (4 - 0.5)(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{1}{2}\text{ in.}) \\ &= 3.72\text{ in.}^2\end{aligned}$$

$$\begin{aligned}A_{nt} &= \left[3\text{ in.} + 1\tfrac{1}{4}\text{ in.} - 1.5(d_h + \tfrac{1}{16}\text{ in.})\right]t \\ &= \left[3\text{ in.} + 1\tfrac{1}{4}\text{ in.} - 1.5(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](\tfrac{1}{2}\text{ in.}) \\ &= 1.47\text{ in.}^2\end{aligned}$$

$$U_{bs} = 0.5$$

and

$$\begin{aligned}R_n &= 0.60(58\text{ ksi})(3.72\text{ in.}^2) + 0.5(58\text{ ksi})(1.47\text{ in.}^2) < 0.60(36\text{ ksi})(5.25\text{ in.}^2) + 0.5(58\text{ ksi})(1.47\text{ in.}^2) \\ &= 172\text{ kips} > 156\text{ kips}\end{aligned}$$

Therefore:

$$R_n = 156\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(156\text{ kips})$ $= 117\text{ kips} > 36.0\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{156\text{ kips}}{2.00}$ $= 78.0\text{ kips} > 24.0\text{ kips}$ **o.k.**

Interaction of Shear Yielding and Flexural Yielding of Plate

From AISC *Manual* Part 10, the plate is checked for the interaction of shear yielding and yielding due to flexure as follows:

LRFD	ASD
$\left(\dfrac{V_r}{V_c}\right)^2+\left(\dfrac{M_r}{M_c}\right)^2\le 1.0$ (*Manual* Eq. 10-5)	$\left(\dfrac{V_r}{V_c}\right)^2+\left(\dfrac{M_r}{M_c}\right)^2\le 1.0$ (*Manual* Eq. 10-5)
From the preceding calculations:	From the preceding calculations:
$V_r = V_u$ $= 36.0$ kips	$V_r = V_a$ $= 24.0$ kips
$V_c = \phi_v V_n$ $= 130$ kips	$V_c = \dfrac{V_n}{\Omega_v}$ $= 86.7$ kips
From AISC *Manual* Part 10:	From AISC *Manual* Part 10:
$M_c = \phi_b M_n$ $= \phi_b F_y Z_{pl}$ $= 0.90(36\text{ ksi})\left[\dfrac{(\tfrac{1}{2}\text{ in.})(12\text{ in.})^2}{4}\right]$ $= 583$ kip-in.	$M_c = \dfrac{M_n}{\Omega_b}$ $= \dfrac{F_y Z_{pl}}{\Omega_b}$ $= \left(\dfrac{36\text{ ksi}}{1.67}\right)\left[\dfrac{(\tfrac{1}{2}\text{ in.})(12\text{ in.})^2}{4}\right]$ $= 388$ kip-in.
$M_r = M_u$ $= V_u a$ $= (36.0\text{ kips})(9\text{ in.})$ $= 324$ kip-in.	$M_r = M_a$ $= V_a a$ $= (24.0\text{ kips})(9\text{ in.})$ $= 216$ kip-in.
$\left(\dfrac{36.0\text{ kips}}{130\text{ kips}}\right)^2+\left(\dfrac{324\text{ kip-in.}}{583\text{ kip-in.}}\right)^2 = 0.386 < 1.0$ **o.k.**	$\left(\dfrac{24.0\text{ kips}}{86.7\text{ kips}}\right)^2+\left(\dfrac{216\text{ kip-in.}}{388\text{ kip-in.}}\right)^2 = 0.387 < 1.0$ **o.k.**

Lateral-Torsional Buckling of Plate

The plate is checked for the limit state of buckling using the double-coped beam procedure as given in AISC *Manual* Part 9, where the unbraced length for lateral-torsional buckling, L_b, is taken as the distance from the first column of bolts to the supporting column web and the top cope dimension, d_{ct}, is conservatively taken as the distance from the top of the beam to the first row of bolts.

$$C_b = \left[3+\ln\left(\frac{L_b}{d}\right)\right]\left(1-\frac{d_{ct}}{d}\right)\ge 1.84 \qquad (\textit{Manual}\text{ Eq. 9-15})$$

$$= \left[3+\ln\left(\frac{9\text{ in.}}{12\text{ in.}}\right)\right]\left(1-\frac{3\text{ in.}}{12\text{ in.}}\right)\ge 1.84$$

$$= 2.03 > 1.84$$

Therefore:

$$C_b = 2.03$$

From AISC *Specification* Section F11, the flexural strength of the plate for the limit state of lateral-torsional buckling is determined as follows:

$$\frac{L_b d}{t^2} = \frac{(9 \text{ in.})(12 \text{ in.})}{(\text{½ in.})^2}$$
$$= 432$$

$$\frac{0.08E}{F_y} = \frac{0.08(29{,}000 \text{ ksi})}{36 \text{ ksi}}$$
$$= 64.4$$

$$\frac{1.9E}{F_y} = \frac{1.9(29{,}000 \text{ ksi})}{36 \text{ ksi}}$$
$$= 1{,}530$$

Because $\frac{0.08E}{F_y} < \frac{L_b d}{t^2} \leq \frac{1.9E}{F_y}$, use AISC *Specification* Section F11.2(b):

$$M_p = F_y Z_x$$
$$= (36 \text{ ksi})\left[\frac{(\text{½ in.})(12 \text{ in.})^2}{4}\right]$$
$$= 648 \text{ kip-in.}$$

$$M_y = F_y S_x$$
$$= (36 \text{ ksi})\left[\frac{(\text{½ in.})(12 \text{ in.})^2}{6}\right]$$
$$= 432 \text{ kip-in.}$$

$$M_n = C_b\left[1.52 - 0.274\left(\frac{L_b d}{t^2}\right)\frac{F_y}{E}\right]M_y \leq M_p$$ (*Spec*. Eq. F11-2)

$$= (2.03)\left[1.52 - 0.274(432)\left(\frac{36 \text{ ksi}}{29{,}000 \text{ ksi}}\right)\right](432 \text{ kip-in.}) > 648 \text{ kip-in.}$$
$$= 1{,}200 \text{ kip-in.} > 648 \text{ kip-in.}$$

Therefore:

$$M_n = 648 \text{ kip-in.}$$

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(648 \text{ kip-in.})$ $= 583 \text{ kip-in.} > 324 \text{ kip-in.}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{648 \text{ kip-in}}{1.67}$ $= 388 \text{ kip-in.} > 216 \text{ kip-in.}$ **o.k.**

Flexural Rupture of Plate

The net plastic section modulus of the plate, Z_{net}, is determined from AISC *Manual* Table 15-3:

$$Z_{net} = 12.8 \text{ in.}^3$$

From AISC *Manual* Equation 9-4:

$$\begin{aligned} M_n &= F_u Z_{net} \\ &= (58 \text{ ksi})(12.8 \text{ in.}^3) \\ &= 742 \text{ kip-in.} \end{aligned}$$
(*Manual* Eq. 9-4)

LRFD	ASD
$\phi_b = 0.75$ $\phi_b M_n = 0.75(742 \text{ kip-in.})$ $= 557 \text{ kip-in.} > 324 \text{ kip-in.}$ **o.k.**	$\Omega_b = 2.00$ $\dfrac{M_n}{\Omega} = \dfrac{742 \text{ kip-in.}}{2.00}$ $= 371 \text{ kip-in.} > 216 \text{ kip-in.}$ **o.k.**

Weld Between Plate and Column Web (AISC Manual *Part 10)*

From AISC *Manual* Part 10, a weld size of (⅝)t_p is used to develop the strength of the shear plate.

$$\begin{aligned} w &= \tfrac{5}{8} t_p \\ &= \tfrac{5}{8}(\tfrac{1}{2} \text{ in.}) \\ &= \tfrac{5}{16} \text{ in.} \end{aligned}$$

Use a two-sided 5⁄16-in. fillet weld.

Strength of Column Web at Weld

The minimum column web thickness to match the shear rupture strength of the weld is determined as follows:

$$\begin{aligned} t_{min} &= \frac{3.09D}{F_u} \\ &= \frac{3.09(5 \text{ sixteenths})}{65 \text{ ksi}} \\ &= 0.238 \text{ in.} < 0.440 \text{ in.} \quad \textbf{o.k.} \end{aligned}$$
(*Manual* Eq. 9-2)

Conclusion

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.A-19B EXTENDED SINGLE-PLATE CONNECTION SUBJECT TO AXIAL AND SHEAR LOADING

Given:

Verify the available strength of an extended single-plate connection for an ASTM A992 W18×60 beam to the web of an ASTM A992 W14×90 column, as shown in Figure II.A-19B-1, to support the following beam end reactions:

LRFD	ASD
Shear, V_u = 75 kips Axial tension, N_u = 60 kips	Shear, V_a = 50 kips Axial tension, N_a = 40 kips

Use 70-ksi electrodes and ASTM A572 Grade 50 plate.

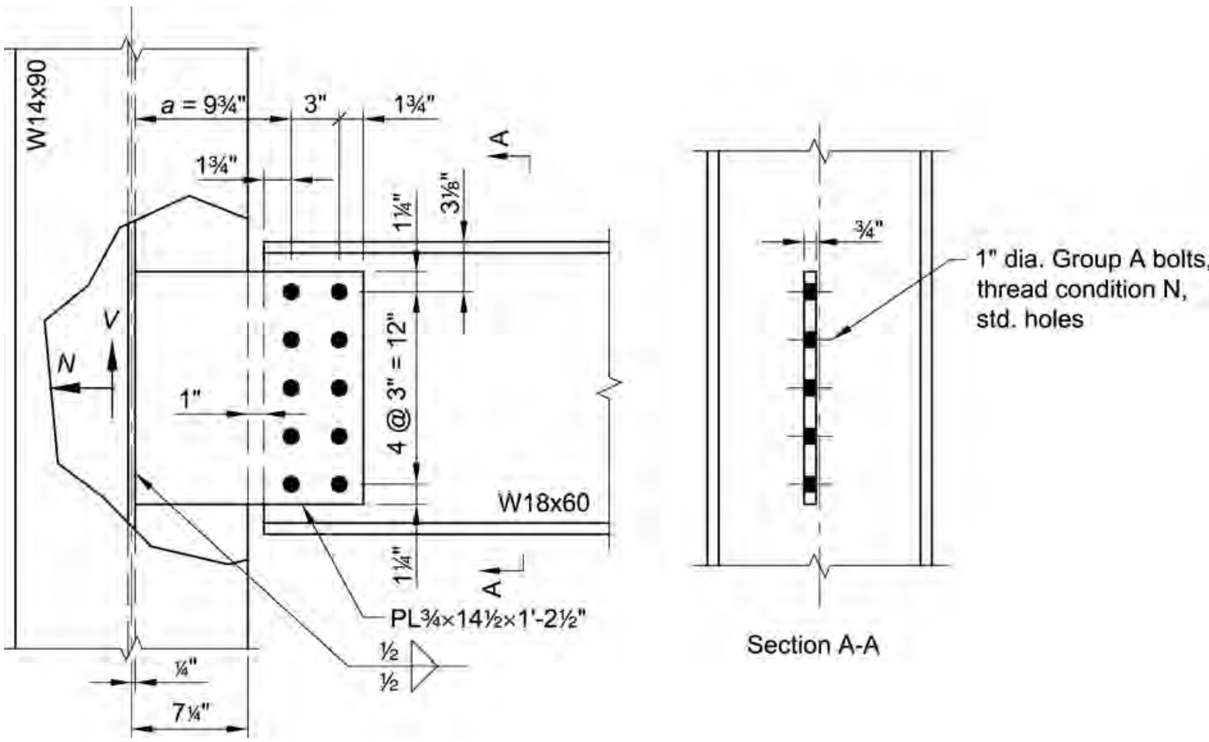


Fig. II.A-19B-1. Connection geometry for Example II.A-19B.

Solution:

From AISC *Manual* Table 2-4 and Table 2-5, the material properties are as follows:

Beam, column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A572 Grade 50
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×60
A_g = 17.6 in.^{2}
d = 18.2 in.
t_w = 0.415 in.
b_f = 7.56 in.
t_f = 0.695 in.

Column
W14×90
d = 14.0 in.
t_w = 0.440 in.
k_{des} = 1.31 in.

From AISC *Specification* Table J3.3, for 1-in.-diameter bolts with standard holes:

$d_h = 1\tfrac{1}{8}$ in.

Per AISC *Specification* Section J3.2, standard holes are required for both the plate and beam web because the beam axial force acts longitudinally to the direction of a slotted hole and bolts are designed for bearing.

The resultant load is determined as follows:

LRFD	ASD
$R_u = \sqrt{V_u^2 + N_u^2}$ $= \sqrt{(75\text{ kips})^2 + (60\text{ kips})^2}$ $= 96.0\text{ kips}$	$R_a = \sqrt{V_a^2 + N_a^2}$ $= \sqrt{(50\text{ kips})^2 + (40\text{ kips})^2}$ $= 64.0\text{ kips}$

The resultant load angle is determined as follows:

LRFD	ASD
$\theta = \tan^{-1}\left(\dfrac{60\text{ kips}}{75\text{ kips}}\right)$ $= 38.7°$	$\theta = \tan^{-1}\left(\dfrac{40\text{ kips}}{50\text{ kips}}\right)$ $= 38.7°$

Strength of Bolted Connection—Beam Web

The strength of the bolt group is determined by interpolating AISC *Manual* Table 7-7 for Angle = 30° and n = 5. Note that 30° is used conservatively in order to employ AISC *Manual* Table 7-7. A direct analysis can be performed to obtain an accurate value using the instantaneous center of rotation method.

$$\begin{aligned} e_x &= a + 0.5s \\ &= 9\tfrac{3}{4}\text{ in.} + 0.5(3\text{ in.}) \\ &= 11.3\text{ in.} \end{aligned}$$

C = 3.53 by interpolation

From AISC *Manual* Table 7-1, the available shear strength per bolt for 1-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) is:

LRFD	ASD
$\phi r_n = 31.8$ kips/bolt	$\dfrac{r_n}{\Omega} = 21.2$ kips/bolt

The available bearing strength of the beam web is determined from AISC *Specification* Equation J3-6b. This equation is applicable in lieu of Equation J3-6a, because plowing of the bolts in the beam web is desirable to provide some flexibility in the connection:

$$\begin{aligned} r_n &= 3.0 d t_w F_u \\ &= 3.0(1 \text{ in.})(0.415 \text{ in.})(65 \text{ ksi}) \\ &= 80.9 \text{ kips/bolt} \end{aligned}$$ (*Spec*. Eq. J3-6b)

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(80.9 \text{ kips/bolt})$ $= 60.7$ kips/bolt	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{80.9 \text{ kips/bolt}}{2.00}$ $= 40.5$ kips/bolt

The available tearout strength of the beam web is determined from *Specification* Equation J3-6d. Similar to the bearing strength determination, this equation is used to allow plowing of the bolts in the beam web, and thus provide some flexibility in the connection.

Because the direction of load on the bolt is unknown, the minimum bolt edge distance is used to determine a worst case available tearout strength (including a ¼-in. tolerance to account for possible beam underrun). If a computer program is available, the true l_e can be calculated based on the instantaneous center of rotation.

$$\begin{aligned} l_c &= l_{eh} - 0.5 d_h \\ &= (1\tfrac{3}{4} \text{ in.} - \tfrac{1}{4} \text{ in.}) - 0.5(1\tfrac{1}{8} \text{ in.}) \\ &= 0.938 \text{ in.} \end{aligned}$$

$$\begin{aligned} r_n &= 1.5 l_c t_w F_u \\ &= 1.5(0.938 \text{ in.})(0.415 \text{ in.})(65 \text{ ksi}) \\ &= 38.0 \text{ kips/bolt} \end{aligned}$$ (*Spec*. Eq. J3-6d)

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(38.0 \text{ kips/bolt})$ $= 28.5$ kips/bolt	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{38.0 \text{ kips/bolt}}{2.00}$ $= 19.0$ kips/bolt

The tearout strength controls for bolts in the beam web.

The available strength of the bolted connection is determined using the minimum available strength calculated for bolt shear, bearing on the beam web and tearout on the beam web. From AISC *Manual* Equation 7-16, the bolt group eccentricity is accounted for by multiplying the minimum available bolt strength by the bolt coefficient *C*.

LRFD	ASD
$\phi R_n = C\phi r_n$ $= 3.53(28.5\text{ kips/bolt})$ $= 101\text{ kips} > 96.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = C\dfrac{r_n}{\Omega}$ $= 3.53(19.0\text{ kips/bolt})$ $= 67.1\text{ kips} > 64.0\text{ kips}$ **o.k.**

Strength of Bolted Connection—Plate

Note that bolt bearing on the beam web controls over bearing on the plate because the beam web is thinner than the plate; therefore, this limit state will not control.

As was discussed for the beam web, the available tearout strength of the plate is determined from *Specification* Equation J3-6d. The bolt edge distance in the vertical direction controls for this design.

$$\begin{aligned} l_c &= l_{ev} - 0.5d_h \\ &= 1\tfrac{1}{4}\text{ in.} - 0.5(1\tfrac{1}{8}\text{ in.}) \\ &= 0.688\text{ in.} \end{aligned}$$

$$\begin{aligned} r_n &= 1.5l_c tF_u \\ &= 1.5(0.688\text{ in.})(\tfrac{3}{4}\text{ in.})(65\text{ ksi}) \\ &= 50.3\text{ kips/bolt} \end{aligned}$$ (*Spec*. Eq. J3-6d)

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(50.3\text{ kips/bolt})$ $= 37.7\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{50.3\text{ kips/bolt}}{2.00}$ $= 25.2\text{ kips/bolt}$

Therefore, the available strength of the bolted connection at the beam web, as determined previously, controls.

Shear Yielding Strength of Beam

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the beam is determined as follows:

$$\begin{aligned} A_{gv} &= dt_w \\ &= (18.2\text{ in.})(0.415\text{ in.}) \\ &= 7.55\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_n &= 0.60F_y A_{gv} \\ &= 0.60(50\text{ ksi})(7.55\text{ in.}^2) \\ &= 227\text{ kips} \end{aligned}$$ (*Spec*. Eq. J4-3)

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(227\text{ kips})$ $= 227\text{ kips} > 75\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{227\text{ kips}}{1.50}$ $= 151\text{ kips} > 50\text{ kips}$ **o.k.**

Tensile Yielding Strength of Beam

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the beam web is determined as follows:

$$R_n = F_y A_g \tag{Spec. Eq. J4-1}$$
$$= (50 \text{ ksi})(17.6 \text{ in.}^2)$$
$$= 880 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(880 \text{ kips})$ $= 792 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\frac{R_n}{\Omega} = \frac{880 \text{ kips}}{1.67}$ $= 527 \text{ kips} > 40 \text{ kips}$ **o.k.**

Tensile Rupture Strength of Beam

From AISC *Specification* Section J4.1, determine the available tensile rupture strength of the beam. The effective net area is $A_e = A_nU$, where U is determined from AISC *Specification* Table D3.1, Case 2.

$$\bar{x} = \frac{2b_f{}^2t_f + t_w{}^2(d - 2t_f)}{8b_ft_f + 4t_w(d - 2t_f)}$$
$$= \frac{2(7.56 \text{ in.})^2(0.695 \text{ in.}) + (0.415 \text{ in.})^2[18.2 \text{ in.} - 2(0.695 \text{ in.})]}{8(7.56 \text{ in.})(0.695 \text{ in.}) + 4(0.415 \text{ in.})[18.2 \text{ in.} - 2(0.695 \text{ in.})]}$$
$$= 1.18 \text{ in.}$$

$$U = 1 - \frac{\bar{x}}{l}$$
$$= 1 - \frac{1.18 \text{ in.}}{3.00 \text{ in.}}$$
$$= 0.607$$

$$A_n = A_g - n(d_h + 1/16 \text{ in.})t_w$$
$$= 17.6 \text{ in.}^2 - 5(1\tfrac{1}{8} \text{ in.} + 1/16 \text{ in.})(0.415 \text{ in.})$$
$$= 15.1 \text{ in.}^2$$

$$R_n = F_u A_e \tag{Spec. Eq. J4-2}$$
$$= F_u A_n U$$
$$= (65 \text{ ksi})(15.1 \text{ in.}^2)(0.607)$$
$$= 596 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(596 \text{ kips})$ $= 447 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{596 \text{ kips}}{2.00}$ $= 298 \text{ kips} > 40 \text{ kips}$ **o.k.**

Block Shear Rupture of Beam Web

Block shear rupture is only applicable in the direction of the axial load because the beam is uncoped and the limit state is not applicable for an uncoped beam subject to vertical shear. Assuming a U-shaped tearout relative to the axial load, and assuming a horizontal edge distance of l_{eh} = 1¾ in. − ¼ in. = 1½ in. to account for a possible beam underrun of ¼ in., the block shear rupture strength is:

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad \text{(\textit{Spec.} Eq. J4-5)}$$

where

$$A_{gv} = (2 \text{ shear planes})(s + l_{eh})t_w$$
$$= (2 \text{ shear planes})(3 \text{ in.} + 1\tfrac{1}{2} \text{ in.})(0.415 \text{ in.})$$
$$= 3.74 \text{ in.}^2$$

$$A_{nv} = A_{gv} - (2 \text{ shear planes})(1.5)(d_h + \tfrac{1}{16} \text{ in.})t_w$$
$$= 3.74 \text{ in.}^2 - (2 \text{ shear planes})(1.5)(1\tfrac{1}{8} \text{ in.} + \tfrac{1}{16} \text{ in.})(0.415 \text{ in.})$$
$$= 2.26 \text{ in.}^2$$

$$A_{nt} = [12.0 \text{ in.} - (n-1)(d_h + \tfrac{1}{16} \text{ in.})]t_w$$
$$= [12.0 \text{ in.} - (5-1)(1\tfrac{1}{8} \text{ in.} + \tfrac{1}{16} \text{ in.})](0.415 \text{ in.})$$
$$= 3.01 \text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$R_n = 0.60(65 \text{ ksi})(2.26 \text{ in.}^2) + 1.0(65 \text{ ksi})(3.01 \text{ in.}^2) \leq 0.60(50 \text{ ksi})(3.74 \text{ in.}^2) + 1.0(65 \text{ ksi})(3.01 \text{ in.}^2)$$
$$= 284 \text{ kips} < 308 \text{ kips}$$

Therefore:

$$R_n = 284 \text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture of the beam web is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(284 \text{ kips})$ $= 213 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{284 \text{ kips}}{2.00}$ $= 142 \text{ kips} > 40 \text{ kips}$ **o.k.**

Maximum Plate Thickness

Determine the maximum plate thickness, t_{max}, that will result in the plate yielding before the bolts shear. From AISC *Specification* Table J3.2:

$F_{nv} = 54$ ksi

From AISC *Manual* Table 7-7 for two column of bolts, Angle = 0°, s = 3 in., and n = 5:

$C' = 38.7$ in.

$$M_{max} = \frac{F_{nv}}{0.90}\left(A_b C'\right) \qquad \text{(\textit{Manual} Eq. 10-4)}$$
$$= \frac{54 \text{ ksi}}{0.90}\left(0.785 \text{ in.}^2\right)\left(38.7 \text{ in.}\right)$$
$$= 1{,}820 \text{ kip-in.}$$

$$t_{max} = \frac{6M_{max}}{F_y l^2} \qquad \text{(\textit{Manual} Eq. 10-3)}$$
$$= \frac{6\left(1{,}820 \text{ kip-in.}\right)}{\left(50 \text{ ksi}\right)\left(14\tfrac{1}{2} \text{ in.}\right)^2}$$
$$= 1.04 \text{ in.} > \tfrac{3}{4} \text{ in.} \quad \textbf{o.k.}$$

Flexure Strength of Plate

The required flexural strength of the plate is determined as follows:

LRFD	ASD
$M_u = V_u a$ $= (75 \text{ kips})(9\tfrac{3}{4} \text{ in.})$ $= 731$ kip-in.	$M_a = V_a a$ $= (50 \text{ kips})(9\tfrac{3}{4} \text{ in.})$ $= 488$ kip-in.

The plate is checked for the limit state of buckling using the double-coped beam procedure as given in AISC *Manual* Part 9, where the unbraced length for lateral-torsional buckling, L_b, is taken as the distance from the first column of bolts to the supporting column web and the top cope dimension, d_{ct}, is conservatively taken as the distance from the top of the beam to the first row of bolts.

$$C_b = \left[3 + \ln\left(\frac{L_b}{d}\right)\right]\left(1 - \frac{d_{ct}}{d}\right) \geq 1.84$$
$$= \left[3 + \ln\left(\frac{9\tfrac{3}{4} \text{ in.}}{14\tfrac{1}{2} \text{ in.}}\right)\right]\left(1 - \frac{3\tfrac{1}{8} \text{ in.}}{14\tfrac{1}{2} \text{ in.}}\right) \geq 1.84$$
$$= 2.04 > 1.84$$

Therefore:

$C_b = 2.04$

The available flexural strength of the plate is determined using AISC *Specification* Section F11 as follows:

For yielding of the plate:

$$M_n = M_p = F_y Z \le 1.6 F_y S_x \qquad (\textit{Spec}. \text{ Eq. F11-1})$$

$$= (50 \text{ ksi})\left[\frac{(\tfrac{3}{4} \text{ in.})(14\tfrac{1}{2} \text{ in.})^2}{4}\right] \le 1.6(50 \text{ ksi})\left[\frac{(\tfrac{3}{4} \text{ in.})(14\tfrac{1}{2} \text{ in.})^2}{6}\right]$$

$$= 1{,}970 \text{ kip-in.} < 2{,}100 \text{ kip-in.}$$

$$= 1{,}970 \text{ kip-in.}$$

For lateral-torsional buckling of the plate:

$$\frac{L_b d}{t^2} = \frac{(9\tfrac{3}{4} \text{ in.})(14\tfrac{1}{2} \text{ in.})}{(\tfrac{3}{4} \text{ in.})^2}$$

$$= 251$$

$$\frac{0.08E}{F_y} = \frac{0.08(29{,}000 \text{ ksi})}{50 \text{ ksi}}$$

$$= 46.4$$

$$\frac{1.9E}{F_y} = \frac{1.9(29{,}000 \text{ ksi})}{50 \text{ ksi}}$$

$$= 1{,}100$$

Because $\frac{0.08E}{F_y} < \frac{L_b d}{t^2} \le \frac{1.9E}{F_y}$, use AISC *Specification* Section F11.2(b):

$$M_y = F_y S_x$$

$$= (50 \text{ ksi})\left[\frac{(\tfrac{3}{4} \text{ in.})(14\tfrac{1}{2} \text{ in.})^2}{6}\right]$$

$$= 1{,}310 \text{ kip-in.}$$

$$M_n = C_b\left[1.52 - 0.274\left(\frac{L_b d}{t^2}\right)\frac{F_y}{E}\right]M_y \le M_p \qquad (\textit{Spec}. \text{ Eq. F11-2})$$

$$= 2.04\left[1.52 - 0.274(251)\left(\frac{50 \text{ ksi}}{29{,}000 \text{ ksi}}\right)\right](1{,}310 \text{ kip-in.}) \le 1{,}970 \text{ kip-in.}$$

$$= 3{,}750 \text{ kip-in.} > 1{,}970 \text{ kip-in.}$$

Therefore:

$$M_n = 1{,}970 \text{ kip-in.}$$

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(1{,}970 \text{ kip-in.})$ $= 1{,}770 \text{ kip-in.} > 731 \text{ kip-in.}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{1{,}970 \text{ kip-in.}}{1.67}$ $= 1{,}180 \text{ kip-in.} > 488 \text{ kip-in.}$ **o.k.**

Shear Yielding Strength of Plate

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the plate is determined as follows:

$$\begin{aligned} A_{gv} &= lt \\ &= (14\tfrac{1}{2} \text{ in.})(\tfrac{3}{4} \text{ in.}) \\ &= 10.9 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_{nv} &= 0.60F_y A_{gv} \\ &= 0.60(50 \text{ ksi})(10.9 \text{ in.}^2) \\ &= 327 \text{ kips} \end{aligned}$$
(*Spec*. Eq. J4-3)

LRFD	ASD
$\phi = 1.00$ $\phi R_{nv} = 1.00(327 \text{ kips})$ $= 327 \text{ kips} > 75 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_{nv}}{\Omega} = \dfrac{327 \text{ kips}}{1.50}$ $= 218 \text{ kips} > 50 \text{ kips}$ **o.k.**

Tension Yielding Strength of Plate

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the plate is determined as follows:

$$\begin{aligned} A_g &= lt \\ &= (14\tfrac{1}{2} \text{ in.})(\tfrac{3}{4} \text{ in.}) \\ &= 10.9 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_{np} &= F_y A_g \\ &= (50 \text{ ksi})(10.9 \text{ in.}) \\ &= 545 \text{ kips} \end{aligned}$$
(from *Spec*. Eq. J4-1)

LRFD	ASD
$\phi = 0.90$ $\phi R_{np} = 0.90(545 \text{ kips})$ $= 491 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_{np}}{\Omega} = \dfrac{545 \text{ kips}}{1.67}$ $= 326 \text{ kips} > 40 \text{ kips}$ **o.k.**

Interaction of Axial, Flexure and Shear Yielding in Plate

AISC *Specification* Chapter H does not address combined flexure and shear. The method employed here is derived from Chapter H in conjunction with AISC *Manual* Equation 10-5, as follows:

LRFD	ASD
$\dfrac{N_u}{\phi R_{np}} = \dfrac{60\text{ kips}}{491\text{ kips}}$ $= 0.122$ Because $\dfrac{N_u}{\phi R_{np}} < 0.2$: $\left(\dfrac{N_u}{2\phi R_{np}} + \dfrac{V_u a}{\phi M_n}\right)^2 + \left(\dfrac{V_u}{\phi R_{nv}}\right)^2 \leq 1$ $= \left[\dfrac{60\text{ kips}}{2(491\text{ kips})} + \dfrac{(75\text{ kips})(9\tfrac{3}{4}\text{ in.})}{1{,}770\text{ kip-in.}}\right]^2$ $+ \left(\dfrac{75\text{ kips}}{327\text{ kips}}\right)^2 \leq 1$ $= 0.278 < 1$ **o.k.**	$\dfrac{\Omega N_a}{R_{np}} = \dfrac{40\text{ kips}}{326\text{ kips}}$ $= 0.123$ Because $\dfrac{\Omega N_a}{R_{np}} < 0.2$: $\left(\dfrac{\Omega N_a}{2R_{np}} + \dfrac{\Omega V_a a}{M_n}\right)^2 + \left(\dfrac{\Omega V_a}{R_{nv}}\right)^2 \leq 1$ $= \left[\dfrac{40\text{ kips}}{2(326\text{ kips})} + \dfrac{(50\text{ kips})(9\tfrac{3}{4}\text{ in.})}{1{,}180\text{ kip-in.}}\right]^2$ $+ \left(\dfrac{50\text{ kips}}{218\text{ kips}}\right)^2 \leq 1$ $= 0.278 < 1$ **o.k.**

Tensile Rupture Strength of Plate

From AISC *Specification* Section J4.1(b), the available tensile rupture strength of the plate is determined as follows:

$$\begin{aligned} A_n &= \left[l - n\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]t \\ &= \left[14\tfrac{1}{2}\text{ in.} - (5\text{ bolts})\left(1\tfrac{1}{8}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]\left(\tfrac{3}{4}\text{ in.}\right) \\ &= 6.42\text{ in.}^2 \end{aligned}$$

AISC *Specification* Table D3.1, Case 1, applies in this case because the tension load is transmitted directly to the cross-sectional element by fasteners; therefore, $U = 1.0$.

$$\begin{aligned} A_e &= A_n U \\ &= \left(6.42\text{ in.}^2\right)(1.0) \\ &= 6.42\text{ in.}^2 \end{aligned} \qquad (\textit{Spec.}\text{ Eq. D3-1})$$

$$\begin{aligned} R_{np} &= F_u A_e \\ &= (65\text{ ksi})\left(6.42\text{ in.}^2\right) \\ &= 417\text{ kips} \end{aligned} \qquad (\textit{Spec.}\text{ Eq. J4-2})$$

LRFD	ASD
$\phi = 0.75$ $\phi R_{np} = 0.75(417\text{ kips})$ $= 313\text{ kips} > 60\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_{np}}{\Omega} = \dfrac{417\text{ kips}}{2.00}$ $= 209\text{ kips} > 40\text{ kips}$ **o.k.**

Flexural Rupture of the Plate

The available flexural rupture strength of the plate is determined as follows:

$$Z_{net} = \frac{tl^2}{4} - \frac{t}{4}\left[(d_h + \text{1/16 in.})(s)(n^2 - 1) + (d_h + \text{1/16 in.})^2\right]$$
$$= \frac{(\text{3/4 in.})(\text{14}\tfrac{1}{2}\text{ in.})^2}{4} - \left(\frac{\text{3/4 in.}}{4}\right)\left\{(\text{1}\tfrac{1}{8}\text{ in.} + \text{1/16 in.})(\text{3 in.})\left[(5)^2 - 1\right] + (\text{1}\tfrac{1}{8}\text{ in.} + \text{1/16 in.})^2\right\}$$
$$= 23.1 \text{ in.}^3$$

$$M_n = F_u Z_{net} \qquad \text{(\textit{Manual} Eq. 9-4)}$$
$$= (65 \text{ ksi})(23.1 \text{ in.}^3)$$
$$= 1{,}500 \text{ kip-in.}$$

LRFD	ASD
$\phi = 0.75$ $\phi M_n = 0.75(1{,}500 \text{ kip-in.})$ $= 1{,}130 \text{ kip-in.} > 731 \text{ kip-in.}$ **o.k.**	$\Omega = 2.00$ $\frac{M_n}{\Omega} = \frac{1{,}500 \text{ kip-in.}}{2.00}$ $= 750 \text{ kip-in.} > 488 \text{ kip-in.}$ **o.k.**

Shear Rupture Strength of Plate

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the plate is determined as follows:

$$A_{nv} = \left[l - n(d_h + \text{1/16 in.})\right]t_p$$
$$= \left[\text{14}\tfrac{1}{2}\text{ in.} - 5(\text{1}\tfrac{1}{8}\text{ in.} + \text{1/16 in.})\right](\text{3/4 in.})$$
$$= 6.42 \text{ in.}^2$$

$$R_{nv} = 0.60 F_u A_{nv} \qquad \text{(\textit{Spec}. Eq. J4-4)}$$
$$= 0.60(65 \text{ ksi})(6.42 \text{ in.}^2)$$
$$= 250 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_{nv} = 0.75(250 \text{ kips})$ $= 188 \text{ kips} > 75 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\frac{R_{nv}}{\Omega} = \frac{250 \text{ kips}}{2.00}$ $= 125 \text{ kips} > 50 \text{ kips}$ **o.k.**

Interaction of Axial, Flexure and Shear Rupture in Plate

AISC *Specification* Chapter H does not address combined flexure and shear. The method employed here is derived from Chapter H in conjunction with AISC *Manual* Equation 10-5, as follows:

LRFD	ASD
$\frac{N_u}{\phi R_{np}} = \frac{60 \text{ kips}}{313 \text{ kips}}$ $= 0.192$	$\frac{\Omega N_a}{R_{np}} = \frac{40 \text{ kips}}{209 \text{ kips}}$ $= 0.191$

LRFD	ASD
Because $\dfrac{N_u}{\phi R_{np}} < 0.2$:	Because $\dfrac{\Omega N_a}{R_{np}} < 0.2$:
$\left(\dfrac{N_u}{2\phi R_{np}} + \dfrac{V_u a}{\phi M_n}\right)^2 + \left(\dfrac{V_u}{\phi R_{nv}}\right)^2 \le 1$	$\left(\dfrac{\Omega N_a}{2R_{np}} + \dfrac{\Omega V_a a}{M_n}\right)^2 + \dfrac{\Omega V_a}{R_{nv}} \le 1$
$\left[\dfrac{60 \text{ kips}}{2(313 \text{ kips})} + \dfrac{(75 \text{ kips})(9\frac{3}{4} \text{ in.})}{1{,}130 \text{ kip-in.}}\right]^2 + \left(\dfrac{75 \text{ kips}}{188 \text{ kips}}\right)^2 \le 1$	$\left[\dfrac{40 \text{ kips}}{2(209 \text{ kips})} + \dfrac{(50 \text{ kips})(9\frac{3}{4} \text{ in.})}{750 \text{ kip-in.}}\right]^2 + \left(\dfrac{50 \text{ kips}}{125 \text{ kips}}\right)^2 \le 1$
$0.711 < 1$ **o.k.**	$0.716 < 1$ **o.k.**

Block Shear Rupture Strength of Plate—Beam Shear Direction

The nominal strength for the limit state of block shear rupture of the plate, assuming an L-shaped tearout due to the shear load only as shown in Figure II.A-19B-2(a), is determined as follows:

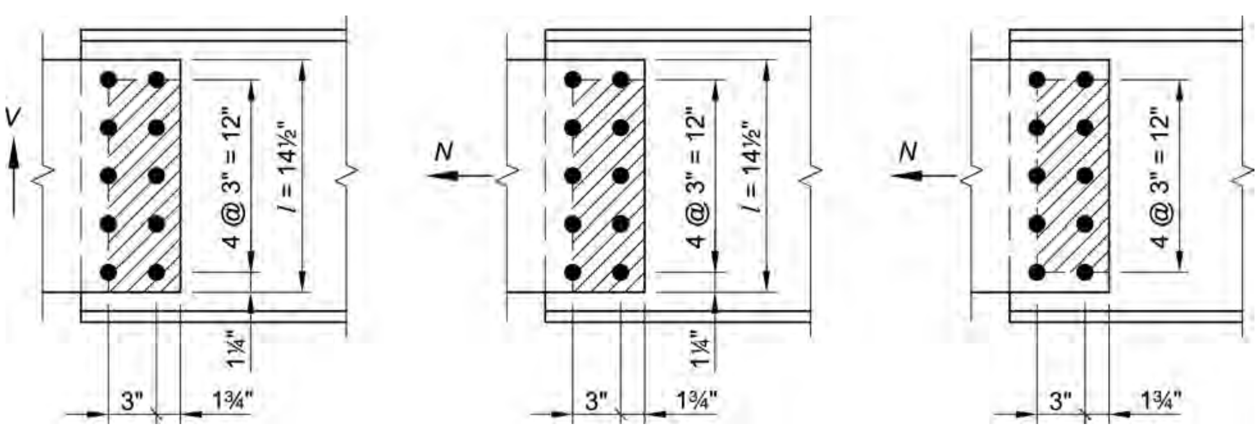
$$R_{bsv} = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad \text{(Spec. Eq. J4-5)}$$

where

$$\begin{aligned} A_{gv} &= (l - l_{ev})t \\ &= (14\tfrac{1}{2} \text{ in.} - 1\tfrac{1}{4} \text{ in.})(\tfrac{3}{4} \text{ in.}) \\ &= 9.94 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nv} &= A_{gv} - (n_v - 0.5)(d_h + \tfrac{1}{16} \text{ in.})t \\ &= 9.94 \text{ in.}^2 - (5 - 0.5)(1\tfrac{1}{8} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{3}{4} \text{ in.}) \\ &= 5.93 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nt} &= \left[l_{eh} + (n_h - 1)s - (n_h - 0.5)(d_h + \tfrac{1}{16} \text{ in.})\right]t \\ &= \left[1\tfrac{3}{4} \text{ in.} + (2 - 1)(3 \text{ in.}) - (2 - 0.5)(1\tfrac{1}{8} \text{ in.} + \tfrac{1}{16} \text{ in.})\right](\tfrac{3}{4} \text{ in.}) \\ &= 2.23 \text{ in.}^2 \end{aligned}$$



(a) Beam shear direction *(b) Beam axial direction—L-shaped* *(c) Beam axial direction—U-shaped*

Fig. II.A-19B-2. Block shear rupture of plate.

Because stress is not uniform along the net tensile area, U_{bs} = 0.5.

$$R_{bsv} = 0.60(65 \text{ ksi})(5.93 \text{ in.}^2) + 0.5(65 \text{ ksi})(2.23 \text{ in.}^2) \le 0.60(50 \text{ ksi})(9.94 \text{ in.}^2) + 0.5(65 \text{ ksi})(2.23 \text{ in.}^2)$$
$$= 304 \text{ kips} < 371 \text{ kips}$$

Therefore:

$$R_{bsv} = 304 \text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture on the plate is:

LRFD	ASD
$\phi = 0.75$ $\phi R_{bsv} = 0.75(304 \text{ kips})$ $= 228 \text{ kips} > 75 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_{bsv}}{\Omega} = \dfrac{304 \text{ kips}}{2.00}$ $= 152 \text{ kips} > 50 \text{ kips}$ **o.k.**

Block Shear Rupture Strength of the Plate—Beam Axial Direction

The plate block shear rupture failure path due to axial load only could occur as an L- or U-shape. Assuming an L-shaped failure path due to axial load only, as shown in Figure II.A-19B-2(b), the available block shear rupture strength of the plate is:

$$R_{bsn} = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \quad (Spec. \text{ Eq. J4-5})$$

where

$$A_{gv} = \left[(n_h - 1)s + l_{eh}\right]t$$
$$= \left[(2-1)(3 \text{ in.}) + 1\tfrac{3}{4} \text{ in.}\right](\tfrac{3}{4} \text{ in.})$$
$$= 3.56 \text{ in.}^2$$

$$A_{nv} = A_{gv} - (n_h - 0.5)(d_h + \tfrac{1}{16} \text{ in.})t$$
$$= 3.56 \text{ in.}^2 - (2-0.5)(1\tfrac{1}{8} \text{ in.} + \tfrac{1}{16} \text{ in})(\tfrac{3}{4} \text{ in.})$$
$$= 2.22 \text{ in.}^2$$

$$A_{nt} = \left[l_{ev} + (n_v - 1)s - (n_v - 0.5)(d_h + \tfrac{1}{16} \text{ in.})\right]t$$
$$= \left[1\tfrac{1}{4} \text{ in.} + (5-1)(3 \text{ in.}) - (5-0.5)(1\tfrac{1}{8} \text{ in.} + \tfrac{1}{16} \text{ in})\right](\tfrac{3}{4} \text{ in.})$$
$$= 5.93 \text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$R_{bsn} = 0.60(65 \text{ ksi})(2.22 \text{ in.}^2) + 1.0(65 \text{ ksi})(5.93 \text{ in.}^2) \le 0.60(50 \text{ ksi})(3.56 \text{ in.}^2) + 1.0(65 \text{ ksi})(5.93 \text{ in.}^2)$$
$$= 472 \text{ kips} < 492 \text{ kips}$$

Therefore:

$R_{bsn} = 472 \text{ kips}$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture on the plate is:

LRFD	ASD
$\phi = 0.75$ $\phi R_{bsn} = 0.75(472 \text{ kips})$ $= 354 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_{bsn}}{\Omega} = \dfrac{472 \text{ kips}}{2.00}$ $= 236 \text{ kips} > 40 \text{ kips}$ **o.k.**

Assuming a U-shaped failure path in the plate due to axial load, as shown in Figure II.A-19B-2(c), the available block shear rupture strength of the plate is:

$$R_{bsn} = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \quad \text{(Spec. Eq. J4-5)}$$

where

$$A_{gv} = (2 \text{ shear planes})\left[l_{eh} + (n_h - 1)s\right]t$$
$$= (2 \text{ shear planes})\left[1\tfrac{3}{4} \text{ in.} + (2-1)(3 \text{ in.})\right](\tfrac{3}{4} \text{ in.})$$
$$= 7.13 \text{ in.}^2$$

$$A_{nv} = A_{gv} - (2 \text{ shear planes})(n_h - 0.5)(d_h + \tfrac{1}{16} \text{ in.})t$$
$$= 7.13 \text{ in.}^2 - (2 \text{ shear planes})(2 - 0.5)(1\tfrac{1}{8} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{3}{4} \text{ in.})$$
$$= 4.46 \text{ in.}^2$$

$$A_{nt} = \left[(n_v - 1)s - (n_v - 1)(d_h + \tfrac{1}{16} \text{ in.})\right]t$$
$$= \left[(5-1)(3 \text{ in.}) - (5-1)(1\tfrac{1}{8} \text{ in.} + \tfrac{1}{16} \text{ in.})\right](\tfrac{3}{4} \text{ in.})$$
$$= 5.44 \text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$R_{bsn} = 0.60(65 \text{ ksi})(4.46 \text{ in.}^2) + 1.0(65 \text{ ksi})(5.44 \text{ in.}^2) \leq 0.60(50 \text{ ksi})(7.13 \text{ in.}^2) + 1.0(65 \text{ ksi})(5.44 \text{ in.}^2)$$
$$= 528 \text{ kips} < 568 \text{ kips}$$

Therefore:

$$R_{bsn} = 528 \text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture on the plate is:

LRFD	ASD
$\phi = 0.75$ $\phi R_{bsn} = 0.75(528 \text{ kips})$ $= 396 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_{bsn}}{\Omega} = \dfrac{528 \text{ kips}}{2.00}$ $= 264 \text{ kips} > 40 \text{ kips}$ **o.k.**

Block Shear Rupture Strength of Plate—Combined Shear and Axial Interaction

The same L-shaped block shear rupture failure path is loaded by forces in both the shear and axial directions. The interaction of loading in both directions is determined as follows:

LRFD	ASD
$\left(\dfrac{V_u}{\phi R_{bsv}}\right)^2+\left(\dfrac{N_u}{\phi R_{bsn}}\right)^2\leq 1$ $\left(\dfrac{75\text{ kips}}{228\text{ kips}}\right)^2+\left(\dfrac{60\text{ kips}}{354\text{ kips}}\right)^2=0.137<1$ **o.k.**	$\left(\dfrac{\Omega V_a}{R_{bsv}}\right)^2+\left(\dfrac{\Omega N_a}{R_{bsn}}\right)^2\leq 1$ $\left(\dfrac{50\text{ kips}}{152\text{ kips}}\right)^2+\left(\dfrac{40\text{ kips}}{236\text{ kips}}\right)^2=0.137<1$ **o.k.**

Shear Rupture Strength of Column Web at Weld

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the column web is determined as follows:

$$\begin{aligned}A_{nv}&=2lt_w\\&=2(14\tfrac{1}{2}\text{ in.})(0.440\text{ in.})\\&=12.8\text{ in.}^2\end{aligned}$$

$$\begin{aligned}R_n&=0.60F_uA_v\\&=0.60(65\text{ ksi})(12.8\text{ in.}^2)\\&=499\text{ kips}\end{aligned}$$ (*Spec*. Eq. J4-4)

LRFD	ASD
$\phi=0.75$ $\phi R_n=0.75(499\text{ kips})$ $=374\text{ kips}>75\text{ kips}$ **o.k.**	$\Omega=2.00$ $\dfrac{R_n}{\Omega}=\dfrac{499\text{ kips}}{2.00}$ $=250\text{ kips}>50\text{ kips}$ **o.k.**

Yield Line Analysis on Supporting Column Web

A yield line analysis is used to determine the strength of the column web in the direction of the axial tension load. The yield line and associated dimensions are shown in Figure II.A-19B-3 and the available strength is determined as follows:

$$\begin{aligned}T&=d-2k_{des}\\&=14.0\text{ in.}-2(1.31\text{ in.})\\&=11.4\text{ in.}\end{aligned}$$

$$\begin{aligned}a&=\frac{d}{2}-k_{des}+\frac{t_w}{2}\\&=\frac{14.0\text{ in.}}{2}-1.31\text{ in.}+\frac{0.415\text{ in.}}{2}\\&=5.90\text{ in.}\end{aligned}$$

$$b = \frac{d}{2} - k_{des} - \frac{t_w}{2} - t_p$$
$$= \frac{14.0 \text{ in.}}{2} - 1.31 \text{ in.} - \frac{0.415 \text{ in.}}{2} - 3/4 \text{ in.}$$
$$= 4.73 \text{ in.}$$

$$c = t_p$$
$$= 3/4 \text{ in.}$$

$$R_n = \frac{t_w^2 F_y}{4}\left[\frac{4\sqrt{2Tab(a+b)} + l(a+b)}{ab}\right] \qquad (\textit{Manual} \text{ Eq. 9-31})$$
$$= \frac{(0.440 \text{ in.})^2 (50 \text{ ksi})}{4}\left[\frac{4\sqrt{2(11.4 \text{ in.})(5.90 \text{ in.})(4.73 \text{ in.})(5.90 \text{ in.} + 4.73 \text{ in.})} + (14\tfrac{1}{2} \text{ in.})(5.90 \text{ in.} + 4.73 \text{ in.})}{(5.90 \text{ in.})(4.73 \text{ in.})}\right]$$
$$= 41.9 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(41.9 \text{ kips})$ $= 41.9 \text{ kips} < 60 \text{ kips}$ **n.g.**	$\Omega = 1.50$ $\frac{R_n}{\Omega} = \frac{41.9 \text{ kips}}{1.50}$ $= 27.9 \text{ kips} < 40 \text{ kips}$ **n.g.**

The available column web strength is not adequate to resist the axial force in the beam. The column may be increased in size for an adequate web thickness or reinforced with stiffeners or web doubler plates. For example, a W14×120 column, with $t_w = 0.590$ in., has adequate strength to resist the given forces.

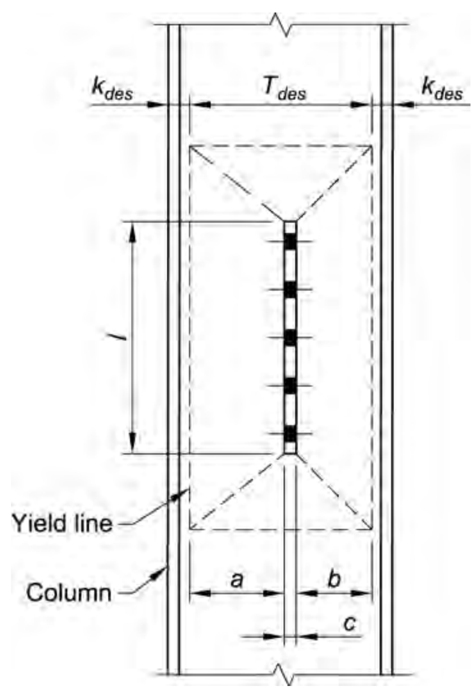


Fig II.A-19B-3. Yield line for column web.

Strength of Weld

A two-sided fillet weld with size of (⅝)t_p = 0.469 in. (use ½-in. fillet welds) is used. As discussed in AISC *Manual* Part 10, this weld size will develop the strength of the shear plate used because the moment generated by this connection is indeterminate.

The available weld strength is determined using AISC *Manual* Equation 8-2a or 8-2b, incorporating the directional strength increase from AISC *Specification* Equation J2-5, as follows:

$$\begin{aligned}\mu &= 1.0 + 0.50\sin^{1.5}\theta \\ &= 1.0 + 0.50\sin^{1.5}(38.7°) \\ &= 1.25\end{aligned}$$

LRFD	ASD
$R_n = (1.392 \text{ kip/in.})Dl\mu(2 \text{ sides})$ $= (1.392 \text{ kip/in.})(8)(14\tfrac{1}{2} \text{ in.})(1.25)(2 \text{ sides})$ $= 404 \text{ kips} > 96.0 \text{ kips}$ **o.k.**	$R_n = (0.928 \text{ kip/in.})Dl\mu(2 \text{ sides})$ $= (0.928 \text{ kip/in.})(8)(14\tfrac{1}{2} \text{ in.})(1.25)(2 \text{ sides})$ $= 269 \text{ kips} > 64.0 \text{ kips}$ **o.k.**

Conclusion

The configuration given does not work due to the inadequate column web. The column would need to be increased in size or reinforced as discussed previously.

Comments: If the applied axial load were in compression, the connection plate would need to be checked for compressive flexural buckling strength as follows. This is required in the case of the extended configuration of a single-plate connection and would not be required for the conventional configuration.

From AISC *Specification* Table C-A-7.1, Case c:

$$K = 1.2$$

$$\begin{aligned}\frac{L_c}{r} &= \frac{KL}{r} \\ &= \frac{1.2(9\tfrac{3}{4} \text{ in.})}{\tfrac{3}{4} \text{ in.}/\sqrt{12}} \\ &= 54.0\end{aligned}$$

As stated in AISC *Specification* Section J4.4, if L_c/r is greater than 25, Chapter E applies. The available critical stress of the plate, ϕF_{cr} or F_{cr}/Ω, is determined using AISC *Manual* Table 4-14 as follows:

LRFD	ASD
$\phi F_{cr} = 36.4 \text{ ksi}$ $\phi R_n = \phi F_{cr} l t_p$ $= (36.4 \text{ ksi})(14\tfrac{1}{2} \text{ in.})(\tfrac{3}{4} \text{ in.})$ $= 396 \text{ kips} > 60 \text{ kips}$ **o.k.**	$\dfrac{F_{cr}}{\Omega} = 24.2 \text{ ksi}$ $\dfrac{R_n}{\Omega} = \dfrac{F_{cr}}{\Omega} l t_p$ $= (24.2 \text{ ksi})(14\tfrac{1}{2} \text{ in.})(\tfrac{3}{4} \text{ in.})$ $= 263 \text{ kips} > 40 \text{ kips}$ **o.k.**

Column Reinforcement

As mentioned there are three options to correct the column web failure. These options are as follows:

1) Use a heavier column. This may not be practical because the steel may have been purchased and perhaps detailed and fabricated before the problem is found.

2) Use a web doubler plate. This plate would be fitted about the shear plate on the same side of the column web as the shear plate. This necessitates a lot of cutting, fitting and welding, and is therefore expensive.

3) Use stiffener or stabilizer plates—also called continuity plates. This is probably the most viable option, but changes the nature of the connection, because the stiffener plates will cause the column to be subjected to a moment. This cannot be avoided, but may be used advantageously.

Option 3 Solution

Because the added stiffeners cause the column to pick-up moment, the moment for which the connection is designed can be reduced.

The connection is designed as a conventional configuration shear plate with axial force for everything to the right of Section A-A as shown in Figure II.A-19B-4. The design to the left of Section A-A is performed following a procedure for Type II stabilizer plates presented in Fortney and Thornton (2016).

As shown in Figure II.A-19B-5, the moment in the shear plate to the left of Section A-A is uncoupled between the stabilizer plates.

$$V_s = \frac{Va'}{l}$$

where

$a' = 7$ in.

$l = 14\frac{1}{2}$ in.

$g = 2\frac{3}{4}$ in.

LRFD	ASD
$V_{us} = \frac{V_u a'}{l}$ $= \frac{(75 \text{ kips})(7 \text{ in.})}{14\frac{1}{2} \text{ in.}}$ $= 36.2 \text{ kips}$	$V_{as} = \frac{V_a a'}{L}$ $= \frac{(50 \text{ kips})(7 \text{ in.})}{14\frac{1}{2} \text{ in.}}$ $= 24.1 \text{ kips}$

The force between the shear plate and stabilizer plate is determined as follows:

LRFD	ASD
$F_{up} = V_{us} + \frac{N_u}{2}$ $= 36.2 \text{ kips} + \frac{60 \text{ kips}}{2}$ $= 66.2 \text{ kips}$	$F_{ap} = V_{as} + \frac{N_a}{2}$ $= 24.1 \text{ kips} + \frac{40 \text{ kips}}{2}$ $= 44.1 \text{ kips}$

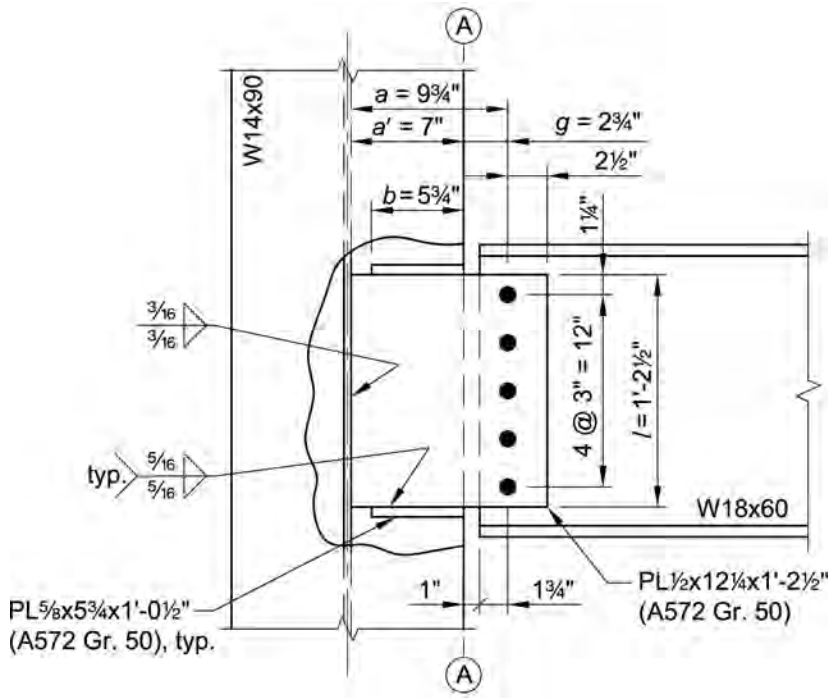


Fig. II.A-19B-4. Design of shear plate with stabilizer plates.

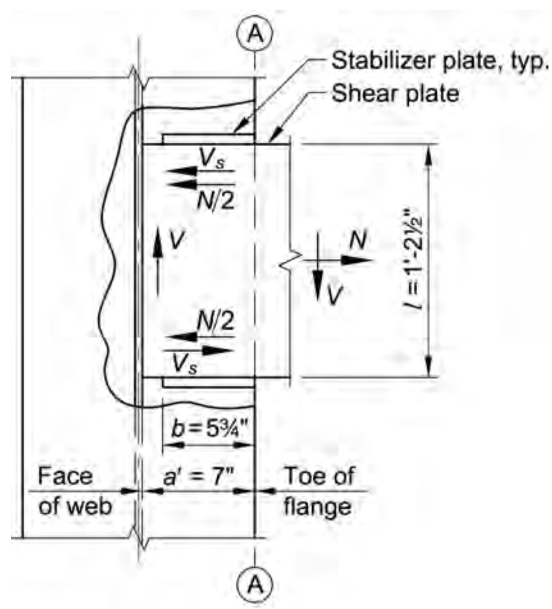


Fig. II.A-19B-5. Forces acting on shear plate.

Stabilizer Plate Design

The stabilizer plate design is shown in Figure II.A-19B-6. The forces in the stabilizer plate are calculated as follows:

LRFD	ASD
Shear: $V_u = \dfrac{F_{up}}{2} = \dfrac{66.2 \text{ kips}}{2} = 33.1 \text{ kips}$	Shear: $V_a = \dfrac{F_{ap}}{2} = \dfrac{44.1 \text{ kips}}{2} = 22.1 \text{ kips}$
Moment: $M_u = \dfrac{F_{up} w}{4} = \dfrac{(66.2 \text{ kips})(12\frac{1}{2} \text{in.})}{4} = 207 \text{ kip-in.}$	Moment: $M_a = \dfrac{F_{ap} w}{4} = \dfrac{(44.1 \text{ kips})(12\frac{1}{2} \text{in.})}{4} = 138 \text{ kip-in.}$

Try ⅝-in.-thick stabilizer plates. The available shear strength of the stabilizer plate is determined using AISC *Specification* Section J4.2 as follows:

$$A_{nv} = bt$$
$$= (5\tfrac{3}{4} \text{ in.})(\tfrac{5}{8} \text{ in.})$$
$$= 3.59 \text{ in.}^2$$

$$R_n = 0.60 F_u A_{nv}$$ (*Spec*. Eq. J4-4)
$$= 0.60(65 \text{ ksi})(3.59 \text{ in.}^2)$$
$$= 140 \text{ kips}$$

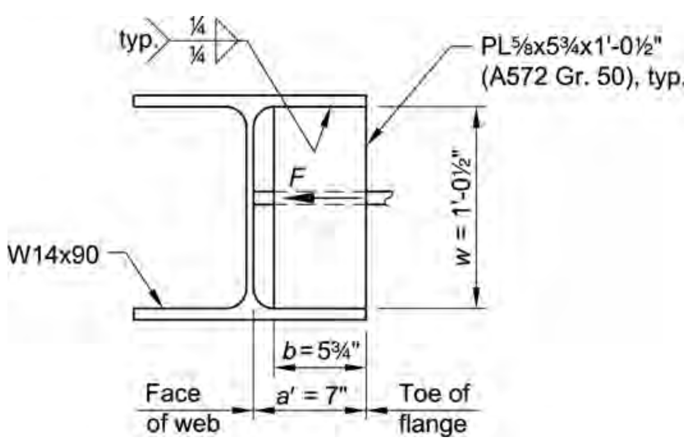


Fig. II.A-19B-6. Stabilizer plate design.

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(140 \text{ kips})$ $= 105 \text{ kips} > 33.1 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{140 \text{ kips}}{2.00}$ $= 70.0 \text{ kips} > 22.1 \text{ kips}$ **o.k.**

The available flexural strength of the stabilizer plate is determined as follows:

$$M_n = F_y Z_x$$
$$= (50 \text{ ksi})\left[\frac{(\text{⅝ in.})(5\text{¾ in.})^2}{4}\right]$$
$$= 258 \text{ kip-in.}$$

LRFD	ASD
$\phi = 0.90$ $\phi M_n = 0.90(258 \text{ kip-in.})$ $= 232 \text{ kip-in.} > 207 \text{ kip-in.}$ **o.k.**	$\Omega = 1.67$ $\dfrac{M_n}{\Omega} = \dfrac{258 \text{ kip-in.}}{1.67}$ $= 154 \text{ kip-in.} > 138 \text{ kip-in.}$ **o.k.**

Stabilizer Plate to Column Weld Design

The required weld size between the stabilizer plate and column flanges is determined using AISC *Manual* Equations 8-2a or 8-2b as follows:

LRFD	ASD
$D_{req} = \dfrac{F_{up}/2}{(2 \text{ welds})(1.392 \text{ kip/in.})b}$ $= \dfrac{(66.2 \text{ kips}/2)}{(2 \text{ welds})(1.392 \text{ kip/in.})(5\text{¾ in.})}$ $= 2.07 \text{ sixteenths}$	$D_{req} = \dfrac{F_{ap}/2}{(2 \text{ welds})(0.928 \text{ kip/in.})b}$ $= \dfrac{(44.1 \text{ kips}/2)}{(2 \text{ welds})(0.928 \text{ kip/in.})(5\text{¾ in.})}$ $= 2.07 \text{ sixteenths}$

The minimum weld size per AISC *Specification* Table J2.4 controls. Use ¼-in. fillet welds.

Shear Plate to Stabilizer Plate Weld Design

The required weld size between the shear plate and stabilizer plates is determined using AISC *Manual* Equations 8-2a or 8-2b as follows:

LRFD	ASD
$D_{req} = \dfrac{F_{up}}{(2 \text{ welds})(1.392 \text{ kip/in.})l_w}$ $= \dfrac{66.2 \text{ kips}}{(2 \text{ welds})(1.392 \text{ kip/in.})(5\text{¾ in.})}$ $= 4.14 \text{ sixteenths}$	$D_{req} = \dfrac{F_{ap}}{(2 \text{ welds})(0.928 \text{ kip/in.})l_w}$ $= \dfrac{44.1 \text{ kips}}{(2 \text{ welds})(0.928 \text{ kip/in.})(5\text{¾ in.})}$ $= 4.13 \text{ sixteenths}$

Use 5⁄16-in. fillet welds.

Strength of Shear Plate at Stabilizer Plate Welds

The minimum shear plate thickness that will match the shear rupture strength of the weld is:

$$t_{min} = \frac{6.19D}{F_u} \qquad \text{(}\textit{Manual}\text{ Eq. 9-3)}$$
$$= \frac{6.19(4.14)}{65 \text{ ksi}}$$
$$= 0.394 \text{ in.} < \tfrac{1}{2} \text{ in.} \quad \textbf{o.k.}$$

Shear Plate to Column Web Weld Design

The shear plate to stabilizer plate welds act as "crack arrestors" for the shear plate to column web welds. As shown in Figure II.A-19B-7, the required shear force is V. The required weld size is determined using AISC *Manual* Equations 8-2a or 8-2b as follows:

LRFD	ASD
$V_u = 75 \text{ kips}$	$V_a = 50 \text{ kips}$
$D_{req} = \dfrac{V_u}{(2 \text{ welds})(1.392 \text{ kip/in.})l}$ $= \dfrac{75 \text{ kips}}{(2 \text{ welds})(1.392 \text{ kip/in.})(14\tfrac{1}{2} \text{ in.})}$ $= 1.86 \text{ sixteenths}$	$D_{req} = \dfrac{V_a}{(2 \text{ welds})(0.928 \text{ kip/in.})l}$ $= \dfrac{50 \text{ kips}}{(2 \text{ welds})(0.928 \text{ kip/in.})(14\tfrac{1}{2} \text{ in.})}$ $= 1.86 \text{ sixteenths}$

The minimum weld size per AISC *Specification* Table J2.4 controls. Use $\tfrac{3}{16}$-in. fillet welds.

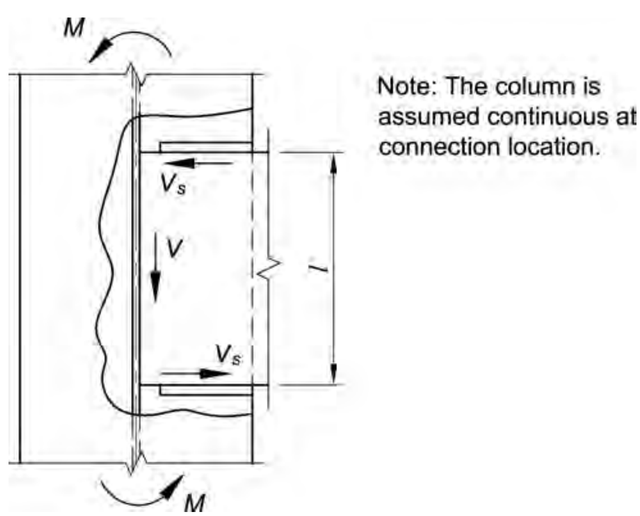


Fig. II.A-19B-7. Moment induced in column.

Strength of Shear Plate at Column Web Welds

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the shear plate is determined as follows:

$$A_{nv} = lt$$
$$= (14\tfrac{1}{2} \text{ in.})(\tfrac{1}{2} \text{ in.})$$
$$= 7.25 \text{ in.}^2$$

$$R_n = 0.60 F_u A_{nv} \qquad (\textit{Spec.} \text{ Eq. J4-4})$$
$$= 0.60(65 \text{ ksi})(7.25 \text{ in.}^2)$$
$$= 283 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(283 \text{ kips})$ $= 212 \text{ kips} > 75 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{283 \text{ kips}}{2.00}$ $= 142 \text{ kips} > 50 \text{ kips}$ **o.k.**

Moment in Column

The moment in the column is determined as follows:

LRFD	ASD
$2M_u = V_{us} l$ $= (36.2 \text{ kips})(14\tfrac{1}{2} \text{ in.})$ $= 525 \text{ kip-in.}$ $M_u = 263 \text{ kip-in.}$	$2M_a = V_{as} l$ $= (24.1 \text{ kips})(14\tfrac{1}{2} \text{ in.})$ $= 349 \text{ kip-in.}$ $M_a = 175 \text{ kip-in.}$

The column design needs to be reviewed to ensure that this moment does not overload the column.

Reference

Fortney, P. and Thornton, W. (2016), “Analysis and Design of Stabilizer Plates in Single-Plate Shear Connections,” *Engineering Journal*, AISC, Vol. 53, No. 1, pp. 1–18.

EXAMPLE II.A-20 ALL-BOLTED SINGLE-PLATE SHEAR SPLICE

Given:

Verify an all-bolted single-plate shear splice between two ASTM A992 beams, as shown in Figure II.A-20-1, to support the following beam end reactions:

R_D = 10 kips
R_L = 30 kips

Use ASTM A36 plate.

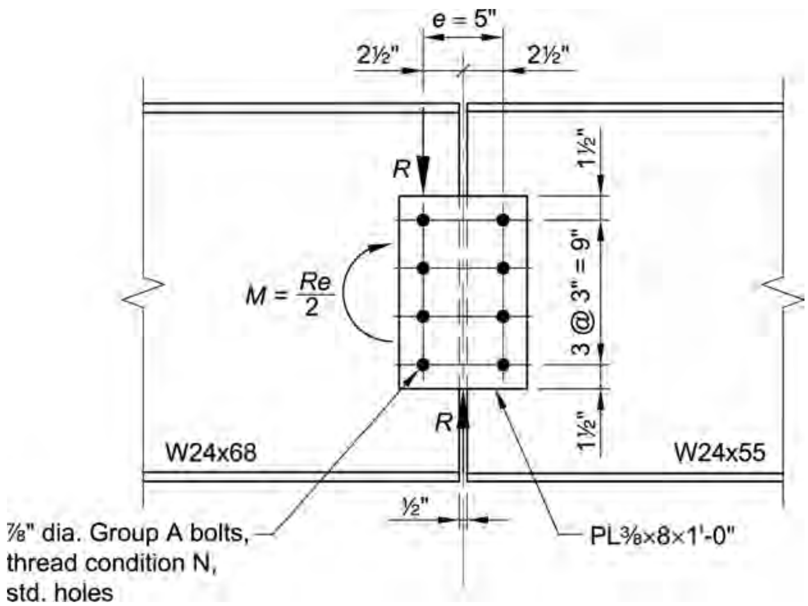


Fig. II.A-20-1. Connection geometry for Example II.A-20.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam and column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W24×55
t_w = 0.395 in.

Beam
W24×68
t_w = 0.415 in.

From AISC *Specification* Table J3.3, for ⅞-in.-diameter bolts with standard holes:

d_h = 15⁄16 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(10\text{ kips}) + 1.6(30\text{ kips})$ $= 60.0\text{ kips}$	$R_a = 10\text{ kips} + 30\text{ kips}$ $= 40.0\text{ kips}$

Strength of the Bolted Connection—Plate

Note: When the splice is symmetrical, the eccentricity of the shear to the center of gravity of either bolt group is equal to half the distance between the centroids of the bolt groups. Therefore, each bolt group can be designed for the shear, R_u or R_a, and one-half the eccentric moment, $R_u e$ or $R_a e$.

Using a symmetrical splice, each bolt group will carry one-half the eccentric moment. Thus, the eccentricity on each bolt group is determined as follows:

$$\frac{e}{2} = \frac{5\text{ in.}}{2}$$
$$= 2.50\text{ in.}$$

From the Commentary to AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the invividual strengths of the individual fasteners, which may be taken as the lesser of the fastener shear strength per AISC *Specification* Section J3.6, the bearing strength at the bolt hole per AISC *Specification* Section J3.10 or the tearout strength at the bolt hole per AISC *Specification* Section J3.10.

From AISC *Manual* Table 7-1, the available shear strength per bolt for ⅞-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) is:

LRFD	ASD
$\phi r_n = 24.3\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = 16.2\text{ kips/bolt}$

The available bearing strength of the plate per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$r_n = 2.4dtF_u \qquad (\textit{Spec.}\text{ Eq. J3-6a})$$
$$= (2.4)(\tfrac{7}{8}\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi})$$
$$= 45.7\text{ kips/bolt}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(45.7\text{ kips/bolt})$ $= 34.3\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{45.7\text{ kips/bolt}}{2.00}$ $= 22.9\text{ kips/bolt}$

The available tearout strength of the plate per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration. Note: The available tearout strength based on edge distance will conservatively be used for all of the bolts.

$$\begin{aligned} l_c &= l_{ev} - 0.5(d_h) \\ &= 1\tfrac{1}{2}\text{ in.} - 0.5(\tfrac{15}{16}\text{ in.}) \\ &= 1.03\text{ in.} \end{aligned}$$

$$\begin{aligned} r_n &= 1.2 l_c t F_u \\ &= 1.2(1.03\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi}) \\ &= 26.9\text{ kips/bolt} \end{aligned}$$ (*Spec*. Eq. J3-6c)

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(26.9\text{ kips/bolt})$ $= 20.2\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{26.9\text{ kips/bolt}}{2.00}$ $= 13.5\text{ kips/bolt}$

The tearout strength controls over bearing and shear for bolts in the plate.

The available strength of the bolt group is determined by interpolating AISC *Manual* Table 7-6, with n = 4, Angle = 0°, and $e_x = 2\tfrac{1}{2}$ in.

$$C = 3.07$$

LRFD	ASD
$C_{min} = \dfrac{R_u}{\phi r_n}$ $= \dfrac{60.0\text{ kips}}{20.2\text{ kips/bolt}}$ $= 2.97 < 3.07$ **o.k.**	$C_{min} = \dfrac{R_a}{r_n / \Omega}$ $= \dfrac{40.0\text{ kips}}{13.5\text{ kips/bolt}}$ $= 2.96 < 3.07$ **o.k.**

Strength of the Bolted Connection—Beam Web

By inspection, bearing and tearout on the webs of the beams will not govern.

Flexural Yielding of Plate

The required flexural strength is determined as follows:

LRFD	ASD
$M_u = \dfrac{R_u e}{2}$ $= \dfrac{(60.0\text{ kips})(5\text{ in.})}{2}$ $= 150\text{ kip-in.}$	$M_a = \dfrac{R_a e}{2}$ $= \dfrac{(40.0\text{ kips})(5\text{ in.})}{2}$ $= 100\text{ kip-in.}$

The available flexural strength is determined as follows:

LRFD	ASD

$\phi = 0.90$ $\phi M_n = \phi F_y Z_x$ $= 0.90(36\text{ ksi})\left[\dfrac{(3/8\text{ in.})(12\text{ in.})^2}{4}\right]$ $= 437\text{ kip-in.} > 150\text{ kip-in.}$ **o.k.**	$\Omega = 1.67$ $\dfrac{M_n}{\Omega} = \dfrac{F_y Z_x}{\Omega}$ $= \dfrac{36\text{ ksi}}{1.67}\left[\dfrac{(3/8\text{ in.})(12\text{ in.})^2}{4}\right]$ $= 291\text{ kip-in.} > 100\text{ kip-in.}$ **o.k.**

Flexural Rupture of Plate

The net plastic section modulus of the plate, Z_{net}, is determined from AISC *Manual* Table 15-3:

$$Z_{net} = 9.00\text{ in.}^3$$

$$\begin{aligned} M_n &= F_u Z_{net} \\ &= (58\text{ ksi})(9.00\text{ in.}^3) \\ &= 522\text{ kip-in.} \end{aligned}$$ (*Manual* Eq. 9-4)

LRFD	ASD
$\phi = 0.75$ $\phi M_n = 0.75(522\text{ kip-in.})$ $= 392\text{ kip-in.} > 150\text{ kip-in.}$ **o.k.**	$\Omega = 2.00$ $\phi M_n = \dfrac{522\text{ kip-in.}}{2.00}$ $= 261\text{ kip-in.} > 100\text{ kip-in.}$ **o.k.**

Shear Strength of Plate

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the plate is determined as follows:

$$\begin{aligned} A_{gv} &= lt \\ &= (12\text{ in.})(3/8\text{ in.}) \\ &= 4.50\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_n &= 0.60 F_y A_{gv} \\ &= 0.60(36\text{ ksi})(4.50\text{ in.}^2) \\ &= 97.2\text{ kips} \end{aligned}$$ (*Spec.* Eq. J4-3)

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(97.2\text{ kips})$ $= 97.2\text{ kips} > 60.0\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{97.2\text{ kips}}{1.50}$ $= 64.8\text{ kips} > 40.0\text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the plate is determined using the net area determined in accordance with AISC *Specification* Section B4.3b.

$$A_{nv} = \left[d - n\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]t$$
$$= \left[12\text{ in.} - 4\left(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]\left(\tfrac{3}{8}\text{ in.}\right)$$
$$= 3.00\text{ in.}^2$$

$$R_n = 0.60F_u A_{nv}$$ (*Spec.* Eq. J4-4)
$$= 0.60\left(58\text{ ksi}\right)\left(3.00\text{ in.}^2\right)$$
$$= 104\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75\left(104\text{ kips}\right)$ $= 78.0\text{ kips} > 60.0\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{104\text{ kips}}{2.00}$ $= 52.0\text{ kips} > 40.0\text{ kips}$ **o.k.**

Block Shear Rupture of Plate

The nominal strength for the limit state of block shear rupture is given by AISC *Specification* Section J4.3.

$$R_n = 0.60F_u A_{nv} + U_{bs}F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs}F_u A_{nt}$$ (*Spec.* Eq. J4-5)

The available block shear rupture strength of the plate is determined as follows, using AISC *Manual* Tables 9-3a, 9-3b and 9-3c, and AISC *Specification* Equation J4-5, with $n = 4$, $l_{eh} = l_{ev} = 1\frac{1}{2}$ in., and $U_{bs} = 1.0$.

LRFD	ASD
Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{\phi F_u A_{nt}}{t} = 43.5\text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{\phi 0.60F_y A_{gv}}{t} = 170\text{ kip/in.}$ Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{\phi 0.60F_u A_{nv}}{t} = 183\text{ kip/in.}$	Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{F_u A_{nt}}{\Omega t} = 29.0\text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{0.60F_y A_{gv}}{\Omega t} = 113\text{ kip/in.}$ Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{0.60F_u A_{nv}}{\Omega t} = 122\text{ kip/in.}$

LRFD	ASD
$\phi R_n = \phi 0.60 F_u A_{nv} + \phi U_{bs} F_u A_{nt}$ $\leq \phi 0.60 F_y A_{gv} + \phi U_{bs} F_u A_{nt}$ $= (3/8 \text{ in.})[183 \text{ kip/in.} + (1.0)(43.5 \text{ kip/in.})]$ $\leq (3/8 \text{ in.})[170 \text{ kip/in.} + (1.0)(43.5 \text{ kip/in.})]$ $= 84.9 \text{ kips} > 80.1 \text{ kips}$ Therefore: $\phi R_n = 80.1 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega} + \dfrac{U_{bs} F_u A_{nt}}{\Omega}$ $\leq \dfrac{0.60 F_y A_{gv}}{\Omega} + \dfrac{U_{bs} F_u A_{nt}}{\Omega}$ $= (3/8 \text{ in.})[122 \text{ kip/in.} + (1.0)(29.0 \text{ kip/in.})]$ $\leq (3/8 \text{ in.})[113 \text{ kip/in.} + (1.0)(29.0 \text{ kip/in.})]$ $= 56.6 \text{ kips} > 53.3 \text{ kips}$ Therefore: $\dfrac{R_n}{\Omega} = 53.3 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

Conclusion

The connection is found to be adequate as given for the applied force.

EXAMPLE II.A-21 BOLTED/WELDED SINGLE-PLATE SHEAR SPLICE

Given:

Verify a single-plate shear splice between between two ASTM A992 beams, as shown in Figure II.A-21-1, to support the following beam end reactions:

R_D = 8 kips
R_L = 24 kips

Use an ASTM A36 plate and 70-ksi electrodes.

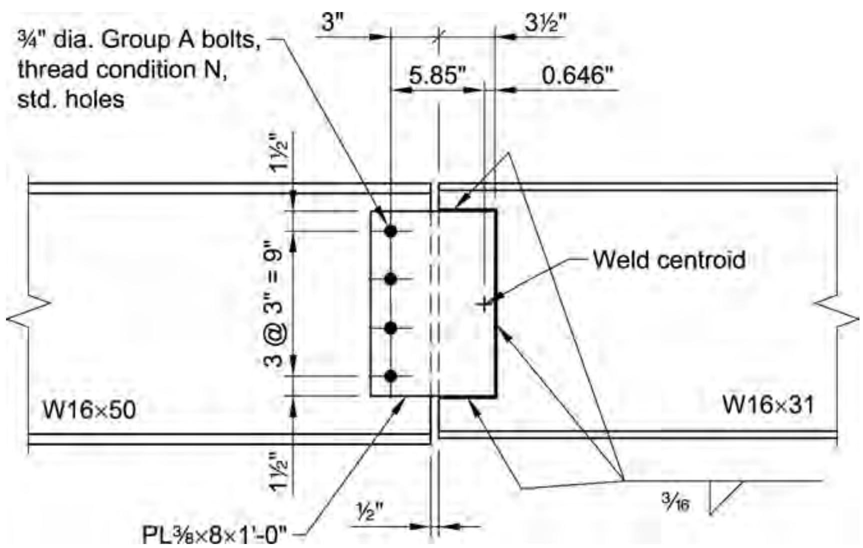


Fig. II.A-21-1. Connection geometry for Example II.A-21.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W16×31
t_w = 0.275 in.

Beam
W16×50
t_w = 0.380 in.

From AISC *Specification* Table J3.3, for ¾-in.-diameter bolts with standard holes:

d_h = 13⁄16 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(8 \text{ kips}) + 1.6(24 \text{ kips})$ $= 48.0 \text{ kips}$	$R_a = 8 \text{ kips} + 24 \text{ kips}$ $= 32.0 \text{ kips}$

Strength of the Welded Connection—Plate

Because the splice is unsymmetrical and the weld group is more rigid, it will be designed for the full moment from the eccentric shear.

Use a PL⅜ in.×8 in.×1 ft 0 in. This plate size meets the dimensional and other limitations of a single-plate connection with a conventional configuration from AISC *Manual* Part 10.

Use AISC *Manual* Table 8-8 to determine the weld size.

$$\begin{aligned} k &= \frac{kl}{l} \\ &= \frac{3\tfrac{1}{2} \text{ in.}}{12 \text{ in.}} \\ &= 0.292 \end{aligned}$$

Interpolating from AISC *Manual* Table 8-8, with Angle = 0°, and k = 0.292:

$$x = 0.0538$$

$$\begin{aligned} xl &= (0.0538)(12 \text{ in.}) \\ &= 0.646 \text{ in.} \end{aligned}$$

$$\begin{aligned} e_x &= al \\ &= 6.50 \text{ in.} - 0.646 \text{ in.} \\ &= 5.85 \text{ in.} \end{aligned}$$

$$\begin{aligned} a &= \frac{al}{l} \\ &= \frac{5.85 \text{ in.}}{12 \text{ in.}} \\ &= 0.488 \end{aligned}$$

By interpolating AISC *Manual* Table 8-8, with Angle = 0°:

$$C = 2.15$$

From AISC *Manual* Equation 8-21, with C_1 = 1.00 from AISC *Manual* Table 8-3, the required weld size is determined as follows:

LRFD	ASD
$D_{min} = \dfrac{R_u}{\phi C C_1 l}$ $= \dfrac{48.0 \text{ kips}}{0.75(2.15)(1.00)(12 \text{ in.})}$ $= 2.48 \rightarrow 3$ sixteenths	$D_{min} = \dfrac{\Omega R_a}{C C_1 l}$ $= \dfrac{(2.00)(32.0 \text{ kips})}{2.15(1.00)(12 \text{ in.})}$ $= 2.48 \rightarrow 3$ sixteenths

The minimum required weld size from AISC *Specification* Table J2.4 is 3⁄16 in.

Use a 3⁄16-in. fillet weld.

Shear Rupture of W16×31 Beam Web at Weld

For fillet welds with F_{EXX} = 70 ksi on one side of the connection, the minimum thickness required to match the available shear rupture strength of the connection element to the available shear rupture strength of the base metal is:

$$t_{min} = \frac{3.09D}{F_u} \qquad \text{(\textit{Manual} Eq. 9-2)}$$
$$= \frac{3.09(2.48)}{65 \text{ ksi}}$$
$$= 0.118 \text{ in.} < 0.275 \text{ in.} \quad \textbf{o.k.}$$

Strength of the Bolted Connection—Plate

From the Commentary to AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the invividual strengths of the individual fasteners, which may be taken as the lesser of the fastener shear strength per AISC *Specification* Section J3.6, the bearing strength at the bolt hole per AISC *Specification* Section J3.10, or the tearout strength at the bolt hole per AISC *Specification* Section J3.10.

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) is:

LRFD	ASD
$\phi r_n = 17.9$ kips/bolt	$\dfrac{r_n}{\Omega} = 11.9$ kips/bolt

The available bearing strength of the plate per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$r_n = 2.4dtF_u \qquad \text{(\textit{Spec.} Eq. J3-6a)}$$
$$= (2.4)(3/4 \text{ in.})(3/8 \text{ in.})(58 \text{ ksi})$$
$$= 39.2 \text{ kips/bolt}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(39.2 \text{ kips/bolt})$ $= 29.4$ kips/bolt	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{39.2 \text{ kips/bolt}}{2.00}$ $= 19.6$ kips/bolt

The available tearout strength of the plate per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration. Note: The available tearout strength based on edge distance will conservatively be used for all of the bolts.

$$\begin{aligned} l_c &= l_{ev} - 0.5(d_h) \\ &= 1\tfrac{1}{2} \text{ in.} - 0.5(\tfrac{13}{16} \text{ in.}) \\ &= 1.09 \text{ in.} \end{aligned}$$

$$\begin{aligned} r_n &= 1.2 l_c t F_u \\ &= 1.2(1.09 \text{ in.})(\tfrac{3}{8} \text{ in.})(58 \text{ ksi}) \\ &= 28.4 \text{ kips/bolt} \end{aligned}$$ (*Spec*. Eq. J3-6c)

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(28.4 \text{ kips/bolt})$ $= 21.3 \text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{28.4 \text{ kips/bolt}}{2.00}$ $= 14.2 \text{ kips/bolt}$

The bolt shear strength controls for bolts in the plate.

Because the weld group is designed for the full eccentric moment, the bolt group is designed for shear only.

LRFD	ASD
$n_{min} = \dfrac{R_u}{\phi r_n}$ $= \dfrac{48.0 \text{ kips}}{17.9 \text{ kips/bolt}}$ $= 2.68 \text{ bolts} < 4 \text{ bolts}$ **o.k.**	$n_{min} = \dfrac{R_a}{r_n / \Omega}$ $= \dfrac{32.0 \text{ kips}}{11.9 \text{ kips/bolt}}$ $= 2.69 \text{ bolts} < 4 \text{ bolts}$ **o.k.**

Strength of the Bolted Connection—Beam Web

By inspection, bearing and tearout on the W16×50 beam web will not govern.

Flexural Yielding of Plate

The required flexural strength of the plate is determined as follows:

LRFD	ASD
$M_u = R_u e_x$ $= (48.0 \text{ kips})(5.85 \text{ in.})$ $= 281 \text{ kip-in.}$	$M_a = R_a e_x$ $= (32.0 \text{ kips})(5.85 \text{ in.})$ $= 187 \text{ kip-in.}$

The available flexural strength of the plate is determined as follows:

LRFD	ASD
$\phi = 0.90$ $\phi M_n = \phi F_y Z_x$ $= 0.90(36\text{ ksi})\left[\dfrac{(\tfrac{3}{8}\text{ in.})(12\text{ in.})^2}{4}\right]$ $= 437\text{ kip-in.} > 281\text{ kip-in.}$ **o.k.**	$\Omega = 1.67$ $\dfrac{M_n}{\Omega} = \dfrac{F_y Z_x}{\Omega}$ $= \dfrac{36\text{ ksi}}{1.67}\left[\dfrac{(\tfrac{3}{8}\text{ in.})(12\text{ in.})^2}{4}\right]$ $= 291\text{ kip-in.} > 187\text{ kip-in.}$ **o.k.**

Shear Strength of Plate

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the plate is determined as follows:

$$\begin{aligned} A_{gv} &= lt \\ &= (12\text{ in.})(\tfrac{3}{8}\text{ in.}) \\ &= 4.50\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_n &= 0.60F_y A_{gv} \\ &= 0.60(36\text{ ksi})(4.50\text{ in.}^2) \\ &= 97.2\text{ kips} \end{aligned}$$ (*Spec*. Eq. J4-3)

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(97.2\text{ kips})$ $= 97.2\text{ kips} > 48.0\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{97.2\text{ kips}}{1.50}$ $= 64.8\text{ kips} > 32.0\text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2, the available shear rupture strength of the plate is determined using the net area determined in accordance with AISC *Specification* Section B4.3b.

$$\begin{aligned} A_{nv} &= [d - n(d_h + \tfrac{1}{16}\text{ in.})]t \\ &= [12\text{ in.} - 4(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})](\tfrac{3}{8}\text{ in.}) \\ &= 3.19\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_n &= 0.60F_u A_{nv} \\ &= 0.60(58\text{ ksi})(3.19\text{ in.}^2) \\ &= 111\text{ kips} \end{aligned}$$ (*Spec*. Eq. J4-4)

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(111\text{ kips})$ $= 83.3\text{ kips} > 48.0\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{111\text{ kips}}{2.00}$ $= 55.5\text{ kips} > 32.0\text{ kips}$ **o.k.**

Block Shear Rupture of Plate

The nominal strength for the limit state of block shear rupture is given by AISC *Specification* Section J4.3.

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \qquad (Spec.\ \text{Eq. J4-5})$$

The available block shear rupture strength of the plate is determined as follows, using AISC *Manual* Tables 9-3a, 9-3b and 9-3c, and AISC *Specification* Equation J4-5, with $n = 4$, $l_{eh} = l_{ev} = 1\frac{1}{2}$ in., and $U_{bs} = 1.0$.

LRFD	ASD
Tension rupture component from AISC *Manual* Table 9-3a:	Tension rupture component from AISC *Manual* Table 9-3a:
$\dfrac{\phi F_uA_{nt}}{t} = 46.2 \text{ kip/in.}$	$\dfrac{F_uA_{nt}}{\Omega t} = 30.8 \text{ kip/in.}$
Shear yielding component from AISC *Manual* Table 9-3b:	Shear yielding component from AISC *Manual* Table 9-3b:
$\dfrac{\phi 0.60F_yA_{gv}}{t} = 170 \text{ kip/in.}$	$\dfrac{0.60F_yA_{gv}}{\Omega t} = 113 \text{ kip/in.}$
Shear rupture component from AISC *Manual* Table 9-3c:	Shear rupture component from AISC *Manual* Table 9-3c:
$\dfrac{\phi 0.60F_uA_{nv}}{t} = 194 \text{ kip/in.}$	$\dfrac{0.60F_uA_{nv}}{\Omega t} = 129 \text{ kip/in.}$
$\phi R_n = \phi 0.60F_uA_{nv} + \phi U_{bs}F_uA_{nt} \le \phi 0.60F_yA_{gv} + \phi U_{bs}F_uA_{nt}$ $= (\frac{3}{8}\text{ in.})[194 \text{ kip/in.} + (1.0)(46.2 \text{ kip/in.})] \le (\frac{3}{8}\text{ in.})[170 \text{ kip/in.} + (1.0)(46.2 \text{ kip/in.})]$ $= 90.1 \text{ kips} > 81.1 \text{ kips}$	$\dfrac{R_n}{\Omega} = \dfrac{0.60F_uA_{nv}}{\Omega} + \dfrac{U_{bs}F_uA_{nt}}{\Omega} \le \dfrac{0.60F_yA_{gv}}{\Omega} + \dfrac{U_{bs}F_uA_{nt}}{\Omega}$ $= (\frac{3}{8}\text{ in.})[129 \text{ kip/in.} + (1.0)(30.8 \text{ kip/in.})] \le (\frac{3}{8}\text{ in.})[113 \text{ kip/in.} + (1.0)(30.8 \text{ kip/in.})]$ $= 59.9 \text{ kips} > 53.9 \text{ kips}$
Therefore:	Therefore:
$\phi R_n = 81.1 \text{ kips} > 48.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 53.9 \text{ kips} > 32.0 \text{ kips}$ **o.k.**

Conclusion

The connection is found to be adequate as given for the applied force.

EXAMPLE II.A-22 BOLTED BRACKET PLATE DESIGN

Given:

Verify the bracket plate to support the loads as shown in Figure II.A-22-1 (loads are per bracket plate). Use ASTM A36 plate. Assume the column has sufficient available strength for the connection.

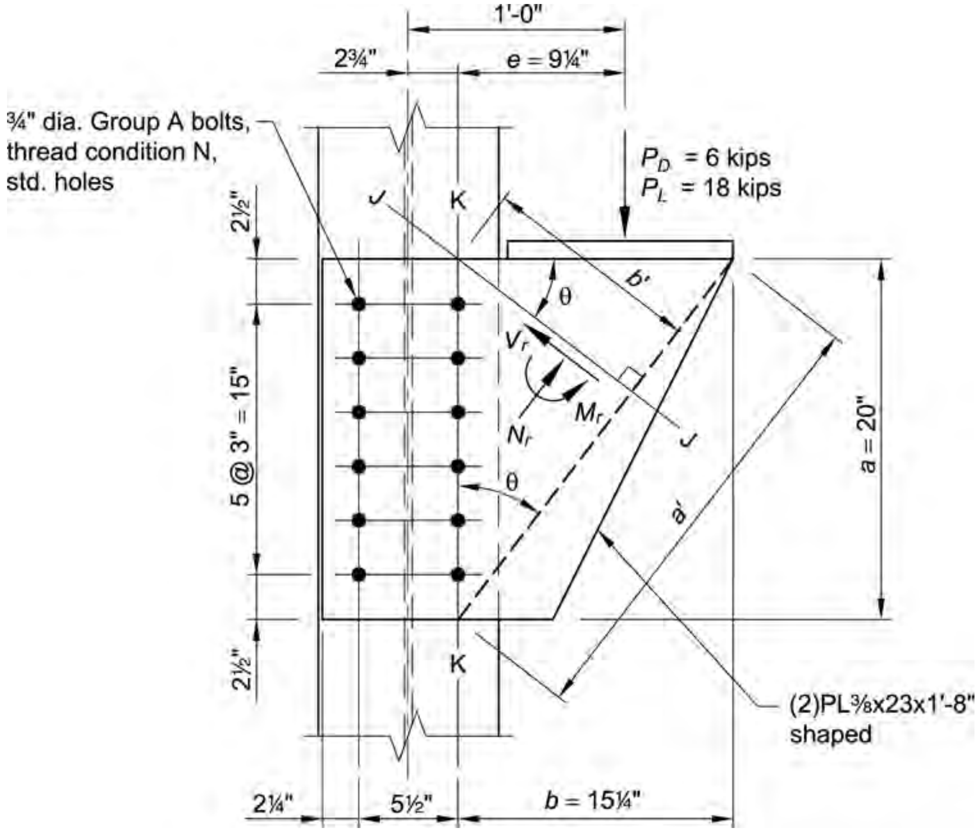


Fig. II.A-22-1. Connection geometry for Example II.A-22.

Solution:

For discussion of the design of a bracket plate, see AISC *Manual* Part 15.

From AISC *Manual* Table 2-5, the material properties are as follows:

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_u = 1.2(6 \text{ kips}) + 1.6(18 \text{ kips})$ $= 36.0 \text{ kips}$	$P_a = 6 \text{ kips} + 18 \text{ kips}$ $= 24.0 \text{ kips}$

From the geometry shown in Figure II.A-22-1 and AISC *Manual* Figure 15-2(b):

$a = 20$ in.
$b = 15\frac{1}{4}$ in.
$e = 9\frac{1}{4}$ in.

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$
$$= \tan^{-1}\left(\frac{15\frac{1}{4}\text{ in.}}{20\text{ in.}}\right)$$
$$= 37.3°$$

$$a' = \frac{a}{\cos\theta} \quad \text{(\textit{Manual} Eq. 15-17)}$$
$$= \frac{20\text{ in.}}{\cos 37.3°}$$
$$= 25.1\text{ in.}$$

$$b' = a\sin\theta$$
$$= (20\text{ in.})(\sin 37.3°)$$
$$= 12.1\text{ in.}$$

Strength of the Bolted Connection—Plate

From the Commentary to AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the invividual strengths of the individual fasteners, which may be taken as the lesser of the fastener shear strength per AISC *Specification* Section J3.6, the bearing strength at the bolt hole per AISC *Specification* Section J3.10, or the tearout strength at the bolt hole per AISC *Specification* Section J3.10.

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) is:

LRFD	ASD
$\phi r_n = 17.9\text{ kips/bolt}$	$\frac{r_n}{\Omega} = 11.9\text{ kips/bolt}$

The available bearing and tearout strength of the plate is determined using AISC *Manual* Table 7-5 conservatively using l_e = 2 in. Note: The available bearing and tearout strength based on edge distance will conservatively be used for all of the bolts.

LRFD	ASD
$\phi r_n = (78.3\text{ kip/in.})(\frac{3}{8}\text{ in.})$ $= 29.4\text{ kips/bolt}$	$\frac{r_n}{\Omega} = (52.2\text{ kip/in.})(\frac{3}{8}\text{ in.})$ $= 19.6\text{ kips/bolt}$

Bolt shear strength controls for bolts in the plate.

The strength of the bolt group is determined by interpolating AISC *Manual* Table 7-8 with Angle = 0°, a 5½ in. gage with s = 3 in., e_x = 12 in. and n = 6:

$C = 4.53$

LRFD	ASD
$C_{min} = \frac{P_u}{\phi r_n}$ $= \frac{36.0 \text{ kips}}{17.9 \text{ kips/bolt}}$ $= 2.01 < 4.53$ **o.k.**	$C_{min} = \frac{\Omega P_a}{r_n}$ $= \frac{24.0 \text{ kips}}{11.9 \text{ kips/bolt}}$ $= 2.02 < 4.53$ **o.k.**

Flexural Yielding of Bracket Plate on Section K-K

The required flexural yielding strength of the plate at Section K-K is determined from AISC *Manual* Equation 15-1a or 15-1b as follows:

LRFD	ASD
$M_u = P_u e$ $= (36.0 \text{ kips})(9\frac{1}{4} \text{ in.})$ $= 333 \text{ kip-in.}$	$M_a = P_a e$ $= (24.0 \text{ kips})(9\frac{1}{4} \text{ in.})$ $= 222 \text{ kip-in.}$

The available flexural yielding strength of the bracket plate is determined as follows:

$$M_n = F_y Z \qquad \text{(\textit{Manual} Eq. 15-2)}$$
$$= (36 \text{ ksi})\left[\frac{(\frac{3}{8} \text{ in.})(20 \text{ in.})^2}{4}\right]$$
$$= 1{,}350 \text{ kip-in.}$$

LRFD	ASD
$\phi = 0.90$ $\phi M_n = 0.90(1{,}350 \text{ kip-in.})$ $= 1{,}220 \text{ kip-in.} > 333 \text{ kip-in.}$ **o.k.**	$\Omega = 1.67$ $\frac{M_n}{\Omega} = \frac{1{,}350 \text{ kip-in.}}{1.67}$ $= 808 \text{ kip-in.} > 222 \text{ kip-in.}$ **o.k.**

Flexural Rupture of Bracket Plate on Section K-K

From AISC *Manual* Table 15-3, for a ⅜-in.-thick bracket plate, with ¾-in. bolts and six bolts in a row, Z_{net} = 21.5 in.^{3} Note that AISC *Manual* Table 15-3 conservatively considers l_{ev} = 1½ in. for holes spaced at 3 in.

The available flexural yielding rupture of the bracket plate at Section K-K is determined as follows:

$$M_n = F_u Z_{net} \qquad \text{(\textit{Manual} Eq. 15-3)}$$
$$= (58 \text{ ksi})(21.5 \text{ in.}^3)$$
$$= 1{,}250 \text{ kip-in.}$$

LRFD	ASD
$\phi = 0.75$ $\phi M_n = 0.75(1{,}250 \text{ kip-in.})$ $= 938 \text{ kip-in.} > 333 \text{ kip-in.}$ **o.k.**	$\Omega = 2.00$ $\frac{M_n}{\Omega} = \frac{1{,}250 \text{ kip-in.}}{2.00}$ $= 625 \text{ kip-in.} > 222 \text{ kip-in.}$ **o.k.**

Shear Yielding of Bracket Plate on Section J-J

The required shear strength of the bracket plate on Section J-J is determined from AISC *Manual* Equation 15-6a or 15-6b as follows:

LRFD	ASD
$V_u = P_u \sin\theta$ $= (36.0 \text{ kips})(\sin 37.3°)$ $= 21.8 \text{ kips}$	$V_a = P_a \sin\theta$ $= (24.0 \text{ kips})(\sin 37.3°)$ $= 14.5 \text{ kips}$

The available shear yielding strength of the plate is determined as follows:

$$V_n = 0.6F_y tb' \qquad \text{(Manual Eq. 15-7)}$$
$$= 0.6(36 \text{ ksi})(\tfrac{3}{8} \text{ in.})(12.1 \text{ in.})$$
$$= 98.0 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi V_n = 1.00(98.0 \text{ kips})$ $= 98.0 \text{ kips} > 21.8 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{V_n}{\Omega} = \dfrac{98.0 \text{ kips}}{1.50}$ $= 65.3 \text{ kips} > 14.5 \text{ kips}$ **o.k.**

Local Yielding and Local Buckling of Bracket Plate on Section J-J (see Figure II.A-22-1)

For local yielding:

$$F_{cr} = F_y \qquad \text{(Manual Eq. 15-13)}$$
$$= 36 \text{ ksi}$$

For local buckling:

$$F_{cr} = QF_y \qquad \text{(Manual Eq. 15-14)}$$

where

$$\lambda = \frac{\left(\frac{b'}{t}\right)\sqrt{F_y}}{5\sqrt{475 + 1{,}120\left(\frac{b'}{a'}\right)^2}} \qquad \text{(Manual Eq. 15-18)}$$
$$= \frac{\left(\frac{12.1 \text{ in.}}{\tfrac{3}{8} \text{ in.}}\right)\sqrt{36 \text{ ksi}}}{5\sqrt{475 + 1{,}120\left(\frac{12.1 \text{ in.}}{25.1 \text{ in.}}\right)^2}}$$
$$= 1.43$$

Because $1.41 < \lambda$:

$$Q = \frac{1.30}{\lambda^2} = \frac{1.30}{(1.43)^2} = 0.636$$ (*Manual* Eq. 15-16)

$$F_{cr} = QF_y = 0.636(36 \text{ ksi}) = 22.9 \text{ ksi}$$ (*Manual* Eq. 15-14)

Local buckling controls over local yielding.

Interaction of Normal and Flexural Strengths

Check that *Manual* Equation 15-10 is satisfied:

LRFD	ASD
$N_u = P_u \cos\theta$ (*Manual* Eq. 15-9a) $= (36.0 \text{ kips})(\cos 37.3°)$ $= 28.6 \text{ kips}$	$N_a = P_a \cos\theta$ (*Manual* Eq. 15-9b) $= (24.0 \text{ kips})(\cos 37.3°)$ $= 19.1 \text{ kips}$
$N_n = F_{cr} t b'$ (*Manual* Eq. 15-11) $= (22.9 \text{ ksi})(3/8 \text{ in.})(12.1 \text{ in.})$ $= 104 \text{ kips}$	$N_n = F_{cr} t b'$ (*Manual* Eq. 15-11) $= (22.9 \text{ ksi})(3/8 \text{ in.})(12.1 \text{ in.})$ $= 104 \text{ kips}$
$\phi = 0.90$	$\Omega = 1.67$
$N_c = \phi N_n$ $= 0.90(104 \text{ kips})$ $= 93.6 \text{ kips}$	$N_c = \dfrac{N_n}{\Omega}$ $= \dfrac{104 \text{ kips}}{1.67}$ $= 62.3 \text{ kips}$
$M_u = P_u e - N_u \left(\dfrac{b'}{2}\right)$ (*Manual* Eq. 15-8a) $= (36.0 \text{ kips})(9\tfrac{1}{4} \text{ in.}) - (28.6 \text{ kips})\left(\dfrac{12.1 \text{ in.}}{2}\right)$ $= 160 \text{ kip-in.}$	$M_a = P_a e - N_a \left(\dfrac{b'}{2}\right)$ (*Manual* Eq. 15-8b) $= (24.0 \text{ kips})(9\tfrac{1}{4} \text{ in.}) - (19.1 \text{ kips})\left(\dfrac{12.1 \text{ in.}}{2}\right)$ $= 106 \text{ kip-in.}$
$M_n = \dfrac{F_{cr} t b'^2}{4}$ (*Manual* Eq. 15-12) $= \dfrac{(22.9 \text{ ksi})(3/8 \text{ in.})(12.1 \text{ in.})^2}{4}$ $= 314 \text{ kip-in.}$	$M_n = \dfrac{F_{cr} t b'^2}{4}$ (*Manual* Eq. 15-12) $= \dfrac{(22.9 \text{ ksi})(3/8 \text{ in.})(12.1 \text{ in.})^2}{4}$ $= 314 \text{ kip-in.}$

LRFD	ASD
$M_c = \phi M_n$ $= 0.90(314 \text{ kip-in.})$ $= 283 \text{ kip-in.}$	$M_c = \dfrac{M_n}{\Omega}$ $= \dfrac{314 \text{ kip-in.}}{1.67}$ $= 188 \text{ kip-in.}$
$\dfrac{N_r}{N_c} + \dfrac{M_r}{M_c} \leq 1.0$ (*Manual* Eq. 15-10) $\dfrac{28.6 \text{ kips}}{93.6 \text{ kips}} + \dfrac{160 \text{ kip-in.}}{283 \text{ kip-in.}} = 0.871 < 1.0$ **o.k.**	$\dfrac{N_r}{N_c} + \dfrac{M_r}{M_c} \leq 1.0$ (*Manual* Eq. 15-10) $\dfrac{19.1 \text{ kips}}{62.3 \text{ kips}} + \dfrac{106 \text{ kip-in.}}{188 \text{ kip-in.}} = 0.870 < 1.0$ **o.k.**

Shear Strength of Bracket Plate on Section K-K

From AISC *Specification* Section J4.2, the available shear yielding strength of the plate on Section K-K is determined as follows:

$$A_{gv} = at$$
$$= (20 \text{ in.})(3/8 \text{ in.})$$
$$= 7.50 \text{ in.}^2$$

$$R_n = 0.60F_y A_{gv} \qquad \text{(\textit{Spec}. Eq. J4-3)}$$
$$= 0.60(36 \text{ ksi})(7.50 \text{ in.}^2)$$
$$= 162 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(162 \text{ kips})$ $= 162 \text{ kips} > 36.0 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{162 \text{ kips}}{1.50}$ $= 108 \text{ kips} > 24.0 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2, the available shear rupture strength of the plate on Section K-K is determined as follows:

$$A_{nv} = \left[a - n(15/16 \text{ in.} + 1/16 \text{ in.})\right]t$$
$$= \left[20 \text{ in.} - 6(13/16 \text{ in.} + 1/16 \text{ in.})\right](3/8 \text{ in.})$$
$$= 5.53 \text{ in.}^2$$

$$R_n = 0.60F_u A_{nv} \qquad \text{(\textit{Spec}. Eq. J4-4)}$$
$$= 0.60(58 \text{ ksi})(5.53 \text{ in.}^2)$$
$$= 192 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(192 \text{ kips})$ $= 144 \text{ kips} > 36.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{192 \text{ kips}}{2.00}$ $= 96.0 \text{ kips} > 24.0 \text{ kips}$ **o.k.**

Conclusion

The connection is found to be adequate as given for the applied force.

EXAMPLE II.A-23 WELDED BRACKET PLATE DESIGN

Given:

Verify the welded bracket plate to support the loads as shown in Figure II.A-23-1 (loads are resisted equally by the two bracket plates). Use ASTM A36 plate and 70-ksi electrodes. Assume the column has sufficient available strength for the connection.

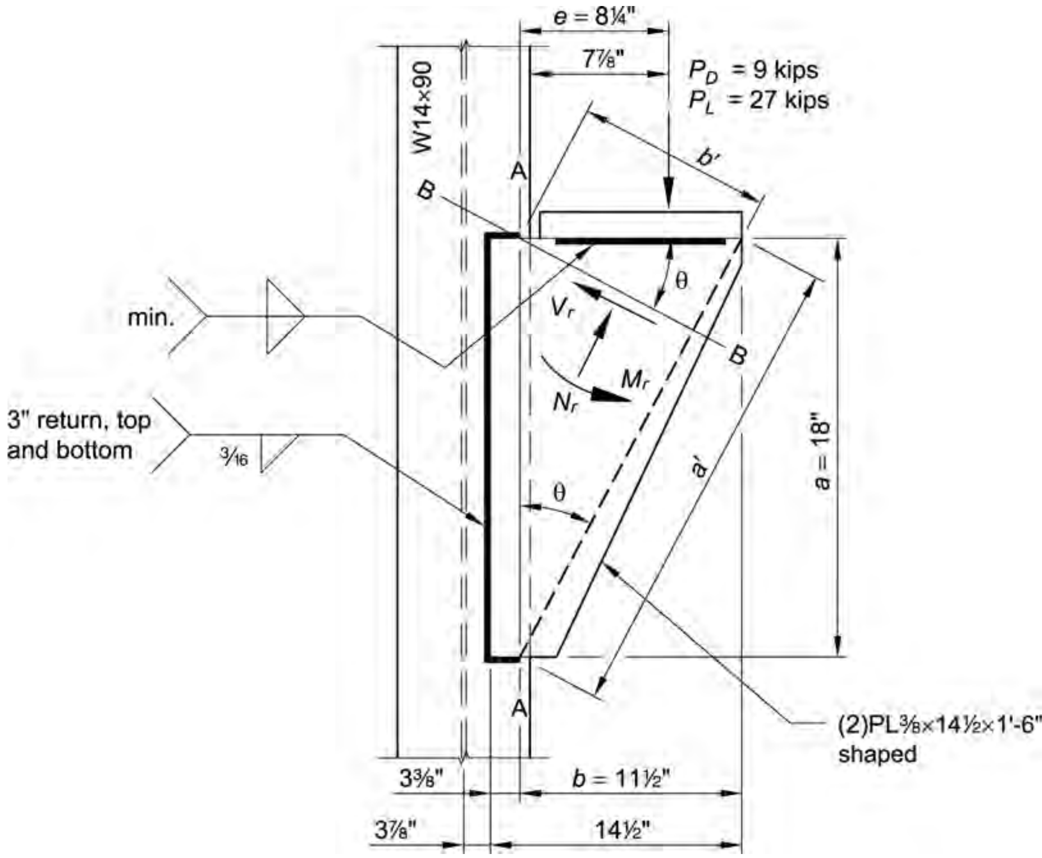


Fig. II.A-23-1. Connection geometry for Example II.A-23.

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From ASCE/SEI 7, Chapter 2, the required strength to be resisted by the bracket plates is:

LRFD	ASD
$P_u = 1.2(9\text{ kips}) + 1.6(27\text{ kips})$ $= 54.0\text{ kips}$	$P_a = 9\text{ kips} + 27\text{ kips}$ $= 36.0\text{ kips}$

From the geometry shown in Figure II.A-23-1 and AISC *Manual* Figure 15-2(b):

$a = 18$ in.

$b = 11\frac{1}{2}$ in.

$e = 8\frac{1}{4}$ in.

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$
$$= \tan^{-1}\left(\frac{11\frac{1}{2}\text{ in.}}{18\text{ in.}}\right)$$
$$= 32.6°$$

$$a' = \frac{a}{\cos\theta}$$ (*Manual* Eq. 15-17)
$$= \frac{18\text{ in.}}{\cos 32.6°}$$
$$= 21.4\text{ in.}$$

$$b' = a\sin\theta$$
$$= (18\text{ in.})(\sin 32.6°)$$
$$= 9.70\text{ in.}$$

Shear Yielding of Bracket Plate at Section A-A

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the bracket plate at Section A-A, is determined as follows:

$$A_{gv} = (2\text{ plates})at$$
$$= (2\text{ plates})(18\text{ in.})(\tfrac{3}{8}\text{ in.})$$
$$= 13.5\text{ in.}^2$$

$$R_n = 0.60F_yA_{gv}$$ (*Spec*. Eq. J4-3)
$$= 0.60(36\text{ ksi})(13.5\text{ in.}^2)$$
$$= 292\text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(292\text{ kips})$ $= 292\text{ kips} > 54.0\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{292\text{ kips}}{1.50}$ $= 195\text{ kips} > 36.0\text{ kips}$ **o.k.**

Shear rupture strength is adequate by insection.

Flexural Yielding of Bracket Plate at Section A-A

The required flexural strength of the bracket plate is determined using AISC *Manual* Equation 15-1a or 15-1b as follows:

LRFD	ASD
$M_u = P_u e$ $= (54.0 \text{ kips})(8\tfrac{1}{4} \text{ in.})$ $= 446 \text{ kip-in.}$	$M_a = P_a e$ $= (36.0 \text{ kips})(8\tfrac{1}{4} \text{ in.})$ $= 297 \text{ kip-in.}$

The available flexural strength of the bracket plate is determined using AISC *Manual* Equation 15-2, as follows:

$$M_n = (2 \text{ plates}) F_y Z \qquad \text{(from } \textit{Manual} \text{ Eq. 15-2)}$$

$$= (2 \text{ plates})(36 \text{ ksi})\left[\frac{(\tfrac{3}{8} \text{ in.})(18 \text{ in.})^2}{4}\right]$$

$$= 2{,}190 \text{ kip-in.}$$

LRFD	ASD
$\phi = 0.90$ $\phi M_n = 0.90(2{,}190 \text{ kip-in.})$ $= 1{,}970 \text{ kip-in.} > 446 \text{ kip-in.}$ **o.k.**	$\Omega = 1.67$ $\dfrac{M_n}{\Omega} = \dfrac{2{,}190 \text{ kip-in.}}{1.67}$ $= 1{,}310 \text{ kip-in.} > 297 \text{ kip-in.}$ **o.k.**

Weld Strength

Try a C-shaped weld with kl = 3 in. and l = 18 in.

$$k = \frac{kl}{l}$$

$$= \frac{3 \text{ in.}}{18 \text{ in.}}$$

$$= 0.167$$

$$xl = \frac{(kl)^2}{2(kl) + l}$$

$$= \frac{(3 \text{ in.})^2}{2(3 \text{ in.}) + 18 \text{ in.}}$$

$$= 0.375 \text{ in.}$$

$$al = 11\tfrac{1}{4} \text{ in.} - 0.375 \text{ in.}$$

$$= 10.9 \text{ in.}$$

$$a = \frac{al}{l}$$

$$= \frac{10.9 \text{ in.}}{18 \text{ in.}}$$

$$= 0.606$$

Interpolate AISC *Manual* Table 8-8 using Angle = 0°, k = 0.167, and a = 0.606.

$$C = 1.46$$

From AISC *Manual* Table 8-3:

$C_1 = 1.00$ (for E70 electrodes)

The required weld size is determined using AISC *Manual* Equation 8-21, as follows:

LRFD	ASD
$\phi = 0.75$ $D_{min} = \dfrac{P_u}{\phi C C_1 l}$ $= \dfrac{54.0 \text{ kips}}{0.75(1.46)(1.00)(18 \text{ in.})(2 \text{ plates})}$ $= 1.37 \rightarrow 3$ sixteenths	$\Omega = 2.00$ $D_{min} = \dfrac{\Omega P_a}{C C_1 l}$ $= \dfrac{2.00(36.0 \text{ kips})}{(1.46)(1.00)(18 \text{ in.})(2 \text{ plates})}$ $= 1.37 \rightarrow 3$ sixteenths

From AISC *Specification* Section J2.2b(b)(2), the maximum weld size is:

$$\begin{aligned} w_{max} &= \tfrac{3}{8} \text{ in.} - \tfrac{1}{16} \text{ in.} \\ &= \tfrac{5}{16} \text{ in.} > \tfrac{3}{16} \text{ in.} \quad \textbf{o.k.} \end{aligned}$$

From AISC *Specification* Table J2.4, the minimum weld size is:

$$w_{min} = \tfrac{3}{16} \text{ in.}$$

Shear Yielding Strength of Bracket at Section B-B

The required shear strength of the bracket plate at Section B-B is determined from AISC *Manual* Equations 15-6a or 15-6b as follows:

LRFD	ASD
$V_u = P_u \sin\theta$ $= (54.0 \text{ kips})(\sin 32.6°)$ $= 29.1 \text{ kips}$	$V_a = P_a \sin\theta$ $= (36.0 \text{ kips})(\sin 32.6°)$ $= 19.4 \text{ kips}$

From AISC *Manual* Part 15, the available shear yielding strength of the bracket plate at Section A-A is determined as follows:

$$\begin{aligned} V_n &= (2 \text{ plates})0.6F_y t b' \\ &= (2 \text{ plates})(0.6)(36 \text{ ksi})(\tfrac{3}{8} \text{ in.})(9.70 \text{ in.}) \\ &= 157 \text{ kips} \end{aligned}$$ (from *Manual* Eq. 15-7)

LRFD	ASD
$\phi = 1.00$ $\phi V_n = 1.00(157 \text{ kips})$ $= 157 \text{ kips} > 29.1 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{V_n}{\Omega} = \dfrac{157 \text{ kips}}{1.50}$ $= 105 \text{ kips} > 19.4 \text{ kips}$ **o.k.**

Bracket Plate Normal and Flexural Strength at Section B-B

From AISC *Manual* Part 15, the required strength of the bracket plate at Section B-B is determined as follows:

LRFD	ASD
$N_u = P_u \cos\theta$ (*Manual* Eq. 15-9a) $= (54.0\text{ kips})(\cos 32.6°)$ $= 45.5\text{ kips}$	$N_a = P_a \cos\theta$ (*Manual* Eq. 15-9b) $= (36.0\text{ kips})(\cos 32.6°)$ $= 30.3\text{ kips}$
$M_u = P_u e - N_u\left(\frac{b'}{2}\right)$ (*Manual* Eq. 15-8a) $= (54.0\text{ kips})(8\tfrac{1}{4}\text{ in.}) - (45.5\text{ kips})\left(\frac{9.70\text{ in.}}{2}\right)$ $= 225\text{ kip-in.}$	$M_a = P_a e - N_a\left(\frac{b'}{2}\right)$ (*Manual* Eq. 15-8b) $= (36.0\text{ kips})(8\tfrac{1}{4}\text{ in.}) - (30.3\text{ kips})\left(\frac{9.70\text{ in.}}{2}\right)$ $= 150\text{ kip-in.}$

For local yielding at the bracket plate:

$$F_{cr} = F_y = 36\text{ ksi} \quad \text{(\textit{Manual} Eq. 15-13)}$$

For local buckling of the bracket plate:

$$F_{cr} = QF_y \quad \text{(\textit{Manual} Eq. 15-14)}$$

where

$$\lambda = \frac{\left(\frac{b'}{t}\right)\sqrt{F_y}}{5\sqrt{475 + 1{,}120\left(\frac{b'}{a'}\right)^2}} \quad \text{(\textit{Manual} Eq. 15-18)}$$

$$= \frac{\left(\frac{9.70\text{ in.}}{\tfrac{3}{8}\text{ in.}}\right)\sqrt{36\text{ ksi}}}{5\sqrt{475 + 1{,}120\left(\frac{9.70\text{ in.}}{21.4\text{ in.}}\right)^2}}$$

$$= 1.17$$

Since $0.70 < \lambda \le 1.41$:

$$Q = 1.34 - 0.486\lambda \quad \text{(\textit{Manual} Eq. 15-15)}$$
$$= 1.34 - 0.486(1.17)$$
$$= 0.771$$

$$F_{cr} = QF_y \quad \text{(\textit{Manual} Eq. 15-14)}$$
$$= 0.771(36\text{ ksi})$$
$$= 27.8\text{ ksi}$$

Therefore; local buckling governs over yielding.

The nominal strength of the bracket plate for the limit states of local yielding and local buckling is:

$$N_n = (2 \text{ plates}) F_{cr} t b'$$ (from *Manual* Eq. 15-11)

$$= (2 \text{ plates})(27.8 \text{ ksi})(3/8 \text{ in.})(9.70 \text{ in.})$$
$$= 202 \text{ kips}$$

The nominal flexural strength of the bracket plate for the limit states of local yielding and local buckling is:

$$M_n = (2 \text{ plates}) \frac{F_{cr} t b'^2}{4}$$ (from *Manual* Eq. 15-12)

$$= (2 \text{ plates}) \frac{(27.8 \text{ ksi})(3/8 \text{ in.})(9.70 \text{ in.})^2}{4}$$
$$= 490 \text{ kip-in.}$$

LRFD	ASD
$M_r = M_u$ $= 225$ kip-in. $\phi = 0.90$ $M_c = \phi M_n$ $= 0.90(490 \text{ kip-in.})$ $= 441 \text{ kip-in.} > 225 \text{ kip-in.}$ **o.k.** $N_r = N_u$ $= 45.5$ kips $N_c = \phi N_n$ $= 0.90(202 \text{ kips})$ $= 182 \text{ kips} > 45.5 \text{ kips}$ **o.k.** $\frac{N_r}{N_c} + \frac{M_r}{M_c} \leq 1.0$ (*Manual* Eq. 15-10) $\frac{45.5 \text{ kips}}{182 \text{ kips}} + \frac{225 \text{ kip-in.}}{441 \text{ kip-in.}} = 0.760 < 1.0$ **o.k.**	$M_r = M_a$ $= 150$ kip-in. $\Omega = 1.67$ $M_c = \frac{M_n}{\Omega}$ $= \frac{490 \text{ kip-in.}}{1.67}$ $= 293 \text{ kip-in.} > 150 \text{ kip-in.}$ **o.k.** $N_r = N_a$ $= 30.3$ kips $N_c = \frac{N_n}{\Omega}$ $= \frac{202 \text{ kips}}{1.67}$ $= 121 \text{ kips} > 30.3 \text{ kips}$ **o.k.** $\frac{N_r}{N_c} + \frac{M_r}{M_c} \leq 1.0$ (*Manual* Eq. 15-10) $\frac{30.3 \text{ kips}}{121 \text{ kips}} + \frac{150 \text{ kip-in.}}{293 \text{ kip-in.}} = 0.762 < 1.0$ **o.k.**

EXAMPLE II.A-24 ECCENTRICALLY LOADED BOLT GROUP (IC METHOD)

Given:

Use AISC *Manual* Table 7-8 to determine the largest eccentric force, acting vertically (0° angle) and at a 15° angle, which can be supported by the available shear strength of the bolts using the instantaneous center of rotation method. Assume that bolt shear controls over bearing and tearout.

Solution A (θ = 0°):

Assume the load is vertical (θ = 0°), as shown in Figure II.A-24-1:

From AISC *Manual* Table 7-1, the available shear strength per bolt for ⅞-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in single shear is:

LRFD	ASD
$\phi r_n = 24.3\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = 16.2\text{ kips/bolt}$

The available strength of the bolt group is determined using AISC *Manual* Table 7-8, with Angle = 0°, a 5½-in. gage with s = 3 in., e_x = 16 in., and n = 6:

$$C = 3.55$$

LRFD	ASD
$\phi R_n = C\phi r_n$ (*Manual* Eq. 7-16) $= 3.55(24.3\text{ kips/bolt})$ $= 86.3\text{ kips}$ Thus, P_u must be less than or equal to 86.3 kips.	$\dfrac{R_n}{\Omega} = C\dfrac{r_n}{\Omega}$ (*Manual* Eq. 7-16) $= 3.55(16.2\text{ kips/bolt})$ $= 57.5\text{ kips}$ Thus, P_a must be less than or equal to 57.5 kips.

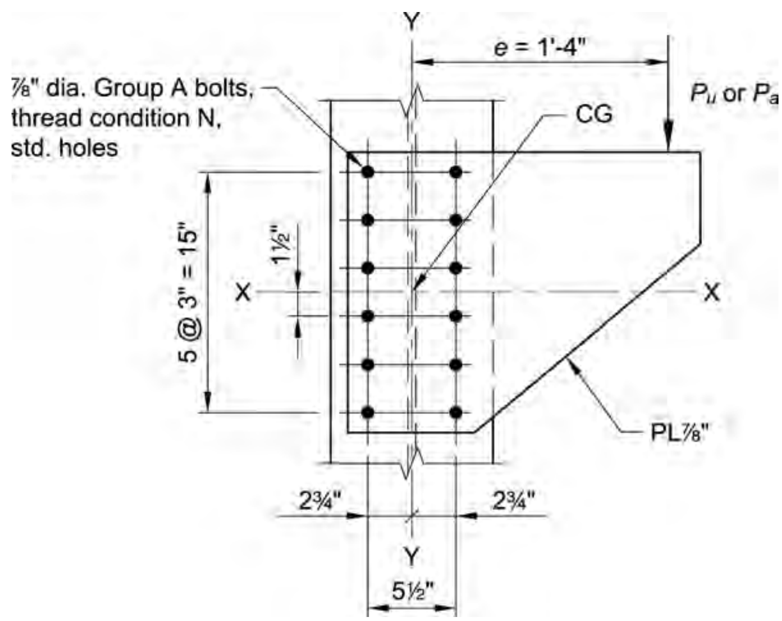


Fig. II.A-24-1. Connection geometry for Example II.A-24—Solution A (θ = 0°).

Note: The eccentricity of the load significantly reduces the shear strength of the bolt group.

Solution B (θ = 15°):

Assume the load acts at an angle of 15° with respect to vertical (θ = 15°), as shown in Figure II.A-24-2:

$$\begin{aligned} e_x &= 16 \text{ in.} + (9 \text{ in.})(\tan 15°) \\ &= 18.4 \text{ in.} \end{aligned}$$

The available strength of the bolt group is determined interpolating from AISC *Manual* Table 7-8, with Angle = 15°, a 5½-in. gage with s = 3 in., e_x = 18.4 in., and n = 6:

$$C = 3.21$$

LRFD	ASD
$\phi R_n = C\phi r_n$ (*Manual* Eq. 7-16) $= 3.21(24.3 \text{ kips/bolt})$ $= 78.0 \text{ kips}$ Thus, P_u must be less than or equal to 78.0 kips.	$\frac{R_n}{\Omega} = C\frac{r_n}{\Omega}$ (*Manual* Eq. 7-16) $= 3.21(16.2 \text{ kips/bolt})$ $= 52.0 \text{ kips}$ Thus, P_a must be less than or equal to 52.0 kips.

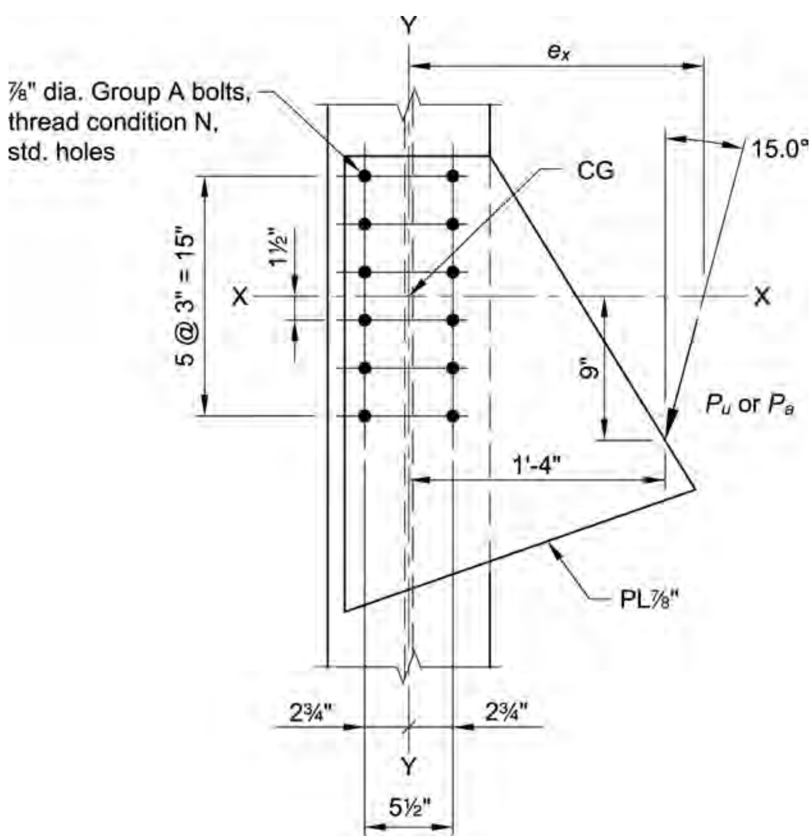


Fig. II.A-24-2. Connection geometry for Example II.A-24—Solution B (θ = 15°).

EXAMPLE II.A-25 ECCENTRICALLY LOADED BOLT GROUP (ELASTIC METHOD)

Given:

Determine the largest eccentric force that can be supported by the available shear strength of the bolts using the elastic method for θ = 0°, as shown in Figure II.A-25-1. Compare the result with that of Example II.A-24. Assume that bolt shear controls over bearing and tearout.

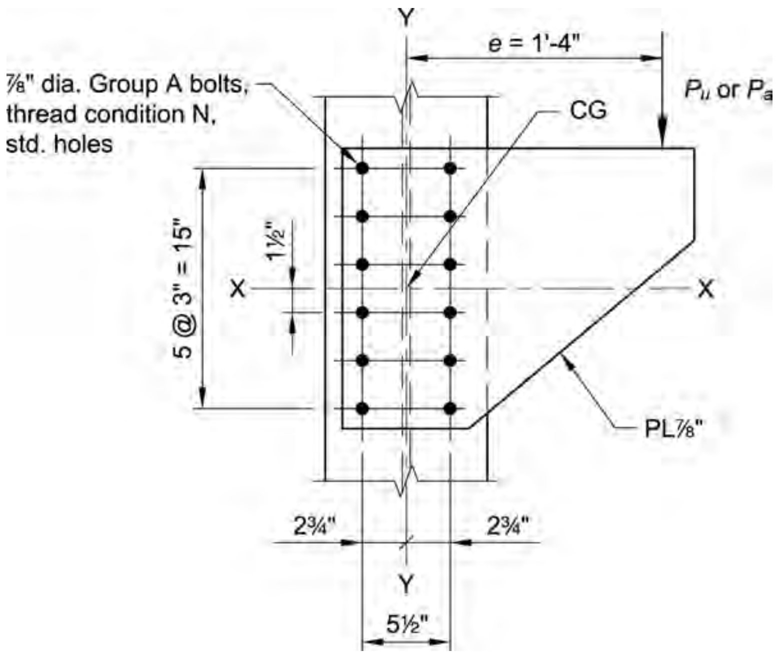


Fig. II.A-25-1. Connection geometry for Example II.A-25.

Solution:

From AISC *Manual* Table 7-1, the available shear strength per bolt for ⅞-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in single shear is:

LRFD	ASD
$\phi r_n = 24.3 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = 16.2 \text{ kips/bolt}$

The direct shear force per bolt is determined as follows:

LRFD	ASD
$r_{pxu} = 0$ $r_{pyu} = \dfrac{P_u}{n}$ (from *Manual* Eq. 7-2a) $= \dfrac{P_u}{12}$	$r_{pxa} = 0$ $r_{pya} = \dfrac{P_a}{n}$ (from *Manual* Eq. 7-2b) $= \dfrac{P_a}{12}$

Additional shear force due to eccentricity is determined as follows:

The polar moment of inertia of the bolt group is:

$$I_x \approx \Sigma y^2$$
$$= 4(7.50 \text{ in.})^2 + 4(4.50 \text{ in.})^2 + 4(1.50 \text{ in.})^2$$
$$= 315 \text{ in.}^4/\text{in.}^2$$

$$I_y \approx \Sigma x^2$$
$$= 12(2.75 \text{ in.})^2$$
$$= 90.8 \text{ in.}^4/\text{in.}^2$$

$$I_p = I_x + I_y$$
$$= 315 \text{ in.}^4/\text{in.}^2 + 90.8 \text{ in.}^4/\text{in.}^2$$
$$= 406 \text{ in.}^4/\text{in.}^2$$

LRFD	ASD
$r_{mxu} = \dfrac{P_u e c_y}{I_p}$ (*Manual* Eq. 7-6a) $= \dfrac{P_u (16.0 \text{ in.})(7.50 \text{ in.})}{406 \text{ in.}^4/\text{in.}^2}$ $= 0.296P_u$	$r_{mxa} = \dfrac{P_a e c_y}{I_p}$ (*Manual* Eq. 7-6b) $= \dfrac{P_a (16.0 \text{ in.})(7.50 \text{ in.})}{406 \text{ in.}^4/\text{in.}^2}$ $= 0.296P_a$
$r_{myu} = \dfrac{P_u e c_x}{I_p}$ (*Manual* Eq. 7-7a) $= \dfrac{P_u (16.0 \text{ in.})(2.75 \text{ in.})}{406 \text{ in.}^4/\text{in.}^2}$ $= 0.108P_u$	$r_{mya} = \dfrac{P_a e c_x}{I_p}$ (*Manual* Eq. 7-7b) $= \dfrac{P_a (16.0 \text{ in.})(2.75 \text{ in.})}{406 \text{ in.}^4/\text{in.}^2}$ $= 0.108P_a$
The resultant shear force is determined from AISC *Manual* Equation 7-8a:	The resultant shear force is determined from AISC *Manual* Equation 7-8b:
$r_u = \sqrt{(r_{pxu} + r_{mxu})^2 + (r_{pyu} + r_{myu})^2}$ $= \sqrt{(0 + 0.296P_u)^2 + \left(\dfrac{P_u}{12} + 0.108P_u\right)^2}$ $= 0.352P_u$	$r_a = \sqrt{(r_{pxa} + r_{mxa})^2 + (r_{pya} + r_{mya})^2}$ $= \sqrt{(0 + 0.296P_a)^2 + \left(\dfrac{P_a}{12} + 0.108P_a\right)^2}$ $= 0.352P_a$
Because r_u must be less than or equal to the available strength:	Because r_a must be less than or equal to the available strength:
$P_u \le \dfrac{\phi r_n}{0.352}$ $= \dfrac{24.3 \text{ kips/bolt}}{0.352}$ $= 69.0 \text{ kips}$	$P_a \le \dfrac{r_n/\Omega}{0.352}$ $= \dfrac{16.2 \text{ kips/bolt}}{0.352}$ $= 46.0 \text{ kips}$

Note: The elastic method, shown here, is more conservative than the instantaneous center of rotation method, shown in Example II.A-24.

EXAMPLE II.A-26 ECCENTRICALLY LOADED WELD GROUP (IC METHOD)

Given:

Use AISC *Manual* Table 8-8 to determine the largest eccentric force, acting vertically and at a 75° angle, that can be supported by the available shear strength of the weld group, using the instantaneous center of rotation method. Use a ⅜-in. fillet weld and 70-ksi electrodes.

Solution A (θ = 0°):

Assume that the load is vertical (θ = 0°), as shown in Figure II.A-26-1.

$$\begin{aligned}k &= \frac{kl}{l}\\ &= \frac{5\text{ in.}}{10\text{ in.}}\\ &= 0.500\end{aligned}$$

$$\begin{aligned}xl &= \frac{(kl)^2}{2(kl)+l}\\ &= \frac{(5\text{ in.})^2}{2(5\text{ in.})+10\text{ in.}}\\ &= 1.25\text{ in.}\end{aligned}$$

$$\begin{aligned}&xl + al = 10.0\text{ in.}\\ &1.25\text{ in.} + a(10\text{ in.}) = 10\text{ in.}\\ &a = 0.875\end{aligned}$$

$$\begin{aligned}e_x &= al\\ &= 0.875(10\text{ in.})\\ &= 8.75\text{ in.}\end{aligned}$$

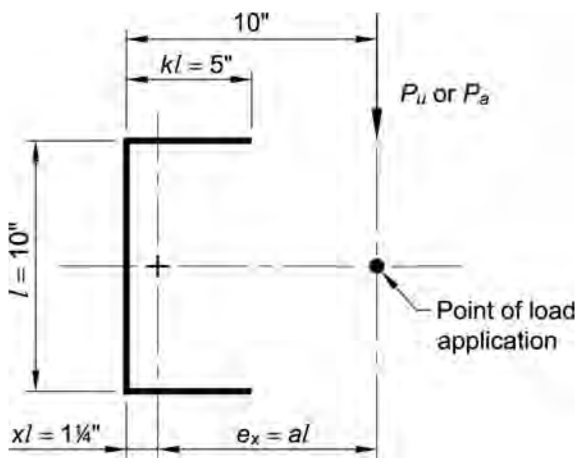


Fig. II.A-26-1. Weld geometry—Solution A (θ = 0°).

The available weld strength is determined using AISC *Manual* Equation 8-21 and interpolating AISC *Manual* Table 8-8, with Angle = 0°, a = 0.875, and k = 0.5:

$$C = 1.88$$

$$C_1 = 1.00 \text{ (from AISC } \textit{Manual} \text{ Table 8-3)}$$

$$R_n = CC_1Dl \qquad (\textit{Manual} \text{ Eq. 8-21})$$
$$= 1.88(1.00)(6)(10 \text{ in.})$$
$$= 113 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(113 \text{ kips})$ $= 84.8 \text{ kips}$ Thus, P_u must be less than or equal to 84.8 kips.	$\phi = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{113 \text{ kips}}{2.00}$ $= 56.5 \text{ kips}$ Thus, P_a must be less than or equal to 56.5 kips.

Note: The eccentricity of the load significantly reduces the shear strength of this weld group as compared to the concentrically loaded case.

Solution B (θ = 75°):

Assume that the load acts at the same point as in Solution A, but at an angle of 75° with respect to vertical (θ = 75°) as shown in Figure II.A-26-2.

As determined in Solution A:

$$k = 0.500$$
$$a = 0.875$$

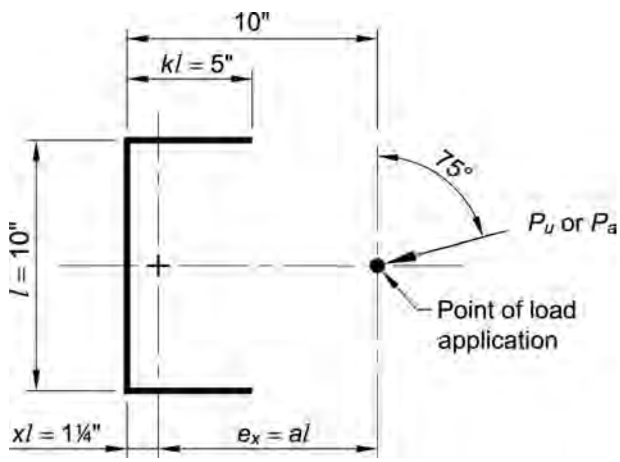


Fig. II.A-26-2. Weld geometry—Solution B (θ = 75°).

The available weld strength is determined using AISC *Manual* Equation 8-21 and interpolating AISC *Manual* Table 8-8, with Angle = 75°, a = 0.875, and k = 0.5:

$$C = 3.45$$

$C_1 = 1.00$ (from AISC *Manual* Table 8-3)

$$\begin{aligned} R_n &= CC_1Dl \\ &= 3.45(1.00)(6)(10 \text{ in.}) \\ &= 207 \text{ kips} \end{aligned}$$ (*Manual* Eq. 8-21)

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(207 \text{ kips})$ $= 155 \text{ kips}$ Thus, P_u must be less than or equal to 155 kips.	$\phi = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{207 \text{ kips}}{2.00}$ $= 104 \text{ kips}$ Thus, P_a must be less than or equal to 104 kips.

EXAMPLE II.A-27 ECCENTRICALLY LOADED WELD GROUP (ELASTIC METHOD)

Given:

Using the elastic method determine the largest eccentric force that can be supported by the available shear strength of the welds in the connection shown in Figure II.A-27-1. Compare the result with that of Example II.A-26. Use ⅜-in. fillet welds and 70-ksi electrodes.

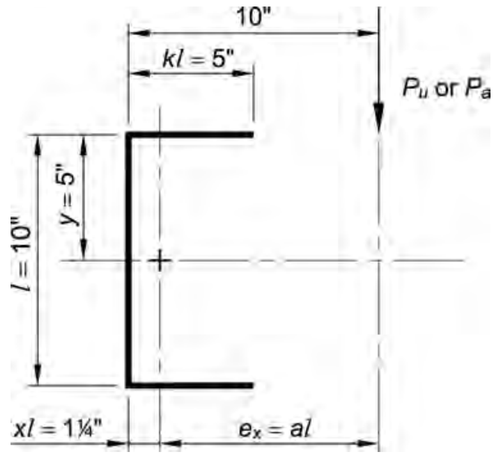


Fig. II.A-27-1. Weld geometry for Example II.A-27.

Solution:

From the weld geometry shown in Figure II.A-27-1 and AISC *Manual* Table 8-8:

$$\begin{aligned} k &= \frac{kl}{l} \\ &= \frac{5 \text{ in.}}{10 \text{ in.}} \\ &= 0.500 \end{aligned}$$

$$\begin{aligned} xl &= \frac{(kl)^2}{2(kl)+l} \\ &= \frac{(5 \text{ in.})^2}{2(5 \text{ in.})+10 \text{ in.}} \\ &= 1.25 \text{ in.} \end{aligned}$$

$$xl + al = 10.0 \text{ in.}$$
$$1.25 \text{ in.} + a(10 \text{ in.}) = 10 \text{ in.}$$
$$a = 0.875$$

$$\begin{aligned} e_x &= al \\ &= 0.875(10 \text{ in.}) \\ &= 8.75 \text{ in.} \end{aligned}$$

Direct Shear Force Per Inch of Weld

LRFD	ASD
$r_{pux} = 0$ $r_{puy} = \dfrac{P_u}{l_{total}}$ (from *Manual* Eq. 8-5a) $= \dfrac{P_u}{20.0 \text{ in.}}$ $= \dfrac{0.0500P_u}{\text{in.}}$	$r_{pax} = 0$ $r_{pay} = \dfrac{P_a}{l_{total}}$ (from *Manual* Eq. 8-5b) $= \dfrac{P_a}{20.0 \text{ in.}}$ $= \dfrac{0.0500P_a}{\text{in.}}$

Additional Shear Force due to Eccentricity

Determine the polar moment of inertia referring to the AISC *Manual* Figure 8-6:

$$I_x = \frac{l^3}{12} + 2(kl)(y)^2$$
$$= \frac{(10 \text{ in.})^3}{12} + 2(5 \text{ in.})(5 \text{ in.})^2$$
$$= 333 \text{ in.}^4/\text{in.}$$

$$I_y = 2\left[\frac{(kl)^3}{12} + (kl)\left(\frac{kl}{2} - xl\right)^2\right] + l(xl)^2$$
$$= 2\left[\frac{(5 \text{ in.})^3}{12} + (5 \text{ in.})(2.50 \text{ in.} - 1\tfrac{1}{4} \text{ in.})^2\right] + (10 \text{ in.})(1\tfrac{1}{4} \text{ in.})^2$$
$$= 52.1 \text{ in.}^4/\text{in.}$$

$$I_p = I_x + I_y$$
$$= 333 \text{ in.}^4/\text{in.} + 52.1 \text{ in.}^4/\text{in.}$$
$$= 385 \text{ in.}^4/\text{in.}$$

LRFD	ASD
$r_{mux} = \dfrac{P_u e_x c_y}{I_p}$ (from *Manual* Eq. 8-9a) $= \dfrac{P_u(8.75 \text{ in.})(5 \text{ in.})}{385 \text{ in.}^4/\text{in.}}$ $= \dfrac{0.114P_u}{\text{in.}}$	$r_{max} = \dfrac{P_a e_x c_y}{I_p}$ (from *Manual* Eq. 8-9b) $= \dfrac{P_a(8.75 \text{ in.})(5 \text{ in.})}{385 \text{ in.}^4/\text{in.}}$ $= \dfrac{0.114P_a}{\text{in.}}$

LRFD	ASD
$r_{muy} = \dfrac{P_u e_x c_x}{I_p}$ (from *Manual* Eq. 8-10a) $= \dfrac{P_u(8.75\text{ in.})(3.75\text{ in.})}{385\text{ in.}^4/\text{in.}}$ $= \dfrac{0.0852P_u}{\text{in.}}$	$r_{may} = \dfrac{P_a e_x c_x}{I_p}$ (from *Manual* Eq. 8-10b) $= \dfrac{P_a(8.75\text{ in.})(3.75\text{ in.})}{385\text{ in.}^4/\text{in.}}$ $= \dfrac{0.0852P_a}{\text{in.}}$
The resultant shear force is determined using AISC *Manual* Equation 8-11a: $r_u = \sqrt{(r_{pux}+r_{mux})^2+(r_{puy}+r_{muy})^2}$ $= \sqrt{\left(0+\dfrac{0.114P_u}{\text{in.}}\right)^2+\left(\dfrac{0.0500P_u}{\text{in.}}+\dfrac{0.0852P_u}{\text{in.}}\right)^2}$ $= \dfrac{0.177P_u}{\text{in.}}$	The resultant shear force is determined using *Manual* Equation 8-11b: $r_a = \sqrt{(r_{pax}+r_{max})^2+(r_{pay}+r_{may})^2}$ $= \sqrt{\left(0+\dfrac{0.114P_a}{\text{in.}}\right)^2+\left(\dfrac{0.0500P_a}{\text{in.}}+\dfrac{0.0852P_a}{\text{in.}}\right)^2}$ $= \dfrac{0.177P_a}{\text{in.}}$
Because r_u must be less than or equal to the available strength: $r_u = \dfrac{0.177P_u}{\text{in.}} \le \phi r_n$	Because r_a must be less than or equal to the available strength: $r_a = \dfrac{0.177P_a}{\text{in.}} \le \dfrac{r_n}{\Omega}$
Solving for P_u and using AISC *Manual* Equation 8-2a: $P_u \le \phi r_n\left(\dfrac{\text{in.}}{0.177}\right)$ $\le (1.392\text{ kip/in.})(6)\left(\dfrac{\text{in.}}{0.177}\right)$ $\le 47.2\text{ kips}$	Solving for P_a and using AISC *Manual* Equation 8-2b: $P_a \le \dfrac{r_n}{\Omega}\left(\dfrac{\text{in.}}{0.177}\right)$ $\le (0.928\text{ kip/in.})(6)\left(\dfrac{\text{in.}}{0.177}\right)$ $\le 31.5\text{ kips}$

Note: The strength of the weld group calculated using the elastic method, as shown here, is significantly less than that calculated using the instantaneous center of rotation method in Example II.A-26.

EXAMPLE II.A-28A ALL-BOLTED SINGLE-ANGLE CONNECTION (BEAM-TO-GIRDER WEB)

Given:

Verify an all-bolted single-angle connection (Case I in AISC *Manual* Table 10-11) between an ASTM A992 W18×35 beam and an ASTM A992 W21×62 girder web, as shown in Figure II.A-28A-1, to support the following beam end reactions:

R_D = 6.5 kips
R_L = 20 kips

The top flange is coped 2 in. deep by 4 in. long, l_{ev} = 1½ in., and l_{eh} = 1¾ in. Use ASTM A36 angle. Use standard angle gages.

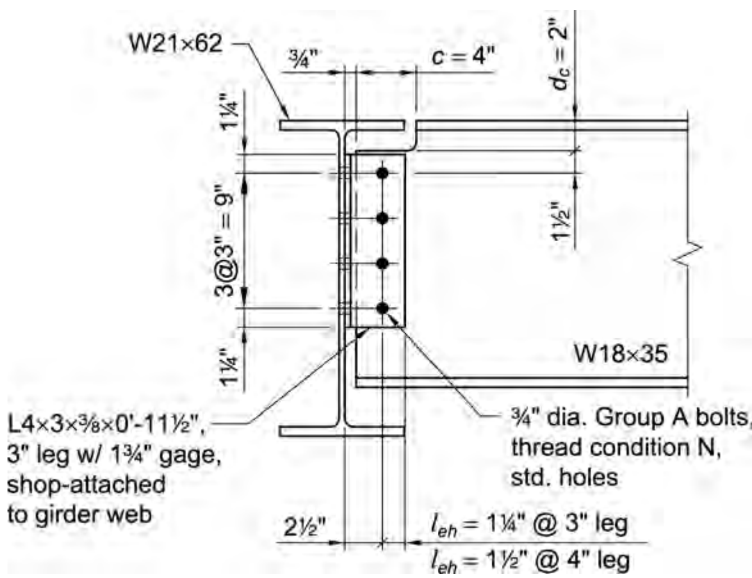


Fig. II.A-28A-1. Connection geometry for Example II.A-28A.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and girder
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angle
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1 the geometric properties are as follows:

Beam
W18×35
t_w = 0.300 in.
d = 17.7 in.

t_f = 0.425 in.

Girder
W21×62
t_w = 0.400 in.

From AISC *Specification* Table J3.3, for ¾-in.-diameter bolts with standard holes:

d_h = 13⁄16 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(6.5 \text{ kips}) + 1.6(20 \text{ kips})$ $= 39.8 \text{ kips}$	$R_a = 6.5 \text{ kips} + 20 \text{ kips}$ $= 26.5 \text{ kips}$

Strength of the Bolted Connection—Angle

Check eccentricity of connection.

For the 4-in. angle leg attached to the supported beam (W18×35):

e = 2½ in. < 3.00 in., therefore, eccentricity does not need to be considered for this leg. (See AISC *Manual* Figure 10-14)

For the 3-in. angle leg attached to the supporting girder (W21×62):

$$e = 1\tfrac{3}{4} \text{ in.} + \frac{0.300 \text{ in.}}{2}$$
$$= 1.90 \text{ in.}$$

Because e = 1.90 in. < 2½ in., AISC *Manual* Table 10-11 may be conservatively used for bolt shear. From Table 10-11, Case I, with n = 4:

$C = 3.07$

From the Commentary to AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the invividual strengths of the individual fasteners, taken as the lesser of the fastener shear strength per AISC *Specification* Section J3.6, the bearing strength at the bolt hole per AISC *Specification* Section J3.10, or the tearout strength at the bolt hole per AISC *Specification* Section J3.10. In this case, the 3-in. angle leg attached to the supporting girder will control because eccentricity must be taken into consideration and the available strength will be determined based on the bolt group using the eccentrically loaded bolt coefficient, C.

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) is:

LRFD	ASD
$\phi r_n = 17.9 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = 11.9 \text{ kips/bolt}$

The available bearing and tearout strength of the angle at the bottom edge bolt is determined using AISC *Manual* Table 7-5, with l_e = 1¼ in., as follows:

LRFD	ASD
$\phi r_n = (44.0 \text{ kip/in.})(\tfrac{3}{8} \text{ in.})$ $= 16.5 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (29.4 \text{ kip/in.})(\tfrac{3}{8} \text{ in.})$ $= 11.0 \text{ kips/bolt}$

The available bearing and tearout strength of the angle at the interior bolts (not adjacent to the edge) is determined using AISC *Manual* Table 7-4 with $s = 3$ in.

LRFD	ASD
$\phi r_n = (78.3 \text{ kip/in.})(\tfrac{3}{8} \text{ in.})$ $= 29.4 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (52.2 \text{ kip/in.})(\tfrac{3}{8} \text{ in.})$ $= 19.6 \text{ kips/bolt}$

The available strength of the bolted connection at the angle is conservatively determined using the minimum available strength calculated for bolt shear, bearing on the angle, and tearout on the angle. The bolt group eccentricity is accounted for by multiplying the minimum available strength by the bolt coefficient C.

LRFD	ASD
$\phi R_n = C\phi r_n$ $= 3.07(16.5 \text{ kips/bolt})$ $= 50.7 \text{ kips} > 39.8 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = C\dfrac{r_n}{\Omega}$ $= 3.07(11.0 \text{ kips/bolt})$ $= 33.8 \text{ kips} > 26.5 \text{ kips}$ **o.k.**

Strength of the Bolted Connection—W18×35 Beam Web

The available bearing and tearout strength of the beam web at the top edge bolt is determined using AISC *Manual* Table 7-5, conservatively using $l_e = 1\tfrac{1}{4}$ in., as follows:

LRFD	ASD
$\phi r_n = (49.4 \text{ kip/in.})(0.300 \text{ in.})$ $= 14.8 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (32.9 \text{ kip/in.})(0.300 \text{ in.})$ $= 9.87 \text{ kips/bolt}$

The available bearing and tearout strength of the beam web at the interior bolts (not adjacent to the edge) is determined using AISC *Manual* Table 7-4 with $s = 3$ in.

LRFD	ASD
$\phi r_n = (87.8 \text{ kip/in.})(0.300 \text{ in.})$ $= 26.3 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (58.5 \text{ kip/in.})(0.300 \text{ in.})$ $= 17.6 \text{ kips/bolt}$

The available strength of the bolted connection at the beam web is determined by summing the effective strength for each bolt using the minimum available strength calculated for bolt shear, bearing on the web, and tearout on the web.

LRFD	ASD
$\phi R_n = n\phi r_n$ $= (1\text{ bolt})(14.8\text{ kips/bolt})$ $+ (3\text{ bolts})(17.9\text{ kips/bolt})$ $= 68.5\text{ kips} > 39.8\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = n\dfrac{r_n}{\Omega}$ $= (1\text{ bolt})(9.87\text{ kips/bolt})$ $+ (3\text{ bolts})(11.9\text{ kips/bolt})$ $= 45.6\text{ kips} > 26.5\text{ kips}$ **o.k.**

Strength of the Bolted Connection—W21×62 Girder Web

The available bearing and tearout strength of the girder web is determined using AISC *Manual* Table 7-4 with s = 3 in.

LRFD	ASD
$\phi r_n = (87.8\text{ kip/in.})(0.400\text{ in.})$ $= 35.1\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (58.5\text{ kip/in.})(0.400\text{ in.})$ $= 23.4\text{ kips/bolt}$

Therefore; bolt shear controls over bearing or tearout on the girder web and is adequate based on previous calculations.

Shear Strength of Angle

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the angle is determined as follows:

$$\begin{aligned} A_{gv} &= lt \\ &= (11\tfrac{1}{2}\text{ in.})(\tfrac{3}{8}\text{ in.}) \\ &= 4.31\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_n &= 0.60F_y A_{gv} \\ &= 0.60(36\text{ ksi})(4.31\text{ in.}^2) \\ &= 93.1\text{ kips} \end{aligned}$$ (*Spec*. Eq. J4-3)

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(93.1\text{ kips})$ $= 93.1\text{ kips} > 39.8\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{93.1\text{ kips}}{1.50}$ $= 62.1\text{ kips} > 26.5\text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the angle is determined using the net area determined in accordance with AISC *Specification* Section B4.3b.

$$\begin{aligned} A_{nv} &= \left[l - n\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]t \\ &= \left[11\tfrac{1}{2}\text{ in.} - 4\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right](\tfrac{3}{8}\text{ in.}) \\ &= 3.00\text{ in.}^2 \end{aligned}$$

$$R_n = 0.60F_uA_{nv} \qquad \text{(Spec. Eq. J4-4)}$$
$$= 0.60(58 \text{ ksi})(3.00 \text{ in.}^2)$$
$$= 104 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(104 \text{ kips})$ $= 78.0 \text{ kips} > 39.8 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{104 \text{ kips}}{2.00}$ $= 52.0 \text{ kips} > 26.5 \text{ kips}$ **o.k.**

Block Shear Rupture of Angle

The nominal strength for the limit state of block shear rupture is given by AISC *Specification* Section J4.3.

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \qquad \text{(Spec. Eq. J4-5)}$$

The available block shear rupture strength of the 3-in. leg is determined as follows, using AISC *Manual* Tables 9-3a, 9-3b and 9-3c, and AISC *Specification* Equation J4-5, with $n = 4$, $l_{ev} = l_{eh} = 1\frac{1}{4}$ in., and $U_{bs} = 1.0$.

LRFD	ASD
Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{\phi F_uA_{nt}}{t} = 35.3 \text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{\phi 0.6F_yA_{gv}}{t} = 166 \text{ kip/in.}$ Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{\phi 0.6F_uA_{nv}}{t} = 188 \text{ kip/in.}$ $\phi R_n = \phi 0.60F_uA_{nv} + \phi U_{bs}F_uA_{nt}$ $\le \phi 0.60F_yA_{gv} + \phi U_{bs}F_uA_{nt}$ $= (3/8 \text{ in.})[188 \text{ kip/in.} + (1.0)(35.3 \text{ kip/in.})]$ $\le (3/8 \text{ in.})[166 \text{ kip/in.} + (1.0)(35.3 \text{ kip/in.})]$ $= 83.7 \text{ kips} > 75.5 \text{ kips}$	Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{F_uA_{nt}}{\Omega t} = 23.6 \text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{0.6F_yA_{gv}}{\Omega t} = 111 \text{ kip/in.}$ Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{0.6F_uA_{nv}}{\Omega t} = 125 \text{ kip/in.}$ $\dfrac{R_n}{\Omega} = \dfrac{0.60F_uA_{nv}}{\Omega} + \dfrac{U_{bs}F_uA_{nt}}{\Omega}$ $\le \dfrac{0.60F_yA_{gv}}{\Omega} + \dfrac{U_{bs}F_uA_{nt}}{\Omega}$ $= (3/8 \text{ in.})[125 \text{ kip/in.} + (1.0)(23.6 \text{ kip/in.})]$ $\le (3/8 \text{ in.})[111 \text{ kip/in.} + (1.0)(23.6 \text{ kip/in.})]$ $= 55.7 \text{ kips} > 50.5 \text{ kips}$

LRFD	ASD
Therefore: $\phi R_n = 75.5 \text{ kips} > 39.8 \text{ kips}$ **o.k.**	Therefore: $\dfrac{R_n}{\Omega} = 50.5 \text{ kips} > 26.5 \text{ kips}$ **o.k.**

Because the edge distance is smaller, block shear rupture is governed by the 3-in. leg.

Flexural Yielding Strength of Angle

The required flexural strength of the support leg of the angle is determined as follows:

LRFD	ASD
$M_u = R_u e$ $= (39.8 \text{ kips})\left(1\tfrac{3}{4} \text{ in.} + \dfrac{0.300 \text{ in.}}{2}\right)$ $= 75.6 \text{ kip-in.}$	$M_a = R_a e$ $= (26.5 \text{ kips})\left(1\tfrac{3}{4} \text{ in.} + \dfrac{0.300 \text{ in.}}{2}\right)$ $= 50.4 \text{ kip-in.}$

The available flexural yielding strength of the support leg of the angle is determined as follows:

$$M_n = F_y Z_x$$
$$= (36 \text{ ksi})\left[\frac{(\tfrac{3}{8} \text{ in.})(11\tfrac{1}{2} \text{ in.})^2}{4}\right]$$
$$= 446 \text{ kip-in.}$$

LRFD	ASD
$\phi = 0.90$ $\phi M_n = 0.90(446 \text{ kip-in.})$ $= 401 \text{ kip-in.} > 75.6 \text{ kip-in.}$ **o.k.**	$\Omega = 1.67$ $\dfrac{M_n}{\Omega} = \dfrac{446 \text{ kip-in.}}{1.67}$ $= 267 \text{ kip-in.} > 50.4 \text{ kip-in.}$ **o.k.**

Flexural Rupture Strength of Angle

The available flexural rupture strength of the support leg of the angle is determined as follows:

$$Z_{net} = (\tfrac{3}{8} \text{ in.})\left[\frac{(11\tfrac{1}{2} \text{ in.})^2}{4} - 2(\tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(4.50 \text{ in.}) - 2(\tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(1.50 \text{ in.})\right]$$
$$= 8.46 \text{ in.}^3$$

$$M_n = F_u Z_{net} \qquad \text{(}Manual\text{ Eq. 9-4)}$$
$$= (58 \text{ ksi})(8.46 \text{ in.}^3)$$
$$= 491 \text{ kip-in.}$$

LRFD	ASD
$\phi_b = 0.75$ $\phi_b M_n = 0.75(491 \text{ kip-in.})$ $= 368 \text{ kip-in.} > 75.6 \text{ kip-in.}$ **o.k.**	$\Omega_b = 2.00$ $\dfrac{M_n}{\Omega_b} = \dfrac{491 \text{ kip-in.}}{2.00}$ $= 246 \text{ kip-in.} > 50.4 \text{ kip-in.}$ **o.k.**

Flexural Yielding and Buckling of Coped Beam Web

The required flexural strength of the coped section of the beam web is determined using AISC *Manual* Equation 9-5a or 9-5b, as follows:

$$\begin{aligned} e &= c + setback \\ &= 4 \text{ in.} + \tfrac{3}{4} \text{ in.} \\ &= 4.75 \text{ in.} \end{aligned}$$

LRFD	ASD
$M_u = R_u e$ $= (39.8 \text{ kips})(4.75 \text{ in.})$ $= 189 \text{ kip-in.}$	$M_a = R_a e$ $= (26.5 \text{ kips})(4.75 \text{ in.})$ $= 126 \text{ kip-in.}$

The minimum length of the connection elements is one-half of the reduced beam depth, h_o:

$$\begin{aligned} h_o &= d - d_c \text{ (from AISC } \textit{Manual} \text{ Figure 9-2)} \\ &= 17.7 \text{ in.} - 2 \text{ in.} \\ &= 15.7 \text{ in.} \end{aligned}$$

$$\begin{aligned} l &> 0.5h_o \\ 11\tfrac{1}{2} \text{ in.} &> 0.5(15.7 \text{ in.}) \\ 11\tfrac{1}{2} \text{ in.} &> 7.85 \text{ in.} \quad \textbf{o.k.} \end{aligned}$$

The available flexural local buckling strength of a beam coped at the top flange is determined as follows:

$$\begin{aligned} \lambda &= \frac{h_o}{t_w} \\ &= \frac{15.7 \text{ in.}}{0.300 \text{ in.}} \\ &= 52.3 \end{aligned} \qquad (\textit{Manual} \text{ Eq. 9-11})$$

$$\begin{aligned} \frac{c}{h_o} &= \frac{4 \text{ in.}}{15.7 \text{ in.}} \\ &= 0.255 \end{aligned}$$

Because $\dfrac{c}{h_o} \le 1.0$, the plate buckling coefficient, k, is calculated as follows:

$$k = 2.2\left(\frac{h_o}{c}\right)^{1.65} \quad \text{(}\textit{Manual}\text{ Eq. 9-13a)}$$
$$= 2.2\left(\frac{15.7\text{ in.}}{4\text{ in.}}\right)^{1.65}$$
$$= 21.0$$

$$\frac{c}{d} = \frac{4\text{ in.}}{17.7\text{ in.}}$$
$$= 0.226$$

Because $\frac{c}{d} \leq 1.0$, the buckling adjustment factor, *f*, is calculated as follows:

$$f = 2\left(\frac{c}{d}\right) \quad \text{(}\textit{Manual}\text{ Eq. 9-14a)}$$
$$= 2(0.226)$$
$$= 0.452$$

$$k_1 = fk \geq 1.61 \quad \text{(}\textit{Manual}\text{ Eq. 9-10)}$$
$$= (0.452)(21.0) \geq 1.61$$
$$= 9.49 > 1.61$$
$$= 9.49$$

$$\lambda_p = 0.475\sqrt{\frac{k_1 E}{F_y}} \quad \text{(}\textit{Manual}\text{ Eq. 9-12)}$$
$$= 0.475\sqrt{\frac{(9.49)(29{,}000\text{ ksi})}{50\text{ ksi}}}$$
$$= 35.2$$

$$2\lambda_p = 2(35.2)$$
$$= 70.4$$

Because $\lambda_p < \lambda \leq 2\lambda_p$, calculate the nominal flexural strength using AISC *Manual* Equation 9-7.

The plastic section modulus of the coped section, Z_{net}, is determined from Table IV-11 (included in Part IV of this document).

$$Z_{net} = 32.1\text{ in.}^3$$

$$M_p = F_y Z_{net}$$
$$= (50\text{ ksi})(32.1\text{ in.}^3)$$
$$= 1{,}610\text{ kip-in.}$$

From AISC *Manual* Table 9-2:

$$S_{net} = 18.2\text{ in.}^3$$

$$
\begin{aligned}
M_y &= F_y S_{net} \\
&= (50\text{ ksi})(18.2\text{ in.}^3) \\
&= 910\text{ kip-in.}
\end{aligned}
$$

$$
\begin{aligned}
M_n &= M_p - (M_p - M_y)\left(\frac{\lambda}{\lambda_p} - 1\right) \\
&= (1{,}610\text{ kip-in.}) - (1{,}610\text{ kip-in.} - 910\text{ kip-in.})\left(\frac{52.3}{35.2} - 1\right) \\
&= 1{,}270\text{ kip-in.}
\end{aligned}
$$
(*Manual* Eq. 9-7)

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(1{,}270\text{ kip-in.})$ $= 1{,}140\text{ kip-in.} > 189\text{ kip-in.}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{1{,}270\text{ kip-in.}}{1.67}$ $= 760\text{ kip-in.} > 126\text{ kip-in.}$ **o.k.**

Shear Strength of Beam Web

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the beam web is determined as follows:

$$
\begin{aligned}
A_{gv} &= h_o t_w \\
&= (15.7\text{ in.})(0.300\text{ in.}) \\
&= 4.71\text{ in.}^2
\end{aligned}
$$

$$
\begin{aligned}
R_n &= 0.60 F_y A_{gv} \\
&= 0.60(50\text{ ksi})(4.71\text{ in.}^2) \\
&= 141\text{ kips}
\end{aligned}
$$
(*Spec.* Eq. J4-3)

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(141\text{ kips})$ $= 141\text{ kips} > 39.8\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{141\text{ kips}}{1.50}$ $= 94.0\text{ kips} > 26.5\text{ kips}$ **o.k.**

Block Shear Rupture of Beam Web

The nominal strength for the limit state of block shear rupture is given by AISC *Specification* Section J4.3.

$$R_n = 0.60 F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60 F_y A_{gv} + U_{bs} F_u A_{nt}$$
(*Spec.* Eq. J4-5)

The available block shear rupture strength of the web is determined as follows, using AISC *Manual* Tables 9-3a, 9-3b and 9-3c, and AISC *Specification* Equation J4-5, with $n = 4$, $l_{ev} = 1\frac{1}{2}$ in., $l_{eh} = 1\frac{1}{2}$ in. (including a ¼-in. tolerance to account for possible beam underrun), and $U_{bs} = 1.0$.

LRFD	ASD
Tension rupture component from AISC *Manual* Table 9-3a: $\frac{\phi F_u A_{nt}}{t} = 51.8 \text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\frac{\phi 0.60 F_y A_{gv}}{t} = 236 \text{ kip/in.}$ Shear rupture component from AISC *Manual* Table 9-3c: $\frac{\phi 0.60 F_u A_{nv}}{t} = 218 \text{ kip/in.}$ $\phi R_n = \phi 0.60 F_u A_{nv} + \phi U_{bs} F_u A_{nt}$ $\leq \phi 0.60 F_y A_{gv} + \phi U_{bs} F_u A_{nt}$ $= (0.300 \text{ in.})[218 \text{ kip/in.} + (1.0)(51.8 \text{ kip/in.})]$ $\leq (0.300 \text{ in.})[236 \text{ kip/in.} + (1.0)(51.8 \text{ kip/in.})]$ $= 80.9 \text{ kips} < 86.3 \text{ kips}$ Therefore: $\phi R_n = 80.9 \text{ kips} > 39.8 \text{ kips}$ **o.k.**	Tension rupture component from AISC *Manual* Table 9-3a: $\frac{F_u A_{nt}}{\Omega t} = 34.5 \text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\frac{0.60 F_y A_{gv}}{\Omega t} = 158 \text{ kip/in.}$ Shear rupture component from AISC *Manual* Table 9-3c: $\frac{0.60 F_u A_{nv}}{\Omega t} = 145 \text{ kip/in.}$ $\frac{R_n}{\Omega} = \frac{0.60 F_u A_{nv}}{\Omega} + \frac{U_{bs} F_u A_{nt}}{\Omega}$ $\leq \frac{0.60 F_y A_{gv}}{\Omega} + \frac{U_{bs} F_u A_{nt}}{\Omega}$ $= (0.300 \text{ in.})[145 \text{ kip/in.} + (1.0)(34.5 \text{ kip/in.})]$ $\leq (0.300 \text{ in.})[158 \text{ kip/in.} + (1.0)(34.5 \text{ kip/in.})]$ $= 53.9 \text{ kips} < 57.8 \text{ kips}$ Therefore: $\frac{R_n}{\Omega} = 53.9 \text{ kips} > 26.5 \text{ kips}$ **o.k.**

Conclusion

The connection is found to be adequate as given for the applied load.

EXAMPLE II.A-28B ALL-BOLTED SINGLE ANGLE CONNECTION—STRUCTURAL INTEGRITY CHECK

Given:

Verify the all-bolted single-angle connection from Example II.A-28A, as shown in Figure II.A-28B-1, for the structural integrity provisions of AISC *Specification* Section B3.9. The connection is verified as a beam end connection. Note that these checks are necessary when design for structural integrity is required by the applicable building code. The angle is ASTM A36 material.

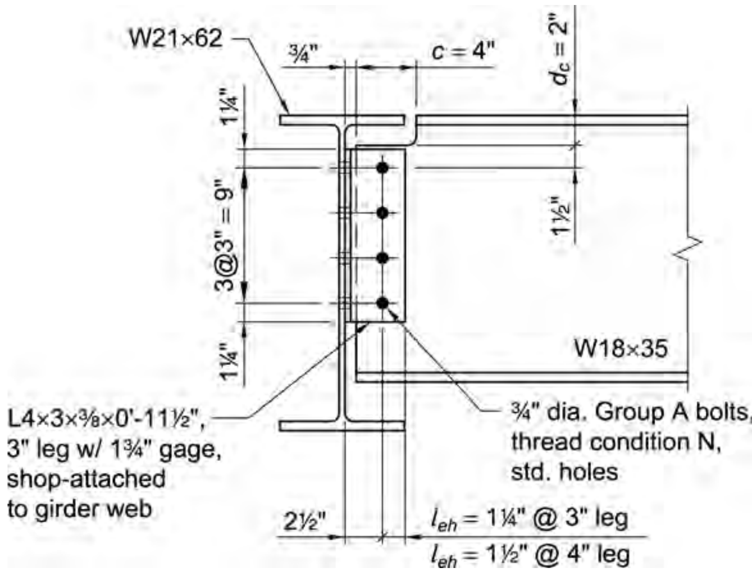


Fig. II.A-28B-1. Connection geometry for Example II.A-28B.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and Girder
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angle
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18x35
t_w = 0.300 in.

Girder
W21x62
t_w = 0.400 in.
d = 21.0 in.
k_{des} = 1.12 in.

From AISC *Specification* Table J3.3, the hole diameter for ¾-in.-diameter bolts with standard holes is:

d_h = 13⁄16 in.

From Example II.A-28A, the required shear strength is:

LRFD	ASD
$V_u = 39.8$ kips	$V_a = 26.5$ kips

From AISC *Specification* Section B3.9(b), the required axial tensile strength is:

LRFD	ASD
$T_u = \frac{2}{3}V_u \geq 10 \text{ kips}$ $= \frac{2}{3}(39.8 \text{ kips}) > 10 \text{ kips}$ $= 26.5 \text{ kips} > 10 \text{ kips}$ $= 26.5 \text{ kips}$	$T_a = V_a \geq 10 \text{ kips}$ $= 26.5 \text{ kips} > 10 \text{ kips}$ $= 26.5 \text{ kips}$

Bolt Shear

From AISC *Specification* Section J3.6, the nominal bolt shear strength is determined as follows:

F_{nv} = 54 ksi, from AISC *Specification* Table J3.2

$$T_n = nF_{nv}A_b \quad \text{(from } Spec. \text{ Eq. J3-1)}$$
$$= (4 \text{ bolts})(54 \text{ ksi})(0.442 \text{ in.}^2)$$
$$= 95.5 \text{ kips}$$

Bolt Tension

From AISC *Specification* Section J3.6, the nominal bolt tensile strength is determined as follows:

F_{nt} = 90 ksi, from AISC *Specification* Table J3.2

$$T_n = nF_{nt}A_b \quad \text{(from } Spec. \text{ Eq. J3-1)}$$
$$= (4 \text{ bolts})(90 \text{ ksi})(0.442 \text{ in.}^2)$$
$$= 159 \text{ kips}$$

Bolt Bearing and Tearout

From AISC *Specification* Section B3.9, for the purpose of satisfying structural integrity requirements inelastic deformations of the connection are permitted; therefore, AISC *Specification* Equations J3-6b and J3-6d are used to determine the nominal bearing and tearout strength.

For bolt bearing on the angle:

$$T_n = (4 \text{ bolts})3.0dtF_u \quad \text{(from } \textit{Spec.} \text{ Eq. J3-6b)}$$
$$= (4 \text{ bolts})(3.0)(\tfrac{3}{4} \text{ in.})(\tfrac{3}{8} \text{ in.})(58 \text{ ksi})$$
$$= 196 \text{ kips}$$

For bolt bearing on the beam web:

$$T_n = (4 \text{ bolts})3.0dt_wF_u \quad \text{(from } \textit{Spec.} \text{ Eq. J3-6b)}$$
$$= (4 \text{ bolts})(3.0)(\tfrac{3}{4} \text{ in.})(0.300 \text{ in.})(65 \text{ ksi})$$
$$= 176 \text{ kips}$$

For bolt tearout on the angle:

$$l_c = l_{eh} - 0.5d_h$$
$$= 1\tfrac{1}{2} \text{ in.} - 0.5(\tfrac{13}{16} \text{ in.})$$
$$= 1.09 \text{ in.}$$

$$T_n = (4 \text{ bolts})1.5l_ctF_u \quad \text{(from } \textit{Spec.} \text{ Eq. J3-6d)}$$
$$= (4 \text{ bolts})(1.5)(1.09 \text{ in.})(\tfrac{3}{8} \text{ in.})(58 \text{ ksi})$$
$$= 142 \text{ kips}$$

For bolt tearout on the beam web (including a ¼-in. tolerance to account for possible beam underrun):

$$l_c = l_{eh} - 0.5d_h$$
$$= (1\tfrac{3}{4} \text{ in.} - \tfrac{1}{4} \text{ in.}) - 0.5(\tfrac{13}{16} \text{ in.})$$
$$= 1.09 \text{ in.}$$

$$T_n = (4 \text{ bolts})1.5l_ct_wF_u \quad \text{(from } \textit{Spec.} \text{ Eq. J3-6d)}$$
$$= (4 \text{ bolts})(1.5)(1.09 \text{ in.})(0.300 \text{ in.})(65 \text{ ksi})$$
$$= 128 \text{ kips}$$

Angle Bending and Prying Action

From AISC *Manual* Part 9, the nominal strength of the angle accounting for prying action is determined as follows:

$$b = gage - \frac{t}{2}$$
$$= 1\tfrac{3}{4} \text{ in.} - \frac{\tfrac{3}{8} \text{ in.}}{2}$$
$$= 1.56 \text{ in.}$$

$$a = \min\{1\tfrac{1}{4}\text{ in., } 1.25b\}$$
$$= \min\{1\tfrac{1}{4}\text{ in., } 1.25(1.56\text{ in.})\}$$
$$= 1.25\text{ in.}$$

$$b' = b - \frac{d_b}{2}$$ (*Manual* Eq. 9-18)
$$= 1.56\text{ in.} - \frac{\tfrac{3}{4}\text{ in.}}{2}$$
$$= 1.19\text{ in.}$$

$$a' = \left(a + \frac{d_b}{2}\right) \le \left(1.25b + \frac{d_b}{2}\right)$$ (*Manual* Eq. 9-23)
$$= 1.25 + \frac{\tfrac{3}{4}\text{ in.}}{2} \le 1.25(1.56\text{ in.}) + \frac{\tfrac{3}{4}\text{ in.}}{2}$$
$$= 1.63\text{ in.} < 2.33\text{ in.}$$
$$= 1.63\text{ in.}$$

$$\rho = \frac{b'}{a'}$$ (*Manual* Eq. 9-22)
$$= \frac{1.19\text{ in.}}{1.63\text{ in.}}$$
$$= 0.730$$

Note that end distances of 1¼ in. are used on the angles, so p is the average pitch of the bolts:

$$p = \frac{l}{n}$$
$$= \frac{11\tfrac{1}{2}\text{ in.}}{4}$$
$$= 2.88\text{ in.}$$

Check:
$p < s = 3.00\text{ in.}$ **o.k.**

$$d' = d_h$$
$$= \tfrac{13}{16}\text{ in.}$$

$$\delta = 1 - \frac{d'}{p}$$ (*Manual* Eq. 9-20)
$$= 1 - \frac{\tfrac{13}{16}\text{ in.}}{2.88\text{ in.}}$$
$$= 0.718$$

$$B_n = F_{nt}A_b$$
$$= (90\text{ ksi})(0.442\text{ in.}^2)$$
$$= 39.8\text{ kips/bolt}$$

$$t_c = \sqrt{\frac{4B_n b'}{pF_u}} \qquad \text{(from } \textit{Manual} \text{ Eq. 9-26)}$$

$$= \sqrt{\frac{4(39.8 \text{ kips/bolt})(1.19 \text{ in.})}{(2.88 \text{ in.})(58 \text{ ksi})}}$$

$$= 1.06 \text{ in.}$$

$$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right] \qquad \text{(} \textit{Manual} \text{ Eq. 9-28)}$$

$$= \frac{1}{0.718(1+0.730)}\left[\left(\frac{1.06 \text{ in.}}{3/8 \text{ in.}}\right)^2 - 1\right]$$

$$= 5.63$$

Because $\alpha' > 1$:

$$Q = \left(\frac{t}{t_c}\right)^2 (1+\delta)$$

$$= \left(\frac{3/8 \text{ in.}}{1.06 \text{ in.}}\right)^2 (1+0.718)$$

$$= 0.215$$

$$T_n = (4 \text{ bolts}) B_n Q \qquad \text{(from } \textit{Manual} \text{ Eq. 9-27)}$$

$$= (4 \text{ bolts})(39.8 \text{ kips/bolt})(0.215)$$

$$= 34.2 \text{ kips}$$

Block Shear Rupture—Angle

From AISC *Specification* Section J4.3, the nominal block shear rupture strength of the angle with a "U" shaped failure plane is determined as follows:

$$T_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad \text{(from } \textit{Spec.} \text{ Eq. J4-5)}$$

where

$$A_{gv} = 2l_{eh}t$$

$$= (2)(1\tfrac{1}{2} \text{ in.})(3/8 \text{ in.})$$

$$= 1.13 \text{ in.}^2$$

$$A_{nv} = (2)\left[l_{eh} - 0.5(d_h + 1/16 \text{ in.})\right]t$$

$$= (2)\left[1\tfrac{1}{2} \text{ in.} - 0.5(13/16 \text{ in.} + 1/16 \text{ in.})\right](3/8 \text{ in.})$$

$$= 0.797 \text{ in.}^2$$

$$A_{nt} = \left[9.00 \text{ in.} - 4(d_h + 1/16 \text{ in.})\right]t$$

$$= \left[9.00 \text{ in.} - 4(13/16 \text{ in.} + 1/16 \text{ in.})\right](3/8 \text{ in.})$$

$$= 2.06 \text{ in.}^2$$

$$U_{bs} = 1.0$$

$$\begin{aligned}T_n &= 0.60(58\text{ ksi})(0.797\text{ in.}^2) + 1.0(58\text{ ksi})(2.06\text{ in.}^2) \le 0.60(36\text{ ksi})(1.13\text{ in.}^2) + 1.0(58\text{ ksi})(2.06\text{ in.}^2)\\ &= 147\text{ kips} > 144\text{ kips}\end{aligned}$$

Therefore:

$$T_n = 144\text{ kips}$$

Tensile Yielding of Angle

From AISC *Specification* Section J4.1, the nominal tensile yielding strength of the angle is determined as follows:

$$\begin{aligned}A_g &= lt\\ &= (11\tfrac{1}{2}\text{ in.})(\tfrac{3}{8}\text{ in.})\\ &= 4.31\text{ in.}^2\end{aligned}$$

$$\begin{aligned}T_n &= F_y A_g\\ &= (36\text{ ksi})(4.31\text{ in.}^2)\\ &= 155\text{ kips}\end{aligned}$$ (from *Spec*. Eq. J4-1)

Tensile Rupture of Angle

From AISC *Specification* Section J4.1, the nominal tensile rupture strength of the angle is determined as follows:

$$\begin{aligned}A_e &= A_n U\\ &= \left[l - n\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]tU\\ &= \left[11\tfrac{1}{2}\text{ in.} - 4\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right](\tfrac{3}{8}\text{ in.})(1.0)\\ &= 3.00\text{ in.}^2\end{aligned}$$ (*Spec*. Eq. D3-1)

$$\begin{aligned}T_n &= F_u A_e\\ &= (58\text{ ksi})(3.00\text{ in.}^2)\\ &= 174\text{ kips}\end{aligned}$$ (from *Spec*. Eq. J4-2)

Block Shear Rupture—Beam Web

From AISC *Specification* Section J4.3, the nominal block shear rupture strength of the beam web with a "U" shaped failure plane is determined as follows (including a ¼-in. tolerance to account for possible beam underrun):

$$T_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt}$$ (from *Spec*. Eq. J4-5)

where

$$\begin{aligned}A_{gv} &= 2l_{eh}t_w\\ &= (2)(1\tfrac{3}{4}\text{ in.} - \tfrac{1}{4}\text{ in.})(0.300\text{ in.})\\ &= 0.900\text{ in.}^2\end{aligned}$$

$$A_{nv} = (2)\left[l_{eh} - 0.5\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]t_w$$
$$= (2)\left[\left(1\tfrac{3}{4}\text{ in.} - \tfrac{1}{4}\text{ in.}\right) - 0.5\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right](0.300\text{ in.})$$
$$= 0.638\text{ in.}^2$$

$$A_{nt} = \left[9.00\text{ in.} - 3\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]t_w$$
$$= \left[9.00\text{ in.} - 3\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right](0.300\text{ in.})$$
$$= 1.91\text{ in.}^2$$

$$U_{bs} = 1.0$$

$$T_n = 0.60(65\text{ ksi})\left(0.638\text{ in.}^2\right) + 1.0(65\text{ ksi})\left(1.91\text{ in.}^2\right) \le 0.60(50\text{ ksi})\left(0.900\text{ in.}^2\right) + 1.0(65\text{ ksi})\left(1.91\text{ in.}^2\right)$$
$$= 149\text{ kips} < 151\text{ kips}$$

Therefore:

$$T_n = 149\text{ kips}$$

Nominal Tensile Strength

The controlling tensile strength, T_n, is the least of those previously calculated:

$$T_n = \min\left\{\begin{array}{l}95.5\text{ kips, }159\text{ kips, }196\text{ kips, }176\text{ kips, }142\text{ kips, }128\text{ kips, }34.2\text{ kips, }144\text{ kips, }155\text{ kips,}\\ 174\text{ kips, }149\text{ kips}\end{array}\right\}$$
$$= 34.2\text{ kips}$$

LRFD	ASD
$T_n = 34.2\text{ kips} > 26.5\text{ kips}$ **o.k.**	$T_n = 34.2\text{ kips} > 26.5\text{ kips}$ **o.k.**

EXAMPLE II.A-29 BOLTED/WELDED SINGLE-ANGLE CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Verify a single-angle connection between an ASTM A992 W16×50 beam and an ASTM A992 W14×90 column flange, as shown in Figure II.A-29-1, to support the following beam end reactions:

R_D = 9 kips
R_L = 27 kips

Use an ASTM A36 single angle. Use 70-ksi electrode welds to connect the single angle to the column flange.

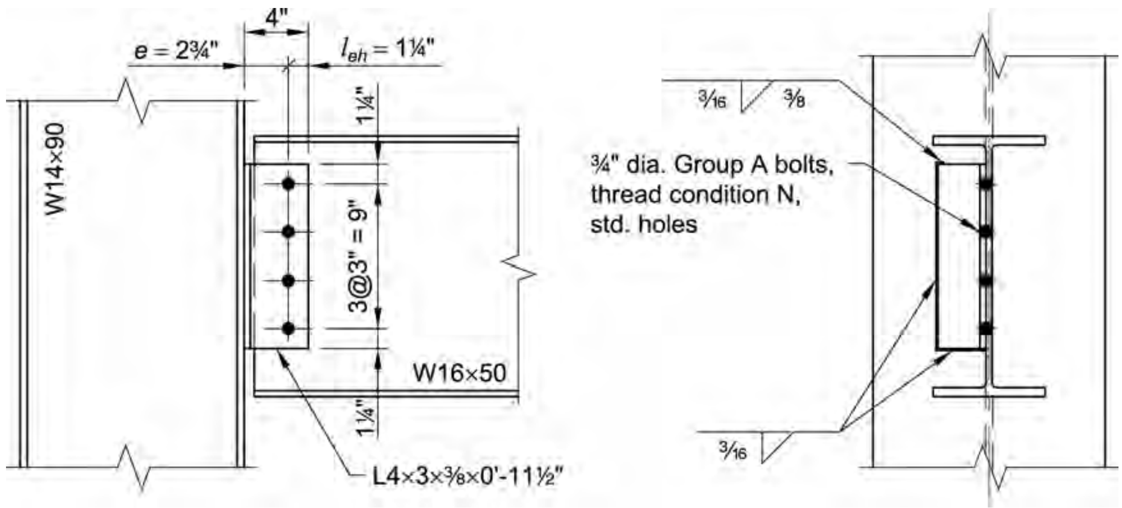


Fig. II.A-29-1. Connection geometry for Example II.A-29.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angle
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W16×50
t_w= 0.380 in.
d = 16.3 in.
t_f = 0.630 in.

Column
W14×90
$t_f = 0.710$

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(9 \text{ kips}) + 1.6(27 \text{ kips})$ $= 54.0 \text{ kips}$	$R_a = 9 \text{ kips} + 27 \text{ kips}$ $= 36.0 \text{ kips}$

Single Angle, Bolts and Welds

Check eccentricity of the connection.

For the 4-in. angle leg attached to the supported beam:

e = 2¾ in. < 3.00 in., therefore, eccentricity does not need to be considered for this leg.

For the 3-in. angle leg attached to the supporting column flange:

Because the half-web dimension of the W16×50 supported beam is less than ¼ in., AISC *Manual* Table 10-12 may conservatively be used.

Use a four-bolt single-angle (L4×3×⅜).

From AISC *Manual* Table 10-12, the bolt and angle available strength is:

LRFD	ASD
$\phi R_n = 71.4 \text{ kips} > 54.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 47.6 \text{ kips} > 36.0 \text{ kips}$ **o.k.**

From AISC *Manual* Table 10-12, the available weld strength for a 3⁄16-in. fillet weld is:

LRFD	ASD
$\phi R_n = 56.6 \text{ kips} > 54.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 37.8 \text{ kips} > 36.0 \text{ kips}$ **o.k.**

Support Thickness

The minimum support thickness that matches the column flange strength to the 3⁄16-in. fillet weld strength is:

$$t_{min} = \frac{3.09D}{F_u} \qquad \text{(\textit{Manual} Eq. 9-2)}$$

$$= \frac{3.09(3)}{65 \text{ ksi}}$$

$$= 0.143 \text{ in.} < 0.710 \text{ in.} \quad \textbf{o.k.}$$

Note: The minimum thickness values listed in Table 10-12 are for conditions with angles on both sides of the web.

Use a four-bolt single-angle, L4×3×⅜. The 3-in. leg will be shop welded to the column flange and the 4-in. leg will be field bolted to the beam web.

Supported Beam Web

The available bearing and tearout strength of the beam web is determined using AISC *Manual* Table 7-4 with s = 3 in.

LRFD	ASD
$\phi R_n = (4 \text{ bolts})(87.8 \text{ kip/in.})(0.380 \text{ in.})$ $= 133 \text{ kips} > 54.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (4 \text{ bolts})(58.5 \text{ kip/in.})(0.380 \text{ in.})$ $= 88.9 \text{ kips} > 36.0 \text{ kips}$ **o.k.**

Conclusion

The connection is found to be adequate as given for the applied load.

EXAMPLE II.A-30 ALL-BOLTED TEE CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Verify an all-bolted tee connection between an ASTM A992 W16×50 beam and an ASTM A992 W14×90 column flange, as shown in Figure II.A-30-1, to support the following beam end reactions:

R_D = 9 kips
R_L = 27 kips

Use an ASTM A992 WT5×22.5 with a four-bolt connection.

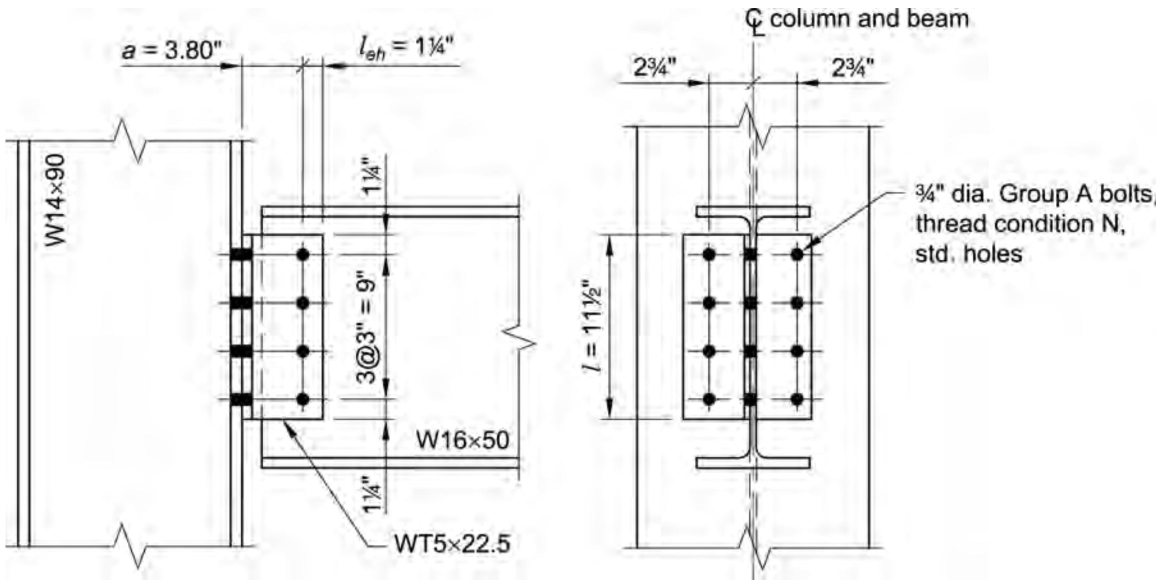


Fig. II.A-30-1. Connection geometry for Example II.A-30.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam, column and tee
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Tables 1-1 and 1-8, the geometric properties are as follows:

Beam
W16×50
t_w= 0.380 in.
d = 16.3 in.
t_f = 0.630 in.

Column
W14×90
t_f = 0.710 in.

Tee
WT5×22.5
d = 5.05 in.
b_f = 8.02 in.
t_f = 0.620 in.
t_{sw} = 0.350 in.
k_1 = 13/16 in. (see W10×45 AISC *Manual* Table 1-1)
k_{des} = 1.12 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(9 \text{ kips}) + 1.6(27 \text{ kips})$ $= 54.0 \text{ kips}$	$R_a = 9 \text{ kips} + 27 \text{ kips}$ $= 36.0 \text{ kips}$

Limitation on Tee Stem or Beam Web Thickness

See rotational ductility discussion at the beginning of the AISC *Manual* Part 9.

For the tee stem, the maximum tee stem thickness is:

$$t_{sw\,max} = \frac{d}{2} + \tfrac{1}{16} \text{ in.} \qquad (\textit{Manual} \text{ Eq. 9-39})$$

$$= \frac{\tfrac{3}{4} \text{ in.}}{2} + \tfrac{1}{16} \text{ in.}$$

$$= 0.438 \text{ in.} > 0.350 \text{ in.} \quad \textbf{o.k.}$$

For W16×50 beam web, the maximum beam web thickness is:

$$t_{w\,max} = \frac{d}{2} + \tfrac{1}{16} \text{ in.} \qquad (\text{from } \textit{Manual} \text{ Eq. 9-39})$$

$$= \frac{\tfrac{3}{4} \text{ in.}}{2} + \tfrac{1}{16} \text{ in.}$$

$$= 0.438 \text{ in.} > 0.380 \text{ in.} \quad \textbf{o.k.}$$

Limitation on Bolt Diameter for Bolts through Tee Flange

Note: The bolts are not located symmetrically with respect to the centerline of the tee.

b = flexible width in connection element (see AISC *Manual* Figure 9-6)

$$= 2\tfrac{3}{4} \text{ in.} - \frac{t_{sw}}{2} - \frac{t_w}{2} - k_1$$

$$= 2\tfrac{3}{4} \text{ in.} - \frac{0.350 \text{ in.}}{2} - \frac{0.380 \text{ in.}}{2} - \tfrac{13}{16} \text{ in.}$$

$$= 1.57 \text{ in.}$$

$$d_{min} = 0.163t_f \sqrt{\frac{F_y}{b}\left(\frac{b^2}{l^2}+2\right)} \le 0.69\sqrt{t_{sw}} \qquad (\textit{Manual}\text{ Eq. 9-38})$$

$$= 0.163(0.620\text{ in.})\sqrt{\left(\frac{50\text{ ksi}}{1.57\text{ in.}}\right)\left[\frac{(1.57\text{ in.})^2}{(11\tfrac{1}{2}\text{ in.})^2}+2\right]} \le 0.69\sqrt{0.350\text{ in.}}$$

$$= 0.810\text{ in.} > 0.408\text{ in.}$$

Therefore:

$$d_{min} = 0.408\text{ in.} < \tfrac{3}{4}\text{ in.} \quad \textbf{o.k.}$$

Because the connection is rigid at the support, the bolts through the tee stem must be designed for shear, but do not need to be designed for an eccentric moment.

Strength of the Bolted Connection—Tee

From the Commentary to AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the invividual strengths of the individual fasteners, taken as the lesser of the fastener shear strength per AISC *Specification* Section J3.6, the bearing strength at the bolt hole per AISC *Specification* Section J3.10, or the tearout strength at the bolt hole per AISC *Specification* Section J3.10.

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) is:

LRFD	ASD
$\phi r_n = 17.9\text{ kips/bolt}$	$\frac{r_n}{\Omega} = 11.9\text{ kips/bolt}$

The available bearing and tearout strength of the tee at the bottom edge bolt is determined using AISC *Manual* Table 7-5, with l_e = 1¼ in., as follows:

LRFD	ASD
$\phi r_n = (49.4\text{ kip/in.})(0.350\text{ in.})$ $= 17.3\text{ kips/bolt}$	$\frac{r_n}{\Omega} = (32.9\text{ kip/in.})(0.350\text{ in.})$ $= 11.5\text{ kips/bolt}$

The bearing or tearout strength controls over bolt shear for the bottom edge bolt in the tee.

The available bearing and tearout strength of the tee at the interior bolts (not adjacent to the edge) is determined using AISC *Manual* Table 7-4 with s = 3 in.

LRFD	ASD
$\phi r_n = (87.8\text{ kip/in.})(0.350\text{ in.})$ $= 30.7\text{ kips/bolt}$	$\frac{r_n}{\Omega} = (58.5\text{ kip/in.})(0.350\text{ in.})$ $= 20.5\text{ kips/bolt}$

The bolt shear strength controls over bearing or tearout for the interior bolts in the tee.

The strength of the bolt group in the beam web is determined by summing the strength of the individual fasteners as follows:

LRFD	ASD
$\phi R_n = (1\text{ bolt})(17.3\text{ kips/bolt}) + (3\text{ bolts})(17.9\text{ kips/bolt})$ $= 71.0\text{ kips} > 54.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (1\text{ bolt})(11.5\text{ kips/bolt}) + (3\text{ bolts})(11.9\text{ kips/bolt})$ $= 47.2\text{ kips} > 36.0\text{ kips}$ **o.k.**

Strength of the Bolted Connection—Beam Web

The available bearing and tearout strength for all bolts in the beam web is determined using AISC *Manual* Table 7-4 with $s = 3$ in.

LRFD	ASD
$\phi r_n = (87.8\text{ kip/in.})(0.380\text{ in.})$ $= 33.4\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (58.5\text{ kip/in.})(0.380\text{ in.})$ $= 22.2\text{ kips/bolt}$

The bolt shear strength controls over bearing or tearout in the beam web; therefore, the beam web is adequate based on previous calculations.

Flexural Yielding of Stem

The flexural yielding strength is checked at the junction of the stem and the fillet. The required flexural strength is determined as follows:

LRFD	ASD
$M_u = P_u e$ $= P_u (a - k_{des})$ $= (54.0\text{ kips})(3.80\text{ in.} - 1.12\text{ in.})$ $= 145\text{ kip-in.}$	$M_a = P_a e$ $= P_a (a - k_{des})$ $= (36.0\text{ kips})(3.80\text{ in.} - 1.12\text{ in.})$ $= 96.5\text{ kip-in.}$

The available flexural strength of the tee stem is determined as follows:

LRFD	ASD
$\phi = 0.90$ $\phi M_n = \phi F_y Z_x$ $= 0.90(50\text{ ksi})\left[\dfrac{(0.350\text{ in.})(11\frac{1}{2}\text{ in.})^2}{4}\right]$ $= 521\text{ kip-in.} > 145\text{ kip-in.}$ **o.k.**	$\Omega = 1.67$ $\dfrac{M_n}{\Omega} = \dfrac{F_y Z_x}{\Omega}$ $= \left(\dfrac{50\text{ ksi}}{1.67}\right)\left[\dfrac{(0.350\text{ in.})(11\frac{1}{2}\text{ in.})^2}{4}\right]$ $= 346\text{ kip-in.} > 96.5\text{ kip-in.}$ **o.k.**

Shear Strength of Stem

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the tee stem is determined as follows:

$$A_{gv} = l t_{sw}$$
$$= (11\tfrac{1}{2}\text{ in.})(0.350\text{ in.})$$
$$= 4.03\text{ in.}^2$$

$$R_n = 0.60F_y A_{gv} \qquad (Spec. \text{ Eq. J4-3})$$
$$= 0.60(50 \text{ ksi})(4.03 \text{ in.}^2)$$
$$= 121 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(121 \text{ kips})$ $= 121 \text{ kips} > 54.0 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{121 \text{ kips}}{1.50}$ $= 80.7 \text{ kips} > 36.0 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2, the available shear rupture strength of the tee stem is determined using the net area determined in accordance with AISC *Specification* Section B4.3b.

$$A_{nv} = \left[l - n\left(d_h + \tfrac{1}{16} \text{ in.}\right)\right]t_{sw}$$
$$= \left[11\tfrac{1}{2} \text{ in.} - 4\left(\tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)\right](0.350 \text{ in.})$$
$$= 2.80 \text{ in.}^2$$

$$R_n = 0.60F_u A_{nv} \qquad (Spec. \text{ Eq. J4-4})$$
$$= 0.60(65 \text{ ksi})(2.80 \text{ in.}^2)$$
$$= 109 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(109 \text{ kips})$ $= 81.8 \text{ kips} > 54.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{109 \text{ kips}}{2.00}$ $= 54.5 \text{ kips} > 36.0 \text{ kips}$ **o.k.**

Block Shear Rupture of Stem

The nominal strength for the limit state of block shear rupture is given by AISC *Specification* Section J4.3.

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (Spec. \text{ Eq. J4-5})$$

The available block shear rupture strength of the tee stem is determined as follows, using AISC *Manual* Tables 9-3a, 9-3b and 9-3c, and AISC *Specification* Equation J4-5, with $n = 4$, $l_{eh} = l_{ev} = 1\tfrac{1}{4}$ in., and $U_{bs} = 1.0$.

LRFD	ASD
Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{\phi F_u A_{nt}}{t} = 39.6 \text{ kip/in.}$	Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{F_u A_{nt}}{\Omega t} = 26.4 \text{ kip/in.}$

LRFD	ASD
Shear yielding component from AISC *Manual* Table 9-3b: $$\frac{\phi 0.60F_y A_{gv}}{t} = 231 \text{ kip/in.}$$ Shear rupture component from AISC *Manual* Table 9-3c: $$\frac{\phi 0.60F_u A_{nv}}{t} = 210 \text{ kip/in.}$$ The design block shear rupture strength is: $$\begin{aligned}\phi R_n &= \phi 0.60F_u A_{nv} + \phi U_{bs} F_u A_{nt} \\ &\le \phi 0.60 F_y A_{gv} + \phi U_{bs} F_u A_{nt} \\ &= (0.350 \text{ in.})[210 \text{ kip/in.} + (1.0)(39.6 \text{ kip/in.})] \\ &\le (0.350 \text{ in.})[231 \text{ kip/in.} + (1.0)(39.6 \text{ kip/in.})] \\ &= 87.4 \text{ kips} < 94.7 \text{ kips}\end{aligned}$$ Therefore: $$\phi R_n = 87.4 \text{ kips} > 54.0 \text{ kips} \quad \textbf{o.k.}$$	Shear yielding component from AISC *Manual* Table 9-3b: $$\frac{0.60F_y A_{gv}}{\Omega t} = 154 \text{ kip/in.}$$ Shear rupture component from AISC *Manual* Table 9-3c: $$\frac{0.60F_u A_{nv}}{\Omega t} = 140 \text{ kip/in.}$$ The allowable block shear rupture strength is: $$\begin{aligned}\frac{R_n}{\Omega} &= \frac{0.60F_u A_{nv}}{\Omega} + \frac{U_{bs} F_u A_{nt}}{\Omega} \\ &\le \frac{0.60F_y A_{gv}}{\Omega} + \frac{U_{bs} F_u A_{nt}}{\Omega} \\ &= (0.350 \text{ in.})[140 \text{ kip/in.} + (1.0)(26.4 \text{ kip/in.})] \\ &\le (0.350 \text{ in.})[154 \text{ kip/in.} + (1.0)(26.4 \text{ kip/in.})] \\ &= 58.2 \text{ kips} < 63.1 \text{ kips}\end{aligned}$$ Therefore: $$\frac{R_n}{\Omega} = 58.2 \text{ kips} > 36.0 \text{ kips} \quad \textbf{o.k.}$$

Because the connection is rigid at the support, the bolts attaching the tee flange to the support must be designed for the shear and the eccentric moment.

Bolt Group at Column

Check bolts for shear and bearing combined with tension due to eccentricity.

The following calculation follows the Case II approach in the Section "Eccentricity Normal to the Plane of the Faying Surface" in Part 7 of the AISC *Manual*. The available shear strength of the bolts is determined as follows:

LRFD	ASD
$\phi r_n = 17.9$ kips/bolt (from AISC *Manual* Table 7-1)	$\frac{r_n}{\Omega} = 11.9$ kips/bolt (from AISC *Manual* Table 7-1)
$$\begin{aligned}r_{uv} &= \frac{P_u}{n} \\ &= \frac{54.0 \text{ kips}}{8 \text{ bolts}} \\ &= 6.75 \text{ kips/bolt} < 17.9 \text{ kips/bolt} \quad \textbf{o.k.}\end{aligned}$$ (*Manual* Eq. 7-13a)	$$\begin{aligned}r_{av} &= \frac{P_a}{n} \\ &= \frac{36.0 \text{ kips}}{8 \text{ bolts}} \\ &= 4.50 \text{ kips/bolt} < 11.9 \text{ kips/bolt} \quad \textbf{o.k.}\end{aligned}$$ (*Manual* Eq. 7-13b)
$A_b = 0.442 \text{ in.}^2$ (from AISC *Manual* Table 7-1)	$A_b = 0.442 \text{ in.}^2$ (from AISC *Manual* Table 7-1)

LRFD	ASD
$f_{rv} = \dfrac{r_{uv}}{A_b}$ $= \dfrac{6.75 \text{ kips/bolt}}{0.442 \text{ in.}^2}$ $= 15.3 \text{ ksi}$	$f_{rv} = \dfrac{r_{av}}{A_b}$ $= \dfrac{4.50 \text{ kips/bolt}}{0.442 \text{ in.}^2}$ $= 10.2 \text{ ksi}$

The nominal tensile stress modified to include the effects of shear stress is determined from AISC *Specification* Section J3.7 as follows. From AISC *Specification* Table J3.2:

$F_{nt} = 90 \text{ ksi}$

$F_{nv} = 54 \text{ ksi}$

LRFD	ASD
Tensile force per bolt, r_{ut}: $r_{ut} = \dfrac{P_u e}{n' d_m}$ (*Manual* Eq. 7-14a) $= \dfrac{(54.0 \text{ kips})(3.80 \text{ in.})}{(4 \text{ bolts})(6.00 \text{ in.})}$ $= 8.55 \text{ kips/bolt}$ $\phi = 0.75$ $F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$ (*Spec.* Eq. J3-3a) $= 1.3(90 \text{ ksi}) - \dfrac{90 \text{ ksi}}{0.75(54 \text{ ksi})}(15.3 \text{ ksi}) \le 90 \text{ ksi}$ $= 83.0 \text{ ksi} < 90 \text{ ksi}$ $= 83.0 \text{ ksi}$ $\phi r_n = \phi F'_{nt} A_b$ (from *Spec.* Eq. J3-2) $= 0.75(83.0 \text{ ksi})(0.442 \text{ in.}^2)$ $= 27.5 \text{ kips/bolt} > 8.55 \text{ kips/bolt}$ **o.k.**	Tensile force per bolt, r_{at}: $r_{at} = \dfrac{P_a e}{n' d_m}$ (*Manual* Eq. 7-14b) $= \dfrac{(36.0 \text{ kips})(3.80 \text{ in.})}{(4 \text{ bolts})(6.00 \text{ in.})}$ $= 5.70 \text{ kips/bolt}$ $\Omega = 2.00$ $F'_{nt} = 1.3F_{nt} - \dfrac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$ (*Spec.* Eq. J3-3b) $= 1.3(90 \text{ ksi}) - \dfrac{2.00(90 \text{ ksi})}{54 \text{ ksi}}(10.2 \text{ ksi}) \le 90 \text{ ksi}$ $= 83.0 \text{ ksi} < 90 \text{ ksi}$ $= 83.0 \text{ ksi}$ $\dfrac{r_n}{\Omega} = \dfrac{F'_{nt} A_b}{\Omega}$ (from *Spec.* Eq. J3-2) $= \dfrac{(83.0 \text{ ksi})(0.442 \text{ in.}^2)}{2.00}$ $= 18.3 \text{ kips/bolt} > 5.70 \text{ kips/bolt}$ **o.k.**

With l_e = 1¼ in. and s = 3 in., the bearing or tearout strength of the tee flange exceeds the single shear strength of the bolts. Therefore, the bearing and tearout strength is adequate.

Prying Action

From AISC *Manual* Part 9, the available tensile strength of the bolts taking prying action into account is determined as follows. By inspection, prying action in the tee will control over prying action in the column.

Note: The bolts are not located symmetrically with respect to the centerline of the tee.

$$a = \frac{b_f}{2} - \frac{t_w}{2} - \frac{t_{sw}}{2} - 2\tfrac{3}{4} \text{ in.}$$
$$= \frac{8.02 \text{ in.}}{2} - \frac{0.380 \text{ in.}}{2} - \frac{0.350 \text{ in.}}{2} - 2\tfrac{3}{4} \text{ in.}$$
$$= 0.895 \text{ in.}$$

$$b = 2\tfrac{3}{4} \text{ in.} + \frac{t_w}{2}$$
$$= 2\tfrac{3}{4} \text{ in.} + \frac{0.380 \text{ in.}}{2}$$
$$= 2.94 \text{ in.}$$

$$a' = \left(a + \frac{d_b}{2}\right) \leq \left(1.25b + \frac{d_b}{2}\right)$$ (*Manual* Eq. 9-23)
$$= 0.895 \text{ in.} + \frac{\tfrac{3}{4} \text{ in.}}{2} \leq 1.25(2.94 \text{ in.}) + \frac{\tfrac{3}{4} \text{ in.}}{2}$$
$$= 1.27 \text{ in.} < 4.05 \text{ in.}$$
$$= 1.27 \text{ in.}$$

$$b' = \left(b - \frac{d_b}{2}\right)$$ (*Manual* Eq. 9-18)
$$= 2.94 \text{ in.} - \frac{\tfrac{3}{4} \text{ in.}}{2}$$
$$= 2.57 \text{ in.}$$

$$\rho = \frac{b'}{a'}$$ (*Manual* Eq. 9-22)
$$= \frac{2.57 \text{ in.}}{1.27 \text{ in.}}$$
$$= 2.02$$

$$p = l_{ev} + 0.5s$$
$$= 1\tfrac{1}{4} \text{ in.} + 0.5(3 \text{ in.})$$
$$= 2.75 \text{ in.}$$

Check:

$p \leq s$

$2.75 \text{ in.} < 3 \text{ in.}$ **o.k.**

$p \leq l_{ev} + 1.75b$

$2.75 \text{ in.} \leq 1\tfrac{1}{4} \text{ in.} + 1.75(2.94 \text{ in.})$

$2.75 \text{ in.} < 6.40 \text{ in.}$ **o.k.**

$$d' = d_h$$
$$= \tfrac{13}{16} \text{ in.}$$

$$\delta = 1 - \frac{d'}{p} \qquad \text{(\textit{Manual} Eq. 9-20)}$$
$$= 1 - \frac{{}^{13}\!/_{16}\text{ in.}}{2.75\text{ in.}}$$
$$= 0.705$$

LRFD	ASD
$T_r = r_{ut}$ $= 8.55\text{ kips/bolt}$ $B_c = \phi r_n$ $= 27.5\text{ kips/bolt}$ $\beta = \frac{1}{\rho}\left(\frac{B_c}{T_r} - 1\right)$ (*Manual* Eq. 9-21) $= \frac{1}{2.02}\left[\left(\frac{27.5\text{ kips/bolt}}{8.55\text{ kips/bolt}}\right) - 1\right]$ $= 1.10$ Because $\beta \geq 1$, set $\alpha' = 1.0$. $\phi = 0.90$ $t_{min} = \sqrt{\frac{4T_u b'}{\phi p F_u (1 + \delta\alpha')}}$ (*Manual* Eq. 9-19a) $= \sqrt{\frac{4(8.55\text{ kips/bolt})(2.57\text{ in.})}{0.90(2.75\text{ in.})(65\text{ ksi})[1 + (0.705)(1.0)]}}$ $= 0.566\text{ in.} < 0.620\text{ in.}$ **o.k.**	$T_r = r_{at}$ $= 5.70\text{ kips/bolt}$ $B_c = \frac{r_n}{\Omega}$ $= 18.3\text{ kips/bolt}$ $\beta = \frac{1}{\rho}\left(\frac{B_c}{T_r} - 1\right)$ (*Manual* Eq. 9-21) $= \frac{1}{2.02}\left[\left(\frac{18.3\text{ kips/bolt}}{5.70\text{ kips/bolt}}\right) - 1\right]$ $= 1.09$ Because $\beta \geq 1$, set $\alpha' = 1.0$. $\Omega = 1.67$ $t_{min} = \sqrt{\frac{\Omega 4T_a b'}{p F_u (1 + \delta\alpha')}}$ (*Manual* Eq. 9-19b) $= \sqrt{\frac{1.67(4)(5.70\text{ kips/bolt})(2.57\text{ in.})}{(2.75\text{ in.})(65\text{ ksi})[1 + (0.705)(1.0)]}}$ $= 0.567\text{ in.} < 0.620\text{ in.}$ **o.k.**

Similarly, checks of the tee flange for shear yielding, shear rupture, and block shear rupture will show that the tee flange is adequate.

Bolt Bearing on Column Flange

The available bearing and tearout strength of the column flange is determined using AISC *Manual* Table 7-4 with s = 3 in.

LRFD	ASD
$\phi R_n = (8\text{ bolts})(87.8\text{ kip/in.})(0.710\text{ in.})$ $= 499\text{ kips} > 54.0\text{ kips}$ **o.k.**	$\frac{R_n}{\Omega} = (8\text{ bolts})(58.5\text{ kip/in.})(0.710\text{ in.})$ $= 332\text{ kips} > 36.0\text{ kips}$ **o.k.**

Note: Although the edge distance (a = 0.895 in.) for one row of bolts in the tee flange does not meet the minimum value indicated in AISC *Specification* Table J3.4, based on footnote [a], the edge distance provided is acceptable because the provisions of AISC *Specification* Section J3.10 and J4.4 have been met in this case.

Conclusion

The connection is found to be adequate as given for the applied load.

EXAMPLE II.A-31 BOLTED/WELDED TEE CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Verify the tee connection bolted to an ASTM A992 W16×50 supported beam and welded to an ASTM A992 W14×90 supporting column flange, as shown in Figure II.A-31-1, to support the following beam end reactions:

R_D = 6 kips
R_L = 18 kips

Use 70-ksi electrodes. Use an ASTM A992 WT5×22.5 with a four-bolt connection to the beam web.

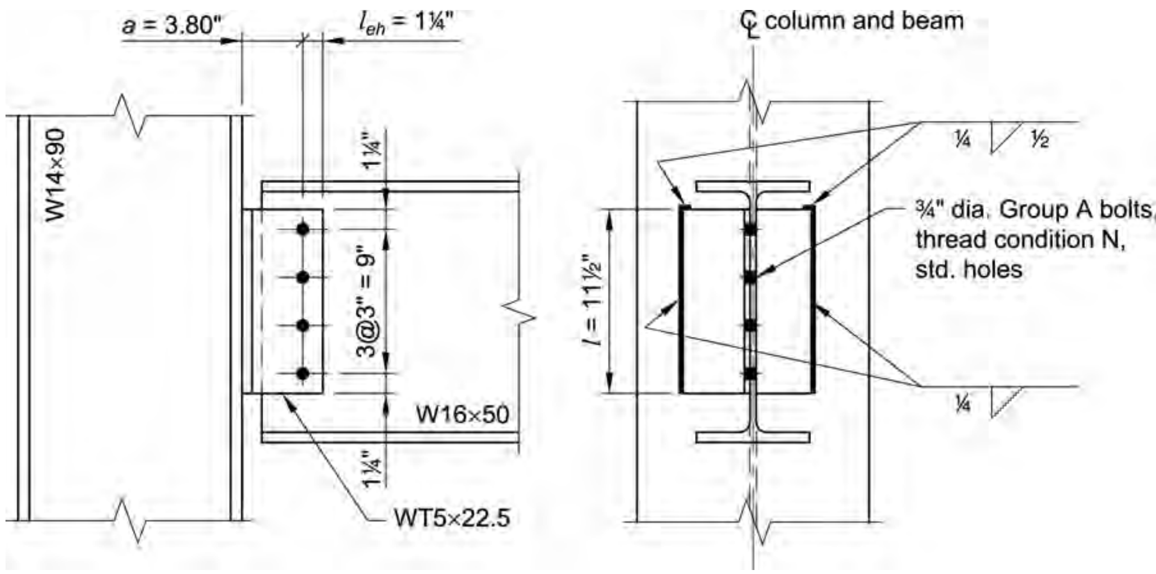


Fig. II.A-31-1. Connection geometry for Example II.A-31.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam, column and tee
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Tables 1-1 and 1-8, the geometric properties are as follows:

Beam
W16×50
t_w= 0.380 in.
d = 16.3 in.
t_f = 0.630 in.

Column
W14×90
t_f = 0.710 in.

Tee
WT5×22.5
d = 5.05 in.
b_f = 8.02 in.
t_f = 0.620 in.
t_{sw} = 0.350 in.
k_1 = 13⁄16 in. (see W10×45, AISC *Manual* Table 1-1)
k_{des} = 1.12 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(6 \text{ kips}) + 1.6(18 \text{ kips})$ $= 36.0 \text{ kips}$	$R_a = 6 \text{ kips} + 18 \text{ kips}$ $= 24.0 \text{ kips}$

Limitation on Tee Stem or Beam Web Thickness

See rotational ductility discussion at the beginning of AISC *Manual* Part 9.

For the tee stem, the maximum tee stem thickness is:

$$t_{sw\,max} = \frac{d}{2} + \tfrac{1}{16} \text{ in.} \qquad \text{(\textit{Manual} Eq. 9-39)}$$

$$= \frac{\tfrac{3}{4} \text{ in.}}{2} + \tfrac{1}{16} \text{ in.}$$

$$= 0.438 \text{ in.} > 0.350 \text{ in.} \quad \textbf{o.k.}$$

For W16×50 beam web, the maximum beam web thickness is:

$$t_{w\,max} = \frac{d}{2} + \tfrac{1}{16} \text{ in.} \qquad \text{(\textit{Manual} Eq. 9-39)}$$

$$= \frac{\tfrac{3}{4} \text{ in.}}{2} + \tfrac{1}{16} \text{ in.}$$

$$= 0.438 \text{ in.} > 0.380 \text{ in.} \quad \textbf{o.k.}$$

Weld Design

b = flexible width in connection element

$$= \frac{b_f - 2k_1}{2}$$

$$= \frac{8.02 \text{ in.} - 2\left(\tfrac{13}{16} \text{ in.}\right)}{2}$$

$$= 3.20 \text{ in.}$$

$$w_{min} = 0.0155\frac{F_y t_f^2}{b}\left(\frac{b^2}{l^2}+2\right) \le \left(\tfrac{5}{8}\right)t_{sw}$$ (*Manual* Eq. 9-37)

$$= 0.0155\left[\frac{(50\text{ ksi})(0.620\text{ in.})^2}{3.20\text{ in.}}\right]\left[\frac{(3.20\text{ in.})^2}{(11\tfrac{1}{2}\text{ in.})^2}+2\right] \le \left(\tfrac{5}{8}\right)(0.350\text{ in.})$$
$$= 0.193\text{ in.} < 0.219\text{ in.}$$
$$= 0.193\text{ in.}$$

The minimum weld size is ¼ in. per AISC *Specification* Table J2.4.

Try ¼-in. fillet welds.

From AISC *Manual* Table 10-2, with n = 4, l = 11½ in., and Welds B = ¼ in.:

LRFD	ASD
$\phi R_n = 79.9\text{ kips} > 36.0\text{ kips}$ **o.k.**	$\frac{R_n}{\Omega} = 53.3\text{ kips} > 24.0\text{ kips}$ **o.k.** .

Use ¼-in. fillet welds.

Supporting Column Flange

From AISC *Manual* Table 10-2, with n = 4, l = 11½ in., and Welds B = ¼ in., the minimum support thickness is 0.190 in.

$$t_f = 0.710\text{ in.} > 0.190\text{ in.} \quad \textbf{o.k.}$$

Strength of Bolted Connection

From the Commentary to AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the invividual strengths of the individual fasteners, taken as the lesser of the fastener shear strength per AISC *Specification* Section J3.6, the bearing strength at the bolt hole per AISC *Specification* Section J3.10, or the tearout strength at the bolt hole per AISC *Specification* Section J3.10. The 3-in. angle leg attached to the supporting girder will control because eccentricity must be taken into consideration.

Because the connection is flexible at the support, the tee stem and bolts must be designed for eccentric shear, where the eccentricity, e_b, is determined as follows:

$$e_b = a$$
$$= d - l_{eh}$$
$$= 5.05\text{ in.} - 1\tfrac{1}{4}\text{ in.}$$
$$= 3.80\text{ in.}$$

From AISC *Manual* Table 7-6 for Angle = 0°, with s = 3 in., $e_x = e_b$ = 3.80 in., and n = 4:

$$C = 2.45$$

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) is:

LRFD	ASD
$\phi r_n = 17.9$ kips/bolt	$\dfrac{r_n}{\Omega} = 11.9$ kips/bolt

The available bearing and tearout strength of the tee at the bottom edge bolt is determined using AISC *Manual* Table 7-5, with l_e = 1¼ in., as follows:

LRFD	ASD
$\phi r_n = (49.4 \text{ kip/in.})(0.350 \text{ in.})$ $= 17.3$ kips/bolt	$\dfrac{r_n}{\Omega} = (32.9 \text{ kip/in.})(0.350 \text{ in.})$ $= 11.5$ kips/bolt

The available bearing and tearout strength of the tee at the interior bolts (not adjacent to the edge) is determined using AISC *Manual* Table 7-4 with s = 3 in.

LRFD	ASD
$\phi r_n = (87.8 \text{ kip/in.})(0.350 \text{ in.})$ $= 30.7$ kips/bolt	$\dfrac{r_n}{\Omega} = (58.5 \text{ kip/in.})(0.350 \text{ in.})$ $= 20.5$ kips/bolt

Note: By inspection, bolt bearing on the beam web does not control.

The available strength of the bolted connection is determined from AISC *Manual* Equation 7-16, conservatively using the minimum available strength calculated for bolt shear, bearing on the tee, and tearout on the tee.

LRFD	ASD
$\phi R_n = C\phi r_n$ $= 2.45(17.3 \text{ kips/bolt})$ $= 42.4 \text{ kips} > 36.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = C\dfrac{r_n}{\Omega}$ $= 2.45(11.5 \text{ kips/bolt})$ $= 28.2 \text{ kips} > 24.0 \text{ kips}$ **o.k.**

Flexural Yielding of Tee Stem

The required flexural strength of the tee stem is determined as follows:

LRFD	ASD
$M_u = P_u e_b$ $= (36.0 \text{ kips})(3.80 \text{ in.})$ $= 137$ kip-in.	$M_a = P_a e_b$ $= (24.0 \text{ kips})(3.80 \text{ in.})$ $= 91.2$ kip-in.

The available flexural yielding strength of the tee stem is determined as follows:

LRFD	ASD
$\phi = 0.90$ $\phi M_n = \phi F_y Z_x$ $= 0.90(50\text{ ksi})\left[\dfrac{(0.350\text{ in.})(11\tfrac{1}{2}\text{ in.})^2}{4}\right]$ $= 521\text{ kip-in.} > 137\text{ kip-in.}$ **o.k.**	$\Omega = 1.67$ $\dfrac{M_n}{\Omega} = \dfrac{F_y Z_x}{\Omega}$ $= \dfrac{50\text{ ksi}}{1.67}\left[\dfrac{(0.350\text{ in.})(11\tfrac{1}{2}\text{ in.})^2}{4}\right]$ $= 346\text{ kip-in.} > 91.2\text{ kip-in.}$ **o.k.**

Flexural Rupture of Tee Stem

The available flexural rupture strength of the plate is determined as follows:

$$Z_{net} = (0.350\text{ in.})\left[\frac{(11\tfrac{1}{2}\text{ in.})^2}{4} - 2(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(4.50\text{ in.}) - 2(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(1.50\text{ in.})\right]$$
$$= 7.90\text{ in.}^3$$

$$M_n = F_u Z_{net} \qquad \text{(\textit{Manual} Eq. 9-4)}$$
$$= (65\text{ ksi})(7.90\text{ in.}^3)$$
$$= 514\text{ kip-in.}$$

LRFD	ASD
$\phi = 0.75$ $\phi M_n = 0.75(514\text{ kip-in.})$ $= 386\text{ kip-in.} > 137\text{ kip-in.}$ **o.k.**	$\Omega = 2.00$ $\dfrac{M_n}{\Omega} = \dfrac{514\text{ kip-in.}}{2.00}$ $= 257\text{ kip-in.} > 91.2\text{ kip-in.}$ **o.k.**

Shear Strength of Stem

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the tee stem is determined as follows:

$$A_{gv} = lt_{sw}$$
$$= (11\tfrac{1}{2}\text{ in.})(0.350\text{ in.})$$
$$= 4.03\text{ in.}^2$$

$$R_n = 0.60F_y A_{gv} \qquad \text{(\textit{Spec.} Eq. J4-3)}$$
$$= 0.60(50\text{ ksi})(4.03\text{ in.}^2)$$
$$= 121\text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(121\text{ kips})$ $= 121\text{ kips} > 36.0\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{121\text{ kips}}{1.50}$ $= 80.7\text{ kips} > 24.0\text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the tee stem is determined using the net area determined in accordance with AISC *Specification* Section B4.3b.

$$A_{nv} = \left[l - n\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]t_{sw}$$
$$= \left[11\tfrac{1}{2}\text{ in.} - 4\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right](0.350\text{ in.})$$
$$= 2.80\text{ in.}^2$$

$$R_n = 0.60F_uA_{nv} \qquad (Spec.\text{ Eq. J4-4})$$
$$= 0.60(65\text{ ksi})(2.80\text{ in.}^2)$$
$$= 109\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(109\text{ kips})$ $= 81.8\text{ kips} > 36.0\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{109\text{ kips}}{2.00}$ $= 54.5\text{ kips} > 24.0\text{ kips}$ **o.k.**

Block Shear Rupture of Stem

The nominal strength for the limit state of block shear rupture is given by AISC *Specification* Section J4.3.

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \qquad (Spec.\text{ Eq. J4-5})$$

The available block shear rupture strength of the tee stem is determined as follows, using AISC *Manual* Tables 9-3a, 9-3b and 9-3c, and AISC *Specification* Equation J4-5, with $n = 4$, $l_{eh} = l_{ev} = 1\tfrac{1}{4}$ in., and $U_{bs} = 1.0$.

LRFD	ASD
Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{\phi F_uA_{nt}}{t} = 39.6\text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{\phi 0.60F_yA_{gv}}{t} = 231\text{ kip/in.}$	Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{F_uA_{nt}}{\Omega t} = 26.4\text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{0.60F_yA_{gv}}{\Omega t} = 154\text{ kip/in.}$

LRFD	ASD
Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{\phi 0.60F_uA_{nv}}{t} = 210 \text{ kip/in.}$ The design block shear rupture strength is: $\phi R_n = \phi 0.60F_uA_{nv} + \phi U_{bs}F_uA_{nt}$ $\quad \le \phi 0.60F_yA_{gv} + \phi U_{bs}F_uA_{nt}$ $\quad = (0.350 \text{ in.})[210 \text{ kip/in.} + (1.0)(39.6 \text{ kip/in.})]$ $\quad \le (0.350 \text{ in.})[231 \text{ kip/in.} + (1.0)(39.6 \text{ kip/in.})]$ $\quad = 87.4 \text{ kips} < 94.7 \text{ kips}$ Therefore: $\phi R_n = 87.4 \text{ kips} > 36.0 \text{ kips}$ **o.k.**	Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{0.60F_uA_{nv}}{\Omega t} = 140 \text{ kip/in.}$ The allowable block shear rupture strength is: $\dfrac{R_n}{\Omega} = \dfrac{0.60F_uA_{nv}}{\Omega} + \dfrac{U_{bs}F_uA_{nt}}{\Omega}$ $\quad \le \dfrac{0.60F_yA_{gv}}{\Omega} + \dfrac{U_{bs}F_uA_{nt}}{\Omega}$ $\quad = (0.350 \text{ in.})[140 \text{ kip/in.} + (1.0)(26.4 \text{ kip/in.})]$ $\quad \le (0.350 \text{ in.})[154 \text{ kip/in.} + (1.0)(26.4 \text{ kip/in.})]$ $\quad = 58.2 \text{ kips} < 63.1 \text{ kips}$ Therefore: $\dfrac{R_n}{\Omega} = 58.2 \text{ kips} > 24.0 \text{ kips}$ **o.k.**

Conclusion

The connection is found to be adequate as given for the applied load.

Chapter IIB
Fully Restrained (FR) Moment Connections

The design of fully restrained (FR) moment connections is covered in Part 12 of the AISC *Manual*.

EXAMPLE II.B-1 BOLTED FLANGE-PLATED FR MOMENT CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Verify a bolted flange-plated FR moment connection between an ASTM A992 W18×50 beam and an ASTM A992 W14×99 column flange, as shown in Figure II.B-1-1, to transfer the following beam end reactions:

Vertical shear:
V_D = 7 kips
V_L = 21 kips

Strong-axis moment:
M_D = 42 kip-ft
M_L = 126 kip-ft

Use 70-ksi electrodes. The flange and web plates are ASTM A36 material. Check the column for stiffening requirements.

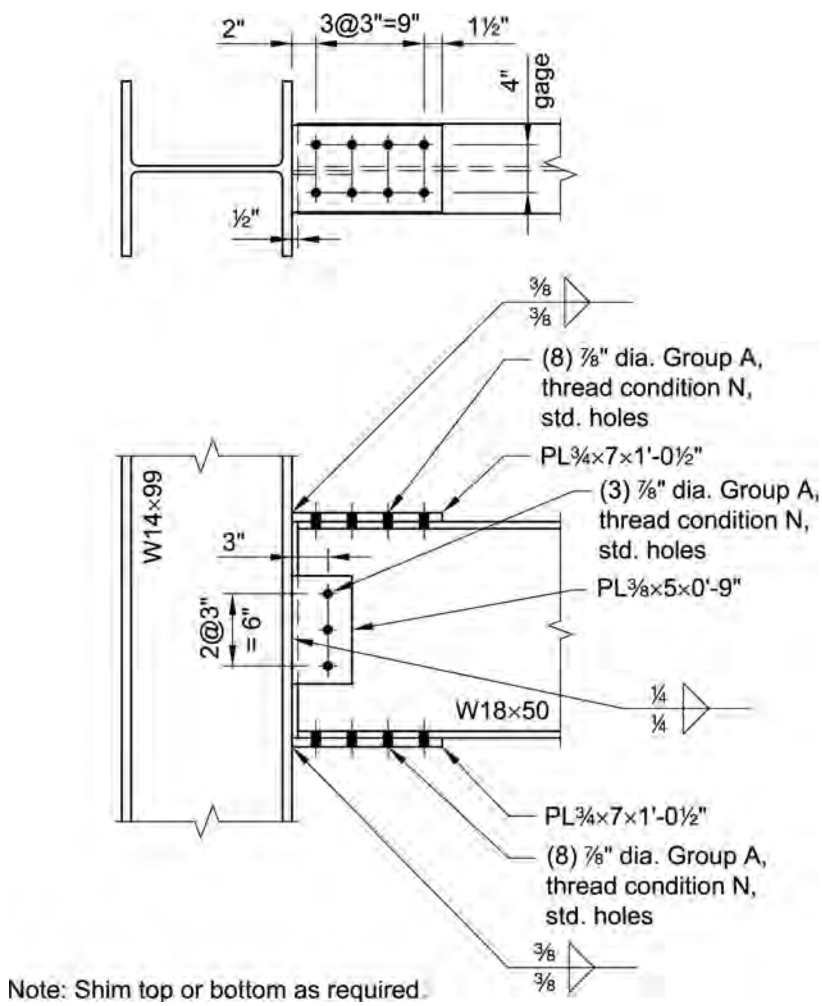


Fig. II.B-1-1. Connection geometry for Example II.B-1.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam and column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plates
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×50
d = 18.0 in.
b_f = 7.50 in.
t_f = 0.570 in.
t_w = 0.355 in.
S_x = 88.9 in.^{3}

Column
W14×99
d = 14.2 in.
b_f = 14.6 in.
t_f = 0.780 in.

From AISC *Specification* Table J3.3, the hole diameter for a ⅞-in.-diameter bolt with standard holes is:

$$d_h = 15/16 \text{ in.}$$

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(7 \text{ kips}) + 1.6(21 \text{ kips}) = 42.0 \text{ kips}$	$R_a = 7 \text{ kips} + 21 \text{ kips} = 28.0 \text{ kips}$
$M_u = 1.2(42 \text{ kip-ft}) + 1.6(126 \text{ kip-ft}) = 252 \text{ kip-ft}$	$M_a = 42 \text{ kip-ft} + 126 \text{ kip-ft} = 168 \text{ kip-ft}$

Flexural Strength of Beam

From AISC *Specification* Section F13.1, the available flexural strength of the beam is limited according to the limit state of tensile rupture of the tension flange.

$$A_{fg} = b_f t_f$$
$$= (7.50 \text{ in.})(0.570 \text{ in.})$$
$$= 4.28 \text{ in.}^2$$

The net area of the flange is determined in accordance with AISC *Specification* Section B4.3b.

$$
\begin{aligned}
A_{fn} &= A_{fg} - (2 \text{ bolts})(d_h + \tfrac{1}{16} \text{ in.})t_f \\
&= 4.28 \text{ in.}^2 - (2 \text{ bolts})(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(0.570 \text{ in.}) \\
&= 3.14 \text{ in.}^2
\end{aligned}
$$

$$
\begin{aligned}
\frac{F_y}{F_u} &= \frac{50 \text{ ksi}}{65 \text{ ksi}} \\
&= 0.769 < 0.8; \text{ therefore, } Y_t = 1.0
\end{aligned}
$$

$$
\begin{aligned}
F_u A_{fn} &= (65 \text{ ksi})(3.14 \text{ in.}^2) \\
&= 204 \text{ kips}
\end{aligned}
$$

$$
\begin{aligned}
Y_t F_y A_{fg} &= 1.0(50 \text{ ksi})(4.28 \text{ in.}^2) \\
&= 214 \text{ kips} > 204 \text{ kips}
\end{aligned}
$$

Therefore, the nominal flexural strength, M_n, at the location of the holes in the tension flange is not greater than:

$$
\begin{aligned}
M_n &= \frac{F_u A_{fn}}{A_{fg}} S_x \qquad \text{(}\textit{Spec}\text{. Eq. F13-1)} \\
&= \left(\frac{204 \text{ kips}}{4.28 \text{ in.}^2}\right)(88.9 \text{ in.}^3) \\
&= 4{,}240 \text{ kip-in. or } 353 \text{ kip-ft}
\end{aligned}
$$

LRFD	ASD
$\phi_b = 0.90$ $\phi M_n = 0.90(353 \text{ kip-ft})$ $= 318 \text{ kip-ft} > 252 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{353 \text{ kip-ft}}{1.67}$ $= 211 \text{ kip-ft} > 168 \text{ kip-ft}$ **o.k.**

Note: The available flexural strength of the beam may be less than that determined based on AISC *Specification* Equation F13-1. Other applicable provisions in AISC *Specification* Chapter F should be checked to possibly determine a lower value for the available flexural strength of the beam.

Single-Plate Web Connection

Strength of the bolted connection—web plate

From the Commentary to AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the strengths of the individual fasteners, which may be taken as the lesser of the fastener shear strength per AISC *Specification* Section J3.6, the bearing strength at the bolt hole per AISC *Specification* Section J3.10, or the tearout strength at the bolt hole per AISC *Specification* Section J3.10.

From AISC *Manual* Table 7-1, the available shear strength per bolt for ⅞-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) is:

LRFD	ASD

$\phi r_n = 24.3$ kips/bolt	$\dfrac{r_n}{\Omega} = 16.2$ kips/bolt

The available bearing strength of the plate per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$\begin{aligned} r_n &= 2.4dtF_u \\ &= 2.4(\tfrac{7}{8}\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi}) \\ &= 45.7\text{ kips/bolt} \end{aligned}$$ (*Spec*. Eq. J3-6a)

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(45.7\text{ kips/bolt})$ $= 34.3$ kips/bolt	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{45.7\text{ kips/bolt}}{2.00}$ $= 22.9$ kips/bolt

The available tearout strength of the plate at the interior bolts is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration.

$$\begin{aligned} l_c &= s - d_h \\ &= 3\text{ in.} - \tfrac{15}{16}\text{ in.} \\ &= 2.06\text{ in.} \end{aligned}$$

$$\begin{aligned} r_n &= 1.2l_ctF_u \\ &= 1.2(2.06\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi}) \\ &= 53.8\text{ kips/bolt} \end{aligned}$$ (*Spec*. Eq. J3-6c)

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(53.8\text{ kips/bolt})$ $= 40.4$ kips/bolt	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{53.8\text{ kips/bolt}}{2.00}$ $= 26.9$ kips/bolt

Therefore, bolt shear controls over bearing or tearout at interior bolts.

The available tearout strength of the plate at the edge bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration.

$$\begin{aligned} l_c &= l_{ev} - 0.5(d_h) \\ &= 1\tfrac{1}{2}\text{ in.} - 0.5(\tfrac{15}{16}\text{ in.}) \\ &= 1.03\text{ in.} \end{aligned}$$

$$\begin{aligned} r_n &= 1.2l_ctF_u \\ &= 1.2(1.03\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi}) \\ &= 26.9\text{ kips/bolt} \end{aligned}$$ (*Spec*. Eq. J3-6c)

LRFD	ASD

$\phi = 0.75$ $\phi r_n = 0.75(26.9 \text{ kips/bolt})$ $= 20.2 \text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{26.9 \text{ kips/bolt}}{2.00}$ $= 13.5 \text{ kips/bolt}$

Therefore, tearout controls over bolt shear or bearing at the edge bolt.

The strength of the bolt group in the plate is determined by summing the strength of the individual fasteners as follows:

LRFD	ASD
$\phi R_n = (1 \text{ bolt})(20.2 \text{ kips/bolt}) + (2 \text{ bolts})(24.3 \text{ kips/bolt})$ $= 68.8 \text{ kips} > 42.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (1 \text{ bolt})(13.5 \text{ kips/bolt}) + (2 \text{ bolts})(16.2 \text{ kips/bolt})$ $= 45.9 \text{ kips} > 28.0 \text{ kips}$ **o.k.**

Strength of the bolted connection—beam web

Because there are no edge bolts, the available bearing and tearout strength of the beam web for all bolts is determined using AISC *Manual* Table 7-4 with $s = 3$ in.

LRFD	ASD
$\phi r_n = (102 \text{ kip/in.})(0.355 \text{ in.})$ $= 36.2 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (68.3 \text{ kip/in.})(0.355 \text{ in.})$ $= 24.2 \text{ kips/bolt}$

Bolt shear strength is the governing limit state for all bolts at the beam web.

The strength of the bolt group in the beam web is determined by summing the strength of the individual fasteners as follows:

LRFD	ASD
$\phi R_n = (3 \text{ bolts})(24.3 \text{ kips/bolt})$ $= 72.9 \text{ kips} > 42.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (3 \text{ bolts})(16.2 \text{ kips/bolt})$ $= 48.6 \text{ kips} > 28.0 \text{ kips}$ **o.k.**

Shear strength of the web plate

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the plate is determined as follows:

$$A_{gv} = lt = (9 \text{ in.})(3/8 \text{ in.}) = 3.38 \text{ in.}^2$$

$$R_n = 0.60F_yA_{gv} \quad \text{(Spec. Eq. J4-3)}$$
$$= 0.60(36 \text{ ksi})(3.38 \text{ in.}^2) = 73.0 \text{ kips}$$

LRFD	ASD

$\phi = 1.00$ $\phi R_n = 1.00(73.0 \text{ kips})$ $= 73.0 \text{ kips} > 42.0 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{73.0 \text{ kips}}{1.50}$ $= 48.7 \text{ kips} > 28.0 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the plate is determined as follows:

$$A_{nv} = \left[l - n\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]t$$
$$= \left[9\text{ in.} - (3\text{ bolts})\left(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]\left(\tfrac{3}{8}\text{ in.}\right)$$
$$= 2.25\text{ in.}^2$$

$$R_n = 0.60F_uA_{nv}$$ (*Spec*. Eq. J4-4)
$$= 0.60(58\text{ ksi})\left(2.25\text{ in.}^2\right)$$
$$= 78.3\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(78.3 \text{ kips})$ $= 58.7 \text{ kips} > 42.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{78.3 \text{ kips}}{2.00}$ $= 39.2 \text{ kips} > 28.0 \text{ kips}$ **o.k.**

Block shear rupture of the web plate

The nominal strength for the limit state of block shear rupture is given by AISC *Specification* Section J4.3.

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \leq 0.60F_yA_{gv} + U_{bs}F_uA_{nt}$$ (*Spec*. Eq. J4-5)

The available block shear rupture strength of the web plate is determined as follows, using AISC *Manual* Tables 9-3a, 9-3b and 9-3c, and AISC *Specification* Equation J4-5, with $n = 3$, $l_{eh} = 2$ in., $l_{ev} = 1\tfrac{1}{2}$ in., and $U_{bs} = 1.0$.

LRFD	ASD
Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{\phi F_uA_{nt}}{t} = 65.3 \text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{\phi 0.60F_yA_{gv}}{t} = 121 \text{ kip/in.}$	Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{F_uA_{nt}}{\Omega t} = 43.5 \text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{0.60F_yA_{gv}}{\Omega t} = 81.0 \text{ kip/in.}$

LRFD	ASD

Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{\phi 0.60 F_u A_{nv}}{t} = 131 \text{ kip/in.}$ The design block shear rupture strength is: $\phi R_n = \phi 0.60 F_u A_{nv} + \phi U_{bs} F_u A_{nt}$ $\leq \phi 0.60 F_y A_{gv} + \phi U_{bs} F_u A_{nt}$ $= (3/8 \text{ in.})[131 \text{ kip/in.} + (1.0)(65.3 \text{ kip/in.})]$ $\leq (3/8 \text{ in.})[121 \text{ kip/in.} + (1.0)(65.3 \text{ kip/in.})]$ $= 73.6 \text{ kips} > 69.9 \text{ kips}$ Therefore: $\phi R_n = 69.9 \text{ kips} > 42.0 \text{ kips}$ **o.k.**	Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{0.60 F_u A_{nv}}{\Omega t} = 87.0 \text{ kip/in.}$ The allowable block shear rupture strength is: $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega} + \dfrac{U_{bs} F_u A_{nt}}{\Omega}$ $\leq \dfrac{0.60 F_y A_{gv}}{\Omega} + \dfrac{U_{bs} F_u A_{nt}}{\Omega}$ $= (3/8 \text{ in.})[87.0 \text{ kip/in.} + (1.0)(43.5 \text{ kip/in.})]$ $\leq (3/8 \text{ in.})[81.0 \text{ kip/in.} + (1.0)(43.5 \text{ kip/in.})]$ $= 48.9 \text{ kips} > 46.7 \text{ kips}$ Therefore: $\dfrac{R_n}{\Omega} = 46.7 \text{ kips} > 28.0 \text{ kips}$ **o.k.**

Weld shear strength of the web plate to the column flange

The available weld strength is determined using AISC *Manual* Equations 8-2a or 8-2b, with the assumption that the weld is in direct shear (the incidental moment in the weld plate due to eccentricity is absorbed by the flange plates).

$D = 4$ (for a ¼-in. fillet weld)

LRFD	ASD
$\phi R_n = (2 \text{ welds})(1.392 \text{ kip/in.})Dl$ $= (2 \text{ welds})(1.392 \text{ kip/in.})(4)(9 \text{ in.})$ $= 100 \text{ kips} > 42.0 \text{ kips}$ **o.k.**	$\phi R_n = (2 \text{ welds})(0.928 \text{ kip/in.})Dl$ $= (2 \text{ welds})(0.928 \text{ kip/in.})(4)(9 \text{ in.})$ $= 66.8 \text{ kips} > 28.0 \text{ kips}$ **o.k.**

Column flange rupture strength at welds

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the column flange is determined as follows:

$$A_{nv} = (2 \text{ welds}) l t_f$$
$$= (2 \text{ welds})(9 \text{ in.})(0.780 \text{ in.})$$
$$= 14.0 \text{ in.}^2$$

$$R_n = 0.60 F_u A_{nv} \quad \text{(Spec. Eq. J4-4)}$$
$$= 0.60(65 \text{ ksi})(14.0 \text{ in.}^2)$$
$$= 546 \text{ kips}$$

LRFD	ASD

$\phi = 0.75$ $\phi R_n = 0.75(546 \text{ kips})$ $= 410 \text{ kips} > 42.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{546 \text{ kips}}{2.00}$ $= 273 \text{ kips} > 28.0 \text{ kips}$ **o.k.**

Flange Plate Connection

Flange force

The moment arm between flange forces, d_m, used for verifying the fastener strength is equal to the depth of the beam. This dimension represents the faying surface between the flange of the beam and the tension plate.

LRFD	ASD
$P_{uf} = \dfrac{M_u}{d_m}$ (*Manual* Eq. 12-1a) $= \dfrac{(252 \text{ kip-ft})(12 \text{ in./ft})}{18.0 \text{ in.}}$ $= 168 \text{ kips}$	$P_{af} = \dfrac{M_a}{d_m}$ (*Manual* Eq. 12-1b) $= \dfrac{(168 \text{ kip-ft})(12 \text{ in./ft})}{18.0 \text{ in.}}$ $= 112 \text{ kips}$

Strength of the bolted connection—flange plate

From the Commentary to AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the strengths of the individual fasteners, which may be taken as the lesser of the fastener shear strength per AISC *Specification* Section J3.6, the bearing strength at the bolt hole per AISC *Specification* Section J3.10, or the tearout strength at the bolt hole per AISC *Specification* Section J3.10.

From AISC *Manual* Table 7-1, the available shear strength per bolt for ⅞-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) is:

LRFD	ASD
$\phi r_n = 24.3 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = 16.2 \text{ kips/bolt}$

The available bearing strength of the plate per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$r_n = 2.4dtF_u \quad (\textit{Spec}.\text{ Eq. J3-6a})$$
$$= 2.4(\tfrac{7}{8} \text{ in.})(\tfrac{3}{4} \text{ in.})(58 \text{ ksi})$$
$$= 91.4 \text{ kips/bolt}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(91.4 \text{ kips/bolt})$ $= 68.6 \text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{91.4 \text{ kips/bolt}}{2.00}$ $= 45.7 \text{ kips/bolt}$

The available tearout strength of the plate at the interior bolts is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration.

$$l_c = s - d_h$$
$$= 3 \text{ in.} - \tfrac{15}{16} \text{ in.}$$
$$= 2.06 \text{ in.}$$

$$r_n = 1.2 l_c t F_u \qquad \text{(\textit{Spec.} Eq. J3-6c)}$$
$$= 1.2(2.06 \text{ in.})(\tfrac{3}{4} \text{ in.})(58 \text{ ksi})$$
$$= 108 \text{ kips/bolt}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(108 \text{ kips/bolt})$ $= 81.0 \text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{108 \text{ kips/bolt}}{2.00}$ $= 54.0 \text{ kips/bolt}$

Therefore, bolt shear controls over bearing or tearout at interior bolts.

The available tearout strength of the plate at the edge bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration.

$$l_c = l_{ev} - 0.5(d_h)$$
$$= 1\tfrac{1}{2} \text{ in.} - 0.5(\tfrac{15}{16} \text{ in.})$$
$$= 1.03 \text{ in.}$$

$$r_n = 1.2 l_c t F_u \qquad \text{(\textit{Spec.} Eq. J3-6c)}$$
$$= 1.2(1.03 \text{ in.})(\tfrac{3}{4} \text{ in.})(58 \text{ ksi})$$
$$= 53.8 \text{ kips/bolt}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(53.8 \text{ kips/bolt})$ $= 40.4 \text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{53.8 \text{ kips/bolt}}{2.00}$ $= 26.9 \text{ kips/bolt}$

Therefore, bolt shear controls over bearing or tearout at edge bolts.

The strength of the bolt group in the beam web is determined by summing the strength of the individual fasteners as follows:

LRFD	ASD
$\phi R_n = (8 \text{ bolts})(24.3 \text{ kips/bolt})$ $= 194 \text{ kips} > 168 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (8 \text{ bolts})(16.2 \text{ kips/bolt})$ $= 130 \text{ kips} > 112 \text{ kips}$ **o.k.**

Strength of the bolted connection—beam flange

The available bearing strength of the flange per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$r_n = 2.4dtF_u$ (*Spec*. Eq. J3-6a)
$= 2.4(\text{⅞ in.})(0.570\text{ in.})(65\text{ ksi})$
$= 77.8\text{ kips/bolt}$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(77.8\text{ kips/bolt})$ $= 58.4\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{77.8\text{ kips/bolt}}{2.00}$ $= 38.9\text{ kips/bolt}$

The available tearout strength of the flange at the interior bolts is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration.

$l_c = s - d_h$
$= 3\text{ in.} - {}^{15}\!/_{16}\text{ in.}$
$= 2.06\text{ in.}$

$r_n = 1.2l_c tF_u$ (*Spec*. Eq. J3-6c)
$= 1.2(2.06\text{ in.})(0.570\text{ in.})(65\text{ ksi})$
$= 91.6\text{ kips/bolt}$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(91.6\text{ kips/bolt})$ $= 68.7\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{91.6\text{ kips/bolt}}{2.00}$ $= 45.8\text{ kips/bolt}$

Therefore, bolt shear controls over bearing or tearout at interior bolts.

The available tearout strength of the flange at the edge bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration.

$l_c = l_{ev} - 0.5(d_h)$
$= 1\text{½ in.} - 0.5({}^{15}\!/_{16}\text{ in.})$
$= 1.03\text{ in.}$

$r_n = 1.2l_c tF_u$ (*Spec*. Eq. J3-6c)
$= 1.2(1.03\text{ in.})(\text{¾ in.})(58\text{ ksi})$
$= 53.8\text{ kips/bolt}$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(53.8\text{ kips/bolt})$ $= 40.4\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{53.8\text{ kips/bolt}}{2.00}$ $= 26.9\text{ kips/bolt}$

Therefore, bolt shear controls over bearing or tearout at edge bolts.

The strength of the bolt group in the flange is determined by summing the strength of the individual fasteners as follows:

LRFD	ASD
$\phi R_n = (8 \text{ bolts})(24.3 \text{ kips/bolt})$ $= 194 \text{ kips} > 168 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (8 \text{ bolts})(16.2 \text{ kips/bolt})$ $= 130 \text{ kips} > 112 \text{ kips}$ **o.k.**

Tensile strength of the flange plate

The moment arm between flange forces, d_m, used for verifying the tensile strength of the flange plate is equal to the depth of the beam plus one plate thickness. This represents the distance between the centerlines of the flange plates at the top and bottom of the beam. From AISC *Manual* Equation 12-1a or 12-1b, the flange force is:

LRFD	ASD
$P_{uf} = \dfrac{M_u}{d_m}$ $= \dfrac{(252 \text{ kip-ft})(12 \text{ in./ft})}{18.0 \text{ in.} + \frac{3}{4} \text{ in.}}$ $= 161 \text{ kips}$	$P_{af} = \dfrac{M_a}{d_m}$ $= \dfrac{(168 \text{ kip-ft})(12 \text{ in./ft})}{18.0 \text{ in.} + \frac{3}{4} \text{ in.}}$ $= 108 \text{ kips}$

From AISC *Specification* Section J4.1(a), the available tensile yield strength of the flange plate is determined as follows:

$$A_g = bt$$
$$= (7 \text{ in.})(\tfrac{3}{4} \text{ in.})$$
$$= 5.25 \text{ in.}^2$$

$$R_n = F_y A_g \qquad \text{(\textit{Spec}. Eq. J4-1)}$$
$$= (36 \text{ ksi})(5.25 \text{ in.}^2)$$
$$= 189 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(189 \text{ kips})$ $= 170 \text{ kips} > 161 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{189 \text{ kips}}{1.67}$ $= 113 \text{ kips} > 108 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.1(b), the available tensile rupture strength of the flange plate is determined as follows:

$$A_n = \left[b - n\left(d_h + \tfrac{1}{16}\text{-in.}\right)\right]t$$
$$= \left[7 \text{ in.} - (2 \text{ bolts})\left(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)\right](\tfrac{3}{4} \text{ in.})$$
$$= 3.75 \text{ in.}^2$$

Table D3.1, Case 1, applies in this case because the tension load is transmitted directly to the cross-sectional element by fasteners; therefore, $U = 1.0$.

$$A_e = A_n U \qquad \text{(Spec. Eq. D3-1)}$$
$$= (3.75 \text{ in.}^2)(1.0)$$
$$= 3.75 \text{ in.}^2$$

$$R_n = F_u A_e \qquad \text{(Spec. Eq. J4-2)}$$
$$= (58 \text{ ksi})(3.75 \text{ in.}^2)$$
$$= 218 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(218 \text{ kips})$ $= 164 \text{ kips} > 161 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{218 \text{ kips}}{2.00}$ $= 109 \text{ kips} > 108 \text{ kips}$ **o.k.**

Flange plate block shear rupture

There are three cases for which block shear rupture of the flange plate must be checked. Case 1, as shown in Figure II.B-1-2(a), involves the tearout of the two blocks outside the two rows of bolt holes in the flange plate; for this case l_{eh} = 1½ in. and l_{ev} = 1½ in. Case 2, as shown in Figure II.B-1-2(b), involves the tearout of the block between the two rows of the holes in the flange plate. AISC *Manual* Tables 9-3a, 9-3b, and 9-3c may be adapted for this calculation by considering the 4 in. width to be comprised of two, 2-in.-wide blocks, where l_{eh} = 2 in. and l_{ev} = 1½ in. Case 1 is more critical than the Case 2 because l_{eh} is smaller. Case 3, as shown in Figure II.B-1-2(c), involves a shear failure through one row of bolts and a tensile failure through the two bolts closest to the column. Therefore, Case 1 and Case 3 will be verified.

Flange plate block shear rupture—Case 1

The nominal strength for the limit state of block shear rupture is given by AISC *Specification* Section J4.3.

$$R_n = 0.60 F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60 F_y A_{gv} + U_{bs} F_u A_{nt} \qquad \text{(Spec. Eq. J4-5)}$$

The available block shear rupture strength of the flange plate is determined as follows, using AISC *Manual* Tables 9-3a, 9-3b and 9-3c, and AISC *Specification* Equation J4-5, with $n = 4$, $l_{eh} = l_{ev}$ = 1½ in., and $U_{bs} = 1.0$.

LRFD	ASD
Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{\phi F_u A_{nt}}{t} = 43.5 \text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{\phi 0.60 F_y A_{gv}}{t} = 170 \text{ kip/in.}$	Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{F_u A_{nt}}{\Omega t} = 29.0 \text{ kip/in.}$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{0.60 F_y A_{gv}}{\Omega t} = 113 \text{ kip/in.}$

TOC Back

LRFD	ASD
Shear rupture component from AISC *Manual* Table 9-3c:	Shear rupture component from AISC *Manual* Table 9-3c:
$\dfrac{\phi 0.60F_u A_{nv}}{t} = 183 \text{ kip/in.}$	$\dfrac{0.60F_u A_{nv}}{\Omega t} = 122 \text{ kip/in.}$
The design block shear rupture strength is:	The allowable block shear rupture strength is:
$\phi R_n = \phi 0.60F_u A_{nv} + \phi U_{bs} F_u A_{nt}$	$\dfrac{R_n}{\Omega} = \dfrac{0.60F_u A_{nv}}{\Omega} + \dfrac{U_{bs} F_u A_{nt}}{\Omega}$
$\leq \phi 0.60F_y A_{gv} + \phi U_{bs} F_u A_{nt}$	$\leq \dfrac{0.60F_y A_{gv}}{\Omega} + \dfrac{U_{bs} F_u A_{nt}}{\Omega}$
$= (2 \text{ planes})(\tfrac{3}{4} \text{ in.})\left[183 \text{ kip/in.} + 1.0(43.5 \text{ kip/in.})\right]$	$= (2 \text{ planes})(\tfrac{3}{4} \text{ in.})\left[122 \text{ kip/in.} + 1.0(29.0 \text{ kip/in.})\right]$
$\leq (2 \text{ planes})(\tfrac{3}{4} \text{ in.})\left[170 \text{ kip/in.} + 1.0(43.5 \text{ kip/in.})\right]$	$\leq (2 \text{ planes})(\tfrac{3}{4} \text{ in.})\left[113 \text{ kip/in.} + 1.0(29.0 \text{ kip/in.})\right]$
$= 340 \text{ kips} > 320 \text{ kips}$	$= 227 \text{ kips} > 213 \text{ kips}$
Therefore:	Therefore:
$\phi R_n = 320 \text{ kips} > 161 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 213 \text{ kips} > 108 \text{ kips}$ **o.k.**

Flange plate block shear rupture—Case 3

Because AISC *Manual* Table 9-3a does not include a large enough edge distance, the nominal strength for the limit state of block shear rupture is calculated by directly applying the provisions of AISC *Specification* Section J4.3.

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \quad \text{(}\textit{Spec.}\text{ Eq. J4-5)}$$

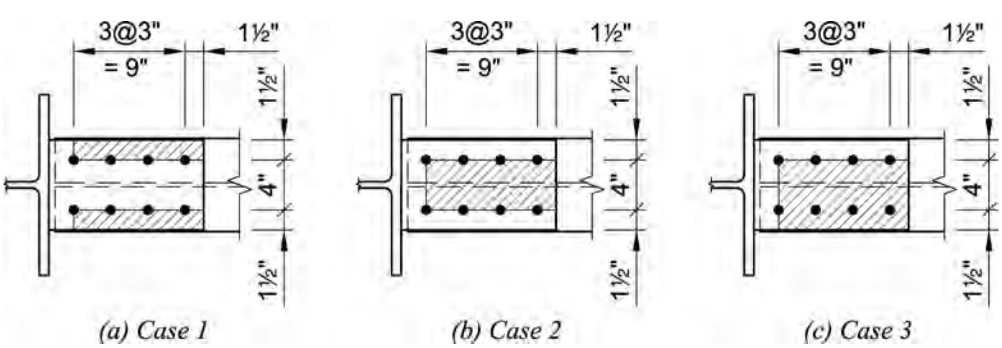


Fig. II.B-1-2. Three cases for block shear rupture.

where

$$A_{gv} = \left[(n-1)s + l_{ev}\right]t$$
$$= \left[(4-1)(3\text{ in.}) + 1\tfrac{1}{2}\text{ in.}\right](\tfrac{3}{4}\text{ in.})$$
$$= 7.88\text{ in.}^2$$

$$A_{nv} = A_{gv} - (n-0.5)(d_h + \tfrac{1}{16}\text{ in.})t$$
$$= 7.88\text{ in.}^2 - (4-0.5)(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{3}{4}\text{ in.})$$
$$= 5.26\text{ in.}^2$$

$$A_{nt} = \left[gage + l_{eh} - 1.5(d_h + \tfrac{1}{16}\text{ in.})\right]t$$
$$= \left[4\text{ in.} + 1\tfrac{1}{2}\text{ in.} - 1.5(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](\tfrac{3}{4}\text{ in.})$$
$$= 3.00\text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$R_n = 0.60(58\text{ ksi})(5.26\text{in.}^2) + 1.0(58\text{ ksi})(3.00\text{ in.}^2) \le 0.60(36\text{ ksi})(7.88\text{ in.}^2) + 1.0(58\text{ ksi})(3.00\text{ in.}^2)$$
$$= 357\text{ kips} > 344\text{ kips}$$

Therefore:

$$R_n = 344\text{ kips}$$

From AISC *Specification* Section J4.3, the available strength for the limit state of block shear rupture on the plate is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(344\text{ kips})$ $= 258\text{ kips} > 161\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{344\text{ kips}}{2.00}$ $= 172\text{ kips} > 108\text{ kips}$ **o.k.**

Beam flange block shear rupture

The nominal strength for the limit state of block shear rupture is given by AISC *Specification* Section J4.3.

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \quad \text{(Spec. Eq. J4-5)}$$

The available block shear rupture strength of the beam flange involves the tearout of the two blocks outside the two rows of bolt holes in the flanges. Conservatively use the flange forces that were found for the fastener checks. From AISC *Manual* Tables 9-3a, 9-3b, and 9-3c, and AISC *Specification* Equation J4-5, with $n = 4$, $l_{eh} = 1\tfrac{3}{4}$ in., $l_{ev} = 1\tfrac{1}{4}$ in. (reduced ¼ in. to account for beam underrun), and $U_{bs} = 1.0$:

LRFD	ASD
Tension rupture component from AISC *Manual* Table 9-3a: $$\frac{\phi F_u A_{nt}}{t} = 60.9 \text{ kip/in.}$$ Shear yielding component from AISC *Manual* Table 9-3b: $$\frac{\phi 0.60 F_y A_{gv}}{t} = 231 \text{ kip/in.}$$ Shear rupture component from AISC *Manual* Table 9-3c: $$\frac{\phi 0.60 F_u A_{nv}}{t} = 197 \text{ kip/in.}$$ The design block shear rupture strength is: $$\begin{aligned}\phi R_n &= \phi 0.60 F_u A_{nv} + \phi U_{bs} F_u A_{nt} \\ &\leq \phi 0.60 F_y A_{gv} + \phi U_{bs} F_u A_{nt} \\ &= (2 \text{ planes})(0.570 \text{ in.})\left[\begin{array}{l}197 \text{ kip/in.} \\ +(1.0)(60.9 \text{ kip/in.})\end{array}\right] \\ &\leq (2 \text{ planes})(0.570 \text{ in.})\left[\begin{array}{l}231 \text{ kip/in.} \\ +(1.0)(60.9 \text{ kip/in.})\end{array}\right] \\ &= 294 \text{ kips} < 333 \text{ kips}\end{aligned}$$ Therefore: $$\phi R_n = 294 \text{ kips} > 168 \text{ kips} \quad \textbf{o.k.}$$	Tension rupture component from AISC *Manual* Table 9-3a: $$\frac{F_u A_{nt}}{\Omega t} = 40.6 \text{ kip/in.}$$ Shear yielding component from AISC *Manual* Table 9-3b: $$\frac{0.60 F_y A_{gv}}{\Omega t} = 154 \text{ kip/in.}$$ Shear rupture component from AISC *Manual* Table 9-3c: $$\frac{0.60 F_u A_{nv}}{\Omega t} = 132 \text{ kip/in.}$$ The allowable block shear rupture strength is: $$\begin{aligned}\frac{R_n}{\Omega} &= \frac{0.60 F_u A_{nv}}{\Omega} + \frac{U_{bs} F_u A_{nt}}{\Omega} \\ &\leq \frac{0.60 F_y A_{gv}}{\Omega} + \frac{U_{bs} F_u A_{nt}}{\Omega} \\ &= (2 \text{ planes})(0.570 \text{ in.})\left[\begin{array}{l}132 \text{ kip/in.} \\ +(1.0)(40.6 \text{ kip/in.})\end{array}\right] \\ &\leq (2 \text{ planes})(0.570 \text{ in.})\left[\begin{array}{l}154 \text{ kip/in.} \\ +(1.0)(40.6 \text{ kip/in.})\end{array}\right] \\ &= 197 \text{ kips} < 222 \text{ kips}\end{aligned}$$ Therefore: $$\frac{R_n}{\Omega} = 197 \text{ kips} > 112 \text{ kips} \quad \textbf{o.k.}$$

Fillet weld to supporting column flange

The applied load is perpendicular to the weld length (θ = 90°); therefore, the directional strength factor is determined from AISC *Specification* Equation J2-5. This increase factor due to directional strength is incorporated into the weld strength calculation.

$$\begin{aligned}1.0 + 0.50 \sin^{1.5} \theta &= 1.0 + 0.50 \sin^{1.5}(90°) \\ &= 1.50\end{aligned}$$

The required fillet weld size is determined using AISC *Manual* Equations 8-2a or 8-2b as follows:

LRFD	ASD

$D_{min} = \dfrac{P_{uf}}{(2\text{ welds})(1.50)(1.392\text{ kip/in.})l}$ $= \dfrac{161\text{ kips}}{(2\text{ welds})(1.50)(1.392\text{ kip/in.})(7\text{ in.})}$ $= 5.51$ Use a ⅜-in. fillet weld on both sides of the flange plate.	$D_{min} = \dfrac{P_{af}}{(2\text{ welds})(1.50)(0.928\text{ kip/in.})l}$ $= \dfrac{108\text{ kips}}{(2\text{ welds})(1.50)(0.928\text{ kip/in.})(7\text{ in.})}$ $= 5.54$ Use a ⅜-in. fillet weld on both sides of the flange plate.

Compression Flange Plate and Connection

From AISC *Specification* Section J4.4, the available strength of the flange plate in compression is determined as follows:

K = 0.65, from AISC *Specification* Commentary Table C-A-7.1
L = 3.00 in. (the distance between adjacent bolt holes)

$$r = \sqrt{\frac{I}{A}}$$
$$= \sqrt{\frac{(7\text{ in.})(\tfrac{3}{4}\text{ in.})^3/12}{(7\text{ in.})(\tfrac{3}{4}\text{ in.})}}$$
$$= 0.217\text{ in.}$$

$$\frac{L_c}{r} = \frac{KL}{r}$$
$$= \frac{0.65(3.00\text{ in.})}{0.217\text{ in.}}$$
$$= 8.99$$

Since $L_c/r \le 25$:

$$P_n = F_y A_g \qquad (Spec.\text{ Eq. J4-6})$$
$$= (36\text{ ksi})(7\text{ in.})(\tfrac{3}{4}\text{ in.})$$
$$= 189\text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi P_n = 0.90(189\text{ kips})$ $= 170\text{ kips} > 161\text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{P_n}{\Omega} = \dfrac{189\text{ kips}}{1.67}$ $= 113\text{ kips} > 108\text{ kips}$ **o.k.**

The compression flange plate will be identical to the tension flange plate; a ¾-in.×7-in. plate with eight bolts in two rows of four bolts on a 4-in. gage and ⅜-in. fillet welds to the supporting column flange.

Note: The bolt bearing and shear checks are the same as for the tension flange plate and have found to be adequate in prior calculations. Tension due to load reversal must also be considered in the design of the fillet weld to the supporting column flange. The result is the same as previously calculated for the top flange connection plate.

Flange Local Bending of Column

From AISC *Specification* Section J10.1, the available strength of the column for the limit state of flange local bending is determined as follows:

$$0.15b_f = 0.15(14.6 \text{ in.})$$
$$= 2.19 \text{ in.}$$

The length of loading (i.e., plate width) is 7 in., which is greater than $0.15b_f$. Thus, flange local bending needs to be checked.

Assume the concentrated force to be resisted is applied at a distance from the column end greater than $10t_f$.

$$10t_f = 10(0.780 \text{ in.})$$
$$= 7.80 \text{ in.}$$

$$R_n = 6.25F_{yf}t_f^2 \qquad \text{(Spec. Eq. J10-1)}$$
$$= 6.25(50 \text{ ksi})(0.780 \text{ in.})^2$$
$$= 190 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(190 \text{ kips})$ $= 171 \text{ kips} > 161 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{190 \text{ kips}}{1.67}$ $= 114 \text{ kips} > 108 \text{ kips}$ **o.k.**

Web Local Yielding of Column

Assume the concentrated force to be resisted is applied at a distance from the column end that is greater than the depth of the column. The available strength of the column for the limit state of web local yielding is determined from AISC *Manual* Table 9-4 and AISC *Manual* Equation 9-47a or 9-47b, with $l_b = t = ¾$ in.

LRFD	ASD
$\phi R_1 = 83.7 \text{ kips}$ $\phi R_2 = 24.3 \text{ kip/in.}$ $\phi R_n = 2(\phi R_1) + l_b(\phi R_2)$ $= 2(83.7 \text{ kips}) + (¾ \text{ in.})(24.3 \text{ kip/in.})$ $= 186 \text{ kips} > 161 \text{ kips}$ **o.k.**	$R_1/\Omega = 55.8 \text{ kips}$ $R_2/\Omega = 16.2 \text{ kip/in.}$ $\dfrac{R_n}{\Omega} = 2(R_1/\Omega) + l_b(R_2/\Omega)$ $= 2(55.8 \text{ kips}) + (¾ \text{ in.})(16.2 \text{ kip/in.})$ $= 124 \text{ kips} > 108 \text{ kips}$ **o.k.**

Web Local Crippling

Assume the concentrated force to be resisted is applied at a distance from the column end that is greater than or equal to one-half of the column depth. The available strength of the column for the limit state of web local crippling is determined from AISC *Manual* Table 9-4 and AISC *Manual* Equation 9-50a or 9-50b, with $l_b = t = ¾$ in.

LRFD	ASD

LRFD	ASD
$\phi R_3 = 108 \text{ kips}$ $\phi R_4 = 11.2 \text{ kip/in.}$ $\phi R_n = 2\left[\phi R_3 + l_b\left(\phi R_4\right)\right]$ $= 2\left[108 \text{ kips} + \left(3/4 \text{ in.}\right)\left(11.2 \text{ kip/in.}\right)\right]$ $= 233 \text{ kips} > 161 \text{ kips}$ **o.k.**	$R_3/\Omega = 71.8 \text{ kips}$ $R_4/\Omega = 7.44 \text{ kip/in.}$ $\dfrac{R_n}{\Omega} = 2\left[R_3/\Omega + l_b\left(R_4/\Omega\right)\right]$ $= 2\left[71.8 \text{ kips} + \left(3/4 \text{ in.}\right)\left(7.44 \text{ kip/in.}\right)\right]$ $= 155 \text{ kips} > 108 \text{ kips}$ **o.k.**

Note: Web compression buckling (AISC *Specification* Section J10.5) must be checked if another beam is framed into the opposite side of the column at this location.

Web panel zone shear (AISC *Specification* Section J10.6) should also be checked for this column.

For further information, see AISC Design Guide 13 *Stiffening of Wide-Flange Columns at Moment Connections: Wind and Seismic Applications* (Carter, 1999).

EXAMPLE II.B-2 WELDED FLANGE-PLATED FR MOMENT CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Verify a welded flange-plated FR moment connection between an ASTM A992 W18×50 beam and an ASTM A992 W14×99 column flange, as shown in Figure II.B-2-1, to transfer the following beam end reactions:

Vertical shear:

V_D = 7 kips
V_L = 21 kips

Strong-axis moment:

M_D = 42 kip-ft
M_L = 126 kip-ft

Use 70-ksi electrodes. The flange plates are ASTM A36 material. Assume the top flange of the beam is in the tension condition due to moment.

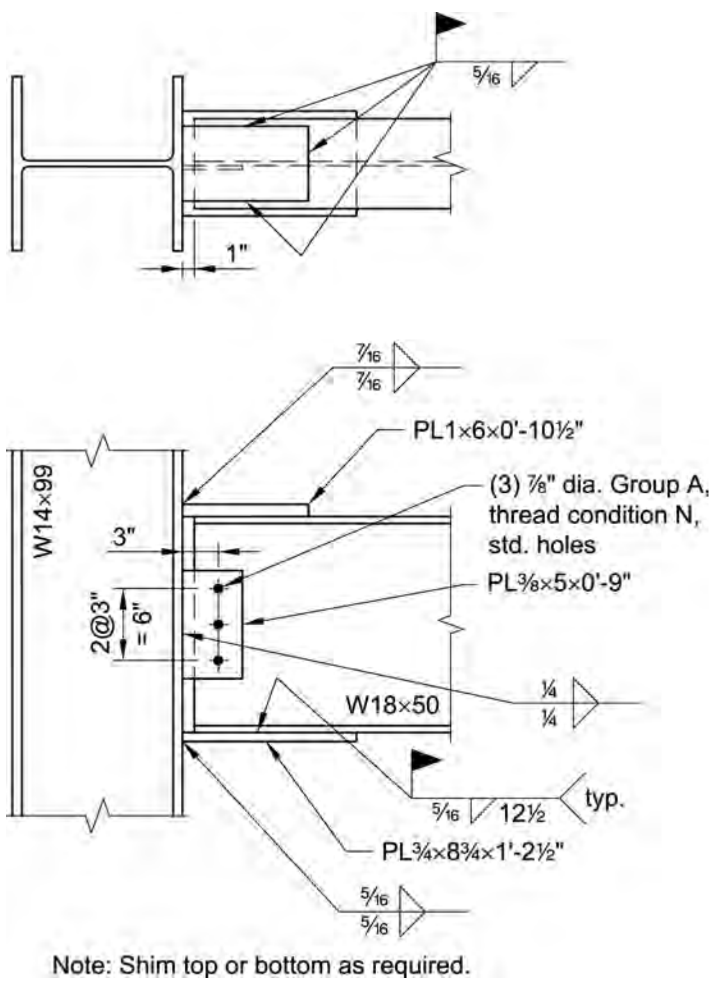


Fig. II.B-2-1. Connection geometry for Example II.B-2.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam and column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plates
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×50
d = 18.0 in.
b_f = 7.50 in.
t_f = 0.570 in.
t_w = 0.355 in.
Z_x = 101 in.^{3}

Column
W14×99
d = 14.2 in.
b_f = 14.6 in.
t_f = 0.780 in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(7 \text{ kips}) + 1.6(21 \text{ kips})$ $= 42.0 \text{ kips}$ $M_u = 1.2(42 \text{ kip-ft}) + 1.6(126 \text{ kip-ft})$ $= 252 \text{ kip-ft}$	$R_a = 7 \text{ kips} + 21 \text{ kips}$ $= 28.0 \text{ kips}$ $M_a = 42 \text{ kip-ft} + 126 \text{ kip-ft}$ $= 168 \text{ kip-ft}$

Single-Plate Web Connection

The single-plate web connection is verified in Example II.B-1.

Note: By inspection, the available effective fastener strength and shear yielding strengths of the beam web are adequate. The beam web is nearly as thick as the web plate and of a higher strength material. Shear rupture and block shear rupture are not limit states for the beam web.

Tension Flange Plate and Connection

Tensile yielding of the flange plate

The top flange plate is specified as a PL1 in. × 6 in. × 0 ft 10½ in. The top beam flange width is b_f = 7.50 in. This provides a shelf dimension of ¾-in. on both sides of the plate for welding.

The moment arm between flange plate forces, d_m, used for verifying the plate strength is equal to the depth of the beam plus one-half the thickness of each of the flange plates. This represents the distance between the centerlines of the flange plates at the top and bottom of the beam.

$$d_m = 18.0 \text{ in.} + \frac{\frac{3}{4}\text{ in.}}{2} + \frac{1 \text{ in.}}{2}$$
$$= 18.9 \text{ in.}$$

From AISC *Manual* Equation 12-1a or 12-1b, the flange force is:

LRFD	ASD
$P_{uf} = \frac{M_u}{d_m}$ $= \frac{(252 \text{ kip-ft})(12 \text{ in./ft})}{18.9 \text{ in.}}$ $= 160 \text{ kips}$	$P_{af} = \frac{M_a}{d_m}$ $= \frac{(168 \text{ kip-ft})(12 \text{ in./ft})}{18.9 \text{ in.}}$ $= 107 \text{ kips}$

From AISC *Specification* Section J4.1(a), the available tensile yield strength of the flange plate is determined as follows:

$$R_n = F_y A_g \quad \text{(}Spec\text{. Eq. J4-1)}$$
$$= (36 \text{ ksi})(6 \text{ in.})(1 \text{ in.})$$
$$= 216 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(216 \text{ kips})$ $= 194 \text{ kips} > 160 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\frac{R_n}{\Omega} = \frac{216 \text{ kips}}{1.67}$ $= 129 \text{ kips} > 107 \text{ kips}$ **o.k.**

Fillet weld strength for top flange plate to beam flange

The moment arm between flange forces, d_m, used for verifying the fillet weld strength is equal to the depth of the beam. This dimension represents the faying surface between the flange of the beam and the tension plate. From AISC *Manual* Equation 12-1a or 12-1b, the flange force is:

LRFD	ASD
$P_{uf} = \frac{M_u}{d_m}$ $= \frac{(252 \text{ kip-ft})(12 \text{ in./ft})}{18.0 \text{ in.}}$ $= 168 \text{ kips}$	$P_{af} = \frac{M_a}{d_m}$ $= \frac{(168 \text{ kip-ft})(12 \text{ in./ft})}{18.0 \text{ in.}}$ $= 112 \text{ kips}$

A 5⁄16-in. fillet weld is specified (D = 5). The available strength may be calculated using the provisions from AISC *Specification* Section J2.4(b)(2). The available shear strength of the fillet weld may be calculated using AISC *Specification* Table J2.5.

The length of the longitudinally loaded welds is determined taking into consideration a ¼-in. tolerance to account for possible beam underrun and a weld termination equal to the weld size.

$$\begin{aligned} l &= 10\tfrac{1}{2}\text{ in.} - 1\text{ in. (setback)} - \tfrac{1}{4}\text{ in. (underrun)} - \tfrac{5}{16}\text{ in. (weld termination)} \\ &= 8.94\text{ in.} \end{aligned}$$

$$\begin{aligned} R_{nwl} &= 0.60F_{EXX}\left(\frac{\sqrt{2}}{2}\right)\left(\frac{D}{16}\right)l \\ &= 0.60(70\text{ ksi})\left(\frac{\sqrt{2}}{2}\right)\left(\frac{5}{16}\right)(8.94\text{ in.})(2\text{ welds}) \\ &= 166\text{ kips} \end{aligned}$$

$$\begin{aligned} R_{nwt} &= 0.60F_{EXX}\left(\frac{\sqrt{2}}{2}\right)\left(\frac{D}{16}\right)l \\ &= 0.60(70\text{ ksi})\left(\frac{\sqrt{2}}{2}\right)\left(\frac{5}{16}\right)(6\text{ in.}) \\ &= 55.7\text{ kips} \end{aligned}$$

The combined strength of the fillet weld group may be taken as the larger of the following:

$$\begin{aligned} R_n &= R_{nwl} + R_{nwt} \\ &= 166\text{ kips} + 55.7\text{ kips} \\ &= 222\text{ kips} \end{aligned}$$ (*Spec*. Eq. J2-6a)

$$\begin{aligned} R_n &= 0.85R_{nwl} + 1.5R_{nwt} \\ &= 0.85(166\text{ kips}) + 1.5(55.7\text{ kips}) \\ &= 225\text{ kips} \end{aligned}$$ (*Spec*. Eq. J2-6b)

Therefore:

$R_n = 225$ kips

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(225\text{ kips})$ $= 169\text{ kips} > 168\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{225\text{ kips}}{2.00}$ $= 113\text{ kips} > 112\text{ kips}$ **o.k.**

Connecting elements rupture strength at top flange welds

At the top flange connection, the minimum base metal thickness to match the shear rupture strength of the weld is determined as follows:

$$\begin{aligned} t_{min} &= \frac{3.09D}{F_u} \\ &= \frac{3.09(5)}{65\text{ ksi}} \\ &= 0.238\text{ in.} < 0.570\text{ in. beam flange} \quad \textbf{o.k.} \end{aligned}$$ (*Manual* Eq. 9-2)

$$t_{min} = \frac{3.09D}{F_u}$$ (*Manual* Eq. 9-2)

$$= \frac{3.09(5)}{58 \text{ ksi}}$$

$$= 0.266 \text{ in.} < 1.00 \text{ in. top flange plate} \quad \textbf{o.k.}$$

Fillet weld at top flange plate to column flange

The applied load is perpendicular to the weld length (θ = 90°), therefore the directional strength factor is determined from AISC *Specification* Equation J2-5. This increase factor due to directional strength is incorporated into the weld strength calculation.

$$1.0 + 0.50\sin^{1.5}\theta = 1.0 + 0.50\sin^{1.5}(90°)$$

$$= 1.50$$

The available strength of fillet welds is determined using AISC *Manual* Equation 8-2a or 8-2b, as follows:

LRFD	ASD
$D_{min} = \frac{P_{uf}}{(2 \text{ welds})(1.50)(1.392 \text{ kip/in.})l}$ $= \frac{160 \text{ kips}}{(2 \text{ welds})(1.50)(1.392 \text{ kip/in.})(6 \text{ in.})}$ $= 6.39$ Use a 7/16-in. fillet weld on both sides of the plate.	$D_{min} = \frac{P_{af}}{(2 \text{ welds})(1.50)(0.928 \text{ kip/in.})l}$ $= \frac{107 \text{ kips}}{(2 \text{ welds})(1.50)(0.928 \text{ kip/in.})(6 \text{ in.})}$ $= 6.41$ Use a 7/16-in. fillet weld on both sides of the plate.

Compression Flange Plate and Connection

Flange plate compressive strength

The bottom flange plate is specified as a PL¾×8¾×1'-2½". The bottom flange width is b_f = 7.50 in. This provides a shelf dimension of ⅝-in. on both sides of the plate for welding.

Assume an underrun dimension of ¼-in. and an additional ½-in. to the start of the weld.

K = 0.65 from AISC *Specification* Commentary Table C-A-7.1
L = 1.75 in.

$$r = \sqrt{\frac{I}{A}}$$

$$= \sqrt{\frac{(8\tfrac{3}{4} \text{ in.})(\tfrac{3}{4} \text{ in.})^3/12}{(8\tfrac{3}{4} \text{ in.})(\tfrac{3}{4} \text{ in.})}}$$

$$= 0.217 \text{ in.}$$

$$\frac{L_c}{r} = \frac{KL}{r}$$

$$= \frac{0.65(1.75 \text{ in.})}{0.217 \text{ in.}}$$

$$= 5.24 < 25$$

Since $L_c/r \le 25$:

$$P_n = F_y A_g \quad \text{(Spec. Eq. J4-6)}$$
$$= (36\text{ ksi})(8\tfrac{3}{4}\text{ in.})(\tfrac{3}{4}\text{ in.})$$
$$= 236\text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi P_n = 0.90(236\text{ kips})$ $= 212\text{ kips} > 160\text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{P_n}{\Omega} = \dfrac{236\text{ kips}}{1.67}$ $= 141\text{ kips} > 107\text{ kips}$ **o.k.**

Fillet weld strength for bottom flange plate to beam flange

The required weld length is determined using AISC *Manual* Equation 8-2a or 8-2b, as follows:

LRFD	ASD
$l_{min} = \dfrac{P_{fu}}{(2\text{ welds})(1.392\text{ kip/in.})D}$ $= \dfrac{168\text{ kips}}{(2\text{ welds})(1.392\text{ kip/in.})(5)}$ $= 12.1\text{ in.}$ Use 12½-in.-long 5⁄16-in. fillet welds.	$l_{min} = \dfrac{P_{fa}}{(2\text{ welds})(0.928\text{ kip/in.})D}$ $= \dfrac{112\text{ kips}}{(2\text{ welds})(0.928\text{ kip/in.})(5)}$ $= 12.1\text{ in.}$ Use 12½-in.-long 5⁄16-in. fillet welds.

Beam bottom flange rupture strength at welds

$$A_{nv} = (2\text{ welds})t_f l$$
$$= (2\text{ welds})(0.570\text{ in.})(12\tfrac{1}{2}\text{ in.})$$
$$= 14.3\text{ in.}^3$$

$$R_n = 0.60F_u A_{nv} \quad \text{(Spec. Eq. J4-4)}$$
$$= 0.60(65\text{ ksi})(14.3\text{ in.}^2)$$
$$= 558\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(558\text{ kips})$ $= 419\text{ kips} > 168\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{558\text{ kips}}{2.00}$ $= 279\text{ kips} > 112\text{ kips}$ **o.k.**

Fillet weld at bottom flange plate to column flange

The applied load is perpendicular to the weld length ($\theta = 90°$) therefore the directional strength factor is determined from AISC *Specification* Equation J2-5. This increase factor due to directional strength is incorporated into the weld strength calculation.

$$1.0 + 0.50\sin^{1.5}\theta = 1.0 + 0.50\sin^{1.5}(90°)$$
$$= 1.50$$

The available strength of fillet welds is determined using AISC *Manual* Equation 8-2a or 8-2b as follows:

LRFD	ASD
$D_{min} = \dfrac{P_{uf}}{(2\text{ welds})(1.50)(1.392\text{ kip/in.})l}$ $= \dfrac{160\text{ kips}}{(2\text{ welds})(1.50)(1.392\text{ kip/in.})(8\tfrac{3}{4}\text{ in.})}$ $= 4.38$ sixteenths Use 5⁄16-in. fillet welds.	$D_{min} = \dfrac{P_{af}}{(2\text{ welds})(1.50)(0.928\text{ kip/in.})l}$ $= \dfrac{107\text{ kips}}{(2\text{ welds})(1.50)(0.928\text{ kip/in.})(8\tfrac{3}{4}\text{ in.})}$ $= 4.39$ sixteenths Use 5⁄16-in. fillet welds.

See Example II.B-1 for checks of the column under concentrated forces. For further information, see AISC Design Guide 13 *Stiffening of Wide-Flange Columns at Moment Connections: Wind and Seismic Applications*. (Carter, 1999).

Conclusion

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.B-3 DIRECTLY WELDED FLANGE FR MOMENT CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Verify a directly welded flange FR moment connection between an ASTM A992 W18×50 beam and an ASTM A992 W14×99 column flange, as shown in Figure II.B-3-1, to transfer the following beam end reactions:

Vertical shear:
V_D = 7 kips
V_L = 21 kips

Strong-axis moment:
M_D = 42 kip-ft
M_L = 126 kip-ft

Use 70-ksi electrodes. Check the column for stiffening requirements.

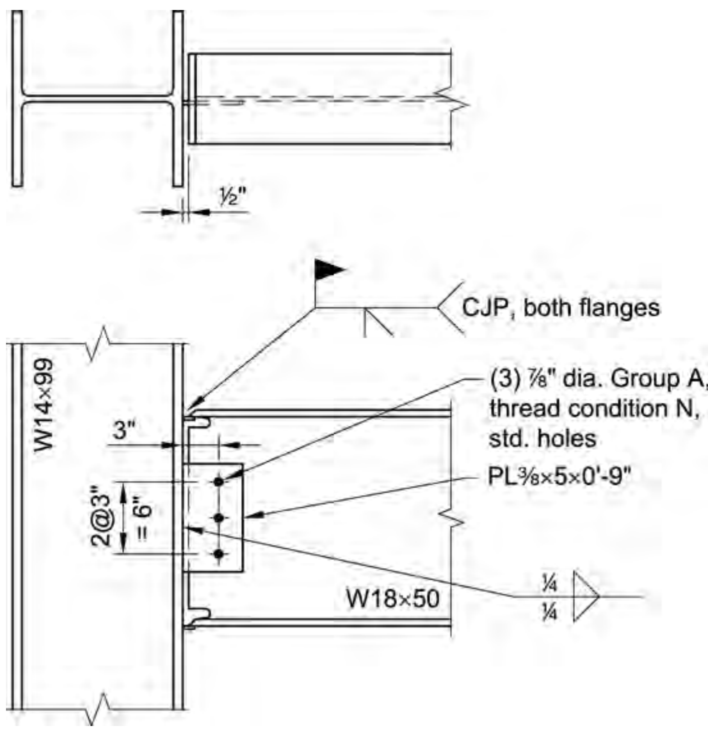


Fig. II.B-3-1. Connection geometry for Example II.B-3.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam and column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From ASCE/SEI 7, Chapter 2 , the required strength is:

LRFD	ASD
$R_u = 1.2(7 \text{ kips}) + 1.6(21 \text{ kips})$ $= 42.0 \text{ kips}$ $M_u = 1.2(42 \text{ kip-ft}) + 1.6(126 \text{ kip-ft})$ $= 252 \text{ kip-ft}$	$R_a = 7 \text{ kips} + 21 \text{ kips}$ $= 28.0 \text{ kips}$ $M_a = 42 \text{ kip-ft} + 126 \text{ kip-ft}$ $= 168 \text{ kip-ft}$

The single-plate web connection is verified in Example II.B-1.

Note: By inspection, the available effective fastener strength and shear yielding strengths of the beam web are adequate. The beam web is nearly as thick as the web plate, and of a higher strength material. Shear rupture and block shear rupture are not limit states for the beam web.

Weld of Beam Flange to Column

A complete-joint-penetration groove weld will transfer the entire flange force in tension and compression. It is assumed that the beam is adequate for the applied moment and will carry the tension and compression forces through the flanges.

See Example II.B-1 for checks of the column under concentrated forces. For further information, see AISC Design Guide 13 *Stiffening of Wide-Flange Columns at Moment Connections: Wind and Seismic Applications*. (Carter, 1999).

Conclusion

The connection is found to be adequate as given for the applied loads.

CHAPTER IIB DESIGN EXAMPLE REFERENCES

Carter, C.J. (1999), *Stiffening of Wide-Flange Columns at Moment Connections: Wind and Seismic Applications*, Design Guide 13, AISC, Chicago, IL.

Chapter IIC
Bracing and Truss Connections

The design of bracing and truss connections is covered in Part 13 of the AISC *Steel Construction Manual*.

EXAMPLE II.C-1 TRUSS SUPPORT CONNECTION

Given:

The truss end connection shown in Figure II.C-1-1 is designed for the required forces shown in Figure II.C-1-2. Verify the following:

a. The connection requirements between the gusset and the column
b. The required gusset size and the weld requirements connecting the diagonal to the gusset

Use 70-ksi electrodes. The top chord and column are ASTM A992 material. The diagonal member, gusset plate and clip angles are ASTM A36 material.

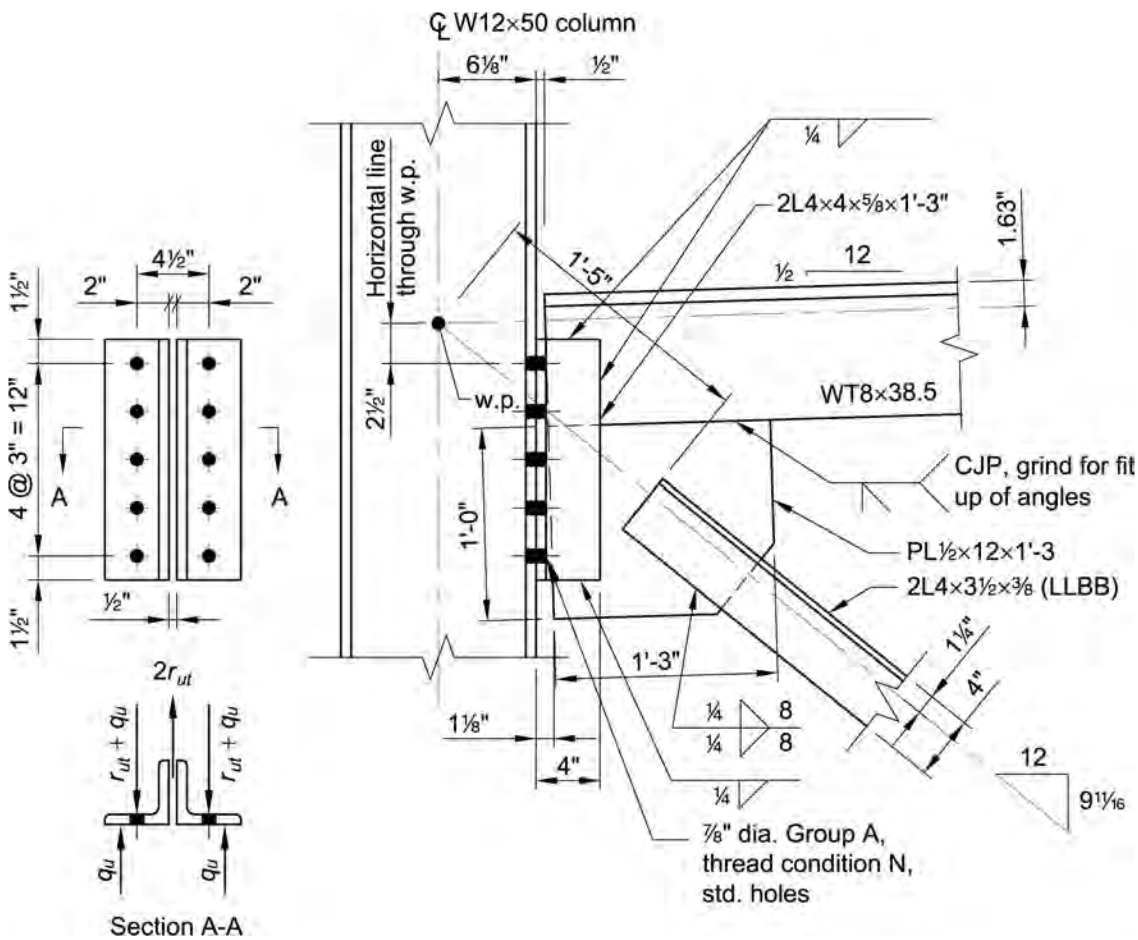


Fig. II.C-1-1. Truss support connection.

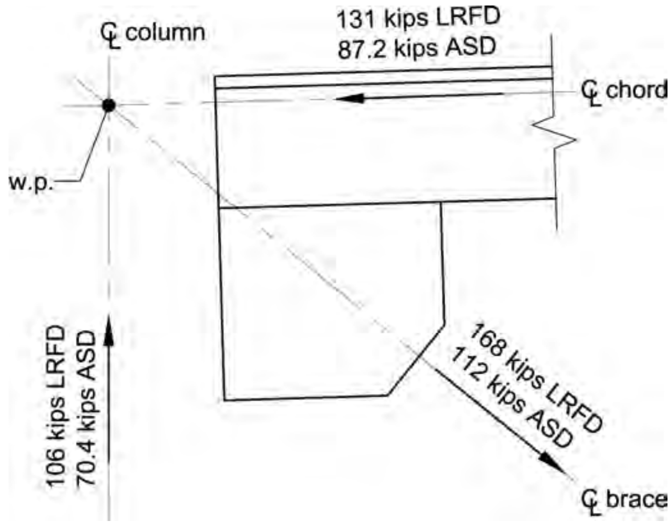


Fig. II.C-1-2. Required forces in members.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Column and top chord
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Diagonal, gusset plate and clip angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1, 1-7, 1-8 and 1-15, the geometric properties are as follows:

Top chord
WT8×38.5
d = 8.26 in.
t_w = 0.455 in.
$\bar{y}$ = 1.63 in.

Column
W12×50
d = 12.2 in.
t_f = 0.640 in.
b_f = 8.08 in.
t_w = 0.370 in.

Diagonal brace
2L4×3½×⅜
t = ⅜ in.
A = 5.36 in.^{2}
$\bar{x}$ = 0.947 in. for single angle

Clip angles
2L4×4×5⁄8
t = 5⁄8 in.

From Figure II.C-1-2 the required strengths are:

LRFD	ASD
Brace axial load:	Brace axial load:
$R_u = 168$ kips	$R_a = 112$ kips
Truss end reaction:	Truss end reaction:
$R_u = 106$ kips	$R_a = 70.4$ kips
Top chord axial load:	Top chord axial load:
$R_u = 131$ kips	$R_a = 87.2$ kips

Weld Connecting the Diagonal to the Gusset Plate

Note: AISC *Specification* Section J1.7, requiring that the center of gravity of the weld group coincide with the center of gravity of the member, does not apply to end connections of statically loaded single-angle, double-angle and similar members.

From AISC *Specification* Table J2.4, the minimum fillet weld size for 3⁄8-in. angles attached to a ½-in.-thick gusset plate is:

$$w_{min} = \tfrac{3}{16}\text{ in.}$$

For ¼-in. fillet welds (D = 4), the required weld length is determined from AISC *Manual* Equations 8-2a or 8-2b, as follows:

LRFD	ASD
$l_{req} = \dfrac{R_u}{(4\text{ welds})(1.392\text{ kip/in.})(D)}$	$l_{req} = \dfrac{R_a}{(4\text{ welds})(0.928\text{ kip/in.})(D)}$
$= \dfrac{168\text{ kips}}{(4\text{ welds})(1.392\text{ kip/in.})(4)}$	$= \dfrac{112\text{ kips}}{(4\text{ welds})(0.928\text{ kip/in.})(4)}$
$= 7.54$ in.	$= 7.54$ in.

Use an 8-in.-long ¼-in. fillet weld at the heel and toe of each angle.

Gusset Shear Rupture at Brace Welds

The minimum plate thickness to match the shear rupture strength of the welds is determined as follows:

$$t_{min} = \frac{6.19D}{F_u} \qquad \text{(\textit{Manual} Eq. 9-3)}$$
$$= \frac{6.19(4)}{58\text{ ksi}}$$
$$= 0.427$$

Try a ½-in.-thick gusset plate.

Tensile Strength of the Brace

From AISC *Specification* Section D2, the available tensile yielding strength of the brace is determined as follows:

$$\begin{aligned} P_n &= F_y A_g \\ &= (36 \text{ ksi})(5.36 \text{ in.}^2) \\ &= 193 \text{ kips} \end{aligned}$$ (*Spec*. Eq. D2-1)

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n = 0.90(193 \text{ kips})$ $= 174 \text{ kips} > 168 \text{ kips}$ **o.k.**	$\Omega_t = 1.67$ $\dfrac{P_n}{\Omega_t} = \dfrac{193 \text{ kips}}{1.67}$ $= 116 \text{ kips} > 112 \text{ kips}$ **o.k.**

From AISC *Specification* Section D2, the available tensile rupture strength of the brace is determined as follows:

$$\begin{aligned} A_n &= A_g \\ &= 5.36 \text{ in.}^2 \end{aligned}$$

The shear lag factor, U, is determined from AISC *Specification* Table D3.1, Case 4:

$$\begin{aligned} U &= \frac{3l^2}{3l^2 + w^2}\left(1 - \frac{\bar{x}}{l}\right) \\ &= \frac{3(8 \text{ in.})^2}{3(8 \text{ in.})^2 + (4 \text{ in.})^2}\left(1 - \frac{0.947 \text{ in.}}{8 \text{ in.}}\right) \\ &= 0.814 \end{aligned}$$

$$\begin{aligned} A_e &= A_n U \\ &= (5.36 \text{ in.}^2)(0.814) \\ &= 4.36 \text{ in.}^2 \end{aligned}$$ (*Spec*. Eq. D3-1)

$$\begin{aligned} P_n &= F_u A_e \\ &= (58 \text{ ksi})(4.36 \text{ in.}^2) \\ &= 253 \text{ kips} \end{aligned}$$ (*Spec*. Eq. D2-2)

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(253 \text{ kips})$ $= 190 \text{ kips} > 168 \text{ kips}$ **o.k.**	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{253 \text{ kips}}{2.00}$ $= 127 \text{ kips} > 112 \text{ kips}$ **o.k.**

Use a ½-in.-thick gusset plate. With the brace-to-gusset welds determined, a gusset plate layout as shown in Figure II.C-1-1 can be made.

Strength of the Bolted Connection—Angles

From the Commentary to AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the individual strengths of the individual fasteners, which may be taken as the lesser of the fastener shear strength per AISC *Specification* Section J3.6, the bearing strength at the bolt hole per AISC *Specification* Section J3.10, or the tearout strength at the bolt hole per AISC *Specification* Section J3.10.

The number of ⅞-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) required for shear only is determined as follows:

LRFD	ASD
From AISC *Manual* Table 7-1, the available bolt shear strength is: $\phi r_n = 24.3 \text{ kips/bolt}$ $n_{min} = \dfrac{R_u}{(2 \text{ bolts/row})\phi r_n}$ $= \dfrac{106 \text{ kips}}{(2 \text{ bolts/row})(24.3 \text{ kips/bolt})}$ $= 2.18 \text{ rows}$	From AISC *Manual* Table 7-1, the available bolt shear strength is: $\dfrac{r_n}{\Omega} = 16.2 \text{ kips/bolt}$ $n_{min} = \dfrac{R_a}{(2 \text{ bolts/row})(r_n/\Omega)}$ $= \dfrac{70.4 \text{ kips}}{(2 \text{ bolts/row})(16.2 \text{ kips/bolt})}$ $= 2.17 \text{ rows}$

Use 2L4×4×⅝ clip angles with five pairs of bolts. Note the number of rows of bolts is increased to "square off" the gusset plate.

The available bearing strength of the angles per bolt is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$\begin{aligned} r_n &= 2.4dtF_u \\ &= 2.4(\tfrac{7}{8} \text{ in.})(\tfrac{5}{8} \text{ in.})(58 \text{ ksi}) \\ &= 76.1 \text{ kips/bolt} \end{aligned}$$ (*Spec*. Eq. J3-6a)

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(76.1 \text{ kips/bolt})$ $= 57.1 \text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{76.1 \text{ kips/bolt}}{2.00}$ $= 38.1 \text{ kips/bolt}$

The available tearout strength of the angles at edge bolts is determined from AISC *Specification* Section J3.10, with d_h = 15⁄16 in. for ⅞-in.-diameter bolts from AISC *Specification* Table J3.3, assuming deformation at service load is a design consideration:

$$\begin{aligned} l_c &= l_e - 0.5d_h \\ &= 1\tfrac{1}{2} \text{ in.} - 0.5(\tfrac{15}{16} \text{ in.}) \\ &= 1.03 \text{ in.} \end{aligned}$$

$$r_n = 1.2 l_c t F_u \qquad \text{(}Spec\text{. Eq. J3-6c)}$$
$$= 1.2(1.03\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})$$
$$= 44.8\text{ kips/bolt}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(44.8\text{ kips/bolt})$ $= 33.6\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{44.8\text{ kips/bolt}}{2.00}$ $= 22.4\text{ kips/bolt}$

Therefore, bolt shear controls over bolt bearing or tearout at the edge bolts.

The available tearout strength of the angles at interior bolts is determined from AISC *Specification* Section J3.10, assuming deformation at service load is a design consideration:

$$l_c = s - d_h$$
$$= 3\text{ in.} - \tfrac{15}{16}\text{ in.}$$
$$= 2.06\text{ in.}$$

$$r_n = 1.2 l_c t F_u \qquad \text{(}Spec\text{. Eq. J3-6c)}$$
$$= 1.2(2.06\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})$$
$$= 89.6\text{ kips/bolt}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(89.6\text{ kips/bolt})$ $= 67.2\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{89.6\text{ kips/bolt}}{2.00}$ $= 44.8\text{ kips/bolt}$

Therefore, bolt shear controls over bolt bearing or tearout at the interior bolts.

Because bolt shear controls for all the bolts, the connection is acceptable based on previous calculations.

Bolt Shear and Tension Interaction—Bolts Connecting Clip Angles to Column

The eccentric moment about the work point (w.p.) at the faying surface (face of column flange) is determined using an eccentricity equal to half of the column depth.

$$e = \frac{d}{2}$$
$$= \frac{12.2\text{ in.}}{2}$$
$$= 6.10\text{ in.}$$

The eccentricity normal to the plane of the faying surface is accounted for using the Case II approach in AISC *Manual* Part 7 for eccentrically loaded bolt groups.

$n' = 4$ bolts (number of bolts above the neutral axis)
$d_m = 9.00$ in. (moment arm between resultant tension force and resultant compressive force)

The maximum tensile force per bolt is determined using AISC *Manual* Equations 7-14a or 7-14b, as follows:

LRFD	ASD
$r_{ut} = \dfrac{P_u e}{n' d_m}$ $= \dfrac{(106 \text{ kips})(6.10 \text{ in.})}{(4 \text{ bolts})(9.00 \text{ in.})}$ $= 18.0 \text{ kips/bolt}$	$r_{at} = \dfrac{P_a e}{n' d_m}$ $= \dfrac{(70.4 \text{ kips})(6.10 \text{ in.})}{(4 \text{ bolts})(9.00 \text{ in.})}$ $= 11.9 \text{ kips/bolt}$

The required shear stress per bolt is determined as follows:

$A_b = 0.601 \text{ in.}^2$ (from AISC *Manual* Table 7-1)
$n = 10$ bolts

LRFD	ASD
$f_{rv} = \dfrac{R_u}{nA_b}$ $= \dfrac{106 \text{ kips}}{(10 \text{ bolts})(0.601 \text{ in.}^2)}$ $= 17.6 \text{ ksi}$	$f_{rv} = \dfrac{R_a}{nA_b}$ $= \dfrac{70.4 \text{ kips}}{(10 \text{ bolts})(0.601 \text{ in.}^2)}$ $= 11.7 \text{ ksi}$

The nominal tensile strength modified to include the effects of shear stress is determined from AISC *Specification* Section J3.7 as follows. From AISC *Specification* Table J3.2:

$F_{nt} = 90$ ksi
$F_{nv} = 54$ ksi

LRFD	ASD
$\phi = 0.75$ $F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3-3a) $= 1.3(90 \text{ ksi}) - \dfrac{90 \text{ ksi}}{0.75(54 \text{ ksi})}(17.6 \text{ ksi}) \le 90 \text{ ksi}$ $= 77.9 \text{ ksi} < 90 \text{ ksi}$ Therefore: $F'_{nt} = 77.9 \text{ ksi}$ $B_c = \phi F'_{nt} A_b$ (from *Spec*. Eq. J3-2) $= 0.75(77.9 \text{ ksi})(0.601 \text{ in.}^2)$ $= 35.1 \text{ kips/bolt} > 18.0 \text{ kips/bolt}$ **o.k.**	$\Omega = 2.00$ $F'_{nt} = 1.3F_{nt} - \dfrac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3-3b) $= 1.3(90 \text{ ksi}) - \dfrac{2.00(90 \text{ ksi})}{54 \text{ ksi}}(11.7 \text{ ksi}) \le 90 \text{ ksi}$ $= 78.0 \text{ ksi} < 90 \text{ ksi}$ Therefore: $F'_{nt} = 78.0 \text{ ksi}$ $B_c = \dfrac{F'_{nt}}{\Omega} A_b$ (from *Spec*. Eq. J3-2) $= \dfrac{78.0 \text{ ksi}}{2.00}(0.601 \text{ in.}^2)$ $= 23.4 \text{ kips/bolt} > 11.9 \text{ kips/bolt}$ **o.k.**

Prying Action on Clip Angles

From AISC *Manual* Part 9, the available tensile strength of the bolts in the outstanding angle legs taking prying action into account is determined as follows:

$$a = \frac{b_f - gage}{2}$$
$$= \frac{8.08 \text{ in.} - 4\tfrac{1}{2} \text{ in.}}{2}$$
$$= 1.79 \text{ in.}$$

Note: a is calculated based on the column flange width in this case because it is less than the double angle width.

$$b = \frac{gage - t_p - t}{2}$$
$$= \frac{4\tfrac{1}{2} \text{ in.} - \tfrac{1}{2} \text{ in.} - \tfrac{5}{8} \text{ in.}}{2}$$
$$= 1.69 \text{ in.}$$

Note: 1¼ in. entering and tightening clearance from AISC *Manual* Table 7-15 is accommodated and the column fillet toe is cleared.

$$a' = \left(a + \frac{d_b}{2}\right) \le \left(1.25b + \frac{d_b}{2}\right) \qquad \text{(}\textit{Manual}\text{ Eq. 9-23)}$$
$$= 1.79 \text{ in.} + \frac{\tfrac{7}{8} \text{ in.}}{2} \le 1.25(1.69 \text{ in.}) + \frac{\tfrac{7}{8} \text{ in.}}{2}$$
$$= 2.23 \text{ in.} < 2.55 \text{ in.} \quad \textbf{o.k.}$$

$$b' = \left(b - \frac{d_b}{2}\right) \qquad \text{(}\textit{Manual}\text{ Eq. 9-18)}$$
$$= 1.69 \text{ in.} - \frac{\tfrac{7}{8} \text{ in.}}{2}$$
$$= 1.25 \text{ in.}$$

$$\rho = \frac{b'}{a'} \qquad \text{(}\textit{Manual}\text{ Eq. 9-22)}$$
$$= \frac{1.25 \text{ in.}}{2.23 \text{ in.}}$$
$$= 0.561$$

$$p = \frac{l}{n}$$
$$= \frac{15 \text{ in.}}{5}$$
$$= 3.00 \text{ in.}$$

Check

$$p \le s$$
$$3.00 \text{ in.} = 3.00 \text{ in.} \quad \textbf{o.k.}$$

$$\delta = 1 - \frac{d'}{p}$$ (*Manual* Eq. 9-20)

$$= 1 - \frac{15/16 \text{ in.}}{3.00 \text{ in.}}$$
$$= 0.688$$

The angle thickness required to develop the available strength of the bolt with no prying action is determined as follows:

LRFD	ASD
$\phi = 0.90$ $B_c = 35.1$ kips/bolt (calculated previously) $t_c = \sqrt{\frac{4B_c b'}{\phi p F_u}}$ (*Manual* Eq. 9-26a) $= \sqrt{\frac{4(35.1 \text{ kips/bolt})(1.25 \text{ in.})}{0.90(3.00 \text{ in.})(58 \text{ ksi})}}$ $= 1.06$ in.	$\Omega = 1.67$ $B_c = 23.4$ kips/bolt (calculated previously) $t_c = \sqrt{\frac{\Omega 4B_c b'}{p F_u}}$ (*Manual* Eq. 9-26b) $= \sqrt{\frac{1.67(4)(23.4 \text{ kips/bolt})(1.25 \text{ in.})}{(3.00 \text{ in.})(58 \text{ ksi})}}$ $= 1.06$ in.

$$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right]$$ (*Manual* Eq. 9-28)

$$= \frac{1}{0.688(1+0.561)}\left[\left(\frac{1.06 \text{ in.}}{5/8 \text{ in.}}\right)^2 - 1\right]$$
$$= 1.75$$

Because $\alpha' > 1$, the angles have insufficient strength to develop the bolt strength, therefore:

$$Q = \left(\frac{t}{t_c}\right)^2 (1+\delta)$$
$$= \left(\frac{5/8 \text{ in.}}{1.06 \text{ in.}}\right)^2 (1+0.688)$$
$$= 0.587$$

The available tensile strength per bolt, taking prying action into account, is determined using AISC *Manual* Equation 9-27, as follows:

LRFD	ASD
$\phi r_n = B_c Q$ $= (35.1 \text{ kips/bolt})(0.587)$ $= 20.6 \text{ kips/bolt} > 18.0 \text{ kips/bolt}$ **o.k.**	$\frac{r_n}{\Omega} = B_c Q$ $= (23.4 \text{ kips/bolt})(0.587)$ $= 13.7 \text{ kips/bolt} > 11.9 \text{ kips/bolt}$ **o.k.**

Shear Strength of Clip Angles

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the angles is determined as follows:

$$A_{gv} = (2 \text{ angles}) lt$$
$$= (2 \text{ angles})(15 \text{ in.})(5/8 \text{ in.})$$
$$= 18.8 \text{ in.}^2$$

$$R_n = 0.60 F_y A_{gv}$$ (*Spec*. Eq. J4-3)
$$= 0.60(36 \text{ ksi})(18.8 \text{ in.}^2)$$
$$= 406 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(406 \text{ kips})$ $= 406 \text{ kips} > 106 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{406 \text{ kips}}{1.50}$ $= 271 \text{ kips} > 70.4 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2, the available shear rupture strength of the angles is determined using the net area determined in accordance with AISC *Specification* Section B4.3b.

$$A_{nv} = (2 \text{ angles})\left[l - n\left(d_h + 1/16 \text{ in.}\right)\right]t$$
$$= (2 \text{ angles})\left[15 \text{ in.} - 5\left(15/16 \text{ in.} + 1/16 \text{ in.}\right)\right](5/8 \text{ in.})$$
$$= 12.5 \text{ in.}^2$$

$$R_n = 0.60 F_u A_{nv}$$ (*Spec*. Eq. J4-4)
$$= 0.60(58 \text{ ksi})(12.5 \text{ in.}^2)$$
$$= 435 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(435 \text{ kips})$ $= 326 \text{ kips} > 106 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{435 \text{ kips}}{2.00}$ $= 218 \text{ kips} > 70.4 \text{ kips}$ **o.k.**

Block Shear Rupture of Clip Angles

The available strength for the limit state of block shear rupture of the angles is determined as follows.

$$R_n = 0.60 F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60 F_y A_{gv} + U_{bs} F_u A_{nt}$$ (*Spec*. Eq. J4-5)

where

$$A_{gv} = (2 \text{ angles})(l - l_{ev})t$$
$$= (2 \text{ angles})(15 \text{ in.} - 1\tfrac{1}{2} \text{ in.})(5/8 \text{ in.})$$
$$= 16.9 \text{ in.}^2$$

$$A_{nv} = A_{gv} - (2\text{ angles})(n - 0.5)(d_h + \tfrac{1}{16}\text{ in.})t$$
$$= 16.9\text{ in.}^2 - (2\text{ angles})(5 - 0.5)(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{5}{8}\text{ in.})$$
$$= 11.3\text{ in.}^2$$

$$A_{nt} = (2\text{ angles})\left[l_{eh} - 0.5(d_h + \tfrac{1}{16}\text{ in.})\right]t$$
$$= (2\text{ angles})\left[2\text{ in.} - 0.5(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](\tfrac{5}{8}\text{ in.})$$
$$= 1.88\text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$R_n = 0.60(58\text{ ksi})(11.3\text{ in.}^2) + 1.0(58\text{ ksi})(1.88\text{ in.}^2) \le 0.60(36\text{ ksi})(16.9\text{ in.}^2) + 1.0(58\text{ ksi})(1.88\text{ in.}^2)$$
$$= 502\text{ kips} > 474\text{ kips}$$

Therefore:

$$R_n = 474\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(474\text{ kips})$ $= 356\text{ kips} > 106\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{474\text{ kips}}{2.00}$ $= 237\text{ kips} > 70.4\text{ kips}$ **o.k.**

Prying Action on Column Flange

Using the same procedure as shown previously for the clip angles, the available tensile strength of the bolts, taking prying action into account, is:

LRFD	ASD
$T_c = 18.7\text{ kips} > 18.0\text{ kips}$ **o.k.**	$T_c = 12.4\text{ kips} > 11.9\text{ kips}$ **o.k.**

Strength of the Bolted Connection—Column Flange

By inspection, the applicable limit states will control for the angles; therefore, the column flange is acceptable.

Clip Angle-to-Gusset Plate Connection

With a top chord slope of ½ in 12, the horizontal welds are unequal length as shown in Figure II.C-1-3. The average horizontal length is used in the following calculations.

$$l = 15\text{ in.}$$

$$kl = \frac{3\tfrac{3}{8}\text{ in.} + 2\tfrac{3}{4}\text{ in.}}{2}$$
$$= 3.06$$

$$k = \frac{kl}{l} = \frac{3.06 \text{ in.}}{15 \text{ in.}} = 0.204$$

$$xl = \frac{(kl)^2}{l + 2(kl)} = \frac{(3.06 \text{ in.})^2}{15 \text{ in.} + 2(3.06 \text{ in.})} = 0.443 \text{ in.}$$

$$al + xl = 6.10 \text{ in.} + 4.00 \text{ in.} = 10.1 \text{ in.}$$

$$a = \frac{10.1 \text{ in.} - xl}{l} = \frac{10.1 \text{ in.} - 0.443 \text{ in.}}{15 \text{ in.}} = 0.644$$

By interpolating AISC *Manual* Table 8-8 with Angle = 0°:

$$C = 1.50$$

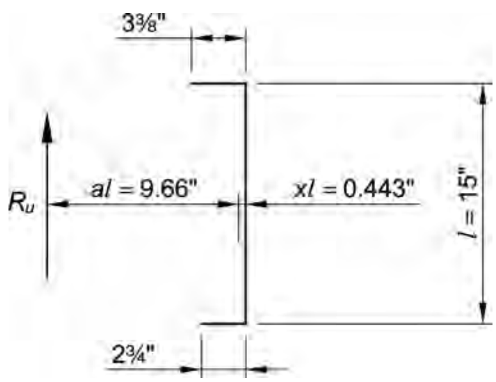


Fig. II.C-1-3. Weld group geometry.

From AISC *Manual* Table 8-8, the minimum required weld size is determined as follows:

LRFD	ASD
$\phi = 0.75$ $D_{min} = \dfrac{R_u}{(2\text{ welds})\phi C C_1 l}$ $= \dfrac{106\text{ kips}}{(2\text{ welds})(0.75)(1.50)(1.0)(15\text{ in.})}$ $= 3.14$ Use ¼-in. fillet welds.	$\Omega = 2.00$ $D_{min} = \dfrac{\Omega R_a}{(2\text{ welds}) C C_1 l}$ $= \dfrac{2.00(70.4\text{ kips})}{2(1.50)(1.0)(15\text{ in.})}$ $= 3.13$ Use ¼-in. fillet welds.

From AISC *Specification* Table J2.4, the minimum weld size for ⅝-in. clip angles attached to a ½-in.-thick gusset plate is:

$$w_{min} = \tfrac{3}{16}\text{ in.} < \tfrac{1}{4}\text{ in.} \quad \textbf{o.k.}$$

Note: Using the average of the horizontal weld lengths provides a reasonable solution when the horizontal welds are close in length. A conservative solution can be determined by using the smaller of the horizontal weld lengths as effective for both horizontal welds. For this example, use kl = 2¾ in., C = 1.43, and D_{min} = 3.29 sixteenths.

Tensile Yielding of Gusset Plate on the Whitmore Section

The gusset plate thickness should match or slightly exceed that of the chord stem. This requirement is satisfied by the ½-in. plate previously selected.

From AISC *Manual* Figure 9-1, the width of the Whitmore section is:

$$\begin{aligned} l_w &= 4.00\text{ in.} + 2(8.00\text{ in.})\tan 30° \\ &= 13.2\text{ in.} \end{aligned}$$

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the gusset plate is determined as follows:

$$\begin{aligned} A_g &= l_w t \\ &= (13.2\text{ in.})(\tfrac{1}{2}\text{ in.}) \\ &= 6.60\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_n &= F_y A_g \\ &= (36\text{ ksi})(6.60\text{ in.}^2) \\ &= 238\text{ kips} \end{aligned} \qquad (\textit{Spec.}\text{ Eq. J4-1})$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(238\text{ kips})$ $= 214\text{ kips} > 168\text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{238\text{ kips}}{1.67}$ $= 143\text{ kips} > 112\text{ kips}$ **o.k.**

Gusset Plate-to-Tee Stem Weld

The interface forces are:

LRFD	ASD
Horizontal shear between gusset and WT: $H_{ub} = 131 \text{ kips} - (4 \text{ bolts})(18.0 \text{ kips/bolt})$ $= 59.0 \text{ kips}$ Vertical tension between gusset and WT: $V_{ub} = (106 \text{ kips})\left(\dfrac{4 \text{ bolts}}{10 \text{ bolts}}\right)$ $= 42.4 \text{ kips}$ Compression between WT and column: $C_{ub} = (4 \text{ bolts})(18.0 \text{ kips/bolt})$ $= 72.0 \text{ kips}$ Summing moments about the face of the column at the workline of the top chord: $M_{ub} = C_{ub}(2\tfrac{1}{2} \text{ in.} + 1.50 \text{ in.}) + H_{ub}(d - \bar{y}) - V_{ub}\left(\dfrac{gusset\ width}{2} + setback\right)$ $= (72.0 \text{ kips})(2\tfrac{1}{2} \text{ in.} + 1.50 \text{ in.}) + (59.0 \text{ kips})(8.26 \text{ in.} - 1.63 \text{ in.}) - (42.4 \text{ kips})\left(\dfrac{15.0 \text{ in.}}{2} + \tfrac{1}{2} \text{ in.}\right)$ $= 340 \text{ kip-in.}$	Horizontal shear between gusset and WT: $H_{ab} = 87.2 \text{ kips} - (4 \text{ bolts})(11.9 \text{ kips/bolt})$ $= 39.6 \text{ kips}$ Vertical tension between gusset and WT: $V_{ab} = (70.4 \text{ kips})\left(\dfrac{4 \text{ bolts}}{10 \text{ bolts}}\right)$ $= 28.2 \text{ kips}$ Compression between WT and column: $C_{ab} = (4 \text{ bolts})(11.9 \text{ kips/bolt})$ $= 47.6 \text{ kips}$ Summing moments about the face of the column at the workline of the top chord: $M_{ab} = C_{ab}(2\tfrac{1}{2} \text{ in.} + 1.50 \text{ in.}) + H_{ab}(d - \bar{y}) - V_{ab}\left(\dfrac{gusset\ width}{2} + setback\right)$ $= (47.6 \text{ kips})(2\tfrac{1}{2} \text{ in.} + 1.50 \text{ in.}) + (39.6 \text{ kips})(8.26 \text{ in.} - 1.63 \text{ in.}) - (28.2 \text{ kips})\left(\dfrac{15.0 \text{ in.}}{2} + \tfrac{1}{2} \text{ in.}\right)$ $= 227 \text{ kip-in.}$

A CJP weld should be used along the interface between the gusset plate and the tee stem. The weld should be ground smooth under the clip angles.

The gusset plate width depends upon the diagonal connection. From a scaled layout, the gusset plate must be 1 ft 3 in. wide.

The gusset plate depth depends upon the connection angles. From a scaled layout, the gusset plate must extend 12 in. below the tee stem.

Use a PL½×12 in.×1 ft 3 in.

Conclusion

The connection is found to be adequate as given for the applied loads.

EXAMPLE II.C-2 TRUSS SUPPORT CONNECTION

Given:

Verify the truss support connections, as shown in Figure II.C-2-1, at the following joints:

A. Joint L_1
B. Joint U_1

Use 70-ksi electrodes, ASTM A36 plate, ASTM A992 bottom and top chords, and ASTM A36 double angles.

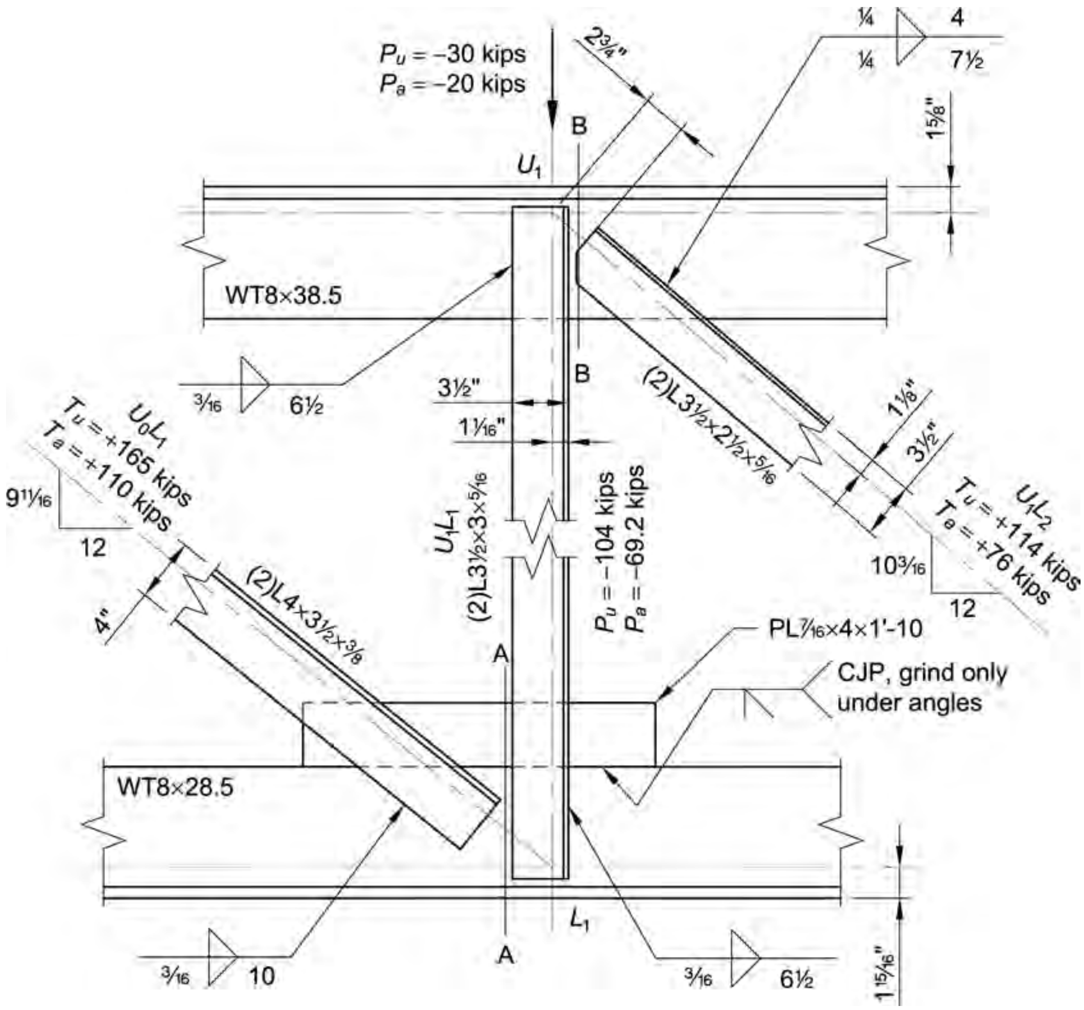


Fig. II.C-2-1. Connection geometry for Example II.C-2.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Top and bottom chord
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Web member, diagonal members and plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-7, 1-8 and 1-15, the geometric properties are as follows:

Top Chord
WT8×38.5
t_w = 0.455 in.
d = 8.26 in.

Bottom Chord
WT8×28.5
t_w = 0.430 in.
d = 8.22 in.

Diagonal U_0L_1
2L4×3½×⅜
A = 5.36 in.^{2}
$\bar{x} = 0.947$ in. (for single angle)

Web U_1L_1
2L3½×3×5⁄16
A = 3.90 in.^{2}

Diagonal U_1L_2
2L3½×2½×5⁄16
A = 3.58 in.^{2}
$\bar{x} = 0.632$ in. (for single angle)

As shown in Figure II.C-2-1, the required forces are:

LRFD	ASD
Web U_1L_1 load: $P_u = -104$ kips Diagonal U_0L_1 load: $T_u = +165$ kips Diagonal U_1L_2 load: $T_u = +114$ kips	Web U_1L_1 load: $P_a = -69.2$ kips Diagonal U_0L_1 load: $T_a = +110$ kips Diagonal U_1L_2 load: $T_a = +76$ kips

Solution A:

Shear Yielding of Bottom Chord Stem

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the bottom chord at Section A-A (see Figure II.C-2-1) is determined as follows:

$$A_{gv} = dt_w$$
$$= (8.22 \text{ in.})(0.430 \text{ in.})$$
$$= 3.53 \text{ in.}^2$$

$$R_n = 0.60F_yA_{gv} \quad (Spec.\ \text{Eq. J4-3})$$
$$= 0.60(50 \text{ ksi})(3.53 \text{ in.}^2)$$
$$= 106 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(106 \text{ kips})$ $= 106 \text{ kips} > 104 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{106 \text{ kips}}{1.50}$ $= 70.7 \text{ kips} > 69.2 \text{ kips}$ **o.k.**

Welds for Member U_1L_1

Note: AISC *Specification* Section J1.7 requiring that the center of gravity of the weld group coincide with the center of gravity of the member does not apply to end connections of statically loaded single angle, double angle and similar members.

From AISC *Specification* Table J2.4, the minimum weld size for a 5⁄16-in.-thick angle is:

$$w_{min} = 3/16 \text{ in.}$$

From AISC *Specification* Section J2.2b(b)(2), the maximum weld size is:

$$w_{max} = t - 1/16 \text{ in.}$$
$$= 5/16 - 1/16 \text{ in.}$$
$$= 1/4 \text{ in.}$$

Try a 3⁄16 in. fillet weld.

The minimum weld length is determined using AISC *Manual* Equation 8-2a or 8-2b:

LRFD	ASD
$l_{min} = \dfrac{R_u}{(2 \text{ sides})(2 \text{ welds})(1.392 \text{ kip/in.})D}$ $= \dfrac{104 \text{ kips}}{(2 \text{ sides})(2 \text{ welds})(1.392 \text{ kip/in.})(3)}$ $= 6.23 \text{ in.}$ Use a 6½-in.-long weld at the heel and toe of the angles.	$l_{min} = \dfrac{R_a}{(2 \text{ sides})(2 \text{ welds})(0.928 \text{ kip/in.})D}$ $= \dfrac{69.2 \text{ kips}}{(2 \text{ sides})(2 \text{ welds})(0.928 \text{ kip/in.})(3)}$ $= 6.21 \text{ in.}$ Use a 6½-in.-long weld at the heel and toe of the angles.

Shear Rupture Strength of Angles at Welds

The minimum angle thickness to match the required shear rupture strength of the welds is determined as follows:

$$t_{min} = \frac{3.09D}{F_u} \quad \text{(\textit{Manual} Eq. 9-2)}$$
$$= \frac{3.09(3)}{58 \text{ ksi}}$$
$$= 0.160 \text{ in.} < \tfrac{5}{16} \text{ in.} \quad \textbf{o.k.}$$

Shear Rupture Strength of Tee-Stem at Welds

The minimum tee-stem thickness to match the required shear rupture strength of the welds is determined as follows:

$$t_{min} = \frac{6.19D}{F_u} \quad \text{(\textit{Manual} Eq. 9-3)}$$
$$= \frac{6.19(3)}{65 \text{ ksi}}$$
$$= 0.286 \text{ in.} < 0.430 \text{ in.} \quad \textbf{o.k.}$$

Note, both the top and bottom chords are acceptable for 3⁄16-in. fillet welds.

Welds for Member U_0L_1

From AISC *Specification* Table J2.4, the minimum weld size for a 3⁄8-in.-thick angle is:

$$w_{min} = \tfrac{3}{16} \text{ in.}$$

From AISC *Specification* Section J2.2b(b)(2), the maximum weld size is:

$$w_{max} = t - \tfrac{1}{16} \text{ in.}$$
$$= \tfrac{3}{8} - \tfrac{1}{16} \text{ in.}$$
$$= \tfrac{5}{16} \text{ in.}$$

Try a 3⁄16 in. fillet weld.

The minimum weld length is determined using AISC *Manual* Equation 8-2a or 8-2b:

LRFD	ASD
$l_{min} = \frac{R_u}{(2 \text{ sides})(2 \text{ welds})(1.392 \text{ kip/in.})D}$ $= \frac{165 \text{ kips}}{(2 \text{ sides})(2 \text{ welds})(1.392 \text{ kip/in.})(3)}$ $= 9.88 \text{ in.}$ Use a 10-in.-long weld at the heel and toe of the angles.	$l_{min} = \frac{R_a}{(2 \text{ sides})(2 \text{ welds})(0.928 \text{ kip/in.})D}$ $= \frac{110 \text{ kips}}{(2 \text{ sides})(2 \text{ welds})(0.928 \text{ kip/in.})(3)}$ $= 9.88 \text{ in.}$ Use a 10-in.-long weld at the heel and toe of the angles.

Note: A plate will be welded to the stem of the WT to provide room for the connection. Based on the preceding calculations for the minimum angle and stem thicknesses, by inspection the angles, stems, and stem plate extension have adequate strength.

Tensile Strength of Diagonal U_0L_1

From AISC *Specification* Section D2, the available tensile yielding strength of the angles is determined as follows:

$$P_n = F_y A_g$$ (*Spec*. Eq. D2-1)

$$= (36 \text{ ksi})(5.36 \text{ in.}^2)$$

$$= 193 \text{ kips}$$

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n = 0.90(193 \text{ kips})$ $= 174 \text{ kips} > 165 \text{ kips}$ **o.k.**	$\Omega_t = 1.67$ $\dfrac{P_n}{\Omega_t} = \dfrac{193 \text{ kips}}{1.67}$ $= 116 \text{ kips} > 110 \text{ kips}$ **o.k.**

From AISC *Specification* Section D2, the available tensile rupture strength of the angles is determined as follows. The shear lag factor, U, is determined using AISC *Specification* Table D3.1, Case 4.

$$U = \frac{3l^2}{3l^2 + w^2}\left(1 - \frac{\bar{x}}{l}\right)$$

$$= \frac{3(10 \text{ in.})^2}{3(10 \text{ in.})^2 + (4 \text{ in.})^2}\left(1 - \frac{0.947 \text{ in.}}{10 \text{ in.}}\right)$$

$$= 0.859$$

$$P_n = F_u A_e$$ (*Spec*. Eq. D2-2)

$$= (58 \text{ ksi})(5.36 \text{ in.}^2)(0.859)$$

$$= 267 \text{ kips}$$

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(267 \text{ kips})$ $= 200 \text{ kips} > 165 \text{ kips}$ **o.k.**	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{267 \text{ kips}}{2.00}$ $= 134 \text{ kips} > 110 \text{ kips}$ **o.k.**

Block Shear Rupture of Bottom Chord

The available strength for the limit state of block shear rupture of the chord is determined as follows.

$$R_n = 0.60 F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60 F_y A_{gv} + U_{bs} F_u A_{nt}$$ (*Spec*. Eq. J4-5)

where

$$A_{gv} = A_{nv}$$

$$= (2 \text{ lines}) l t_w$$

$$= (2 \text{ lines})(10 \text{ in.})(0.430 \text{ in.})$$

$$= 8.60 \text{ in.}^2$$

$$A_{nt} = (angle\ leg)t$$
$$= (4 \text{ in.})(0.430 \text{ in.})$$
$$= 1.72 \text{ in.}^2$$

$$U_{bs} = 1.0$$

Note, because ASTM A36 is used for the stem extension plate, F_y = 36 ksi and F_u = 58 ksi are used for the shear components of AISC *Specification* Equation J4-5.

$$R_n = 0.60(58 \text{ ksi})(8.60 \text{ in.}^2) + 1.0(65 \text{ ksi})(1.72 \text{ in.}^2) \le 0.60(36 \text{ ksi})(8.60 \text{ in.}^2) + 1.0(65 \text{ ksi})(1.72 \text{ in.}^2)$$
$$= 411 \text{ kips} > 298 \text{ kips}$$

Therefore:

$$R_n = 298 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(298 \text{ kips})$ $= 224 \text{ kips} > 165 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{298 \text{ kips}}{2.00}$ $= 149 \text{ kips} > 110 \text{ kips}$ **o.k.**

Solution B:

Shear Yielding of Top Chord Stem

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the top chord at Section B-B (see Figure II.C-2-1) is determined as follows:

$$A_{gv} = dt_w$$
$$= (8.26 \text{ in.})(0.455 \text{ in.})$$
$$= 3.76 \text{ in.}^2$$

$$R_n = 0.60 F_y A_{gv}$$ (*Spec*. Eq. J4-3)
$$= 0.60(50 \text{ ksi})(3.76 \text{ in.}^2)$$
$$= 113 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(113 \text{ kips})$ $= 113 \text{ kips} > 74.0 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{113 \text{ kips}}{1.50}$ $= 75.3 \text{ kips} > 49.2 \text{ kips}$ **o.k.**

Welds for Member U_1L_1

As calculated previously in Solution A, use 6½-in.-long 3⁄16-in. fillet welds at the heel and toe of both angles.

Welds for Member U_1L_2

As determined in previous calculations, the minimum and maximum weld sizes for a 5/16-in.-thick angle are:

$$w_{min} = \tfrac{3}{16}\text{ in.}$$
$$w_{max} = \tfrac{1}{4}\text{ in.}$$

Try a ¼ in. fillet weld.

To avoid having to use a stem extension plate unequal length welds are provided at the heel and toe of the angle. The minimum weld length for each angle is determined using AISC *Manual* Equation 8-2a or 8-2b:

LRFD	ASD
$l_{min} = \dfrac{R_u}{(2\text{ sides})(1.392\text{ kip/in.})D}$ $= \dfrac{114\text{ kips}}{(2\text{ sides})(1.392\text{ kip/in.})(4)}$ $= 10.2\text{ in.}$	$l_{min} = \dfrac{R_a}{(2\text{ sides})(0.928\text{ kip/in.})D}$ $= \dfrac{76\text{ kips}}{(2\text{ sides})(0.928\text{ kip/in.})(4)}$ $= 10.2\text{ in.}$

Try 7½ in. of ¼-in. fillet weld at the heel and 4 in. of ¼-in. fillet weld at the toe of each angle.

$$l = 7\tfrac{1}{2}\text{ in.} + 4\text{ in.}$$
$$= 11.5\text{ in.} > 10.2\text{ in.} \quad \textbf{o.k.}$$

Shear Rupture Strength of Angles at Welds

The minimum angle thickness to match the required shear rupture strength of the welds is determined as follows:

$$t_{min} = \frac{3.09D}{F_u} \qquad (\textit{Manual}\text{ Eq. 9-2})$$
$$= \frac{3.09(4)}{58\text{ ksi}}$$
$$= 0.213\text{ in.} < \tfrac{5}{16}\text{ in.} \quad \textbf{o.k.}$$

Shear Rupture Strength of Tee-Stem at Welds

The minimum tee-stem thickness to match the required shear rupture strength of the welds is determined as follows:

$$t_{min} = \frac{6.19D}{F_u} \qquad (\textit{Manual}\text{ Eq. 9-3})$$
$$= \frac{6.19(4)}{65\text{ ksi}}$$
$$= 0.381\text{ in.} < 0.455\text{ in.} \quad \textbf{o.k.}$$

Tensile Strength of Diagonal U_1L_2

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the angles are determined as follows:

$$R_n = F_y A_g$$ (*Spec*. Eq. J4-1)

$$= (36 \text{ ksi})(3.58 \text{ in.}^2)$$
$$= 129 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(129 \text{ kips})$ $= 116 \text{ kips} > 114 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{129 \text{ kips}}{1.67}$ $= 77.2 \text{ kips} > 76 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.1(b), the available tensile rupture strength of the angles are determined as follows. The shear lag factor, U, is determined using AISC *Specification* Table D3.1, Case 4.

$$l = \frac{l_1 + l_2}{2}$$
$$= \frac{7\tfrac{1}{2} \text{ in.} + 4 \text{ in.}}{2}$$
$$= 5.75 \text{ in.}$$

$$U = \frac{3l^2}{3l^2 + w^2}\left(1 - \frac{\bar{x}}{l}\right)$$
$$= \frac{3(5.75 \text{ in.})^2}{3(5.75 \text{ in.})^2 + (3\tfrac{1}{2} \text{ in.})^2}\left(1 - \frac{0.632 \text{ in.}}{5.75 \text{ in.}}\right)$$
$$= 0.792$$

$$R_n = F_u A_e$$ (*Spec*. Eq. J4-2)

$$= (58 \text{ ksi})(3.58 \text{ in.}^2)(0.792)$$
$$= 164 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(164 \text{ kips})$ $= 123 \text{ kips} > 114 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{164 \text{ kips}}{2.00}$ $= 82.0 \text{ kips} > 76 \text{ kips}$ **o.k.**

Conclusion

Joints L_1 and U_1 are found to be adequate as given for the applied loads.

EXAMPLE II.C-3 HEAVY WIDE-FLANGE COMPRESSION CONNECTION (FLANGES ON THE OUTSIDE)

Given:

The truss shown in Figure II.C-3-1 has been designed with ASTM A992 W14 shapes with flanges to the outside of the truss. Beams framing into the top chord and lateral bracing are not shown but can be assumed to be adequate.

Based on multiple load cases, the critical dead and live load forces for this connection are shown in Figure II.C-3-2. A typical top chord connection is shown in Figure II.C-3-1, Detail A. Design this typical connection using 1-in.-diameter Group A slip-critical bolts in standard holes with threads not excluded from the shear plane (thread condition N) with Class A faying surfaces and ASTM A36 gusset plates.

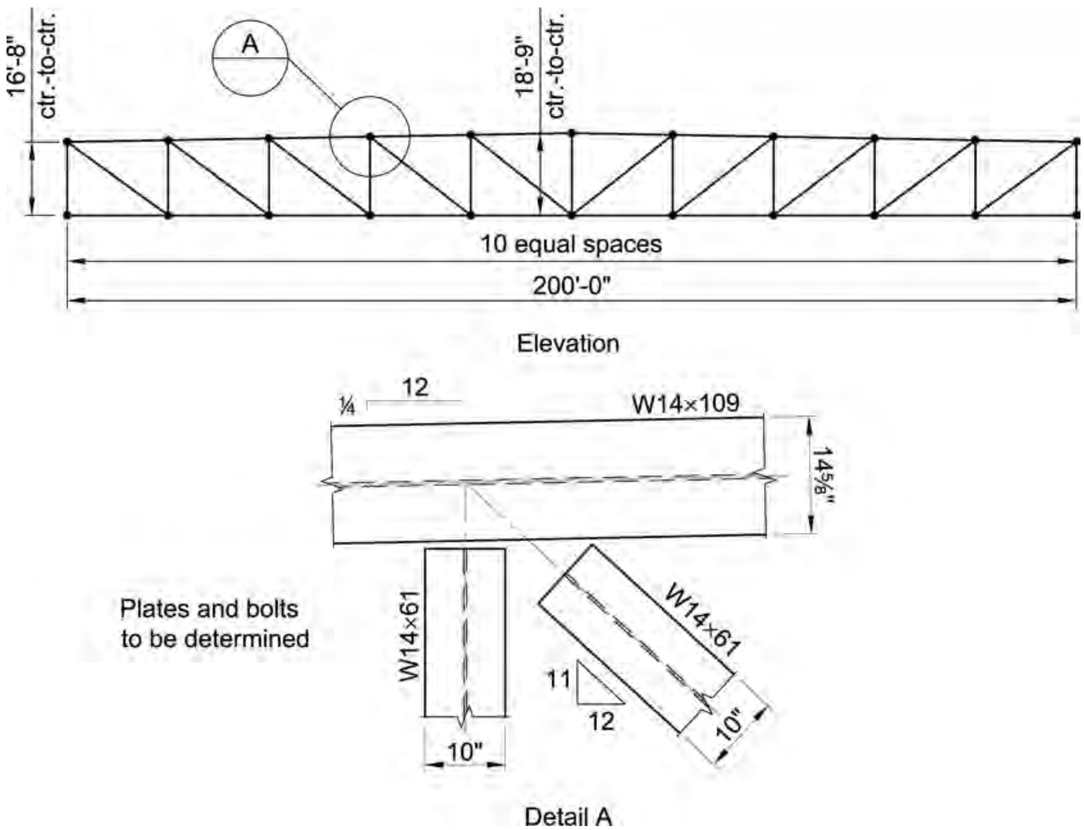


Fig II.C-3-1. Truss layout for Example II.C-3.

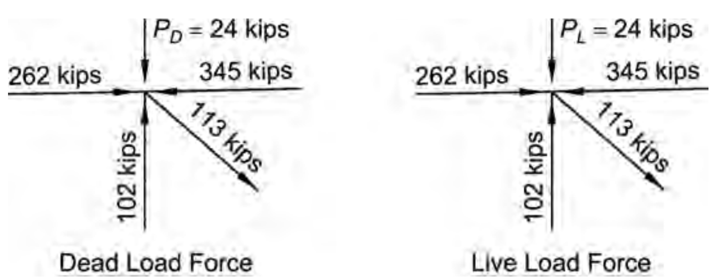


Fig. II.C-3-2. Forces at Detail A.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

W-shapes
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Gusset plates
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Top chord
W14×109
d = 14.3 in.
b_f = 14.6 in.
t_f = 0.860 in.

Web members
W14×61
d = 13.9 in.
b_f = 10.0 in.
t_f = 0.645 in.

From AISC *Specification* Table J3.3, for 1-in.-diameter bolts with standard holes:

$d_h = 1\frac{1}{8}$ in.

From ASCE/SEI 7, Chapter 2, the required strengths are determined as follows and summarized in Figure II.C-3-2.

LRFD	ASD
Left top chord: $P_u = 1.2(262 \text{ kips}) + 1.6(262 \text{ kips})$ $= 734 \text{ kips}$ Right top chord: $P_u = 1.2(345 \text{ kips}) + 1.6(345 \text{ kips})$ $= 966 \text{ kips}$ Vertical Web: $P_u = 1.2(102 \text{ kips}) + 1.6(102 \text{ kips})$ $= 286 \text{ kips}$	Left top chord: $P_a = 262 \text{ kips} + 262 \text{ kips}$ $= 524 \text{ kips}$ Right top chord: $P_a = 345 \text{ kips} + 345 \text{ kips}$ $= 690 \text{ kips}$ Vertical Web: $P_a = 102 \text{ kips} + 102 \text{ kips}$ $= 204 \text{ kips}$

LRFD	ASD
Diagonal Web: $P_u = 1.2(113 \text{ kips}) + 1.6(113 \text{ kips})$ $= 316 \text{ kips}$	Diagonal Web: $P_a = 113 \text{ kips} + 113 \text{ kips}$ $= 226 \text{ kips}$

Note: In checking equilibrium of vertical forces, $\Sigma F_y \neq 0$, due to the external (loading) forces not included. Refer to Figure II.C-3-2 for the magnitude of external load forces. In most truss designs, member forces only are provided and force equilibrium of the internal truss forces will not sum to zero.

Bolt Slip Resistance Strength

From AISC *Specification* Section J3.8(a), the available slip resistance for the limit state of slip for standard size holes is determined as follows:

$\mu = 0.30$ for Class A surface

$D_u = 1.13$

$h_f = 1.0$, no filler is provided

$T_b = 51$ kips, from AISC *Specification* Table J3.1, Group A

$n_s = 1$, number of slip planes

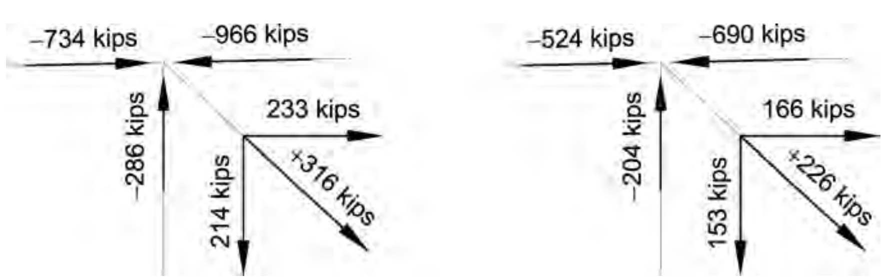
$$r_n = \mu D_u h_f T_b n_s \quad \text{(}\textit{Spec}\text{. Eq. J3-4)}$$

$$= (0.30)(1.13)(1.0)(51 \text{ kips})(1)$$

$$= 17.3 \text{ kips/bolt}$$

LRFD	ASD
$\phi = 1.00$ $\phi r_n = 1.00(17.3 \text{ kips/bolt})$ $= 17.3 \text{ kips/bolt}$	$\Omega = 1.50$ $\dfrac{r_n}{\Omega} = \dfrac{17.3 \text{ kips/bolt}}{1.50}$ $= 11.5 \text{ kips/bolt}$

Note: Standard holes are used in both plies for this example. Other hole sizes may be used and should be considered based on the preferences of the fabricator or erector on a case-by-case basis.



(a) LRFD *(b) ASD*

Fig. II.C-3-2. Required forces at Detail A.

Diagonal Connection

The required number of bolts is determined as follows:

LRFD	ASD
$P_u = 316 \text{ kips}$ $n_{req} = \dfrac{P_u}{\phi r_n}$ $= \dfrac{316 \text{ kips}}{17.3 \text{ kips/bolt}}$ $= 18.3 \text{ bolts}$ For two lines of bolts on both sides, the required number of rows is: $\dfrac{18.3 \text{ bolts}}{(2 \text{ sides})(2 \text{ lines})} = 4.58$ Therefore, use five rows at min. 3-in. spacing.	$P_a = 226 \text{ kips}$ $n_{req} = \dfrac{\Omega P_a}{r_n}$ $= \dfrac{226 \text{ kips}}{11.5 \text{ kips/bolt}}$ $= 19.7 \text{ bolts}$ For two lines of bolts on both sides, the required number of rows is: $\dfrac{19.7 \text{ bolts}}{(2 \text{ sides})(2 \text{ lines})} = 4.93$ Therefore, use five rows at min. 3-in. spacing.

Whitmore section in gusset plate

The width of the Whitmore section, l_w, is determined as shown in AISC *Manual* Figure 9-1.

$$\begin{aligned} l_w &= gage + 2l \tan 30° \\ &= 5\tfrac{1}{2} \text{ in.} + 2(12 \text{ in.})(\tan 30°) \\ &= 19.4 \text{ in.} \end{aligned}$$

Try a ⅜-in.-thick plate.

$$\begin{aligned} A_g &= (2 \text{ plates}) l_w t \\ &= (2 \text{ plates})(19.4 \text{ in.})(\tfrac{3}{8} \text{ in.}) \\ &= 14.6 \text{ in.}^2 \end{aligned}$$

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the gusset plate is determined as follows:

$$\begin{aligned} R_n &= F_y A_g \\ &= (36 \text{ ksi})(14.6 \text{ in.}^2) \\ &= 526 \text{ kips} \end{aligned}$$ (*Spec.* Eq. J4-1)

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(526 \text{ kips})$ $= 473 \text{ kips} > 316 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{526 \text{ kips}}{1.67}$ $= 315 \text{ kips} > 226 \text{ kips}$ **o.k.**

Block shear rupture of gusset plate

The available strength for the limit state of block shear rupture of the gusset plates is determined as follows.

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt}$$ (*Spec.* Eq. J4-5)

where

$$\begin{aligned} A_{gv} &= (2 \text{ plates})(2 \text{ lines})\left[l_{ev} + (n-1)s\right]t \\ &= (2 \text{ plates})(2 \text{ lines})\left[2 \text{ in.} + (5-1)(3 \text{ in.})\right](3/8 \text{ in.}) \\ &= 21.0 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nv} &= A_{gv} - (2 \text{ plates})(2 \text{ lines})(5-0.5)(d_h + 1/16 \text{ in.})t \\ &= 21.0 \text{ in.}^2 - (2 \text{ plates})(2 \text{ lines})(5-0.5)(1 1/8 \text{ in.} + 1/16 \text{ in.})(3/8 \text{ in.}) \\ &= 13.0 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{nt} &= (2 \text{ plates})\left[gage - (d_h + 1/16 \text{ in.})\right]t \\ &= (2 \text{ plates})\left[5 1/2 \text{ in.} - (1 1/8 \text{ in.} + 1/16 \text{ in.})\right](3/8 \text{ in.}) \\ &= 3.23 \text{ in.}^2 \end{aligned}$$

$$U_{bs} = 1.0$$

and

$$\begin{aligned} R_n &= 0.60(58 \text{ ksi})(13.0 \text{ in.}^2) + 1.0(58 \text{ ksi})(3.23 \text{ in.}^2) \le 0.60(36 \text{ ksi})(21.0 \text{ in.}^2) + 1.0(58 \text{ ksi})(3.23 \text{ in.}^2) \\ &= 640 \text{ kips} < 641 \text{ kips} \end{aligned}$$

Therefore:

$$R_n = 640 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(640 \text{ kips})$ $= 480 \text{ kips} > 316 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{640 \text{ kips}}{2.00}$ $= 320 \text{ kips} > 226 \text{ kips}$ **o.k.**

Block shear rupture of diagonal flange

By inspection, block shear rupture on the diagonal flange will not control.

Strength of bolted connection—gusset plate

Slip-critical connections must also be designed for the limit states of bearing-type connections. From the Commentary to AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the individual strengths of the individual fasteners, which may be taken as the lesser of the fastener shear strength per AISC *Specification* Section J3.6, the bearing strength at the bolt hole per AISC *Specification* Section J3.10, or the tearout strength at the bolt hole per AISC *Specification* Section J3.10.

From AISC *Manual* Table 7-1, the available shear strength per bolt for 1-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) is:

LRFD	ASD
$\phi r_n = 31.8 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = 21.2 \text{ kips/bolt}$

The available bearing and tearout strength of the gusset plate at the edge bolts is determined using AISC *Manual* Table 7-5, using l_e = 2 in.

LRFD	ASD
$\phi r_n = (75.0 \text{ kip/in.})(\tfrac{3}{8} \text{ in.})$ $= 28.1 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (50.0 \text{ kip/in.})(\tfrac{3}{8} \text{ in.})$ $= 18.8 \text{ kips/bolt}$

Therefore, the bearing or tearout strength controls over bolt shear at the edge bolts.

The available bearing and tearout strength of the gusset plate at the other bolts is determined using AISC *Manual* Table 7-4 with s = 3 in.

LRFD	ASD
$\phi r_n = (97.9 \text{ kip/in.})(\tfrac{3}{8} \text{ in.})$ $= 36.7 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (65.3 \text{ kip/in.})(\tfrac{3}{8} \text{ in.})$ $= 24.5 \text{ kips/bolt}$

Therefore, bolt shear controls over bearing or tearout at the other bolts.

The strength of the bolt group in the gusset plate is determined by summing the strength of the individual fasteners as follows:

LRFD	ASD
$\phi R_n = (2 \text{ sides})(2 \text{ lines})\begin{bmatrix}(1 \text{ bolt})(28.1 \text{ kips/bolt}) \\ +(4 \text{ bolts})(31.8 \text{ kips/bolt})\end{bmatrix}$ $= 621 \text{ kips} > 316 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (2 \text{ sides})(2 \text{ lines})\begin{bmatrix}(1 \text{ bolt})(18.8 \text{ kips/bolt}) \\ +(4 \text{ bolts})(21.2 \text{ kips/bolt})\end{bmatrix}$ $= 414 \text{ kips} > 226 \text{ kips}$ **o.k.**

Strength of bolted connection—diagonal flange

By inspection the strength of the bolted connection at the gusset plate will control.

Horizontal Connection

The required strength of the gusset plate to horizontal member is determined as follows:

LRFD	ASD
$P_u = 966 \text{ kips} - 734 \text{ kips}$ $= 232 \text{ kips}$	$P_a = 690 \text{ kips} - 524 \text{ kips}$ $= 166 \text{ kips}$

Using the bolt slip resistance strength determined previously, the required number of rows of bolts is determined as follows:

LRFD	ASD
$n_{req} = \dfrac{P_u}{\phi r_n}$ $= \dfrac{232 \text{ kips}}{17.3 \text{ kips/bolt}}$ $= 13.4 \text{ bolts}$ For two lines of bolts on both sides the required number of rows is: $\dfrac{13.4 \text{ bolts}}{(2 \text{ sides})(2 \text{ lines})} = 3.35$	$n_{req} = \dfrac{\Omega P_u}{r_n}$ $= \dfrac{166 \text{ kips}}{11.5 \text{ kips/bolt}}$ $= 14.4 \text{ bolts}$ For two lines of bolts on both sides the required number of rows is: $\dfrac{14.4 \text{ bolts}}{(2 \text{ sides})(2 \text{ lines})} = 3.60$

For members not subject to corrosion the maximum bolt spacing is determined using AISC *Specification* Section J3.5(a):

$$\begin{aligned} 24t &= 24(\tfrac{3}{8} \text{ in.}) \\ &= 9.00 \text{ in.} \end{aligned}$$

Due to the geometry of the gusset plate, the use of 4 rows of bolts in the horizontal connection will exceed the maximum bolt spacing; instead use 5 rows of bolts in two lines.

Shear strength of the gusset plate

From AISC *Specification* Section J4.2(a), the available shear yielding strength of the gusset plates is determined as follows:

$$\begin{aligned} A_{gv} &= (2 \text{ plates})lt \\ &= (2 \text{ plates})(32.0 \text{ in.})(\tfrac{3}{8} \text{ in.}) \\ &= 24.0 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_n &= 0.60 F_y A_{gv} \\ &= 0.60(36 \text{ ksi})(24.0 \text{ in.}^2) \\ &= 518 \text{ kips} \end{aligned}$$ (*Spec*. Eq. J4-3)

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(518 \text{ kips})$ $= 518 \text{ kips} > 232 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{518 \text{ kips}}{1.50}$ $= 345 \text{ kips} > 166 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2(b), the available shear rupture strength of gusset plates is determined as follows:

$$\begin{aligned} A_{nv} &= (2 \text{ plates})\left[l - n(d_h + \tfrac{1}{16} \text{ in.})\right]t \\ &= (2 \text{ plates})\left[32.0 \text{ in.} - 5(1\tfrac{1}{8} \text{ in.} + \tfrac{1}{16} \text{ in.})\right](\tfrac{3}{8} \text{ in.}) \\ &= 19.5 \text{ in.}^2 \end{aligned}$$

$$R_n = 0.60F_u A_{nv}$$ (*Spec.* Eq. J4-4)

$$= 0.60(58 \text{ ksi})(19.5 \text{ in.}^2)$$

$$= 679 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(679 \text{ kips})$ $= 509 \text{ kips} > 232 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{679 \text{ kips}}{1.50}$ $= 453 \text{ kips} > 166 \text{ kips}$ **o.k.**

Strength of bolted connection

By comparison to the preceding calculations for the diagonal connection, bolt bearing or tearout does not control.

Vertical Connection

Using the bolt slip resistance strength determined previously, the required number of bolts is determined as follows:

LRFD	ASD
$P_u = 286 \text{ kips}$ $n_{req} = \dfrac{P_u}{\phi r_n}$ $= \dfrac{286 \text{ kips}}{17.3 \text{ kips/bolt}}$ $= 16.5 \text{ bolts}$ For two lines of bolts on both sides, the required number of rows is: $\dfrac{16.5 \text{ bolts}}{(2 \text{ sides})(2 \text{ lines})} = 4.12$ Therefore, use 5 rows at min. 3-in. spacing.	$P_u = 204 \text{ kips}$ $n_{req} = \dfrac{\Omega P_u}{r_n}$ $= \dfrac{204 \text{ kips}}{11.5 \text{ kips/bolt}}$ $= 17.7 \text{ bolts}$ For two lines of bolts on both sides, the required number of rows is: $\dfrac{17.7 \text{ bolts}}{(2 \text{ sides})(2 \text{ lines})} = 4.43$ Therefore, use 5 rows at min. 3-in. spacing.

Shear strength of the gusset plate

From AISC *Specification* Section J4.2(a), the available shear yielding strength of gusset plates is determined as follows:

$$A_{gv} = (2 \text{ plates})lt$$

$$= (2 \text{ plates})(31\tfrac{3}{4} \text{ in.})(\tfrac{3}{8} \text{ in.})$$

$$= 23.8 \text{ in.}^2$$

$$R_n = 0.60F_y A_{gv}$$ (*Spec.* Eq. J4-3)

$$= 0.60(36 \text{ ksi})(23.8 \text{ in.}^2)$$

$$= 514 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(514 \text{ kips})$ $= 514 \text{ kips} > 286 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{514 \text{ kips}}{1.50}$ $= 343 \text{ kips} > 204 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.2(b), the available shear rupture strength of gusset plates is determined as follows:

$$\begin{aligned} A_{nv} &= (2 \text{ plates})\left[l - n\left(d_h + \tfrac{1}{16} \text{ in.}\right)\right]t \\ &= (2 \text{ plates})\left[31\tfrac{3}{4} \text{ in.} - 7\left(1\tfrac{1}{8} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)\right]\left(\tfrac{3}{8} \text{ in.}\right) \\ &= 17.6 \text{ in.}^2 \end{aligned}$$

$$R_n = 0.60 F_u A_{nv} \qquad \text{(\textit{Spec.} Eq. J4-4)}$$

$$\begin{aligned} &= 0.60(58 \text{ ksi})\left(17.6 \text{ in.}^2\right) \\ &= 612 \text{ kips} \end{aligned}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(612 \text{ kips})$ $= 459 \text{ kips} > 286 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{612 \text{ kips}}{2.00}$ $= 306 \text{ kips} > 204 \text{ kips}$ **o.k.**

Strength of bolted connection

By comparison to the preceding calculations for the diagonal connection, bolt bearing does not control.

Note that because of the difference in depths between the top chord and the vertical and diagonal members, 3⁄16-in. loose shims are required on each side of the shallower members.

The final connection design is shown in Figure II.C-3-4.

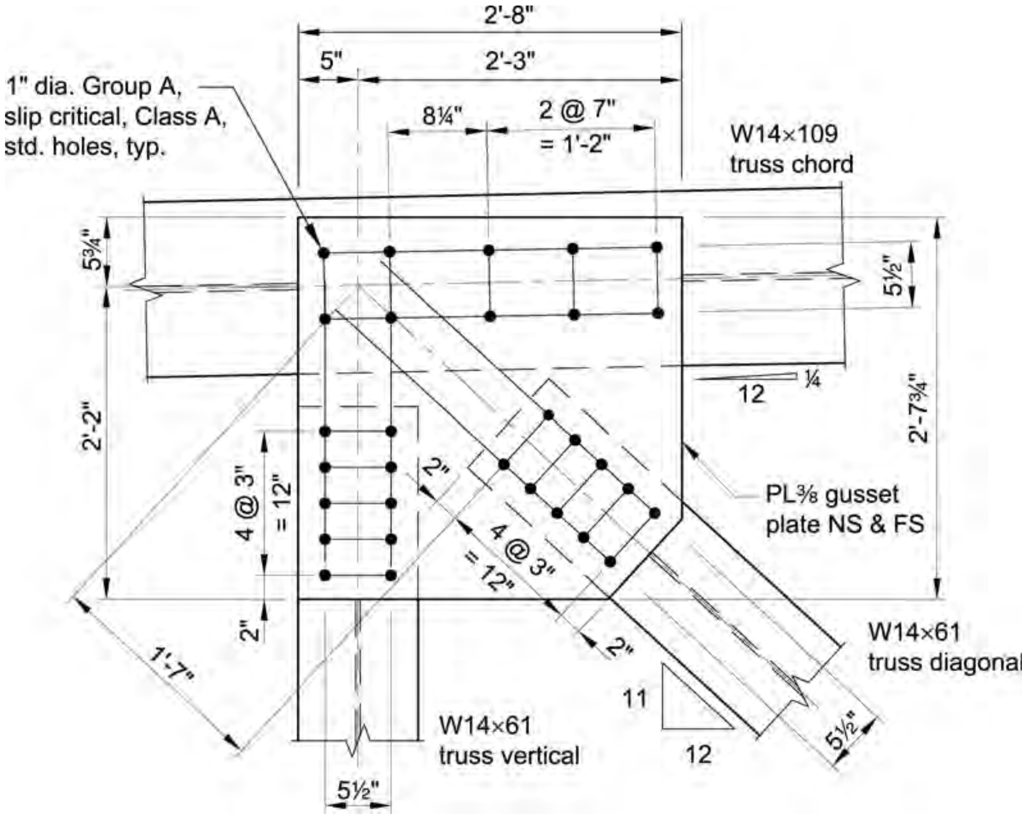


Fig. II.C-3-4. Connection layout for Example II.C-3.

Chapter IID
Miscellaneous Connections

This section contains design examples on connections in the AISC *Steel Construction Manual* that are not covered in other sections of the AISC *Design Examples*.

EXAMPLE II.D-1 WT HANGER CONNECTION

Given:

Design an ASTM A992 WT hanger connection between an ASTM A36 2L3×3×5⁄16 tension member and an ASTM A992 W24×94 beam to support the following loads:

P_D = 13.5 kips
P_L = 40 kips

Use 70-ksi electrodes.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and WT hanger
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1, 1-7 and 1-15, the geometric properties are as follows:

Beam
W24×94
d = 24.3 in.
t_w = 0.515 in.
b_f = 9.07 in.
t_f = 0.875 in.

Angles
2L3×3×5⁄16
A = 3.56 in.^{2}
$\bar{x}$ = 0.860 in. (for single angle)

From AISC *Specification* Table J3.3, the hole diameter for ¾-in.-diameter bolts with standard holes is:

$d_h = {}^{13}\!/_{16}$ in.

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$P_u = 1.2(13.5 \text{ kips}) + 1.6(40 \text{ kips})$ $= 80.2 \text{ kips}$	$P_a = 13.5 \text{ kips} + 40 \text{ kips}$ $= 53.5 \text{ kips}$

Weld Design

Note: AISC *Specification* Section J1.7 requiring that the center of gravity of the weld group coincide with the center of gravity of the member does not apply to end connections of statically loaded single-angle, double-angle and similar members.

From AISC *Specification* Table J2.4, the minimum weld size for a 5⁄16-in.-thick angle is:

$$w_{min} = \tfrac{3}{16} \text{ in.}$$

From AISC *Specification* Section J2.2b(b)(2), the maximum weld size is:

$$\begin{aligned} w_{max} &= t - \tfrac{1}{16} \text{ in.} \\ &= \tfrac{5}{16} - \tfrac{1}{16} \text{ in.} \\ &= \tfrac{1}{4} \text{ in.} \end{aligned}$$

Try ¼-in. fillet welds.

The minimum weld length is determined using AISC *Manual* Equations 8-2a or 8-2b, as follows:

LRFD	ASD
$l_{min} = \dfrac{R_u}{(2 \text{ sides})(2 \text{ welds})(1.392 \text{ kip/in.})D}$ $= \dfrac{80.2 \text{ kips}}{(2 \text{ sides})(2 \text{ welds})(1.392 \text{ kip/in.})(4)}$ $= 3.60 \text{ in.}$ Use a 4-in.-long weld at the heel and toe of the angles.	$l_{min} = \dfrac{R_a}{(2 \text{ sides})(2 \text{ welds})(0.928 \text{ kip/in.})D}$ $= \dfrac{53.5 \text{ kips}}{(2 \text{ sides})(2 \text{ welds})(0.928 \text{ kip/in.})(4)}$ $= 3.60 \text{ in.}$ Use a 4-in.-long weld at the heel and toe of the angles.

Tensile Strength of Angles

From AISC *Specification* Section D2, the available tensile yielding strength of the angles is determined as follows:

$$\begin{aligned} P_n &= F_y A_g \\ &= (36 \text{ ksi})(3.56 \text{ in.}^2) \\ &= 128 \text{ kips} \end{aligned}$$ (*Spec*. Eq. D2-1)

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n = 0.90(128 \text{ kips})$ $= 115 \text{ kips} > 80.2 \text{ kips}$ **o.k.**	$\Omega_t = 1.67$ $\dfrac{P_n}{\Omega_t} = \dfrac{128 \text{ kips}}{1.67}$ $= 76.6 \text{ kips} > 53.5 \text{ kips}$ **o.k.**

From AISC *Specification* Section D2, the available tensile rupture strength of the brace is determined as follows:

$$\begin{aligned} A_n &= A_g \\ &= 3.56 \text{ in.}^2 \end{aligned}$$

The shear lag factor, U, is determined from AISC *Specification* Table D3.1, Case 4:

$$U = \frac{3l^2}{3l^2 + w^2}\left(1 - \frac{\bar{x}}{l}\right)$$
$$= \frac{3(4 \text{ in.})^2}{3(4 \text{ in.})^2 + (3 \text{ in.})^2}\left(1 - \frac{0.860 \text{ in.}}{4 \text{ in.}}\right)$$
$$= 0.661$$

$$A_e = A_n U \qquad \text{(}\textit{Spec}\text{. Eq. D3-1)}$$
$$= (3.56 \text{ in.}^2)(0.661)$$
$$= 2.35 \text{ in.}^2$$

$$P_n = F_u A_e \qquad \text{(}\textit{Spec}\text{. Eq. D2-2)}$$
$$= (58 \text{ ksi})(2.35 \text{ in.}^2)$$
$$= 136 \text{ kips}$$

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(136 \text{ kips})$ $= 102 \text{ kips} > 80.2 \text{ kips}$ **o.k.**	$\Omega_t = 2.00$ $\frac{P_n}{\Omega_t} = \frac{136 \text{ kips}}{2.00}$ $= 68.0 \text{ kips} > 53.5 \text{ kips}$ **o.k.**

Preliminary WT Selection Using Beam Gage

Try four ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N), with a 4-in. gage.

LRFD	ASD
$T_r = r_{ut}$ $= \frac{P_u}{n}$ $= \frac{80.2 \text{ kips}}{4 \text{ bolts}}$ $= 20.1 \text{ kips/bolt}$ From AISC *Manual* Table 7-2: $B_c = \phi r_n$ $= 29.8 \text{ kips/bolt} > 20.1 \text{ kips/bolt}$ **o.k.**	$T_r = r_{at}$ $= \frac{P_a}{n}$ $= \frac{53.5 \text{ kips}}{4 \text{ bolts}}$ $= 13.4 \text{ kips/bolt}$ From AISC *Manual* Table 7-2: $B_c = \frac{r_n}{\Omega}$ $= 19.9 \text{ kips/bolt} > 13.4 \text{ kips/bolt}$ **o.k.**

Determine tributary length per pair of bolts, p, using AISC *Manual* Figure 9-4.

$$p = \frac{4\tfrac{1}{2} \text{ in.}}{2} + \frac{8.00 \text{ in.} - 4\tfrac{1}{2} \text{ in.}}{2}$$
$$= 4.00 \text{ in.}$$

Check:

$$p \le s$$
$$4.00 \text{ in.} < 4\tfrac{1}{2} \text{ in.} \quad \textbf{o.k.}$$

Verify that the tributary length on each side of the bolt conforms to dimensional limits assuming a ½-in. tee stem thickness:

$$\begin{aligned} b &= \frac{\left(4.00 \text{ in.} - \tfrac{1}{2} \text{ in.}\right)}{2} \\ &= 1.75 \text{ in.} \end{aligned}$$

$$\frac{4\tfrac{1}{2} \text{ in.}}{2} \le 1.75b$$
$$2.25 \text{ in.} < 3.06 \text{ in.} \quad \textbf{o.k.}$$

$$\frac{8.00 \text{ in.} - 4\tfrac{1}{2} \text{ in.}}{2} \le 1.75b$$
$$1.75 \text{ in.} < 3.06 \text{ in.} \quad \textbf{o.k.}$$

A preliminary hanger connection is determined using AISC *Manual* Table 15-2b.

LRFD	ASD
$\begin{aligned} 2R_{ut} &= \frac{(rows)B_c}{p} \\ &= \frac{(2)(20.1 \text{ kips/bolt})}{4.00 \text{ in.}} \\ &= 10.1 \text{ kip/in.} \end{aligned}$	$\begin{aligned} 2R_{at} &= \frac{(rows)B_c}{p} \\ &= \frac{(2 \text{ bolts})(13.4 \text{ kips/bolt})}{4.00 \text{ in.}} \\ &= 6.70 \text{ kip/in.} \end{aligned}$

From AISC *Manual* Table 15-2b, with an assumed b = (4.00 in. – ½ in.)/2 = 1.75 in., the flange thickness, $t = t_f$, of the WT hanger should be approximately ⅝ in.

The minimum depth WT that can be used is equal to the sum of the weld length plus the weld size plus the k-dimension for the selected section. From AISC *Manual* Table 1-8 with an assumed b = 1.75 in., $t_f \approx$ ⅝ in., and d_{min} = 4 in. + ¼ in. + $k \approx$ 6 in., appropriate selections include:

WT6×25
WT7×26.5
WT8×25
WT9×27.5

Try a WT6×25.

From AISC *Manual* Table 1-8, the geometric properties are as follows:

b_f = 8.08 in.
t_f = 0.640 in.
t_w = 0.370 in.

Prying Action

From AISC *Manual* Part 9, the available tensile strength of the bolts taking prying action into account is determined as follows. The beam flange is thicker than the WT flange; therefore, prying in the tee flange will control over prying in the beam flange.

$$
\begin{aligned}
a &= \frac{b_f - gage}{2} \\
&= \frac{8.08 \text{ in.} - 4 \text{ in.}}{2} \\
&= 2.04 \text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
b &= \frac{gage - t_w}{2} \\
&= \frac{4 \text{ in.} - 0.370 \text{ in.}}{2} \\
&= 1.82 \text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
b' &= b - \frac{d_b}{2} \\
&= 1.82 \text{ in.} - \left(\frac{3/4 \text{ in.}}{2}\right) \\
&= 1.45 \text{ in.}
\end{aligned}
$$

(*Manual* Eq. 9-18)

$$
\begin{aligned}
a' &= \left(a + \frac{d_b}{2}\right) \leq \left(1.25b + \frac{d_b}{2}\right) \\
&= 2.04 \text{ in.} + \frac{3/4 \text{ in.}}{2} \leq 1.25(1.82 \text{ in.}) + \frac{3/4 \text{ in.}}{2} \\
&= 2.42 \text{ in.} < 2.65 \text{ in.}
\end{aligned}
$$

(*Manual* Eq. 9-23)

$$
\begin{aligned}
\rho &= \frac{b'}{a'} \\
&= \frac{1.45 \text{ in.}}{2.42 \text{ in.}} \\
&= 0.599
\end{aligned}
$$

(*Manual* Eq. 9-22)

From AISC *Manual* Equation 9-21:

LRFD	ASD
$\beta = \frac{1}{\rho}\left(\frac{B_c}{T_r} - 1\right)$ $= \frac{1}{0.599}\left(\frac{29.8 \text{ kips/bolt}}{20.1 \text{ kips/bolt}} - 1\right)$ $= 0.806$	$\beta = \frac{1}{\rho}\left(\frac{B_c}{T_r} - 1\right)$ $= \frac{1}{0.599}\left(\frac{19.9 \text{ kips/bolt}}{13.4 \text{ kips/bolt}} - 1\right)$ $= 0.810$

$$
\begin{aligned}
d' &= d_h \\
&= 13/16 \text{ in.}
\end{aligned}
$$

$$\delta = 1 - \frac{d'}{p}$$ (*Manual* Eq. 9-20)

$$= 1 - \frac{13/16 \text{ in.}}{4.00 \text{ in.}}$$

$$= 0.797$$

Because $\beta < 1.0$:

LRFD	ASD
$\alpha' = \frac{1}{\delta}\left(\frac{\beta}{1-\beta}\right) \le 1.0$ $= \frac{1}{0.797}\left(\frac{0.806}{1-0.806}\right) > 1.0$ $= 5.21 > 1.0$ Therefore, $\alpha' = 1.0$. $\phi = 0.90$ $t_{min} = \sqrt{\frac{4T_u b'}{\phi p F_u (1+\delta\alpha')}}$ (*Manual* Eq. 9-19a) $= \sqrt{\frac{4(20.1 \text{ kips/bolt})(1.45 \text{ in.})}{0.90(4.00 \text{ in.})(65 \text{ ksi})[1+(0.797)(1.0)]}}$ $= 0.527 \text{ in.} < t_f = 0.640 \text{ in.}$ **o.k.**	$\alpha' = \frac{1}{\delta}\left(\frac{\beta}{1-\beta}\right) \le 1.0$ $= \frac{1}{0.797}\left(\frac{0.810}{1-0.810}\right) > 1.0$ $= 5.35 > 1.0$ Therefore, $\alpha' = 1.0$. $\Omega = 1.67$ $t_{min} = \sqrt{\frac{\Omega 4T_a b'}{p F_u (1+\delta\alpha')}}$ (*Manual* Eq. 9-19b) $= \sqrt{\frac{1.67(4)(13.4 \text{ kips/bolt})(1.45 \text{ in.})}{(4.00 \text{ in.})(65 \text{ ksi})[1+(0.797)(1.0)]}}$ $= 0.527 \text{ in.} < t_f = 0.640 \text{ in.}$ **o.k.**

Note: As an alternative to the preceding calculations, the designer can use a simplified procedure to select a WT hanger with a flange thick enough to eliminate prying action. Assuming b' = 1.45 in., the required thickness to eliminate prying action is determined from AISC *Manual* Equation 9-17a or 9-17b, as follows:

LRFD	ASD
$\phi = 0.90$ $t_{np} = \sqrt{\frac{4T_u b'}{\phi p F_u}}$ $= \sqrt{\frac{4(20.1 \text{ kips/bolt})(1.45 \text{ in.})}{0.90(4.00 \text{ in.})(65 \text{ ksi})}}$ $= 0.706 \text{ in.}$	$\Omega = 1.67$ $t_{np} = \sqrt{\frac{\Omega 4T_a b'}{p F_u}}$ $= \sqrt{\frac{1.67(4)(13.4 \text{ kips/bolt})(1.45 \text{ in.})}{(4.00 \text{ in.})(65 \text{ ksi})}}$ $= 0.707 \text{ in.}$

The WT6×25 that was selected does not have a sufficient flange thickness to reduce the effect of prying action to an insignificant amount. In this case, the simplified approach requires a WT section with a thicker flange.

Tensile Yielding of the WT Stem on the Whitmore Section

As shown in AISC *Manual* Figure 9-1, the Whitmore section defines the effective width of the WT stem. Note that the Whitmore section cannot exceed the actual 8 in. width of the WT.

$$l_w = 3.00 \text{ in.} + 2(4.00 \text{ in.})(\tan 30^\circ) \le 8.00 \text{ in.}$$
$$= 7.62 \text{ in.} < 8.00 \text{ in.}$$

Therefore:

$$l_w = 7.62 \text{ in.}$$

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the WT stem is determined as follows:

$$A_g = l_w t_w$$
$$= (7.62 \text{ in.})(0.370 \text{ in.})$$
$$= 2.82 \text{ in.}^2$$

$$R_n = F_y A_g \qquad (\textit{Spec}. \text{ Eq. J4-1})$$
$$= (50 \text{ ksi})(2.82 \text{ in.}^2)$$
$$= 141 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(141 \text{ kips})$ $= 127 \text{ kips} > 80.2 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{141 \text{ kips}}{1.67}$ $= 84.4 \text{ kips} > 53.5 \text{ kips}$ **o.k.**

Shear Rupture of the WT Stem Base Metal

From AISC *Specification* Section J4.2(b), the available shear rupture strength of the WT stem at the welds is determined as follows:

$$R_n = (2 \text{ welds})(2 \text{ planes})0.60F_u l_w t_w$$
$$= (2 \text{ welds})(2 \text{ planes})(0.60)(65 \text{ ksi})(4 \text{ in.})(0.370 \text{ in.}) \qquad (\text{from } \textit{Spec}. \text{ Eq. J4-4})$$
$$= 231 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(231 \text{ kips})$ $= 173 \text{ kips} > 80.2 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{231 \text{ kips}}{2.00}$ $= 116 \text{ kips} > 53.5 \text{ kips}$ **o.k.**

Block Shear Rupture of the WT Stem

The available strength for the limit state of block shear rupture of the stem is determined as follows.

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (\textit{Spec}. \text{ Eq. J4-5})$$

where

$$A_{gv} = A_{nv}$$
$$= (2 \text{ lines}) l t_w$$
$$= (2 \text{ lines})(4 \text{ in.})(0.370 \text{ in.})$$
$$= 2.96 \text{ in.}^2$$
$$A_{nt} = (leg) t_w$$
$$= (3 \text{ in.})(0.370 \text{ in.})$$
$$= 1.11 \text{ in.}^2$$
$$U_{bs} = 1.0$$

and

$$R_n = 0.60(65 \text{ ksi})(2.96 \text{ in.}^2) + 1.0(65 \text{ ksi})(1.11 \text{ in.}^2) \le 0.60(50 \text{ ksi})(2.96 \text{ in.}^2) + 1.0(65 \text{ ksi})(1.11 \text{ in.}^2)$$
$$= 188 \text{ kips} > 161 \text{ kips}$$

Therefore:

$$R_n = 161 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(161 \text{ kips})$ $= 121 \text{ kips} > 80.2 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{161 \text{ kips}}{2.00}$ $= 80.5 \text{ kips} > 53.5 \text{ kips}$ **o.k.**

The final connection design is shown in Figure II.D-1-1.

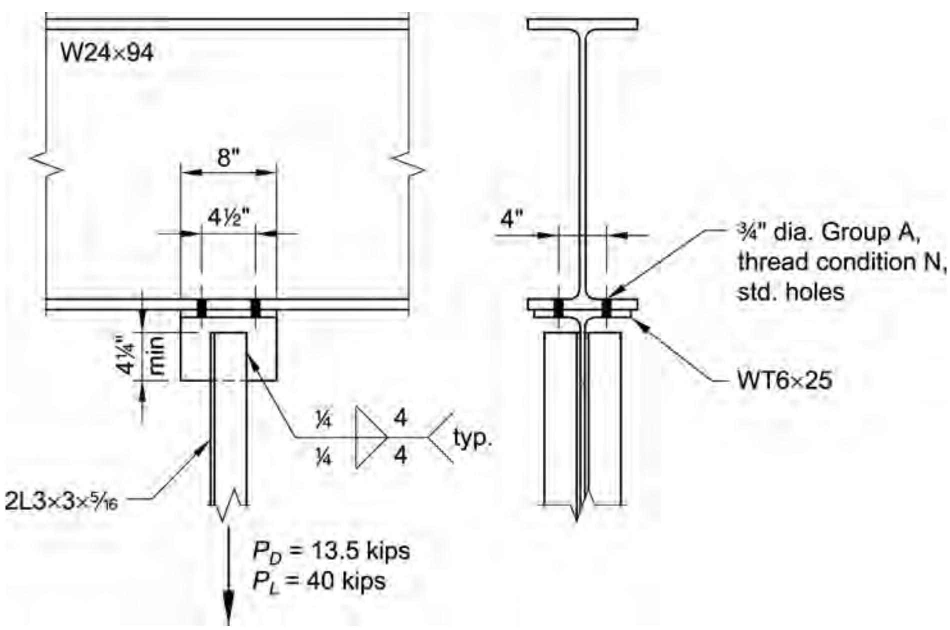


Fig. II.D-1-1. Final hanger design for Example II.D-1

EXAMPLE II.D-2 BEAM BEARING PLATE

Given:

An ASTM A992 W18×50 beam supported by a 10-in.-thick concrete wall, as shown in Figure II.D-2-1, has the following end reactions:

R_D = 15 kips
R_L = 45 kips

Verify the following:

A. If a bearing plate is required when the beam is supported by the full wall thickness ($l_b = h$ = 10 in)
B. The bearing plate required if $l_b = h$ = 10 in. (the full wall thickness)
C. The bearing plate required if l_b = 6½ in. and the bearing plate is centered on the thickness of the wall

The concrete has f'_c = 3 ksi and the bearing plate is ASTM A36 material.

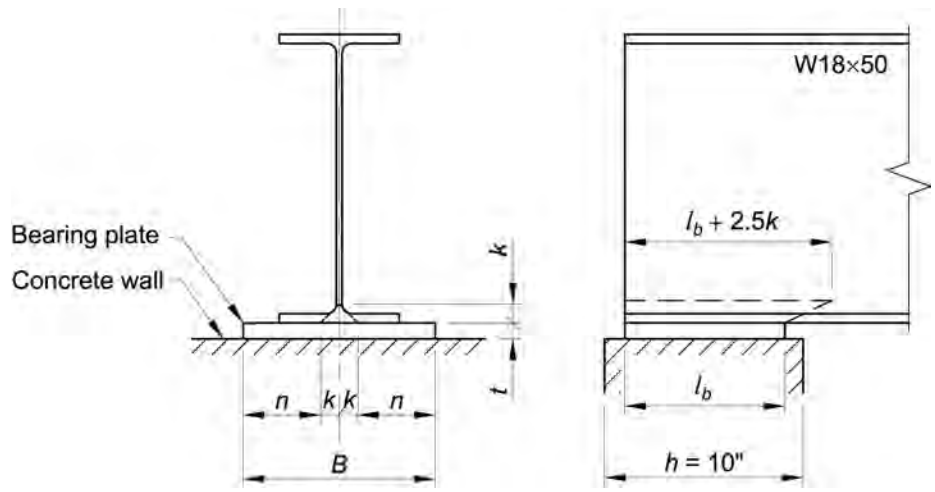


Fig. II.D-2-1. Connection geometry for Example II.D-2.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Bearing plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

Concrete wall
f'_c = 3 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×50
d = 18.0 in.
t_w = 0.355 in.
b_f = 7.50 in.
t_f = 0.570 in.
k_{des} = 0.972 in.
k_1 = 13⁄16 in.

From ASCE/SEI, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(15 \text{ kips}) + 1.6(45 \text{ kips})$ $= 90.0 \text{ kips}$	$R_a = 15 \text{ kips} + 45 \text{ kips}$ $= 60.0 \text{ kips}$

Solution A:

Required Bearing Length

The required bearing length for the limit state of web local yielding is determined using AISC *Manual* Table 9-4 and AISC *Manual* Equation 9-46a or 9-46b, as follows:

LRFD	ASD
$\phi R_1 = 43.1 \text{ kips}$ $\phi R_2 = 17.8 \text{ kip/in.}$ $l_{b\,min} = \dfrac{R_u - \phi R_1}{\phi R_2} \geq k_{des}$ $= \dfrac{90.0 \text{ kips} - 43.1 \text{ kips}}{17.8 \text{ kip/in.}} > 0.972 \text{ in.}$ $= 2.63 \text{ in.} > 0.972 \text{ in.}$ Therefore: $l_{b\,min} = 2.63 \text{ in.} < 10.0 \text{ in.}$ **o.k.**	$R_1/\Omega = 28.8 \text{ kips}$ $R_2/\Omega = 11.8 \text{ kip/in.}$ $l_{b\,min} = \dfrac{R_a - R_1/\Omega}{R_2/\Omega} \geq k_{des}$ $= \dfrac{60.0 \text{ kips} - 28.8 \text{ kips}}{11.8 \text{ kip/in.}} > 0.972 \text{ in.}$ $= 2.64 \text{ in.} > 0.972 \text{ in.}$ Therefore: $l_{b\,min} = 2.64 \text{ in.} < 10.0 \text{ in.}$ **o.k.**

The required bearing length for the limit state of web local crippling is determined using AISC *Manual* Table 9-4.

$$\frac{l_b}{d} = \frac{10.0 \text{ in.}}{18.0 \text{ in.}}$$
$$= 0.556$$

Because $\dfrac{l_b}{d} > 0.2$, use AISC *Manual* Table 9-4 and AISC *Manual* Equation 9-49a or 9-49b, as follows:

LRFD	ASD
$\phi R_5 = 52.0 \text{ kips}$ $\phi R_6 = 6.30 \text{ kip/in.}$	$R_5/\Omega = 34.7 \text{ kips}$ $R_6/\Omega = 4.20 \text{ kip/in.}$

LRFD	ASD
$l_{b\,min} = \dfrac{R_u - \phi R_5}{\phi R_6} \geq k_{des}$ $= \dfrac{90.0 \text{ kips} - 52.0 \text{ kips}}{6.30 \text{ kip/in.}} > 0.972 \text{ in.}$ $= 6.03 \text{ in.} > 0.972 \text{ in.}$ Therefore: $l_{b\,min} = 6.03 \text{ in.} < 10.0 \text{ in.}$ **o.k.** Verify $\dfrac{l_b}{d} > 0.2$: $\dfrac{l_b}{d} = \dfrac{6.03 \text{ in.}}{18.0 \text{ in.}}$ $= 0.335 > 0.2$ **o.k.**	$l_{b\,min} = \dfrac{R_a - R_5/\Omega}{R_6/\Omega} \geq k_{des}$ $= \dfrac{60.0 \text{ kips} - 34.7 \text{ kips}}{4.20 \text{ kip/in.}} > 0.972 \text{ in.}$ $= 6.02 \text{ in.} > 0.972 \text{ in.}$ Therefore: $l_{b\,min} = 6.02 \text{ in.} < 10.0 \text{ in.}$ **o.k.** Verify $\dfrac{l_b}{d} > 0.2$: $\dfrac{l_b}{d} = \dfrac{6.02 \text{ in.}}{18.0 \text{ in.}}$ $= 0.334 > 0.2$ **o.k.**

The bearing strength of the concrete is determined from AISC *Specification* Section J8. Note that AISC *Specification* Equation J8-1 is used because A_2 is not larger than A_1 in this case.

$$\begin{aligned} A_1 &= b_f l_b \\ &= (7.50 \text{ in.})(10.0 \text{ in.}) \\ &= 75.0 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} P_p &= 0.85 f_c' A_1 \\ &= 0.85(3 \text{ ksi})(75.0 \text{ in.}^2) \\ &= 191 \text{ kips} \end{aligned}$$ (*Spec*. Eq. J8-1)

LRFD	ASD
$\phi_c = 0.65$ $\phi_c P_p = 0.65(191 \text{ kips})$ $= 124 \text{ kips} > 90.0 \text{ kips}$ **o.k.**	$\Omega_c = 2.31$ $\dfrac{P_p}{\Omega_c} = \dfrac{191 \text{ kips}}{2.31}$ $= 82.7 \text{ kips} > 60.0 \text{ kips}$ **o.k.**

Beam Flange Thickness

Using the cantilever length from AISC *Manual* Part 14, determine the minimum beam flange thickness required if no bearing plate is provided. The beam flanges along the length, n, are assumed to be fixed end cantilevers with a minimum thickness determined using the limit state of flexural yielding.

$$\begin{aligned} n &= \frac{b_f}{2} - k_{des} \\ &= \frac{7.50 \text{ in.}}{2} - 0.972 \text{ in.} \\ &= 2.78 \text{ in.} \end{aligned}$$ (from *Manual* Eq. 14-1)

LRFD	ASD
The bearing pressure is determined as follows: $f_p = \dfrac{R_u}{A_1}$ The required flexural strength of the flange is: $M_u = \dfrac{f_p n^2}{2} = \dfrac{R_u n^2}{2A_1}$ The available flexural strength of the flange is: $\phi = 0.90$ $\phi M_n = \phi F_y Z = \phi F_y \left(\dfrac{t_f^{\,2}}{4}\right)$ For $\phi R_n = R_u$ and solving for t_f, the minimum flange thickness is determined as follows: $t_{f\,min} = \sqrt{\dfrac{2R_u n^2}{\phi A_1 F_y}} = \sqrt{\dfrac{2(90.0\text{ kips})(2.78\text{ in.})^2}{0.90(75.0\text{ in.}^2)(50\text{ ksi})}} = 0.642\text{ in.} > t_f = 0.570\text{ in.}$ **n.g.** Therefore, a bearing plate is required.	The bearing pressure is determined as follows: $f_p = \dfrac{R_a}{A_1}$ The required flexural strength of the flange is: $M_a = \dfrac{f_p n^2}{2} = \dfrac{R_a n^2}{2A_1}$ The available flexural strength of the flange is: $\Omega = 1.67$ $\dfrac{M_n}{\Omega} = \dfrac{F_y Z}{\Omega} = \dfrac{F_y}{\Omega}\left(\dfrac{t_f^{\,2}}{4}\right)$ For $R_n/\Omega = R_a$ and solving for t_f, the minimum flange thickness is determined as follows: $t_{f\,min} = \sqrt{\dfrac{\Omega 2R_a n^2}{A_1 F_y}} = \sqrt{\dfrac{1.67(2)(60.0\text{ kips})(2.78\text{ in.})^2}{(75.0\text{ in.}^2)(50\text{ ksi})}} = 0.643\text{ in.} > t_f = 0.570\text{ in.}$ **n.g.** Therefore, a bearing plate is required.

Note: The designer may assume a bearing width narrower than the beam flange to justify a thinner flange. In this case, the bearing width is constrained by the lower bound concrete bearing strength and the upper bound 0.570-in. flange thickness.

$$5.43\text{ in.} \le \text{bearing width} \le 6.56\text{ in.}$$

Solution B:

Bearing Length

From Solution A, with l_b = 10 in., the web local yielding and web local crippling strengths for the beam are adequate.

Bearing Plate Design

The required bearing plate width is determined using AISC *Specification* Equation J8-1 as follows:

LRFD	ASD
$\phi_c = 0.65$ $A_{1\,req} = \dfrac{R_u}{\phi_c 0.85 f_c'}$ $= \dfrac{90.0 \text{ kips}}{0.65(0.85)(3 \text{ ksi})}$ $= 54.3 \text{ in.}^2$ $B_{req} = \dfrac{A_{1\,req}}{l_b}$ $= \dfrac{54.3 \text{ in.}^2}{10.0 \text{ in.}}$ $= 5.43 \text{ in.}$ Use B = 8 in. (selected as the least whole-inch dimension that exceeds b_f).	$\Omega_c = 2.31$ $A_{1\,req} = \dfrac{R_a \Omega_c}{0.85 f_c'}$ $= \dfrac{(60.0 \text{ kips})(2.31)}{0.85(3 \text{ ksi})}$ $= 54.4 \text{ in.}^2$ $B_{req} = \dfrac{A_{1\,req}}{l_b}$ $= \dfrac{54.4 \text{ in.}^2}{10.0 \text{ in.}}$ $= 5.44 \text{ in.}$ Use B = 8 in. (selected as the least whole-inch dimension that exceeds b_f).

From AISC *Manual* Part 14, the bearing plate cantilever dimension is determined as follows:

$$n = \frac{B}{2} - k_{des} \qquad \text{(\textit{Manual} Eq. 14-1)}$$
$$= \frac{8 \text{ in.}}{2} - 0.972 \text{ in.}$$
$$= 3.03 \text{ in.}$$

The required thickness of the base plate is determined using the available flexural strength equation previously derived for the required beam flange thickness.

LRFD	ASD
$t_{min} = \sqrt{\dfrac{2R_u n^2}{\phi F_y B l_b}}$ $= \sqrt{\dfrac{2(90.0 \text{ kips})(3.03 \text{ in.})^2}{0.90(36 \text{ ksi})(8 \text{ in.})(10 \text{ in.})}}$ $= 0.798 \text{ in.}$ Use PL⅞ in.×10 in.×0 ft 8 in.	$t_{min} = \sqrt{\dfrac{\Omega 2 R_a n^2}{F_y B l_b}}$ $= \sqrt{\dfrac{1.67(2)(60.0 \text{ kips})(3.03 \text{ in.})^2}{(36 \text{ ksi})(8 \text{ in.})(10 \text{ in.})}}$ $= 0.799 \text{ in.}$ Use PL⅞ in.×10 in.×0 ft 8 in.

Note: The calculations for t_{min} are conservative. Taking the strength of the beam flange into consideration results in a thinner required bearing plate or no bearing plate at all.

Solution C:

From Solution A, with l_b = 6½ in., the web local yielding and web local crippling strengths for the beam are adequate.

Bearing Plate Design

Try B = 8 in.

$$A_1 = Bl_b$$
$$= (8 \text{ in.})(6\tfrac{1}{2} \text{ in.})$$
$$= 52.0 \text{ in.}^2$$

AISC *Specification* Section J8 requires that the area, A_2, be geometrically similar to A_1.

$$N_1 = 6\tfrac{1}{2} \text{ in.} + 2(1.75 \text{ in.})$$
$$= 10.0 \text{ in.}$$

$$B_1 = 8 \text{ in.} + 2(1.75 \text{ in.})$$
$$= 11.5 \text{ in.}$$

$$A_2 = B_1 N_1$$
$$= (11.5 \text{ in.})(10.0 \text{ in.})$$
$$= 115 \text{ in.}^2$$

The bearing strength of the concrete is determined from AISC *Specification* Section J8. Note that AISC *Specification* Equation J8-2 is used because A_2 is larger than A_1 in this case.

$$P_p = 0.85 f_c' A_1 \sqrt{A_2 / A_1} \leq 1.7 f_c' A_1 \qquad \text{(}\textit{Spec.}\text{ Eq. J8-2)}$$
$$= 0.85(3 \text{ ksi})(52.0 \text{ in.}^2)\sqrt{115 \text{ in.}^2 / 52.0 \text{ in.}^2} \leq 1.7(3 \text{ ksi})(52.0 \text{ in.}^2)$$
$$= 197 \text{ kips} < 265 \text{ kips}$$

Therefore:

$$P_p = 197 \text{ kips}$$

LRFD	ASD
$\phi_c = 0.65$ $\phi_c P_p = 0.65(197 \text{ kips})$ $= 128 \text{ kips} > 90.0 \text{ kips}$ **o.k.**	$\Omega_c = 2.31$ $\dfrac{P_p}{\Omega_c} = \dfrac{197 \text{ kips}}{2.31}$ $= 85.3 \text{ kips} > 60.0 \text{ kips}$ **o.k.**

From AISC *Manual* Part 14, the bearing plate cantilever dimension is determined as follows:

$$n = \frac{B}{2} - k_{des} \qquad \text{(}\textit{Manual}\text{ Eq. 14-1)}$$
$$= \frac{8 \text{ in.}}{2} - 0.972 \text{ in.}$$
$$= 3.03 \text{ in.}$$

The required thickness of the base plate is determined using the available flexural strength equation previously derived for the required beam flange thickness.

LRFD	ASD
$t_{min} = \sqrt{\dfrac{2R_u n^2}{\phi F_y B l_b}}$ $= \sqrt{\dfrac{2(90.0 \text{ kips})(3.03 \text{ in.})^2}{0.90(36 \text{ ksi})(8 \text{ in.})(6\frac{1}{2} \text{ in.})}}$ $= 0.990$ in. Use PL1 in.×6½ in.×0 ft 8 in.	$t_{min} = \sqrt{\dfrac{\Omega 2R_a n^2}{F_y B l_b}}$ $= \sqrt{\dfrac{1.67(2)(60.0 \text{ kips})(3.03 \text{ in.})^2}{(36 \text{ ksi})(8 \text{ in.})(6\frac{1}{2} \text{ in.})}}$ $= 0.991$ in. Use PL1 in.×6½ in.×0 ft 8 in

Note: The calculations for t_{min} are conservative. Taking the strength of the beam flange into consideration results in a thinner required bearing plate or no bearing plate at all.

EXAMPLE II.D-3 SLIP-CRITICAL CONNECTION WITH OVERSIZED HOLES

Given:

Verify the connection of an ASTM A36 2L3×3×5⁄16 tension member to an ASTM A36 plate welded to an ASTM A992 beam, as shown in Figure II.D-3-1, for the following loads:

P_D = 15 kips
P_L = 45 kips

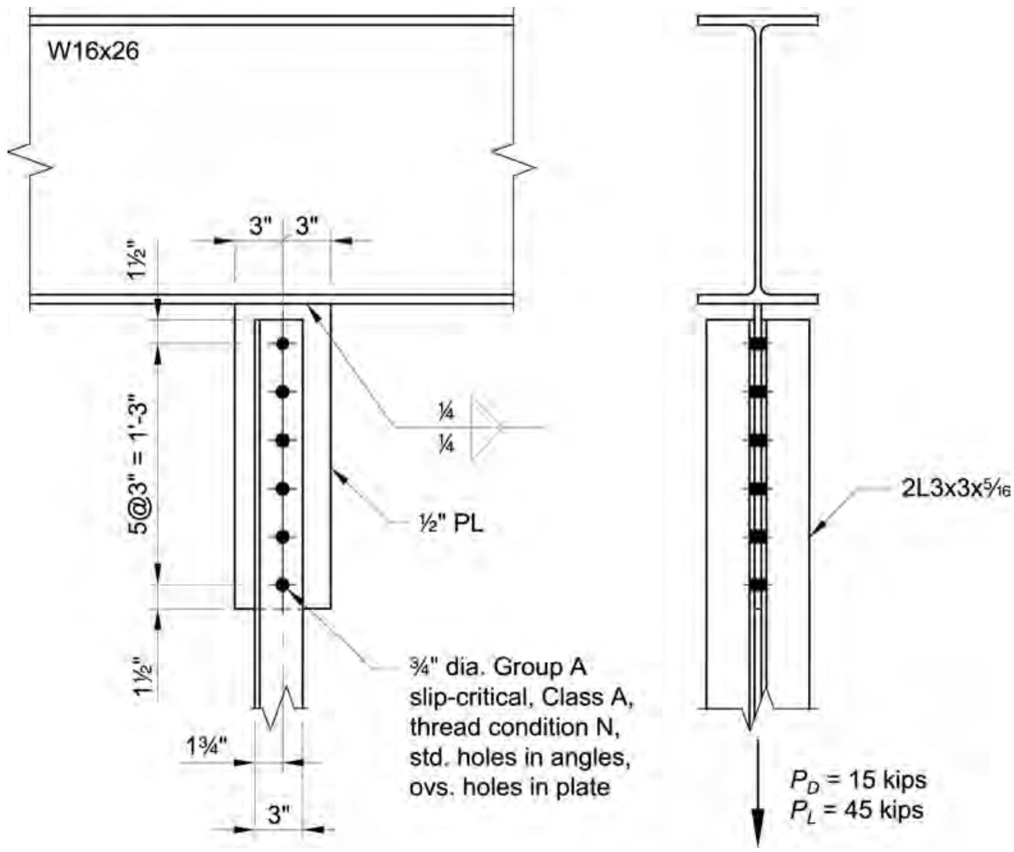


Fig. II.D-3-1. Connection configuration for Example II.D-3.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Hanger and plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1, 1-7 and 1-15, the geometric properties are as follows:

Beam
W16×26
t_f = 0.345 in.
t_w = 0.250 in.
k_{des} = 0.747 in.

Hanger
2L3×3×5⁄16
A = 3.56 in.^{2}
$\bar{x} = 0.860$ in. for single angle

From AISC *Specification* Table J3.3, the hole diameter for ¾-in.-diameter bolts with standard and oversized holes is:

d_h = 13⁄16 in. (standard hole)
d_h = 15⁄16 in. (oversized hole)

From ASCE/SEI 7, Chapter 2, the required strength is:

LRFD	ASD
$R_u = 1.2(15 \text{ kips}) + 1.6(45 \text{ kips})$ $= 90.0 \text{ kips}$	$R_a = 15 \text{ kips} + 45 \text{ kips}$ $= 60.0 \text{ kips}$

Bolt Slip Resistance Strength

From AISC *Manual* Table 7-3, with ¾-in.-diameter Group A slip-critical bolts with Class A faying surfaces in oversized holes and double shear, the available slip resistance strength is:

LRFD	ASD
$\phi r_n = 16.1 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = 10.8 \text{ kips/bolt}$

The required number of bolts is determined as follows:

LRFD	ASD
$n = \dfrac{R_u}{\phi r_n}$ $= \dfrac{90.0 \text{ kips}}{16.1 \text{ kips/bolt}}$ $= 5.59$ Therefore, use 6 bolts.	$n = \dfrac{R_a}{(r_n / \Omega)}$ $= \dfrac{60.0 \text{ kips}}{10.8 \text{ kips/bolt}}$ $= 5.56$ Therefore, use 6 bolts.

Strength of Bolted Connection—Angles

Slip-critical connections must also be designed for the limit states of bearing-type connections. From the Commentary to AISC *Specification* Section J3.6, the strength of the bolt group is taken as the sum of the individual strengths of the individual fasteners, which may be taken as the lesser of the fastener shear strength per AISC *Specification* Section J3.6, the bearing strength at the bolt hole per AISC *Specification* Section J3.10, or the tearout strength at the bolt hole per AISC *Specification* Section J3.10.

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear is:

LRFD	ASD
$\phi r_n = 35.8 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = 23.9 \text{ kips/bolt}$

The available bearing and tearout strength of the angles using standard holes at the edge bolt is determined using AISC *Manual* Table 7-5, conservatively using l_e = 1¼ in.

LRFD	ASD
$\phi r_n = (2 \text{ angles})(44.0 \text{ kip/in.})(\tfrac{5}{16} \text{ in.})$ $= 27.5 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (2 \text{ angles})(29.4 \text{ kip/in.})(\tfrac{5}{16} \text{ in.})$ $= 18.4 \text{ kips/bolt}$

Therefore, the bearing or tearout strength controls over bolt shear at the edge bolts.

The available bearing and tearout strength of the angles using standard holes at the other bolts is determined using AISC *Manual* Table 7-4 with s = 3 in.

LRFD	ASD
$\phi r_n = (2 \text{ angles})(78.3 \text{ kip/in.})(\tfrac{5}{16} \text{ in.})$ $= 48.9 \text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (2 \text{ angles})(52.2 \text{ kip/in.})(\tfrac{5}{16} \text{ in.})$ $= 32.6 \text{ kips/bolt}$

Therefore, bolt shear controls over bearing or tearout at the other bolts.

The strength of the bolt group in the angles is determined by summing the strength of the individual fasteners as follows:

LRFD	ASD
$\phi R_n = (1 \text{ bolt})(27.5 \text{ kips/bolt})$ $+ (5 \text{ bolts})(35.8 \text{ kips/bolt})$ $= 207 \text{ kips} > 90.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (1 \text{ bolt})(18.4 \text{ kips/bolt})$ $+ (5 \text{ bolts})(23.9 \text{ kips/bolt})$ $= 138 \text{ kips} > 60.0 \text{ kips}$ **o.k.**

Tensile Strength of the Angles

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the angles is determined as follows:

$$P_n = F_y A_g \qquad \text{(}Spec\text{. Eq. J4-1)}$$

$$= (36 \text{ ksi})(3.56 \text{ in.}^2)$$

$$= 128 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi P_n = 0.90(128 \text{ kips})$ $= 115 \text{ kips} > 90.0 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{P_n}{\Omega} = \dfrac{128 \text{ kips}}{1.67}$ $= 76.6 \text{ kips} > 60.0 \text{ kips}$ **o.k.**

From AISC *Specification* Section J4.1(b), the available tensile rupture strength of the angles is determined as follows. The shear lag factor, U, is determined using AISC *Specification* Table D3.1, Case 2.

$$U = 1 - \frac{\bar{x}}{l}$$
$$= 1 - \frac{0.860 \text{ in.}}{15.0 \text{ in.}}$$
$$= 0.943$$

$$A_e = A_n U \qquad \text{(}Spec\text{. Eq. D3-1)}$$
$$= \left[A_g - (2 \text{ angles})(d_h + \tfrac{1}{16} \text{ in.})t\right]U$$
$$= \left[3.56 \text{ in.}^2 - (2 \text{ angles})(\tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{5}{16} \text{ in.})\right](0.943)$$
$$= 2.84 \text{ in.}^2$$

$$P_n = F_u A_e \qquad \text{(}Spec\text{. Eq. J4-2)}$$
$$= (58 \text{ ksi})(2.84 \text{ in.}^2)$$
$$= 165 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi P_n = 0.75(165 \text{ kips})$ $= 124 \text{ kips} > 90.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{P_n}{\Omega} = \dfrac{165 \text{ kips}}{2.00}$ $= 82.5 \text{ kips} > 60.0 \text{ kips}$ **o.k.**

Block Shear Rupture Strength of the Angles

The available strength for the limit state of block shear rupture of the angles is determined as follows:

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad \text{(}Spec\text{. Eq. J4-5)}$$

where

$$A_{gv} = (2 \text{ angles})\left[l_{ev} + (n-1)s\right]t$$
$$= (2 \text{ angles})\left[1\tfrac{1}{2} \text{ in.} + (6-1)(3 \text{ in.})\right](\tfrac{5}{16} \text{ in.})$$
$$= 10.3 \text{ in.}^2$$

$$A_{nv} = A_{gv} - (2 \text{ angles})(n - 0.5)(d_h + \tfrac{1}{16} \text{ in.})t$$
$$= 10.3 \text{ in.}^2 - (2 \text{ angles})(6 - 0.5)(\tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{5}{16} \text{ in.})$$
$$= 7.29 \text{ in.}^2$$

$$A_{nt} = (2\text{ angles})\left[l_{eh} - 0.5\left(d_h + \tfrac{1}{16}\text{ in.}\right)\right]t$$
$$= (2\text{ angles})\left[1\tfrac{1}{4}\text{ in.} - 0.5\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]\left(\tfrac{5}{16}\text{ in.}\right)$$
$$= 0.508\text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$R_n = 0.60(58\text{ ksi})\left(7.29\text{ in.}^2\right) + 1.0(58\text{ ksi})\left(0.508\text{ in.}^2\right) \le 0.60(36\text{ ksi})\left(10.3\text{ in.}^2\right) + 1.0(58\text{ ksi})\left(0.508\text{ in.}^2\right)$$
$$= 283\text{ kips} > 252\text{ kips}$$

Therefore:

$$R_n = 252\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(252\text{ kips})$ $= 189\text{ kips} > 90.0\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{252\text{ kips}}{2.00}$ $= 126\text{ kips} > 60.0\text{ kips}$ **o.k.**

Strength of Bolted Connection—Plate

From AISC *Manual* Table 7-1, the available shear strength per bolt for ¾-in.-diameter Group A bolts with threads not excluded from the shear plane (thread condition N) in double shear is:

LRFD	ASD
$\phi r_n = 35.8\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = 23.9\text{ kips/bolt}$

The available bearing and tearout strength of the plate using oversized holes at the edge bolt is determined using AISC *Manual* Table 7-5, conservatively using $l_e = 1\tfrac{1}{4}$ in.

LRFD	ASD
$\phi r_n = (40.8\text{ kip/in.})\left(\tfrac{1}{2}\text{ in.}\right)$ $= 20.4\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (27.2\text{ kip/in.})\left(\tfrac{1}{2}\text{ in.}\right)$ $= 13.6\text{ kips/bolt}$

Therefore, the bearing or tearout strength controls over bolt shear at the edge bolts.

The available bearing and tearout strength of the plate using oversized holes at the other bolts is determined using AISC *Manual* Table 7-4 with $s = 3$ in.

LRFD	ASD
$\phi r_n = (78.3\text{ kip/in.})\left(\tfrac{1}{2}\text{ in.}\right)$ $= 39.2\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = (52.2\text{ kip/in.})\left(\tfrac{1}{2}\text{ in.}\right)$ $= 26.1\text{ kips/bolt}$

Therefore, bolt shear controls over bearing or tearout at the other bolts.

The strength of the bolt group in the plate is determined by summing the strength of the individual fasteners as follows:

LRFD	ASD
$\phi R_n = (1\text{ bolt})(20.4\text{ kips/bolt})$ $+(5\text{ bolts})(35.8\text{ kips/bolt})$ $= 199\text{ kips} > 90.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (1\text{ bolt})(13.6\text{ kips/bolt})$ $+(5\text{ bolts})(23.9\text{ kips/bolt})$ $= 133\text{ kips} > 60.0\text{ kips}$ **o.k.**

Tensile Strength of the Plate

From AISC *Specification* Section J4.1(a), the available tensile yielding strength of the plate is determined as follows. By inspection, the Whitmore section, as defined in AISC *Manual* Figure 9-1, includes the entire width of the ½-in. plate.

$$\begin{aligned} A_g &= bt \\ &= (6\text{ in.})(\tfrac{1}{2}\text{ in.}) \\ &= 3.00\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_n &= F_y A_g \\ &= (36\text{ ksi})(3.00\text{ in.}^2) \\ &= 108\text{ kips} \end{aligned}$$ (*Spec*. Eq. J4-1)

LRFD	ASD
$\phi = 0.90$	$\Omega = 1.67$
$\phi R_n = 0.90(108\text{ kips})$ $= 97.2\text{ kips} > 90.0\text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega} = \dfrac{108\text{ kips}}{1.67}$ $= 64.7\text{ kips} > 60.0\text{ kips}$ **o.k.**

From AISC *Specification* Section J4.1(b), the available tensile rupture strength of the plate is determined as follows:

$$\begin{aligned} A_n &= A_g - (d_h + \tfrac{1}{16}\text{ in.})t \\ &= 3.00\text{ in.}^2 - (\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{1}{2}\text{ in.}) \\ &= 2.50\text{ in.}^2 \end{aligned}$$

AISC *Specification* Table D3.1, Case 1, applies in this case because tension load is transmitted directly to the cross-sectional element by fasteners; therefore, $U = 1.0$.

$$\begin{aligned} A_e &= A_n U \\ &= (2.50\text{ in.}^2)(1.0) \\ &= 2.50\text{ in.}^2 \end{aligned}$$ (*Spec*. Eq. D3-1)

$$R_n = F_u A_e \quad \text{(Spec. Eq. J4-2)}$$
$$= (58 \text{ ksi})(2.50 \text{ in.}^2)$$
$$= 145 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(145 \text{ kips})$ $= 109 \text{ kips} > 90.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{P_n}{\Omega} = \dfrac{145 \text{ kips}}{2.00}$ $= 72.5 \text{ kips} > 60.0 \text{ kips}$ **o.k.**

Block Shear Rupture Strength of the Plate

The available strength for the limit state of block shear rupture of the plate is determined as follows.

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \quad \text{(Spec. Eq. J4-5)}$$

where

$$A_{gv} = \left[l_{ev} + (n-1)s\right]t$$
$$= \left[1\tfrac{1}{2} \text{ in.} + (6-1)(3 \text{ in.})\right](\tfrac{1}{2} \text{ in.})$$
$$= 8.25 \text{ in.}^2$$

$$A_{nv} = A_{gv} - (n-0.5)(d_h + \tfrac{1}{16} \text{ in.})t$$
$$= 8.25 \text{ in.}^2 - (6-0.5)(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{1}{2} \text{ in.})$$
$$= 5.50 \text{ in.}^2$$

$$A_{nt} = \left[l_{eh} - 0.5(d_h + \tfrac{1}{16} \text{ in.})\right]t$$
$$= \left[3 \text{ in.} - 0.5(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})\right](\tfrac{1}{2} \text{ in.})$$
$$= 1.25 \text{ in.}^2$$

$$U_{bs} = 1.0$$

and

$$R_n = 0.60(58 \text{ ksi})(5.50 \text{ in.}^2) + 1.0(58 \text{ ksi})(1.25 \text{ in.}^2) \le 0.60(36 \text{ ksi})(8.25 \text{ in.}^2) + 1.0(58 \text{ ksi})(1.25 \text{ in.}^2)$$
$$= 264 \text{ kips} > 251 \text{ kips}$$

Therefore:

$$R_n = 251 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(251 \text{ kips})$ $= 188 \text{ kips} > 90.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{251 \text{ kips}}{2.00}$ $= 126 \text{ kips} > 60.0 \text{ kips}$ **o.k.**

Plate-to-Beam Weld

The applied load is perpendicular to the weld length (θ = 90°), therefore the directional strength factor is determined from AISC *Specification* Equation J2-5. This increase factor due to directional strength is incorporated into the weld strength calculation.

$$1.0 + 0.50\sin^{1.5}\theta = 1.0 + 0.50\sin^{1.5}(90°)$$
$$= 1.50$$

The required fillet weld size is determined using AISC *Manual* Equation 8-2a or 8-2b, as follows:

LRFD	ASD
$D_{req} = \dfrac{P_u}{(2 \text{ welds})(1.50)(1.392 \text{ kip/in.})l}$ $= \dfrac{90.0 \text{ kips}}{(2 \text{ welds})(1.50)(1.392 \text{ kip/in.})(6 \text{ in.})}$ $= 3.59$ Use ¼-in. fillet welds on each side of the plate.	$D_{req} = \dfrac{P_a}{(2 \text{ welds})(1.50)(0.928 \text{ kip/in.})l}$ $= \dfrac{60.0 \text{ kips}}{(2 \text{ welds})(1.50)(0.928 \text{ kip/in.})(6 \text{ in.})}$ $= 3.59$ Use ¼-in. fillet welds on each side of the plate.

From AISC *Manual* Table J2.4, the minimum fillet weld size is:

$$w_{min} = 3/16 \text{ in.} < 1/4 \text{ in.} \quad \textbf{o.k.}$$

Beam Flange Base Metal Check

The minimum flange thickness to match the required shear rupture strength of the welds is determined as follows:

$$t_{min} = \frac{3.09D}{F_u} \qquad (\textit{Manual} \text{ Eq. 9-2})$$
$$= \frac{3.09(3.59)}{65 \text{ ksi}}$$
$$= 0.171 \text{ in.} < 0.345 \text{ in.} \quad \textbf{o.k.}$$

Beam Concentrated Forces Check

From AISC *Specification* Section J10.2, the beam web is checked for the limit state of web local yielding assuming the connection is at a distance from the member end greater than the depth of the member, *d*.

$$R_n = F_{yw}t_w(5k_{des} + l_b) \qquad (\textit{Spec}. \text{ Eq. J10-2})$$
$$= (50 \text{ ksi})(0.250 \text{ in.})[5(0.747 \text{ in.}) + 6 \text{ in.}]$$
$$= 122 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(122 \text{ kips})$ $= 122 \text{ kips} > 90.0 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{122 \text{ kips}}{1.50}$ $= 81.3 \text{ kips} > 60.0 \text{ kips}$ **o.k.**

Conclusion

The connection is found to be adequate as given for the applied loads.

Part III
System Design Examples

EXAMPLE III-1 DESIGN OF SELECTED MEMBERS AND LATERAL ANALYSIS OF A FOUR-STORY BUILDING

INTRODUCTION

This section illustrates the load determination and selection of representative members that are part of the gravity and lateral frame of a typical four-story building. The design is completed in accordance with the AISC *Specification* and AISC *Manual*. Loading criteria are based on ASCE/SEI 7.

This section includes:

- Analysis and design of a typical steel frame for gravity loads
- Analysis and design of a typical steel frame for lateral loads
- Examples illustrating three methods for satisfying the stability provisions of AISC *Specification* Chapter C

The building being analyzed in this design example is located in a Midwestern city with moderate wind and seismic loads. The loads are given in the description of the design example. All members are ASTM A992 material.

CONVENTIONS

The following conventions are used throughout this example:

1. Beams or columns that have similar, but not necessarily identical, loads are grouped together. This is done because such grouping is generally a more economical practice for design, fabrication and erection.

2. Certain calculations, such as design loads for snow drift, which might typically be determined using a spreadsheet or structural analysis program, are summarized and then incorporated into the analysis. This simplifying feature allows the design example to illustrate concepts relevant to the member selection process.

3. Two commonly used deflection calculations, for uniform loads, have been rearranged so that the conventional units in the problem can be directly inserted into the equation for design. They are as follows:

   Simple beam:

$$\Delta = \frac{5\,(w \text{ kip/in.})(L \text{ in.})^4}{384(29{,}000 \text{ ksi})(I \text{ in.}^4)}$$

$$= \frac{(w \text{ kip/ft})(L \text{ ft})^4}{1{,}290(I \text{ in.}^4)}$$

   Beam fixed at both ends:

$$\Delta = \frac{(w \text{ kip/in.})(L \text{ in.})^4}{384(29{,}000 \text{ ksi})(I \text{ in.}^4)}$$

$$= \frac{(w \text{ kip/ft})(L \text{ ft})^4}{6{,}440(I \text{ in.}^4)}$$

DESIGN SEQUENCE

The design sequence is presented as follows:

1. General description of the building including geometry, gravity loads and lateral loads
2. Roof member design and selection
3. Floor member design and selection
4. Column design and selection for gravity loads
5. Wind load determination
6. Seismic load determination
7. Horizontal force distribution to the lateral frames
8. Preliminary column selection for the moment frames and braced frames
9. Seismic load application to lateral systems
10. Stability (P-Δ) analysis

GENERAL DESCRIPTION OF THE BUILDING

Geometry

The design example is a four-story building, consisting of seven bays at 30 ft in the east-west (numbered grids) direction and bays of 45 ft, 30 ft and 45 ft in the north-south (lettered grids) direction, as shown in Figure III-1. The floor-to-floor height for the four floors is 13 ft 6 in. and the height from the fourth floor to the roof (at the edge of the building) is 14 ft 6 in. Based on discussions with fabricators, the same column size will be used for the whole height of the building.

The plans of these floors and the roof are shown on Sheets S2.1 thru S2.3, found at the end of this Chapter. The exterior of the building is a ribbon window system with brick spandrels supported and back-braced with steel and infilled with metal studs. The spandrel wall extends 2 ft above the elevation of the edge of the roof. The window and spandrel system is shown on design drawing Sheet S4.1.

The roof system is 1½-in. metal deck on open web steel joists. The open web steel joists are supported on steel beams as shown on Sheet S2.3. The roof slopes to interior drains. The middle three bays have a 6-ft-tall screen wall around them and house the mechanical equipment and the elevator over run. This area has steel beams, in place of open web steel joists, to support the mechanical equipment.

The three elevated floors have 3 in. of normal weight concrete over 3-in. composite deck for a total slab thickness of 6 in. The supporting beams are spaced at 10 ft on center. These beams are carried by composite girders in the east-west direction to the columns. There is a 30 ft by 29 ft opening in the second floor, to create a two-story atrium at the entrance. These floor layouts are shown on Sheets S2.1 and S2.2. The first floor is a slab on grade and the foundation consists of conventional spread footings.

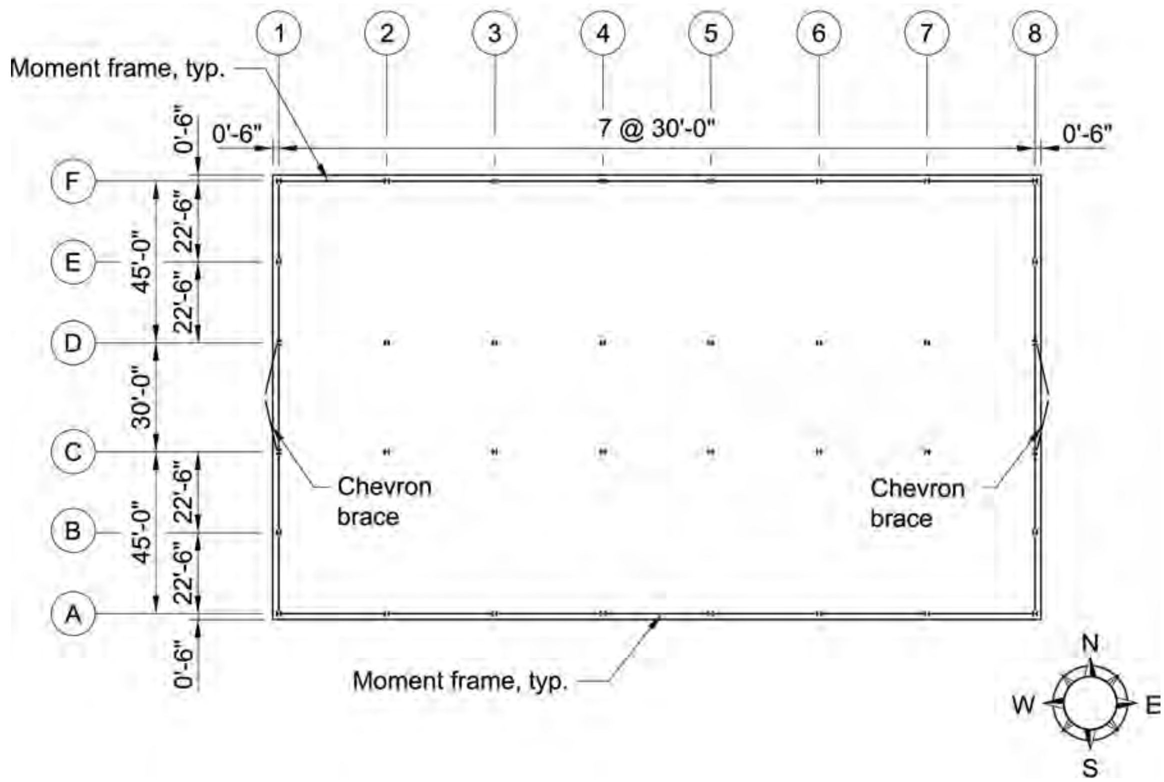


Fig. III-1. Basic building layout.

The building includes both moment frames and braced frames for lateral resistance. The lateral system in the north-south direction consists of chevron braces at the end of the building located adjacent to the stairways. In the east-west direction there are no locations in which chevron braces can be concealed; consequently, the lateral system in the east-west direction is composed of moment frames at the north and south faces of the building.

This building is sprinklered and has large open spaces around it, and consequently does not require fireproofing for the floors.

Wind Forces

The Basic Wind Speed is 107 miles per hour (3-second gust). Because it is sited in an open, rural area, it will be analyzed as Wind Exposure Category C. Because it is an ordinary office occupancy, the building is Risk Category II.

Seismic Forces

The sub-soil has been evaluated and the site class has been determined to be Site Class D. The area has a short period $S_s = 0.121g$ and a one-second period $S_1 = 0.060g$. The Seismic Importance Factor is 1.0, that of an ordinary office occupancy (Risk Category II).

Roof and Floor Loads

Roof Loads

The ground snow load, p_g, is 20 psf. The slope of the roof is ¼ in./ft or more at all locations, but not exceeding ½ in./ft; consequently, 5 psf rain-on-snow surcharge is to be considered, but ponding instability design calculations are beyond the scope of this example. This roof can be designed as a fully exposed roof, but, per ASCE/SEI 7, Section 7.3, cannot be designed for less than $p_f = (I)p_g = 20$ psf uniform snow load. Snow drift will be applied at the edges of the roof and at the screen wall around the mechanical area. The roof live load for this building is 20 psf, but may be reduced per ASCE/SEI 7, Section 4.8, where applicable.

Floor Loads

The basic live load for the floor is 50 psf. An additional partition live load of 20 psf is specified, which exceeds the minimum partition load required by ASCE/SEI 7, Section 4.3.2. Because the locations of partitions and, consequently, corridors are not known, and will be subject to change, the entire floor will be designed for a live load of 80 psf. This live load will be reduced based on type of member and area per the ASCE/SEI 7 provisions for live-load reduction.

Wall Loads

A wall load of 55 psf will be used for the brick spandrels, supporting steel, and metal stud back-up. A wall load of 15 psf will be used for the ribbon window glazing system.

ROOF MEMBER DESIGN AND SELECTION

Calculate dead load and snow load.

Dead load:

Roofing	= 5 psf
Insulation	= 2 psf
Deck	= 2 psf
Beams	= 3 psf
Joists	= 3 psf
Misc.	= 5 psf
Total	= 20 psf

Snow load from ASCE/SEI 7, Sections 7.3 and 7.10:

Snow	= 20 psf
Rain on snow	= 5 psf
Total	= 25 psf

Note: In this design, the rain and snow load is greater than the roof live load.

The deck is 1½ in., wide rib, 22 gage, painted roof deck, placed in a pattern of three continuous spans minimum. The typical joist spacing is 6 ft on center. At 6 ft on center, this deck has an allowable total load capacity of 87 psf (from the manufacturer's catalog). The roof diaphragm and roof loads extend 6 in. past the centerline of grid as shown on Sheet S4.1.

From ASCE/SEI 7, Section 7.7, the following drift loads are calculated:

Flat roof snow load: p_g = 20 psf
Density: γ = 16.6 lb/ft^3
h_b = 1.20 ft

Summary of Drifts

The snow drift at the penthouse was calculated for the maximum effect, using the east-west wind and an upwind fetch from the parapet to the centerline of the columns at the penthouse. This same drift is conservatively used for wind in the north-south direction. The precise location of the drift will depend upon the details of the penthouse construction, but will not affect the final design in this case. A summary of the drift load is given in Table III-1.

Table III-1
Summary of Drifts

	Upwind Roof Length, l_u, ft	Projection Height, ft	Max. Drift Load, psf	Max. Drift Width, W, ft
Side parapet	121	2	13.2	6.36
End parapet	211	2	13.2	6.36
Screen wall	60.5	6	30.5	7.35

SELECT ROOF JOISTS

Layout loads and size joists.

The 45-ft side joist with the heaviest loads is shown in Figure III-2 with end reactions and maximum moment.

Note: Joists may be specified using ASD or LRFD but are most commonly specified by ASD as shown here.

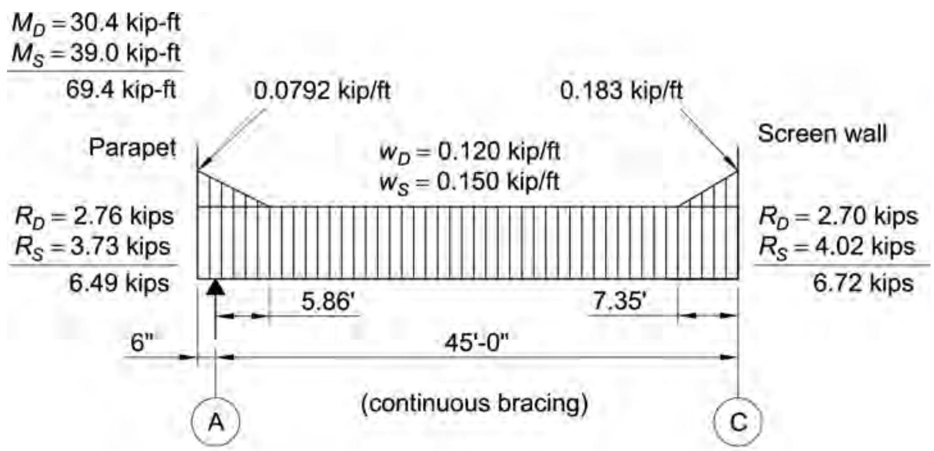


Fig III-2. Joist loading and bracing diagram—ASD.

Because the load is not uniform, select a 24KCS4 joist from the Steel Joist Institute (SJI) *Load Tables and Weight Tables for Steel Joists and Joist Girders* (SJI, 2015). This joist has an allowable moment of 92.3 kip-ft, an allowable shear of 8.40 kips, a gross moment of inertia of 453 in.^{4} and weighs 16.5 plf.

The first joist away from the end of the building is loaded with snow drift along the length of the member. Based on analysis, a 24KCS4 joist is also acceptable for this uniform load case.

As an alternative to directly specifying the joist sizes on the design document, as done in this example, loading diagrams can be included on the design documents to allow the joist manufacturer to economically design the joists.

The typical 30-ft-long joist in the middle bay will have a uniform load of:

$$\begin{aligned} w &= (6 \text{ ft})(20 \text{ psf} + 25 \text{ psf}) \\ &= 270 \text{ plf} \end{aligned}$$

$$\begin{aligned} w_S &= (6 \text{ ft})(25 \text{ psf}) \\ &= 150 \text{ plf} \end{aligned}$$

From the SJI load tables, select an 18K5 joist that weighs approximately 7.7 plf and satisfies both strength and deflection requirements.

Note: the first joist away from the screen wall and the first joist away from the end of the building carry snow drift. Based on analysis, an 18K7 joist will be used in these locations.

SELECT ROOF BEAMS

Calculate loads and select beams in the mechanical area.

For the beams in the mechanical area, the mechanical units could weigh as much as 60 psf. Use 40 psf additional dead load, which will account for the mechanical units and the screen wall around the mechanical area. Use 15 psf additional snow load, which will account for any snow drift that could occur in the mechanical area. The beams in the mechanical area are spaced at 6 ft on center. Loading is calculated as follows and shown in Figure III-3.

$$\begin{aligned} w_D &= (6 \text{ ft})\left(0.020 \text{ kip/ft}^2 + 0.040 \text{ kip/ft}^2\right) \\ &= 0.360 \text{ kip/ft} \end{aligned}$$

$$\begin{aligned} w_S &= (6 \text{ ft})\left(0.025 \text{ kip/ft}^2 + 0.015 \text{ kip/ft}^2\right) \\ &= 0.240 \text{ kip/ft} \end{aligned}$$

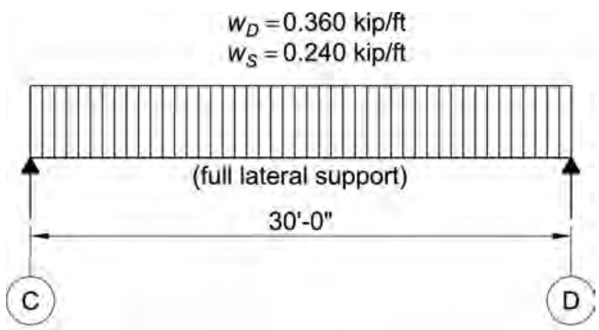


Fig. III-3. Loading and bracing diagram for roof beams in mechanical area.

From ASCE/SEI 7, Chapter 2, calculate the required strength of the beams in the mechanical area.

LRFD	ASD
$w_u = 1.2(0.360 \text{ kip/ft}) + 1.6(0.240 \text{ kip/ft}) = 0.816 \text{ kip/ft}$	$w_a = 0.360 \text{ kip/ft} + 0.240 \text{ kip/ft} = 0.600 \text{ kip/ft}$
$R_u = (0.816 \text{ kip/ft})\left(\frac{30 \text{ ft}}{2}\right) = 12.2 \text{ kips}$	$R_a = (0.600 \text{ kip/ft})\left(\frac{30 \text{ ft}}{2}\right) = 9.00 \text{ kips}$
$M_u = \frac{(0.816 \text{ kip/ft})(30 \text{ ft})^2}{8} = 91.8 \text{ kip-ft}$	$M_a = \frac{(0.600 \text{ kip/ft})(30 \text{ ft})^2}{8} = 67.5 \text{ kip-ft}$

As discussed in AISC Design Guide 3, *Serviceability Design Considerations for Steel Buildings* (West and Fisher, 2003), limit deflection to L/360 because a plaster ceiling will be used in the lobby area.

$$\begin{aligned} \frac{L}{360} &= \frac{(30 \text{ ft})(12 \text{ in./ft})}{360} \\ &= 1.00 \text{ in.} \end{aligned}$$

Using the equation for deflection derived previously, the required moment of inertia, $I_{x\ req}$, can be determined as follows. Use 40 psf as an estimate of the snow load, including some drifting that could occur in this area, for deflection calculations.

$$I_{x\ req} = \frac{(0.240\text{ kip/ft})(30\text{ ft})^4}{1{,}290(1.00\text{ in.})}$$
$$= 151\text{ in.}^4$$

From AISC *Manual* Table 3-3, select a beam size with an adequate moment of inertia. Try a W14×22:

$$I_x = 199\text{ in.}^4 > 151\text{ in.}^4 \quad \textbf{o.k.}$$

From AISC *Manual* Table 6-2, the available flexural strength and shear strength for a W14×22 is determined as follows. Assume the beam has full lateral support; therefore, $L_b = 0$.

LRFD	ASD
$\phi_b M_{nx} = 125\text{ kip-ft} > 91.8\text{ kip-ft}$ **o.k.**	$\dfrac{M_{nx}}{\Omega_b} = 82.8\text{ kip-ft} > 67.5\text{ kip-ft}$ **o.k.**
$\phi_v V_n = 94.5\text{ kips} > 12.2\text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega_v} = 63.0\text{ kips} > 9.00\text{ kips}$ **o.k.**

Note: The beams and supporting girders in this area should be rechecked when the final weights and locations for the mechanical units have been determined.

SELECT ROOF BEAMS AT THE END (EAST & WEST) OF THE BUILDING

The beams at the ends of the building carry the brick spandrel panel and a small portion of roof load. For these beams, the cladding weight exceeds 25% of the total dead load on the beam. Therefore, per AISC Design Guide 3, limit the vertical deflection due to cladding and initial dead load to L/600 or ⅜ in. maximum. In addition, because these beams are supporting brick above and there is continuous glass below, limit the superimposed dead and live load deflection to L/600 or 0.3 in. maximum to accommodate the brick and L/360 or ¼ in. maximum to accommodate the glass. Therefore, combining the two limitations, limit the superimposed dead and live load deflection to L/600 or ¼ in. The superimposed dead load includes all of the dead load that is applied after the cladding has been installed. In calculating the wall loads, the spandrel panel weight is taken as 55 psf. Beam loading is calculated as follows and shown in Figure III-4. Note, the beams are laterally supported by the deck as shown in Detail 4 on Sheet S4.1.

The dead load from the spandrel is:

$$\begin{aligned} w_D &= (7.50 \text{ ft})(0.055 \text{ kip/ft}^2) \\ &= 0.413 \text{ kip/ft} \end{aligned}$$

The dead load from the roof is equal to:

$$\begin{aligned} w_D &= (3.50 \text{ ft})(0.020 \text{ kip/ft}^2) \\ &= 0.070 \text{ kip/ft} \end{aligned}$$

Use 8 psf for the initial dead load, which includes the deck, beams and joists:

$$\begin{aligned} w_{D(initial)} &= (3.50 \text{ ft})(0.008 \text{ kip/ft}^2) \\ &= 0.028 \text{ kip/ft} \end{aligned}$$

Use 12 psf for the superimposed dead load:

$$\begin{aligned} w_{D(super)} &= (3.50 \text{ ft})(0.012 \text{ kip/ft}^2) \\ &= 0.042 \text{ kip/ft} \end{aligned}$$

The snow load from the roof conservatively uses the maximum snow drift as a uniform load, considering both side and end parapet drift pressures:

$$\begin{aligned} w_S &= (3.50 \text{ ft})(0.025 \text{ kip/ft}^2 + 0.0132 \text{ kip/ft}^2) \\ &= 0.134 \text{ kip/ft} \end{aligned}$$

From ASCE/SEI 7, Chapter 2, calculate the required strength of the beams at the east and west ends of the roof.

LRFD	ASD
$w_u = 1.2(0.483 \text{ kip/ft}) + 1.6(0.134 \text{ kip/ft})$ $= 0.794 \text{ kip/ft}$	$w_a = 0.483 \text{ kip/ft} + 0.134 \text{ kip/ft}$ $= 0.617 \text{ kip/ft}$

LRFD	ASD
$R_u = (0.794 \text{ kip/ft})\left(\dfrac{22.5 \text{ ft}}{2}\right)$ $= 8.93 \text{ kips}$	$R_a = (0.617 \text{ kip/ft})\left(\dfrac{22.5 \text{ ft}}{2}\right)$ $= 6.94 \text{ kips}$
$M_u = \dfrac{(0.794 \text{ kip/ft})(22.5 \text{ ft})^2}{8}$ $= 50.2 \text{ kip-ft}$	$M_a = \dfrac{(0.617 \text{ kip/ft})(22.5 \text{ ft})^2}{8}$ $= 39.0 \text{ kip-ft}$

Assume the beams are simple spans of 22.5 ft. Calculate the minimum moment of inertia to limit the superimposed dead and live load deflection after cladding is installed to L/600 or ¼ in.

$$\frac{L}{600} = \frac{(22.5 \text{ ft})(12 \text{ in./ft})}{600} \le \text{¼ in.}$$
$$= 0.450 \text{ in.} > \text{¼ in.}$$

Therefore, limit deflection to ¼ in. Using the equation for deflection derived previously, the required moment of inertia, $I_{x\,req}$, can be determined as follows:

$$I_{x\,req} = \frac{(0.042 \text{ kip/ft} + 0.134 \text{ kip/ft})(22.5 \text{ ft})^4}{1{,}290(\text{¼ in.})}$$
$$= 140 \text{ in.}^4$$

Calculate minimum moment of inertia to limit the cladding and initial dead load deflection to L/600 or ⅜ in.

$$\frac{L}{600} = \frac{(22.5 \text{ ft})(12 \text{ in./ft})}{600} \le \text{⅜ in.}$$
$$= 0.450 \text{ in.} > \text{⅜ in.}$$

Therefore, limit deflection to ⅜ in. Using the equation for deflection derived previously, the required moment of inertia, $I_{x\,req}$, can be determined as follows:

$$I_{x\,req} = \frac{(0.413 \text{ kip/ft} + 0.028 \text{ kip/ft})(22.5 \text{ ft})^4}{1{,}290(\text{⅜ in.})}$$
$$= 234 \text{ in.}^4 \quad \textbf{controls}$$

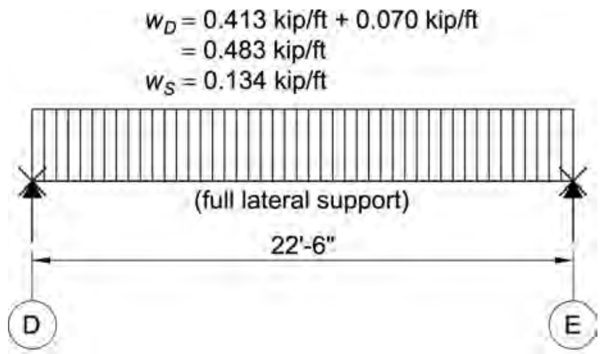


Fig. III-4. Beam loading and bracing diagram for roof beams at east and west ends of building.

From AISC *Manual* Table 3-3, select a beam size with an adequate moment of inertia. Try a W16×26:

$$I_x = 301\text{ in.}^4 > 234\text{ in.}^4 \quad \textbf{o.k.}$$

From AISC *Manual* Table 6-2, the available flexural strength and shear strength for a W16×26 is determined as follows. The beam has full lateral support; therefore, $L_b = 0$.

LRFD	ASD
$\phi_b M_{nx} = 166\text{ kip-ft} > 50.2\text{ kip-ft} \quad \textbf{o.k.}$	$\dfrac{M_{nx}}{\Omega_b} = 110\text{ kip-ft} > 39.0\text{ kip-ft} \quad \textbf{o.k.}$
$\phi_v V_n = 106\text{ kips} > 8.93\text{ kips} \quad \textbf{o.k.}$	$\dfrac{V_n}{\Omega_v} = 70.5\text{ kips} > 6.94\text{ kips} \quad \textbf{o.k.}$

SELECT ROOF BEAMS ALONG THE SIDE (NORTH & SOUTH) OF THE BUILDING

The beams along the side of the building carry the spandrel panel and a substantial roof dead load and live load. For these beams, the cladding weight exceeds 25% of the total dead load on the beam. From AISC Design Guide 3, limit the vertical deflection due to cladding and initial dead load to L/600 or ⅜ in. maximum. In addition, because these beams are supporting brick above and there is continuous glass below, limit the superimposed dead and live load deflection to L/600 or 0.3 in. maximum to accommodate the brick and L/360 or ¼ in. maximum to accommodate the glass. Therefore, combining the two limitations, limit the superimposed dead and live load deflection to L/600 or ¼ in. The superimposed dead load includes all of the dead load that is applied after the cladding has been installed. These beams will be part of the moment frames on the side of the building and therefore will be designed as fixed at both ends. The roof dead load and snow load on this edge beam is equal to the joist end dead load and snow load reaction. Treat this as a uniform load and divide by the joist spacing. (Note: treating this as a uniform load is a convenient and reasonable approximation in this case, resulting in a difference in maximum moment of approximately 4% as compared to the moment calculated using concentrated loading from each of the roof joists acting on the beam). Beam loading is calculated as follows, and shown in Figure III-5.

The dead load from the joist end reaction is:

$$\begin{aligned} w_D &= \frac{2.76 \text{ kips}}{6.00 \text{ ft}} \\ &= 0.460 \text{ kip/ft} \end{aligned}$$

From previous calculations, the dead load from the spandrel is:

$$w_D = 0.413 \text{ kip/ft}$$

The snow load from the joist end reaction is:

$$\begin{aligned} w_S &= \frac{3.73 \text{ kips}}{6.00 \text{ ft}} \\ &= 0.622 \text{ kip/ft} \end{aligned}$$

Use 8 psf for initial dead load and 12 psf for superimposed dead load.

$$\begin{aligned} w_{D(initial)} &= \left(22.5 \text{ ft} + 0.5 \text{ ft}\right)\left(0.008 \text{ kip/ft}^2\right) \\ &= 0.184 \text{ kip/ft} \end{aligned}$$

$$\begin{aligned} w_{D(super)} &= \left(22.5 \text{ ft} + 0.5 \text{ ft}\right)\left(0.012 \text{ kip/ft}^2\right) \\ &= 0.276 \text{ kip/ft} \end{aligned}$$

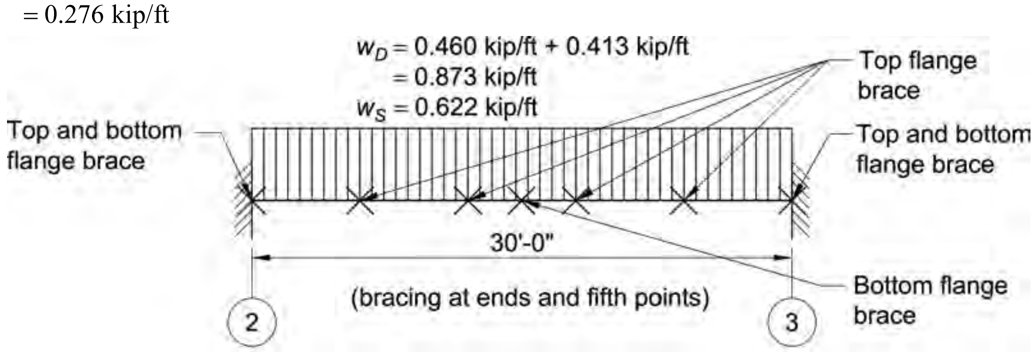


Fig. III-5. Loading and bracing diagram for roof beams at north and south ends of building.

From ASCE/SEI 7, Chapter 2, calculate the required strength of the beams at the roof sides.

LRFD	ASD
$w_u = 1.2(0.873 \text{ kip/ft}) + 1.6(0.622 \text{ kip/ft})$ $= 2.04 \text{ kip/ft}$ $R_u = (2.04 \text{ kip/ft})\left(\dfrac{30 \text{ ft}}{2}\right)$ $= 30.6 \text{ kips}$	$w_a = 0.873 \text{ kip/ft} + 0.622 \text{ kip/ft}$ $= 1.50 \text{ kip/ft}$ $R_a = (1.50 \text{ kip/ft})\left(\dfrac{30 \text{ ft}}{2}\right)$ $= 22.5 \text{ kips}$

Using the equation for deflection derived previously, the required moment of inertia, $I_{x\,req}$, is determined as follows.

To limit the superimposed dead and live load deflection to ¼ in.:

$$I_{x\,req} = \frac{(0.622 \text{ kip/ft} + 0.276 \text{ kip/ft})(30 \text{ ft})^4}{6{,}440(\tfrac{1}{4} \text{ in.})}$$

$$= 452 \text{ in.}^4 \quad \textbf{controls}$$

To limit the cladding and initial dead load deflection to ⅜ in.:

$$I_{req} = \frac{(0.597 \text{ kip/ft})(30.0 \text{ ft})^4}{6{,}440(\tfrac{3}{8} \text{ in.})}$$

$$= 200 \text{ in.}^4$$

From AISC *Manual* Table 3-3, select a beam size with an adequate moment of inertia. Try a W18×35:

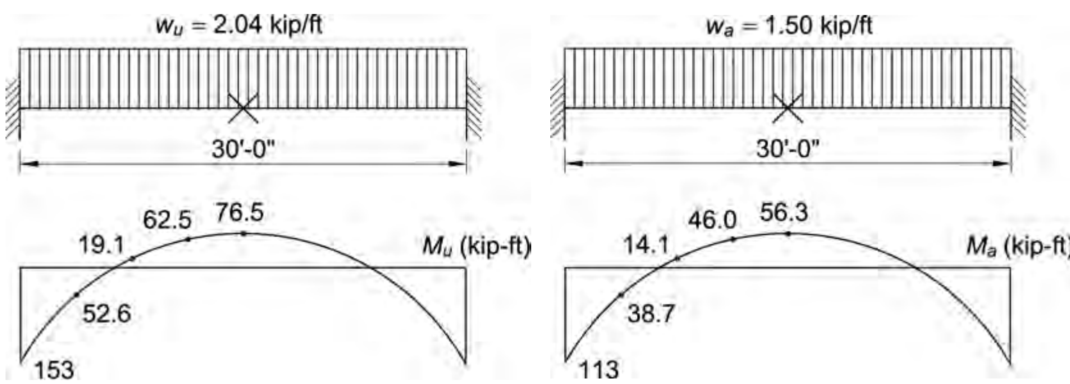
$$I_x = 510 \text{ in.}^4 > 452 \text{ in.}^4 \quad \textbf{o.k.}$$

Calculate C_b for compression in the bottom flange braced at the midpoint and supports using AISC *Specification* Equation F1-1. Moments along the span are summarized in Figure III-6.

LRFD	ASD
From AISC *Manual* Table 3-23, Case 15: $M_{u\,max} = \dfrac{(2.04 \text{ kip/ft})(30 \text{ ft})^2}{12}$ $= 153 \text{ kip-ft (at supports)}$ At midpoint: $M_u = \dfrac{(2.04 \text{ kip/ft})(30 \text{ ft})^2}{24}$ $= 76.5 \text{ kip-ft}$	From AISC *Manual* Table 3-23, Case 15: $M_{a\,max} = \dfrac{(1.50 \text{ kip/ft})(30 \text{ ft})^2}{12}$ $= 113 \text{ kip-ft (at supports)}$ At midpoint: $M_a = \dfrac{(1.50 \text{ kip/ft})(30 \text{ ft})^2}{24}$ $= 56.3 \text{ kip-ft}$

LRFD	ASD

LRFD	ASD
At quarter-point of unbraced length: $M_{uA} = \dfrac{2.04 \text{ kip/ft}}{12}\left[\begin{array}{l}6(30\text{ ft})(3.75\text{ ft})-(30\text{ ft})^2\\-6(3.75\text{ ft})^2\end{array}\right]$ $= 52.6$ kip-ft	At quarter-point of unbraced length: $M_{aA} = \dfrac{1.50 \text{ kip/ft}}{12}\left[\begin{array}{l}6(30\text{ ft})(3.75\text{ ft})-(30\text{ ft})^2\\-6(3.75\text{ ft})^2\end{array}\right]$ $= 38.7$ kip-ft
At midpoint of unbraced length: $M_{uB} = \dfrac{2.04 \text{ kip/ft}}{12}\left[\begin{array}{l}6(30\text{ ft})(7.50\text{ ft})-(30\text{ ft})^2\\-6(7.50\text{ ft})^2\end{array}\right]$ $= 19.1$ kip-ft	At midpoint of unbraced length: $M_{aB} = \dfrac{1.50 \text{ kip/ft}}{12}\left[\begin{array}{l}6(30\text{ ft})(7.50\text{ ft})-(30\text{ ft})^2\\-6(7.50\text{ ft})^2\end{array}\right]$ $= 14.1$ kip-ft
At three-quarter point of unbraced length: $M_{uC} = \dfrac{2.04 \text{ kip/ft}}{12}\left[\begin{array}{l}6(30\text{ ft})(11.3\text{ ft})-(30\text{ ft})^2\\-6(11.3\text{ ft})^2\end{array}\right]$ $= 62.5$ kip-ft	At three-quarter point of unbraced length: $M_{aC} = \dfrac{1.50 \text{ kip/ft}}{12}\left[\begin{array}{l}6(30\text{ ft})(11.3\text{ ft})-(30\text{ ft})^2\\-6(11.3\text{ ft})^2\end{array}\right]$ $= 46.0$ kip-ft
Using AISC *Specification* Equation F1-1: $C_b = \dfrac{12.5M_{max}}{2.5M_{max}+3M_A+4M_B+3M_C}$ $= \dfrac{12.5(153\text{ kip-ft})}{\left[\begin{array}{l}2.5(153\text{ kip-ft})+3(52.6\text{ kip-ft})\\+4(19.1\text{ kip-ft})+3(62.5\text{ kip-ft})\end{array}\right]}$ $= 2.38$	Using AISC *Specification* Equation F1-1: $C_b = \dfrac{12.5M_{max}}{2.5M_{max}+3M_A+4M_B+3M_C}$ $= \dfrac{12.5(113\text{ kip-ft})}{\left[\begin{array}{l}2.5(113\text{ kip-ft})+3(38.7\text{ kip-ft})\\+4(14.1\text{ kip-ft})+3(46.0\text{ kip-ft})\end{array}\right]}$ $= 2.38$



(a) LRFD *(b) ASD*

Fig. III-6. Beam moment diagram.

From AISC *Manual* Table 6-2, with L_b = 6 ft and C_b = 1.0 the available flexural strength is determined as follows:

LRFD	ASD
$\phi_b M_n = 229 \text{ kip-ft} > 76.5 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = 152 \text{ kip-ft} > 56.3 \text{ kip-ft}$ **o.k.**

From AISC *Manual* Table 6-2, with L_b = 15 ft and C_b = 2.38, the available flexural strength is determined as follows:

LRFD	ASD
$\phi_b M_n C_b \le \phi_b M_p$ $(109 \text{ kip-ft})(2.38) > 249 \text{ kip-ft}$ $259 \text{ kip-ft} > 249 \text{ kip-ft}$ Therefore: $\phi_b M_n = 249 \text{ kip-ft} > 153 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} C_b \le \dfrac{M_p}{\Omega_b}$ $(72.4 \text{ kip-ft})(2.38) > 166 \text{ kip-ft}$ $172 \text{ kip-ft} > 166 \text{ kip-ft}$ Therefore: $\dfrac{M_n}{\Omega_b} = 166 \text{ kip-ft} > 113 \text{ kip-ft}$ **o.k.**

From AISC *Manual* Table 6-2, the available shear strength is determined as follows:

LRFD	ASD
$\phi_v V_n = 159 \text{ kips} > 30.6 \text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega_v} = 106 \text{ kips} > 22.5 \text{ kips}$ **o.k.**

Therefore, the W18×35 is acceptable.

Note: This roof beam may need to be upsized during the lateral load analysis to increase the stiffness and strength of the member and improve lateral frame drift performance.

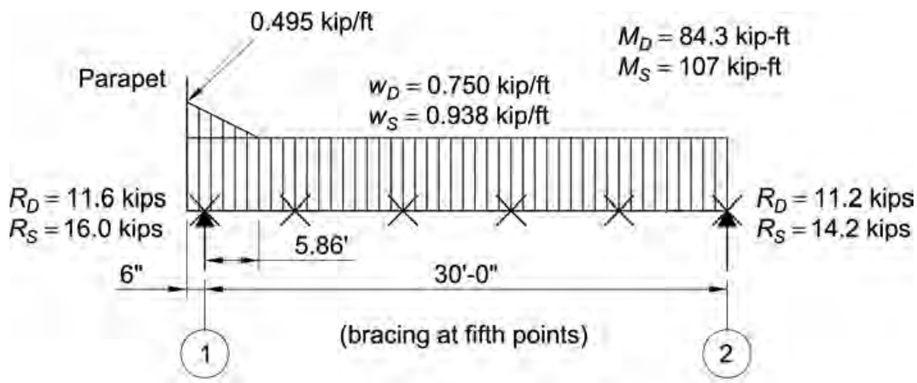
SELECT THE ROOF BEAMS ALONG THE INTERIOR LINES OF THE BUILDING

There are three individual beam loadings that occur along grids C and D. The beams from 1 to 2 and 7 to 8 have a uniform snow load except for the snow drift at the end at the parapet. The snow drift from the far ends of the 45-ft joists is negligible. The beams from 2 to 3 and 6 to 7 are the same as the first group, except they have snow drift at the screen wall. The live load deflection is limited to L/240 (or 1.50 in.). Joist reactions are divided by the joist spacing and treated as a uniform load, just as they were for the side beams.

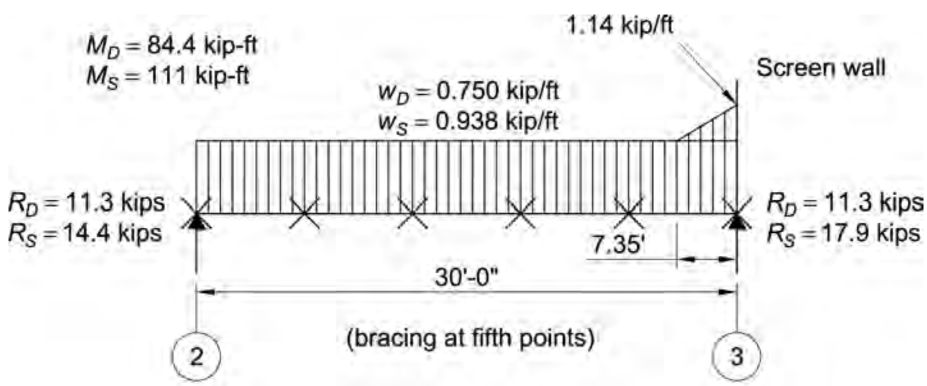
$$w_D = \left(0.020 \text{ kip/ft}^2\right)\left(\frac{45 \text{ ft} + 30 \text{ ft}}{2}\right)$$
$$= 0.750 \text{ kip/ft}$$

$$w_S = \left(0.025 \text{ kip/ft}^2\right)\left(\frac{45 \text{ ft} + 30 \text{ ft}}{2}\right)$$
$$= 0.938 \text{ kip/ft}$$

The loading diagrams with moments and end reactions are shown in Figure III-7.



(a) Grids 1 to 2 and 7 to 8



(b) Grids 2 to 3 and 6 to 7

Fig. III-7. Roof beam loading and bracing diagram.

From ASCE/SEI 7, Chapter 2, the required strength for the beams from grids 1 to 2 and 7 to 8 (opposite hand) is determined as follows:

LRFD	ASD
$R_u\text{ (left end)} = 1.2(11.6\text{ kips}) + 1.6(16.0\text{ kips})$ $= 39.5\text{ kips}$ $R_u\text{ (right end)} = 1.2(11.2\text{ kips}) + 1.6(14.2\text{ kips})$ $= 36.2\text{ kips}$ $M_u = 1.2(84.3\text{ kip-ft}) + 1.6(107\text{ kip-ft})$ $= 272\text{ kip-ft}$	$R_a\text{ (left end)} = 11.6\text{ kips} + 16.0\text{ kips}$ $= 27.6\text{ kips}$ $R_a\text{ (right end)} = 11.2\text{ kips} + 14.2\text{ kips}$ $= 25.4\text{ kips}$ $M_a = 84.3\text{ kip-ft} + 107\text{ kip-ft}$ $= 191\text{ kip-ft}$

Using the equation for deflection derived previously, the minimum moment of inertia, $I_{x\,req}$, to limit the live load deflection to 1.50 in., considering a 30-ft simply supported beam and neglecting the modest snow drift is:

$$I_{x\,req} = \frac{(0.938\text{ kip/ft})(30\text{ ft})^4}{1{,}290(1.50\text{ in.})}$$
$$= 393\text{ in.}^4$$

From AISC *Manual* Table 3-3, select a beam size with an adequate moment of inertia. Try a W21×44:

$$I_x = 843\text{ in.}^4 > 393\text{ in.}^4 \quad \textbf{o.k.}$$

From AISC *Manual* Table 6-2, for a W21×44 with L_b = 6 ft and C_b = 1.0, the available flexural strength and shear strength is determined as follows:

LRFD	ASD
$\phi_b M_n = 332\text{ kip-ft} > 272\text{ kip-ft}$ **o.k.** $\phi_v V_n = 217\text{ kips} > 39.5\text{ kips}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = 221\text{ kip-ft} > 191\text{ kip-ft}$ **o.k.** $\dfrac{V_n}{\Omega_v} = 145\text{ kips} > 27.6\text{ kips}$ **o.k.**

From ASCE/SEI 7, Chapter 2, the required strength for the beams from grids 2 to 3 and 6 to 7 (opposite hand) is determined as follows:

LRFD	ASD
$R_u\text{ (left end)} = 1.2(11.3\text{ kips}) + 1.6(14.4\text{ kips})$ $= 36.6\text{ kips}$ $R_u\text{ (right end)} = 1.2(11.3\text{ kips}) + 1.6(17.9\text{ kips})$ $= 42.2\text{ kips}$ $M_u = 1.2(84.4\text{ kip-ft}) + 1.6(111\text{ kip-ft})$ $= 279\text{ kip-ft}$	$R_a\text{ (left end)} = 11.3\text{ kips} + 14.4\text{ kips}$ $= 25.7\text{ kips}$ $R_a\text{ (right end)} = 11.3\text{ kips} + 17.9\text{ kips}$ $= 29.2\text{ kips}$ $M_a = 84.4\text{ kip-ft} + 111\text{ kip-ft}$ $= 195\text{ kip-ft}$

From AISC *Manual* Table 6-2, for a W21×44 with L_b = 6 ft and C_b = 1.0, the available flexural strength and shear strength is determined as follows:

LRFD	ASD
$\phi_b M_n = 332 \text{ kip-ft} > 279 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = 221 \text{ kip-ft} > 195 \text{ kip-ft}$ **o.k.**
$\phi_v V_n = 217 \text{ kips} > 42.2 \text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega_v} = 145 \text{ kips} > 29.2 \text{ kips}$ **o.k.**

The third individual beam loading occurs at the beams from 3 to 4, 4 to 5, and 5 to 6. For these beams there is a uniform snow load outside the screen walled area, except for the snow drift at the parapet ends and the screen wall ends of the 45-ft-long joists. Inside the screen walled area the beams support the mechanical equipment. The loading diagram is shown in Figure III-8.

$$w_D = \left(\frac{2.70 \text{ kips}}{6 \text{ ft}}\right) + \left(0.360 \text{ kip/ft}^2\right)\left(\frac{15 \text{ ft}}{6 \text{ ft}}\right)$$
$$= 1.35 \text{ kip/ft}$$

$$w_S = \left(\frac{4.02 \text{ kips}}{6 \text{ ft}}\right) + \left(0.240 \text{ kip/ft}^2\right)\left(\frac{15 \text{ ft}}{6 \text{ ft}}\right)$$
$$= 1.27 \text{ kip/ft}$$

From ASCE/SEI 7, Chapter 2, the required strength for the beams from grids 3 to 4, 4 to 5, and 5 to 6 is determined as follows:

LRFD	ASD
$w_u = 1.2(1.35 \text{ kip/ft}) + 1.6(1.27 \text{ kip/ft})$ $= 3.65 \text{ kip/ft}$	$w_a = 1.35 \text{ kip/ft} + 1.27 \text{ kip/ft}$ $= 2.62 \text{ kip/ft}$
$M_u = \dfrac{(3.65 \text{ kip/ft})(30 \text{ ft})^2}{8}$ $= 411 \text{ kip-ft}$	$M_a = \dfrac{(2.62 \text{ kip/ft})(30 \text{ ft})^2}{8}$ $= 295 \text{ kip-ft}$

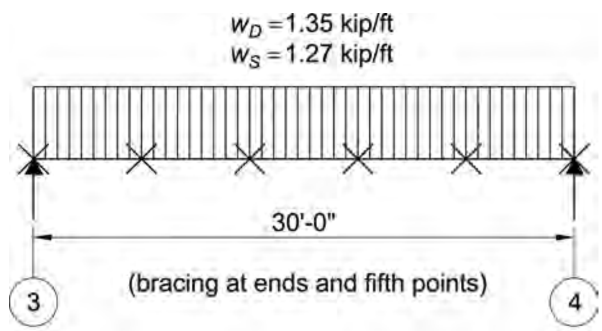


Fig. III-8. Loading and bracing diagram for roof beams from grid 3 to 4, 4 to 5, and 5 to 6.

LRFD	ASD

$R_u = (3.65 \text{ kip/ft})\left(\dfrac{30 \text{ ft}}{2}\right)$ $= 54.8 \text{ kips}$	$R_a = (2.62 \text{ kip/ft})\left(\dfrac{30 \text{ ft}}{2}\right)$ $= 39.3 \text{ kips}$

Using the equation for deflection derived previously, the minimum moment of inertia, $I_{x\,req}$, to limit the live load deflection to 1.50 in. is:

$$I_{x\,req} = \frac{(1.27 \text{ kip/ft})(30 \text{ ft})^4}{1{,}290(1.50 \text{ in.})}$$
$$= 532 \text{ in.}^4$$

From AISC *Manual* Table 3-3, select a beam size with an adequate moment of inertia. Try a W21×55:

$$I_x = 1{,}140 \text{ in.}^4 > 532 \text{ in.}^4 \quad \textbf{o.k.}$$

From AISC *Manual* Table 6-2, for a W21×55 with L_b = 6 ft and C_b = 1.0, the available flexural strength and shear strength is determined as follows:

LRFD	ASD
$\phi_b M_n = 473 \text{ kip-ft} > 411 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = 314 \text{ kip-ft} > 295 \text{ kip-ft}$ **o.k.**
$\phi_v V_n = 234 \text{ kips} > 54.8 \text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega_v} = 156 \text{ kips} > 39.3 \text{ kips}$ **o.k.**

FLOOR MEMBER DESIGN AND SELECTION

Calculate dead load and live load.

Dead load:

Slab and deck	= 57 psf
Beams (est.)	= 8 psf
Misc. (ceiling, mechanical, etc.)	= 10 psf
Total	= 75 psf

Note: The weight of the floor slab and deck was obtained from the manufacturer's literature.

Live load:

Total (can be reduced for area per ASCE/SEI 7) = 80 psf

The floor and deck will be 3 in. of normal weight concrete, $f_c' = 4$ ksi, on 3-in., 20 gage, galvanized, composite deck, laid in a pattern of three or more continuous spans. The total depth of the slab is 6 in. From the Steel Deck Institute *Floor Deck Design Manual* (SDI, 2014), the maximum unshored span for construction with this deck and a three-span condition is 10 ft 6 in. The general layout for the floor beams is 10 ft on center; therefore, the deck does not need to be shored during construction. At 10 ft on center, this deck has an allowable superimposed live load capacity of 143 psf. In addition, it can be shown that this deck can carry a 2,000 pound load over an area of 2.5 ft by 2.5 ft as required by ASCE/SEI 7, Section 4.4. The floor diaphragm and the floor loads extend 6 in. past the centerline of grid as shown on Sheet S4.1.

SELECT FLOOR BEAMS (COMPOSITE AND NONCOMPOSITE)

Note: There are two early and important checks in the design of composite beams. First, select a beam that either does not require camber, or establish a target camber and moment of inertia at the start of the design process. A reasonable approximation of the camber is between *L*/300 minimum and *L*/180 maximum (or a maximum of 1½ to 2 in.).

Second, check that the beam is strong enough to safely carry the wet concrete and a 20 psf construction live load [per *Design Loads on Structures During Construction,* ASCE 37-14 (ASCE, 2014)] when designed by the ASCE/SEI 7 load combinations and the provisions of AISC *Specification* Chapter F.

SELECT TYPICAL 45-FT-LONG INTERIOR COMPOSITE BEAM (10 FT ON CENTER)

Find a target moment of inertia for an unshored beam.

$$\begin{aligned} w_D &= (10 \text{ ft})\left(0.057 \text{ kip/ft}^2 + 0.008 \text{ kip/ft}^2\right) \\ &= 0.650 \text{ kip/ft} \end{aligned}$$

Hold deflection to 2 in. maximum to facilitate concrete placement. Using the equation for deflection derived previously, the required moment of inertia is determined as follows:

$$\begin{aligned} I_{req} &\approx \frac{(0.650 \text{ kip/ft})(45 \text{ ft})^4}{1{,}290(2 \text{ in.})} \\ &= 1{,}030 \text{ in.}^4 \end{aligned}$$

The construction live load is determined as follows:

$$\begin{aligned} w_L &= (10 \text{ ft})\left(0.020 \text{ kip/ft}^2\right) \\ &= 0.200 \text{ kip/ft} \end{aligned}$$

From ASCE/SEI 7, the required flexural strength due to wet concrete only is determined as follows:

LRFD	ASD
$w_u = 1.4(0.650 \text{ kip/ft}) = 0.910 \text{ kip/ft}$ $M_u = \frac{(0.910 \text{ kip/ft})(45 \text{ ft})^2}{8} = 230 \text{ kip-ft}$	$w_a = 0.650 \text{ kip/ft}$ $M_a = \frac{(0.650 \text{ kip/ft})(45 \text{ ft})^2}{8} = 165 \text{ kip-ft}$

From ASCE/SEI 7, the required flexural strength due to wet concrete and construction live load is determined as follows:

LRFD	ASD
$w_u = 1.2(0.650 \text{ kip/ft}) + 1.6(0.200 \text{ kip/ft}) = 1.10 \text{ kip/ft}$	$w_a = 0.650 \text{ kip/ft} + 0.200 \text{ kip/ft} = 0.850 \text{ kip/ft}$

LRFD	ASD
$M_u = \dfrac{(1.10 \text{ kip/ft})(45 \text{ ft})^2}{8}$ $= 278 \text{ kip-ft}$ **controls**	$M_a = \dfrac{(0.850 \text{ kip/ft})(45 \text{ ft})^2}{8}$ $= 215 \text{ kip-ft}$ **controls**

Use AISC *Manual* Table 3-2 to select a beam with $I_x \geq 1{,}030 \text{ in.}^4$ Select W21×50, with $I_x = 984 \text{ in.}^4$, close to the target value.

From AISC *Manual* Table 6-2, the available flexural strength for a fully braced, $L_b = 0$ ft, W21×50 is determined as follows:

LRFD	ASD
$\phi_b M_n = 413 \text{ kip-ft} > 278 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = 274 \text{ kip-ft} > 215 \text{ kip-ft}$ **o.k.**

Check for possible live load reduction due to area in accordance with ASCE/SEI 7, Section 4.7.2.

From ASCE/SEI 7, Table 4.7-1, for interior beams:

$$K_{LL} = 2$$

The beams are at 10 ft on center, therefore the tributary area is:

$$\begin{aligned} A_T &= (45 \text{ ft})(10 \text{ ft}) \\ &= 450 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} K_{LL}A_T &= 2\left(450 \text{ ft}^2\right) \\ &= 900 \text{ ft}^2 \end{aligned}$$

Because $K_{LL}A_T \geq 400 \text{ ft}^2$, a reduced live load can be used.

From ASCE/SEI 7, Equation 4.7-1:

$$\begin{aligned} L &= L_o\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right) \geq 0.50L_o \\ &= (80 \text{ psf})\left(0.25 + \frac{15}{\sqrt{900 \text{ ft}^2}}\right) > 0.50(80 \text{ psf}) \\ &= 60.0 \text{ psf} > 40.0 \text{ psf} \end{aligned}$$

Therefore, use $L = 60.0$ psf.

The beams are at 10 ft on center, therefore the loading is as shown in Figure III-9. Note, the beam is continuously braced by the deck.

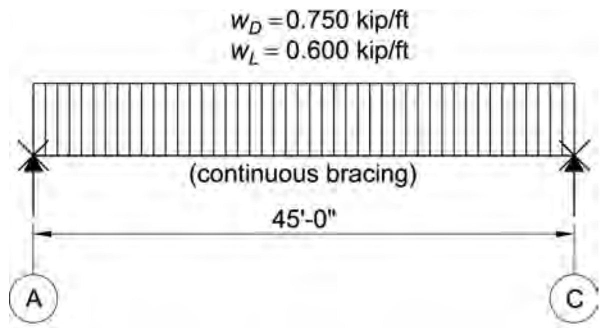


Fig. III-9. Loading and bracing diagram for typical interior composite floor beams.

From ASCE/SEI 7, Chapter 2, the required strengths are determined as follows:

LRFD	ASD
$w_u = 1.2(0.750\text{ kip/ft}) + 1.6(0.600\text{ kip/ft})$ $= 1.86\text{ kip/ft}$ $R_u = (1.86\text{ kip/ft})\left(\dfrac{45\text{ ft}}{2}\right)$ $= 41.9\text{ kips}$ $M_u = \dfrac{(1.86\text{ kip/ft})(45\text{ ft})^2}{8}$ $= 471\text{ kip-ft}$	$w_a = 0.750\text{ kip/ft} + 0.600\text{ kip/ft}$ $= 1.35\text{ kip/ft}$ $R_a = (1.35\text{ kip/ft})\left(\dfrac{45\text{ ft}}{2}\right)$ $= 30.4\text{ kips}$ $M_a = \dfrac{(1.35\text{ kip/ft})(45\text{ ft})^2}{8}$ $= 342\text{ kip-ft}$

The available flexural strength for the composite beam is determined using AISC *Manual* Part 3. Assume initially $a = 1$ in.

$$Y2 = Y_{con} - \frac{a}{2} \qquad \text{(}\textit{Manual}\text{ Eq. 3-6)}$$
$$= 6.00\text{ in.} - \frac{1\text{ in.}}{2}$$
$$= 5.50\text{ in.}$$

Enter AISC *Manual* Table 3-19 for a W21×50 with $Y2$ = 5.50 in. Selecting PNA location 7, with ΣQ_n = 184 kips, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = 598\text{ kip-ft} > 471\text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = 398\text{ kip-ft} > 342\text{ kip-ft}$ **o.k.**

Determine effective width, b

The effective width of the concrete slab is the sum of the effective widths for each side of the beam centerline as determined by the minimum value of the three widths set forth in AISC *Specification* Section I3.1a:

1. one-eighth of the span of the beam, center-to-center of the supports

$$\left(\frac{45\text{ ft}}{8}\right)(2\text{ sides}) = 11.3\text{ ft}$$

2. one-half the distance to the centerline of the adjacent beam

$$\left(\frac{10\text{ ft}}{2}\right)(2\text{ sides}) = 10.0\text{ ft} \quad \textbf{controls}$$

3. distance to the edge of the slab

The latter is not applicable for an interior member.

Determine the height of the compression block, a.

$$a = \frac{\sum Q_n}{0.85 f_c{}'b} \qquad \text{(}\textit{Manual}\text{ Eq. 3-7)}$$

$$= \frac{184\text{ kips}}{0.85(4\text{ ksi})(10\text{ ft})(12\text{ in./ft})}$$

$$= 0.451\text{ in.} < 1.00\text{ in.} \quad \textbf{o.k.}$$

From AISC *Manual* Table 6-2, the available shear strength of the W21×50 bare steel beam is determined as follows:

LRFD	ASD
$\phi_v V_n = 237\text{ kips} > 41.9\text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega_v} = 158\text{ kips} > 30.4\text{ kips}$ **o.k.**

Check live load deflection

$$\frac{L}{360} = \frac{(45\text{ ft})(12\text{ in./ft})}{360}$$

$$= 1.50\text{ in.}$$

Entering AISC *Manual* Table 3-20 for a W21×50, with PNA location 7 and $Y2$ = 5.50 in., provides a lower bound moment of inertia of I_{LB} = 1,730 in.^{4} From the equation previously derived, the live load deflection is determined as follows:

$$\Delta_{LL} = \frac{w_L L^4}{1,290 I_{LB}}$$

$$= \frac{(0.600\text{ kip/ft})(45\text{ ft})^4}{1,290(1,730\text{ in.}^4)}$$

$$= 1.10\text{ in.} < 1.50\text{ in.} \quad \textbf{o.k.}$$

From AISC Design Guide 3 limit the live load deflection, using 50% of the (unreduced) design live load, to L/360 with a maximum absolute value of 1 in. across the bay. From the equation previously derived, the deflection is determined as follows:

$$\Delta_{LL} = \frac{0.5\left(0.800 \text{ kip/ft}\right)\left(45 \text{ ft}\right)^4}{1,290\left(1,730 \text{ in.}^4\right)} \leq 1 \text{ in.}$$
$$= 0.735 \text{ in.} < 1 \text{ in.}$$
$$= 0.735 \text{ in.}$$

$$1 \text{ in.} - 0.735 \text{ in.} = 0.265 \text{ in.}$$

Note: Limit the supporting girders to 0.265 in. deflection under the same load case at the connection point of the beam.

Determine the required number of shear stud connectors

From AISC *Manual* Table 3-21, using perpendicular deck with one ¾-in.-diameter anchor per rib in normal weight concrete with f'_c = 4 ksi in the weak position:

$$Q_n = 17.2 \text{ kips/anchor}$$

$$n = \frac{\Sigma Q_n}{Q_n}$$
$$= \frac{184 \text{ kips}}{17.2 \text{ kips/anchor}}$$
$$= 10.7 \text{ anchors (on each side of maximum moment)}$$

Therefore, 22 studs are required to satisfy strength requirements. However, per AISC *Specification* Commentary Section I3.2d.1, 44 studs are specified to provide sufficient deformation capacity by ensuring a degree of composite action of at least 50%.

From AISC Design Guide 3, limit the wet concrete deflection in a bay to *L*/360, not to exceed 1 in. From the equation previously derived, the wet concrete deflection is determined as follows:

$$\Delta_{DL(wet\ conc)} = \frac{\left(0.650 \text{ kip/ft}\right)\left(45 \text{ ft}\right)^4}{1,290\left(984 \text{ in.}^4\right)}$$
$$= 2.10 \text{ in.}$$

Camber the beam for 80% of the calculated wet deflection.

$$Camber = 0.80\left(2.10 \text{ in.}\right)$$
$$= 1.68 \text{ in.}$$

Round the calculated value down to the nearest ¼ in.; therefore, specify 1½ in. of camber.

$$2.10 \text{ in.} - 1\tfrac{1}{2} \text{ in.} = 0.600 \text{ in.}$$

$$1 \text{ in.} - 0.600 \text{ in.} = 0.400 \text{ in.}$$

Note: Limit the supporting girders to 0.400 in. deflection under the same load combination at the connection point of the beam.

SELECT TYPICAL 30-FT INTERIOR COMPOSITE (OR NONCOMPOSITE) BEAM (10 FT ON CENTER)

Find a target moment of inertia for an unshored beam.

Determine the required strength to carry wet concrete and construction live load. The dead load from the slab and deck is:

$$\begin{aligned} w_D &= (10\text{ ft})\left(0.057\text{ kip/ft}^2 + 0.008\text{ kip/ft}^2\right) \\ &= 0.650\text{ kip/ft} \end{aligned}$$

Hold deflection to 1½ in. maximum to facilitate concrete placement. Using the equation for deflection derived previously, the required moment of inertia is determined as follows:

$$\begin{aligned} I_{req} &\approx \frac{(0.650\text{ kip/ft})(30\text{ ft})^4}{1{,}290(1\tfrac{1}{2}\text{ in.})} \\ &= 272\text{ in.}^4 \end{aligned}$$

The construction live load is:

$$\begin{aligned} w_L &= (10\text{ ft})\left(0.020\text{ kip/ft}^2\right) \\ &= 0.200\text{ kip/ft} \end{aligned}$$

From ASCE/SEI 7, Chapter 2, determine the required flexural strength due to wet concrete only.

LRFD	ASD
$w_u = 1.4(0.650\text{ kip/ft}) = 0.910\text{ kip/ft}$	$w_a = 0.650\text{ kip/ft}$
$M_u = \frac{(0.910\text{ kip/ft})(30\text{ ft})^2}{8} = 102\text{ kip-ft}$	$M_a = \frac{(0.650\text{ kip/ft})(30\text{ ft})^2}{8} = 73.1\text{ kip-ft}$

From ASCE/SEI 7, Chapter 2, determine the required flexural strength due to wet concrete and construction live load.

LRFD	ASD
$w_u = 1.2(0.650\text{ kip/ft}) + 1.6(0.200\text{ kip/ft}) = 1.10\text{ kip/ft}$	$w_a = 0.650\text{ kip/ft} + 0.200\text{ kip/ft} = 0.850\text{ kip/ft}$
$M_u = \frac{(1.10\text{ kip/ft})(30\text{ ft})^2}{8} = 124\text{ kip-ft}$ **controls**	$M_a = \frac{(0.850\text{ kip/ft})(30\text{ ft})^2}{8} = 95.6\text{ kip-ft}$ **controls**

Use AISC *Manual* Table 3-2 to find a beam with an $I_x \geq 272\text{ in.}^4$ Select W16×26, with $I_x = 301\text{ in.}^4$, which exceeds the target value.

From AISC *Manual* Table 6-2, the available flexural strength for a fully braced, L_b = 0 ft, W16×26 is determined as follows:

LRFD	ASD
$\phi_b M_n = 166 \text{ kip-ft} > 124 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = 110 \text{ kip-ft} > 95.6 \text{ kip-ft}$ **o.k.**

Check for possible live load reduction due to area in accordance with ASCE/SEI 7, Section 4.7.2.

From ASCE/SEI 7, Table 4.7-1, for interior beams:

$$K_{LL} = 2$$

The beams are at 10 ft on center, therefore the tributary area is:

$$\begin{aligned} A_T &= (30 \text{ ft})(10 \text{ ft}) \\ &= 300 \text{ ft}^2 \end{aligned}$$

$$\begin{aligned} K_{LL}A_T &= 2\left(300 \text{ ft}^2\right) \\ &= 600 \text{ ft}^2 \end{aligned}$$

Because $K_{LL}A_T \geq 400 \text{ ft}^2$, a reduced live load can be used.

From ASCE/SEI 7, Equation 4.7-1:

$$\begin{aligned} L &= L_o\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right) \geq 0.50L_o \\ &= (80 \text{ psf})\left(0.25 + \frac{15}{\sqrt{600 \text{ ft}^2}}\right) > 0.50(80 \text{ psf}) \\ &= 69.0 \text{ psf} > 40.0 \text{ psf} \end{aligned}$$

Therefore, use L = 69.0 psf.

The beams are at 10 ft on center, therefore the loading is as shown in Figure III-10.

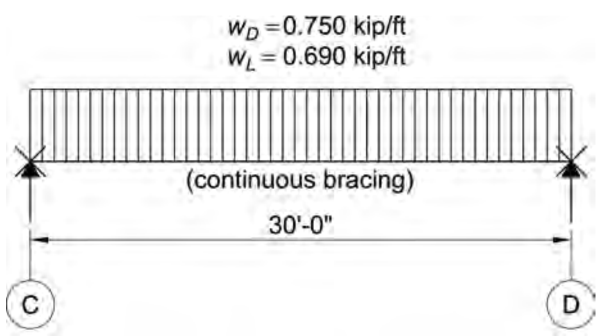


Fig. III-10. Loading and bracing diagram for typical 30-ft interior floor beams.

From ASCE/SEI 7, Chapter 2, calculate the required strength.

LRFD	ASD
$w_u = 1.2(0.750 \text{ kip/ft}) + 1.6(0.690 \text{ kip/ft})$ $= 2.00 \text{ kip/ft}$ $R_u = (2.00 \text{ kip/ft})\left(\frac{30 \text{ ft}}{2}\right)$ $= 30.0 \text{ kips}$ $M_u = \frac{(2.00 \text{ kip/ft})(30 \text{ ft})^2}{8}$ $= 225 \text{ kip-ft}$	$w_a = 0.750 \text{ kip/ft} + 0.690 \text{ kip/ft}$ $= 1.44 \text{ kip/ft}$ $R_a = (1.44 \text{ kip/ft})\left(\frac{30 \text{ ft}}{2}\right)$ $= 21.6 \text{ kips}$ $M_a = \frac{(1.44 \text{ kip/ft})(30 \text{ ft})^2}{8}$ $= 162 \text{ kip-ft}$

The available flexural strength for the composite beam is determined from AISC *Manual* Part 3 as follows. Assume initially that a = 1 in.

$$Y2 = Y_{con} - \frac{a}{2} \qquad (\textit{Manual} \text{ Eq. 3-6})$$
$$= 6.00 \text{ in.} - \frac{1 \text{ in.}}{2}$$
$$= 5.50 \text{ in.}$$

Enter AISC *Manual* Table 3-19 for a W16×26 with $Y2$ = 5.50 in. Selecting PNA location 7, with ΣQ_n = 96.0 kips, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = 248 \text{ kip-ft} > 225 \text{ kip-ft}$ **o.k.**	$\frac{M_n}{\Omega_b} = 165 \text{ kip-ft} > 162 \text{ kip-ft}$ **o.k.**

Determine effective width, b

The effective width of the concrete slab is the sum of the effective widths for each side of the beam centerline as determined by the minimum value of the three widths set forth in AISC *Specification* Section I3.1a:

1. one-eighth of the span of the beam, center-to-center of the supports

$$\left(\frac{30 \text{ ft}}{8}\right)(2 \text{ sides}) = 7.50 \text{ ft} \quad \textbf{controls}$$

2. one-half the distance to the centerline of the adjacent beam

$$\left(\frac{10 \text{ ft}}{2}\right)(2 \text{ sides}) = 10.0 \text{ ft}$$

3. distance to the edge of the slab

The latter is not applicable for an interior member.

Determine the height of the compression block, a.

$$a = \frac{\sum Q_n}{0.85 f_c{}'b}$$ (*Manual* Eq. 3-7)

$$= \frac{96.0 \text{ kips}}{0.85(4 \text{ ksi})(7.50 \text{ ft})(12 \text{ in./ft})}$$

$$= 0.314 \text{ in.} < 1.00 \text{ in.} \quad \textbf{o.k.}$$

From AISC *Manual* Table 6-2, the available shear strength of the W16×26 bare steel beam is determined as follows:

LRFD	ASD
$\phi_v V_n = 106 \text{ kips} > 30.0 \text{ kips}$ **o.k.**	$\frac{V_n}{\Omega_v} = 70.5 \text{ kips} > 21.6 \text{ kips}$ **o.k.**

Check live load deflection

$$\frac{L}{360} = \frac{(30 \text{ ft})(12 \text{ in./ft})}{360}$$

$$= 1.00 \text{ in.}$$

Entering AISC *Manual* Table 3-20 for a W16×26, with PNA location 7 and $Y2$ = 5.50 in., provides a lower bound moment of inertia of I_{LB} = 575 in.^{4} From the equation previously derived, the live load deflection is determined as follows:

$$\Delta_{LL} = \frac{w_L L^4}{1{,}290 I_{LB}}$$

$$= \frac{(0.690 \text{ kip/ft})(30 \text{ ft})^4}{1{,}290(575 \text{ in.}^4)}$$

$$= 0.753 \text{ in.} < 1.00 \text{ in.} \quad \textbf{o.k.}$$

From AISC Design Guide 3, limit the live load deflection, using 50% of the (unreduced) design live load, to L/360 with a maximum absolute value of 1 in. across the bay. From the equation previously derived, the deflection is determined as follows:

$$\Delta_{LL} = \frac{0.5(0.800 \text{ kip/ft})(30 \text{ ft})^4}{1{,}290(575 \text{ in.}^4)}$$

$$= 0.437 \text{ in.} < 1 \text{ in.} \quad \textbf{o.k.}$$

$$1 \text{ in.} - 0.437 \text{ in.} = 0.563 \text{ in.}$$

Note: Limit the supporting girders to 0.563 in. deflection under the same load combination at the connection point of the beam.

Determine the required number of shear stud connectors

From AISC *Manual* Table 3-21, using perpendicular deck with one ¾-in.-diameter anchor per rib in normal weight concrete with $f_c' = 4$ ksi in the weak position:

$$Q_n = 17.2 \text{ kips/anchor}$$

$$n = \frac{\Sigma Q_n}{Q_n}$$
$$= \frac{96.0 \text{ kips}}{17.2 \text{ kips/anchor}}$$
$$= 5.58 \text{ anchors (on each side of maximum moment)}$$

Note: Per AISC *Specification* Section I8.2d, there is a maximum spacing limit of 8(6 in.) = 48 in. (not to exceed 36 in.) between anchors.

Therefore use 12 anchors, uniformly spaced at no more than 36 in. on center. Per AISC *Specification* Commentary Section I3.2d.1, beams with spans not exceeding 30 ft are not susceptible to connector failure due to insufficient connector capacity.

Note: Although the studs may be placed up to 36 in. on center, the steel deck must still be anchored to the supporting member at a spacing not to exceed 18 in. per AISC *Specification* Section I3.2c.

From AISC Design Guide 3, limit the wet concrete deflection in a bay to L/360, not to exceed 1 in. From the equation previously derived, the wet concrete deflection is determined as follows:

$$\Delta_{DL(wet\ conc)} = \frac{(0.650 \text{ kip/ft})(30 \text{ ft})^4}{1{,}290\left(301 \text{ in.}^4\right)}$$
$$= 1.36 \text{ in.}$$

Camber the beam for 80% of the calculated wet concrete dead load deflection.

$$Camber = 0.80(1.36 \text{ in.})$$
$$= 1.09 \text{ in.}$$

Round the calculated value down to the nearest ¼ in. Therefore, specify 1 in. of camber.

$$1.36 \text{ in.} - 1 \text{ in.} = 0.360 \text{ in.}$$

$$1.00 \text{ in.} - 0.360 \text{ in.} = 0.640 \text{ in.}$$

Note: Limit the supporting girders to 0.640 in. deflection under the same load combination at the connection point of the beam.

This beam could also be designed as a noncomposite beam.

Try a W18×35. From AISC *Manual* Table 6-2 the available flexural strength for a fully braced beam, L_b = 0 ft, and shear strength are determined as follows:

LRFD	ASD
$\phi_b M_n = 249 \text{ kip-in.} > 225 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = 166 \text{ kip-in.} > 162 \text{ kip-ft}$ **o.k.**
$\phi_v V_n = 159 \text{ kips} > 30.0 \text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega_v} = 106 \text{ kips} > 21.6 \text{ kips}$ **o.k.**

Check beam deflections

Check live load deflection. From AISC *Manual* Table 3-2 for a W18×35:

$$I_x = 510 \text{ in.}^4$$

$$\begin{aligned}\Delta_{LL} &= \frac{(0.690 \text{ kip/ft})(30 \text{ ft})^4}{1{,}290(510 \text{ in.}^4)} \\ &= 0.850 \text{ in.} < 1 \text{ in.} \quad \textbf{o.k.}\end{aligned}$$

Based on AISC Design Guide 3, limit the live load deflection, using 50% of the (unreduced) design live load, to L/360 with a maximum absolute value of 1 in. across the bay. From the equation previously derived, the deflection is determined as follows:

$$\begin{aligned}\Delta_{LL} &= \frac{0.5(0.800 \text{ kip/ft})(30 \text{ ft})^4}{1{,}290(510 \text{ in.}^4)} \\ &= 0.492 \text{ in.} < 1 \text{ in.} \quad \textbf{o.k.}\end{aligned}$$

$$1 \text{ in.} - 0.492 \text{ in.} = 0.508 \text{ in.}$$

Note: Limit the supporting girders to 0.508 in. deflection under the same load combination at the connection point of the beam.

Note: Because this beam is stronger than the W16×26 composite beam, no wet concrete strength checks are required in this example.

From AISC Design Guide 3, limit the wet concrete deflection in a bay to L/360, not to exceed 1 in. From the equation previously derived, the wet concrete deflection is determined as follows:

$$\begin{aligned}\Delta_{DL(wet\ conc)} &= \frac{(0.650 \text{ kip/ft})(30 \text{ ft})^4}{1{,}290(510 \text{ in.}^4)} \\ &= 0.800 \text{ in.} < 1 \text{ in.} \quad \textbf{o.k.}\end{aligned}$$

Camber the beam for 80% of the calculated wet concrete deflection.

$$\begin{aligned}Camber &= 0.80(0.800 \text{ in.}) \\ &= 0.640 \text{ in.}\end{aligned}$$

A good break point to eliminate camber is ¾ in.; therefore, do not specify a camber for this beam.

$$1 \text{ in.} - 0.800 \text{ in.} = 0.200 \text{ in.}$$

Note: Limit the supporting girders to 0.200 in. deflection under the same load case at the connection point of the beam.

Therefore, selecting a W18×35 will eliminate both shear studs and cambering. The cost of the extra steel weight may be offset by the elimination of studs and cambering. Local labor and material costs should be checked to make this determination.

SELECT TYPICAL NORTH-SOUTH EDGE BEAM

The influence area, $K_{LL}A_T$, for these beams is less than 400 ft^2; therefore, no live load reduction can be taken per ASCE/SEI 7, Section 4.7.2.

These beams carry 5.5 ft of dead load and live load as well as a wall load.

The floor dead load is:

$$\begin{aligned} w &= (5.5\text{ ft})\left(0.075\text{ kip/ft}^2\right) \\ &= 0.413\text{ kip/ft} \end{aligned}$$

Use 65 psf for the initial dead load due to the wet concrete:

$$\begin{aligned} w_{D(initial)} &= (5.5\text{ ft})\left(0.065\text{ kip/ft}^2\right) \\ &= 0.358\text{ kip/ft} \end{aligned}$$

Use 10 psf for the superimposed dead load:

$$\begin{aligned} w_{D(super)} &= (5.5\text{ ft})\left(0.010\text{ kip/ft}^2\right) \\ &= 0.055\text{ kip/ft} \end{aligned}$$

The dead load of the wall system at the floor is:

$$\begin{aligned} w &= (7.50\text{ ft})\left(0.055\text{ kip/ft}^2\right) + (6.00\text{ ft})\left(0.015\text{ kip/ft}^2\right) \\ &= 0.413\text{ kip/ft} + 0.090\text{ kip/ft} \\ &= 0.503\text{ kip/ft} \end{aligned}$$

The total dead load is:

$$\begin{aligned} w_D &= 0.413\text{ kip/ft} + 0.503\text{ kip/ft} \\ &= 0.916\text{ kip/ft} \end{aligned}$$

The live load is:

$$\begin{aligned} w_L &= (5.5\text{ ft})\left(0.080\text{ kip/ft}^2\right) \\ &= 0.440\text{ kip/ft} \end{aligned}$$

Beam loading is shown in Figure III-11.

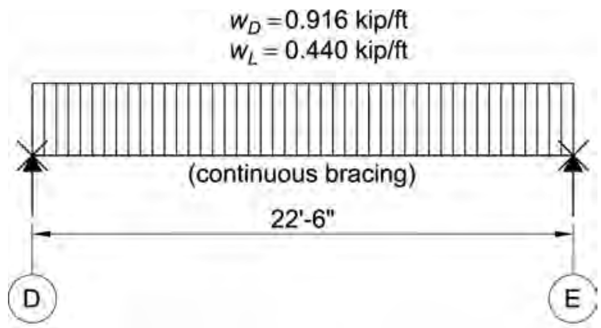


Fig. III-11. Loading and bracing diagram for typical north-south floor beams.

Calculate the required strengths from ASCE/SEI 7, Chapter 2:

LRFD	ASD
$w_u = 1.2(0.916\text{ kip/ft}) + 1.6(0.440\text{ kip/ft})$ $= 1.80\text{ kip/ft}$	$w_a = 0.916\text{ kip/ft} + 0.440\text{ kip/ft}$ $= 1.36\text{ kip/ft}$
$R_u = (1.80\text{ kip/ft})\left(\dfrac{22.5\text{ ft}}{2}\right)$ $= 20.3\text{ kips}$	$R_a = (1.36\text{ kip/ft})\left(\dfrac{22.5\text{ ft}}{2}\right)$ $= 15.3\text{ kips}$
$M_u = \dfrac{(1.80\text{ kip/ft})(22.5\text{ ft})^2}{8}$ $= 114\text{ kip-ft}$	$M_a = \dfrac{(1.36\text{ kip/ft})(22.5\text{ ft})^2}{8}$ $= 86.1\text{ kip-ft}$

Because these beams are less than 25 ft long, they will be most efficient as noncomposite beams. The beams at the edges of the building carry a brick spandrel panel. For these beams, the cladding weight exceeds 25% of the total dead load on the beam. From AISC Design Guide 3, limit the vertical deflection due to cladding and initial dead load to L/600 or ⅜ in. maximum. In addition, because these beams are supporting brick above and there is continuous glass below, limit the superimposed dead and live load deflection to L/600 or 0.3 in. maximum to accommodate the brick and L/360 or ¼ in. maximum to accommodate the glass. Therefore, combining the two limitations, limit the superimposed dead and live load deflection to L/600 or ¼ in. The superimposed dead load includes all of the dead load that is applied after the cladding has been installed. Note that it is typically not recommended to camber beams supporting spandrel panels.

Using the equation for deflection derived previously, the minimum moment of inertia, $I_{x\,req}$, to limit the superimposed dead and live load deflection to ¼ in.

$$I_{x\,req} = \frac{(0.055\text{ kip/ft} + 0.440\text{ kip/ft})(22.5\text{ ft})^4}{1{,}290(¼\text{ in.})}$$
$$= 393\text{ in.}^4$$

Using the equation for deflection derived previously, the minimum moment of inertia, $I_{x\,req}$, to limit the cladding and initial dead load deflection to ⅜ in.

$$I_{x\,req} = \frac{(0.358\text{ kip/ft} + 0.503\text{ kip/ft})(22.5\text{ ft})^4}{1{,}290(⅜\text{ in.})}$$
$$= 456\text{ in.}^4 \quad \textbf{controls}$$

From AISC *Manual* Table 3-2, find a beam with $I_x \geq 456$ in.^{4} Select a W18×35 with $I_x = 510$ in.^{4}

From AISC *Manual* Table 6-2, the available flexural strength for a fully braced beam, $L_b = 0$ ft, and shear strength are determined as follows:

LRFD	ASD
$\phi_b M_n = 249 \text{ kip-in.} > 114 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = 166 \text{ kip-in.} > 86.1 \text{ kip-ft}$ **o.k.**
$\phi_v V_n = 159 \text{ kips} > 20.3 \text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega_v} = 106 \text{ kips} > 15.3 \text{ kips}$ **o.k.**

SELECT TYPICAL EAST-WEST EDGE GIRDER

The beams along the sides of the building carry the spandrel panel and glass, and dead load and live load from the intermediate floor beams. For these beams, the cladding weight exceeds 25% of the total dead load on the beam. Therefore, per AISC Design Guide 3, limit the vertical deflection due to cladding and initial dead load to L/600 or ⅜ in. maximum. In addition, because these beams are supporting brick above and there is continuous glass below, limit the superimposed dead and live load deflection to L/600 or 0.3 in. maximum to accommodate the brick and L/360 or ¼ in. maximum to accommodate the glass. Therefore, combining the two limitations, limit the superimposed dead and live load deflection to L/600 or ¼ in. The superimposed dead load includes all of the dead load that is applied after the cladding has been installed. These beams will be part of the moment frames on the north and south sides of the building and therefore will be designed as fixed at both ends.

Establish the loading.

The dead load reaction from the floor beams is:

$$P_D = (0.750\text{ kip/ft})\left(\frac{45\text{ ft}}{2}\right)$$
$$= 16.9\text{ kips}$$

$$P_{D(initial)} = (0.650\text{ kip/ft})\left(\frac{45\text{ ft}}{2}\right)$$
$$= 14.6\text{ kips}$$

$$P_{D(super)} = (0.100\text{ kip/ft})\left(\frac{45\text{ ft}}{2}\right)$$
$$= 2.25\text{ kips}$$

The uniform dead load along the beam is:

$$w_D = (0.5\text{ ft})\left(0.075\text{ kip/ft}^2\right) + 0.503\text{ kip/ft}$$
$$= 0.541\text{ kip/ft}$$

$$w_{D(initial)} = (0.5\text{ ft})\left(0.065\text{ kip/ft}^2\right)$$
$$= 0.033\text{ kip/ft}$$

$$w_{D(super)} = (0.5\text{ ft})\left(0.010\text{ kip/ft}^2\right)$$
$$= 0.005\text{ kip/ft}$$

Select typical 30-ft composite (or noncomposite) girders.

Check for possible live load reduction due to area in accordance with ASCE/SEI 7, Section 4.7.2.

From ASCE/SEI 7, Table 4.7-1, for edge beams with cantilevered slabs:

$$K_{LL} = 1$$

However, it is also permissible to calculate the value of K_{LL} based upon influence area. Because the cantilever dimension is small, K_{LL} will be closer to 2 than 1. The calculated value of K_{LL} based upon the influence area is:

$$K_{LL} = \frac{(45.5 \text{ ft})(30 \text{ ft})}{\left(\dfrac{45 \text{ ft}}{2} + 0.5 \text{ ft}\right)(30 \text{ ft})}$$
$$= 1.98$$

$$A_T = (30 \text{ ft})(22.5 \text{ ft} + 0.5 \text{ ft})$$
$$= 690 \text{ ft}^2$$

From ASCE/SEI 7, Equation 4.7-1:

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL} A_T}}\right) \geq 0.50 L_o$$
$$= (80 \text{ psf})\left(0.25 + \frac{15}{\sqrt{1.98\left(690 \text{ ft}^2\right)}}\right) > 0.50(80 \text{ psf})$$
$$= 52.5 \text{ psf} > 40.0 \text{ psf}$$

Therefore, use L = 52.5 psf.

The live load from the floor beams is:

$$P_L = (0.525 \text{ kip/ft})\left(\frac{45 \text{ ft}}{2}\right)$$
$$= 11.8 \text{ kips}$$

The uniform live load along the beam is:

$$w_L = (0.5 \text{ ft})\left(0.0526 \text{ kip/ft}^2\right)$$
$$= 0.0263 \text{ kip/ft}$$

The loading diagram is shown in Figure III-12.

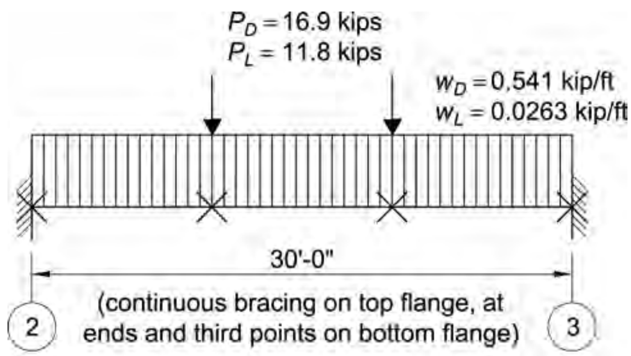


Fig. III-12. Loading and bracing diagram for typical east-west edge girders.

The required moment and end reactions at the floor side beams are determined from a structural analysis of a fixed-end beam and summarized as follows:

LRFD	ASD
Typical side beam: $R_u = 49.5$ kips M_u (at ends) $= 313$ kip-ft M_u (at center) $= 156$ kip-ft	Typical side beam: $R_a = 37.2$ kips M_a (at ends) $= 234$ kip-ft M_a (at center) $= 117$ kip-ft

The maximum moment occurs at the support with compression in the bottom flange. The bottom flange is laterally braced at 10 ft on center by the intermediate beams.

Note: During concrete placement, because the deck is parallel to the beam, the beam will not have continuous lateral support. It will be braced at 10 ft on center by the intermediate beams. By inspection, this condition will not control because the maximum moment under full loading causes compression in the bottom flange, which is braced at 10 ft on center.

LRFD	ASD
Calculate C_b for compression in the bottom flange braced at 10 ft on center. C_b = 2.21 (from computer output) Select a W21×44. With continuous bracing, L_b = 0 ft, from AISC *Manual* Table 6-2: $\phi_b M_n = 358 \text{ kip-ft} > 156 \text{ kip-ft}$ **o.k.** From AISC *Manual* Table 6-2 with L_b = 10 ft and C_b = 2.21: $\phi_b M_n C_b = (264 \text{ kip-ft})(2.21)$ $= 583 \text{ kip-ft}$ From AISC *Specification* Section F2.2, the nominal flexural strength is limited to M_p. $\phi_b M_n \le \phi_b M_p$ $583 \text{ kip-ft} > 358 \text{ kip-ft}$	Calculate C_b for compression in the bottom flange braced at 10 ft on center. C_b = 2.22 (from computer output) Select a W21×44. With continuous bracing, L_b = 0 ft, from AISC *Manual* Table 6-2: $\dfrac{M_n}{\Omega_b} = 238 \text{ kip-ft} > 117 \text{ kip-ft}$ **o.k.** From AISC *Manual* Table 6-2 with L_b = 10 ft and C_b = 2.22: $\dfrac{M_n}{\Omega_b} C_b = (176 \text{ kip-ft})(2.22)$ $= 391 \text{ kip-ft}$ From AISC *Specification* Section F2.2, the nominal flexural strength is limited to M_p. $\dfrac{M_n}{\Omega_b} \le \dfrac{M_p}{\Omega_b}$ $391 \text{ kip-ft} > 238 \text{ kip-ft}$

LRFD	ASD
Therefore: $\phi_b M_n = 358 \text{ kip-ft} > 313 \text{ kip-ft}$ **o.k.**	Therefore: $\dfrac{M_n}{\Omega_b} = 238 \text{ kip-ft} > 234 \text{ kip-ft}$ **o.k.**

From AISC *Manual* Table 6-2, the available shear strength is determined as follows:

LRFD	ASD
$\phi_v V_n = 217 \text{ kips} > 49.5 \text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega_v} = 145 \text{ kips} > 37.2 \text{ kips}$ **o.k.**

Deflections are determined from a structural analysis of a fixed-end beam. For deflection due to cladding and initial dead load:

$\Delta = 0.295 \text{ in.} < 3/8 \text{ in.}$ **o.k.**

For deflection due to superimposed dead and live loads:

$\Delta = 0.212 \text{ in.} < 1/4 \text{ in.}$ **o.k.**

Note that both of the deflection criteria stated previously for the girder and for the locations on the girder where the floor beams are supported have also been met.

Also, as noted previously, it is not typically recommended to camber beams supporting spandrel panels. The W21×44 is adequate for strength and deflection, but may be increased in size to help with moment frame strength or drift control.

SELECT TYPICAL EAST-WEST INTERIOR GIRDER

Establish loads

The dead load reaction from the floor beams is:

$$P_D = (0.750 \text{ kip/ft})\left(\frac{45 \text{ ft} + 30 \text{ ft}}{2}\right)$$
$$= 28.1 \text{ kips}$$

Check for live load reduction due to area in accordance with ASCE/SEI 7, Section 4.7.2.

From ASCE/SEI 7, Table 4.7-1, for interior beams:

$$K_{LL} = 2$$

$$A_T = (30 \text{ ft})\left(\frac{45 \text{ ft} + 30 \text{ ft}}{2}\right)$$
$$= 1{,}130 \text{ ft}^2$$

Using ASCE/SEI 7, Equation 4.7-1:

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right) \geq 0.50L_o$$
$$= (80 \text{ psf})\left(0.25 + \frac{15}{\sqrt{(2)(1{,}130 \text{ ft}^2)}}\right) \geq 0.50(80 \text{ psf})$$
$$= 45.2 \text{ psf} > 40.0 \text{ psf}$$

Therefore, use $L = 45.2$ psf.

The live load from the floor beams is:

$$P_L = (0.0452 \text{ kip/ft}^2)\left(\frac{45 \text{ ft} + 30 \text{ ft}}{2}\right)(10 \text{ ft})$$
$$= 17.0 \text{ kips}$$

The loading is shown in Figure III-13.

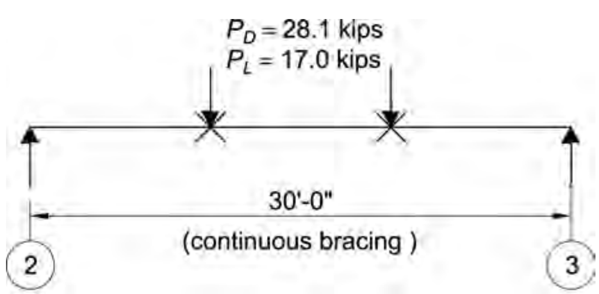


Fig. III-13. Loading and bracing diagram for typical interior girder.

Note: The dead load for this beam is included in the assumed overall dead load.

From ASCE/SEI 7, Chapter 2, the required strengths are determined as follows:

LRFD	ASD
$R_u = 1.2(28.1 \text{ kips}) + 1.6(17.0 \text{ kips})$ $= 60.9 \text{ kips}$ $M_u = (60.9 \text{ kips})(10 \text{ ft})$ $= 609 \text{ kip-ft}$	$R_a = 28.1 \text{ kips} + 17.0 \text{ kips}$ $= 45.1 \text{ kips}$ $M_a = (45.1 \text{ kips})(10 \text{ ft})$ $= 451 \text{ kip-ft}$

Check for beam requirements when carrying wet concrete. Limit wet concrete deflection to 1½ in.

$$P_D = (0.650 \text{ kip/ft})\left(\frac{45 \text{ ft} + 30 \text{ ft}}{2}\right)$$
$$= 24.4 \text{ kips}$$

$$P_L = (0.200 \text{ kip/ft})\left(\frac{45 \text{ ft} + 30 \text{ ft}}{2}\right)$$
$$= 7.50 \text{ kips}$$

Note: During concrete placement, because the deck is parallel to the beam, the beam will not have continuous lateral support. It will be braced at 10 ft on center by the intermediate beams. Also, during concrete placement, a construction live load of 20 psf will be present. The loading is shown in Figure III-14.

From ASCE/SEI 7, Chapter 2, the required strengths for the typical interior beams with wet concrete only is determined as follows:

LRFD	ASD
$R_u = 1.4(24.4 \text{ kips})$ $= 34.2 \text{ kips}$ $M_u = (34.2 \text{ kips})(10 \text{ ft})$ $= 342 \text{ kip-ft}$	$R_a = 24.4 \text{ kips}$ $M_a = (24.4 \text{ kips})(10 \text{ ft})$ $= 244 \text{ kip-ft}$

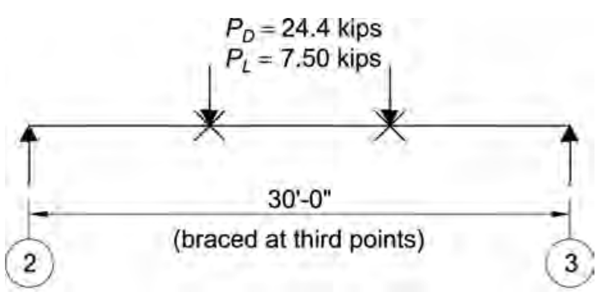


Fig. III-14. Loading and bracing diagram for typical interior girder with wet concrete and construction loads.

From ASCE/SEI 7, Chapter 2, the required strengths for the typical interior beams with wet concrete and construction live load is determined as follows:

LRFD	ASD
$R_u = 1.2(24.4 \text{ kips}) + 1.6(7.50 \text{ kips})$ $= 41.3 \text{ kips}$ $M_u(\text{midspan}) = (41.3 \text{ kips})(10 \text{ ft})$ $= 413 \text{ kip-ft}$	$R_a = 24.4 \text{ kips} + 7.50 \text{ kips}$ $= 31.9 \text{ kips}$ $M_a(\text{midspan}) = (31.9 \text{ kips})(10 \text{ ft})$ $= 319 \text{ kip-ft}$

Assume $I_x \geq 935$ in.^{4}, which is determined based on a wet concrete deflection of 1½ in. From AISC *Manual* Table 3-2, select a W21×68 with I_x = 1,480 in.^{4}.

From AISC *Manual* Table 6-2, verify the available flexural strength and shear strength using L_b = 10 ft, and C_b = 1.0.

LRFD	ASD
$\phi_b M_n = 532 \text{ kip-ft} > 413 \text{ kip-ft}$ **o.k.** $\phi_v V_n = 272 \text{ kips} > 41.3 \text{ kips}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = 354 \text{ kip-ft} > 319 \text{ kip-ft}$ **o.k.** $\dfrac{V_n}{\Omega_v} = 181 \text{ kips} > 31.9 \text{ kips}$ **o.k.**

Check W21×68 as a composite beam.

From previous calculations:

LRFD	ASD
$R_u = 60.9 \text{ kips}$ $M_u(\text{midspan}) = 609 \text{ kip-ft}$	$R_a = 45.1 \text{ kips}$ $M_a(\text{midspan}) = 451 \text{ kip-ft}$

From previous calculations, assuming a = 1 in.:

$$Y2 = 5.50 \text{ in.}$$

Enter AISC *Manual* Table 3-19 for a W21×68 with $Y2$ = 5.50 in. Selecting PNA location 7 with ΣQ_n = 250 kips provides an available flexural strength of:

LRFD	ASD
$\phi_b M_n = 844 \text{ kip-ft} > 609 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = 561 \text{ kip-ft} > 451 \text{ kip-ft}$ **o.k.**

From AISC Design Guide 3, limit the wet concrete deflection in a bay to L/360, not to exceed 1 in. From AISC *Manual* Table 3-23, Case 9:

$$\Delta_{DL(wet\ conc)} = \frac{23P_L L^3}{648EI}$$
$$= \frac{23(24.4\text{ kips})(30\text{ ft})^3(12\text{ in./ft})^3}{648(29{,}000\text{ ksi})(1{,}480\text{ in.}^4)}$$
$$= 0.941\text{ in.}$$

Camber the beam for 80% of the calculated wet concrete deflection.

$$Camber = 0.80(0.941\text{ in.})$$
$$= 0.753\text{ in.}$$

Round the calculated value down to the nearest ¼ in,: therefore, specify ¾-in. of camber.

$$0.941\text{ in.} - ¾\text{ in.} = 0.191\text{ in.} < 0.400\text{ in.}$$

Therefore, the total deflection limit of 1 in. for the bay has been met.

Determine the effective width, b

From AISC *Specification* Section I3.1a, the effective width of the concrete slab is the sum of the effective widths for each side of the beam centerline, which shall not exceed:

1. one-eighth of the span of the beam, center-to-center of supports

$$\left(\frac{30\text{ ft}}{8}\right)(2\text{ sides}) = 7.50\text{ ft} \quad \textbf{controls}$$

2. one-half the distance to the centerline of the adjacent beam

$$\left(\frac{45\text{ ft}}{2} + \frac{30\text{ ft}}{2}\right) = 37.5\text{ ft}$$

3. the distance to the edge of the slab

   The latter is not applicable for an interior member.

Determine the height of the compression block, a

$$a = \frac{\sum Q_n}{0.85 f_c{}'b} \qquad (Manual\text{ Eq. 3-7})$$
$$= \frac{250\text{ kips}}{0.85(4\text{ ksi})(7.50\text{ ft})(12\text{ in./ft})}$$
$$= 0.817\text{ in.} < 1\text{ in.} \quad \textbf{o.k.}$$

From AISC *Manual* Table 6-2, the available shear strength of the W21×68 is determined as follows.

LRFD	ASD
$\phi_v V_n = 272\text{ kips} > 60.9\text{ kips}$ **o.k.**	$\dfrac{V_n}{\Omega_v} = 181\text{ kips} > 45.1\text{ kips}$ **o.k.**

Check live load deflection.

$$\Delta_{LL} = \frac{L}{360} = \frac{(30\text{ ft})(12\text{ in./ft})}{360} = 1.00\text{ in.}$$

Entering AISC *Manual* Table 3-20 for a W21×68, with PNA location 7 and $Y2$ = 5.50 in., provides a lower bound moment of inertia of $I_{LB} = 2{,}510\text{ in.}^4$

$$\Delta_{LL} = \frac{23P_L L^3}{648EI_{LB}} = \frac{23(17.0\text{ kips})(30\text{ ft})^3(12\text{ in./ft})^3}{648(29{,}000\text{ ksi})(2{,}510\text{ in.}^4)} = 0.387\text{ in.} < 1.00\text{ in.} \quad \textbf{o.k.}$$

From AISC Design Guide 3, limit the live load deflection, using 50% of the (unreduced) design live load, to L/360 with a maximum absolute value of 1 in. across the bay.

The maximum deflection is:

$$\Delta_{LL} = \frac{23(0.5)(30.0\text{ kips})(30\text{ ft})^3(12\text{ in./ft})^3}{648(29{,}000\text{ ksi})(2{,}510\text{ in.}^4)} = 0.341\text{ in.} < 1.00\text{ in.} \quad \textbf{o.k.}$$

Check the deflection at the location where the floor beams are supported.

$$\Delta_{LL} = \frac{0.5(30.0\text{ kips})(120\text{ in.})}{6(29{,}000\text{ ksi})(2{,}510\text{ in.}^4)}\left[3(360\text{ in.})(120\text{ in.}) - 4(120\text{ in.})^2\right] = 0.297\text{ in.} > 0.265\text{ in.} \quad \textbf{o.k.}$$

Therefore, the total deflection in the bay is 0.297 in. + 0.735 in. = 1.03 in., which is acceptably close to the limit of 1 in, where Δ_{LL} = 0.735 in. is from the 45 ft interior composite beam running north-south.

Determine the required shear stud connectors

Using *Manual* Table 3-21, for parallel deck with, $w_r/h_r \geq 1.5$, one ¾-in.-diameter stud in normal weight, 4-ksi concrete:

$$Q_n = 21.5\text{ kips/anchor}$$

$$\frac{\sum Q_n}{Q_n} = \frac{250\text{ kips}}{21.5\text{ kips/anchor}} = 11.6\text{ anchors (on each side of maximum moment)}$$

Therefore, use a minimum of 24 studs for horizontal shear.

Per AISC *Specification* Section I8.2d, the maximum stud spacing is 36 in.

Since the load is concentrated at ⅓ points, the studs are to be arranged as follows:

Use 12 studs between supports and supported beams at third points. Between supported beams (middle third of span), use 4 studs to satisfy minimum spacing requirements.

Therefore, 28 studs are required in a 12:4:12 arrangement.

Notes: Although the studs may be placed up to 36 in. on center, the steel deck must still be anchored to be the supporting member at a spacing not to exceed 18 in. in accordance with AISC *Specification* Section I3.2c.

This W21×68 beam, with full lateral support, is very close to having sufficient available strength to support the imposed loads without composite action. A larger noncomposite beam might be a better solution.

COLUMN DESIGN AND SELECTION FOR GRAVITY LOADS

Estimate column loads

Roof loads (from previous calculations):

Dead load	= 20 psf
Snow load	= 25 psf
Total	= 45 psf

The snow drift loads at the perimeter of the roof and at the mechanical screen wall are developed from previous calculations.

Reaction to column (side parapet):

$$w = \left(\frac{3.73\text{ kips}}{6.00\text{ ft}}\right) - \left(0.025\text{ kip/ft}^2\right)\left(23.0\text{ ft}\right)$$
$$= 0.0467\text{ kip/ft}$$

where 3.73 kips is the snow load reaction, including drift, from the 24KCS4 roof joist at the side parapet.

Reaction to column (end parapet):

$$w = \left(\frac{16.0\text{ kips}}{37.5\text{ ft}}\right) - \left(0.025\text{ kip/ft}^2\right)\left(15.5\text{ ft}\right)$$
$$= 0.0392\text{ kip/ft}$$

where 16.0 kips is the snow load reaction, including drift, from the W21×44 roof beam along the interior lines of the building.

Reaction to column (screen wall along lines C & D):

$$w = \left(\frac{4.02\text{ kips}}{6\text{ ft}}\right) - \left(0.025\text{ kip/ft}^2\right)\left(22.5\text{ ft}\right)$$
$$= 0.108\text{ kip/ft}$$

where 4.02 kips is the snow load reaction, including drift, from the 24KCS4 joist at the screen wall.

Mechanical equipment and screen wall (average):

$w = 40$ psf

The spandrel panel weight was calculated as 0.413 kip/ft as part of the selection process for the W16×26 roof beams at the east and west ends of the building.

The mechanical room dead load of 0.060 kip/ft^2 and snow load of 0.040 kip/ft^2 was determined as part of the selection process for the W14×22 roof beams at the mechanical area.

A summary of the column loads at the roof is given in Table III-2.

Table III-2
Summary of Column Loads at the Roof

Column	Loading Width, ft	Loading Length, ft	Area, ft^2	DL, kip/ft^2	P_D, kips	SL, kip/ft^2	P_S, kips
2A, 2F, 3A, 3F, 4A, 4F, 5A, 5F, 6A ,6F, 7A, 7F	23.0	30.0	690	0.020	13.8	0.025	17.3
Snow drifting at side		30.0				0.0467 kip/ft	1.40
Exterior wall		30.0		0.413 kip/ft	12.4		
					26.2		**18.7**
1B, 1E, 8B, 8E	3.50	22.5	78.8	0.020	1.58	0.025	1.97
Snow drifting at side		22.5				0.0392 kip/ft	0.882
Exterior wall		22.5		0.413 kip/ft	9.29		
					10.9		**2.85**
1A, 1F, 8A, 8F	23.0	15.5	357 $-\dfrac{78.8\text{ ft}^2}{2}$ = 318	0.020	6.36	0.025	7.95
Snow drifting at end		11.8				0.0392 kip/ft	0.463
Snow drifting at side		15.5				0.0467 kip/ft	0.724
Exterior wall		27.3		0.413 kip/ft	11.3		
					17.7		**9.14**
1C, 1D, 8C, 8D	37.5	15.5	581 $-\dfrac{78.8\text{ ft}^2}{2}$ = 542	0.020	10.8	0.025	13.6
Snow drifting at end		26.3				0.0392 kip/ft	1.03
Exterior wall		26.3		0.413 kip/ft	10.9		
					21.7		**14.6**
2C, 2D, 7C, 7D	37.5	30.0	1,125	0.020	**22.5**	0.025	**28.1**
3C, 3D, 4C, 4D, 5C, 5D, 6C, 6D	22.5	30.0	675	0.020	13.5	0.025	16.9
Snow drifting		30.0				0.108 kip/ft	3.24
Mechanical area	15.0	30.0	450	0.060	27.0	0.040	18.0
					40.5		**38.1**

Floor loads (from previous calculations):

Dead load	=	75 psf
Snow load	=	80 psf
Total	=	155 psf

Calculate reduction in live loads, analyzed at the base of three floors (n = 3) using ASCE/SEI 7, Section 4.7.2. Note that the 6-in. cantilever of the floor slab has been ignored for the calculation of K_{LL} for columns in this building because it has a negligible effect.

Columns: 2A, 2F, 3A, 3F, 4A, 4F, 5A, 5F, 6A, 6F, 7A, 7F
Exterior column without cantilever slabs
K_{LL} = 4 (ASCE/SEI 7, Table 4.7-1)
L_o = 80 psf
n = 3 (three floors supported)

$$A_T = (22.5\text{ ft} + 0.5\text{ ft})(30\text{ ft})$$
$$= 690\text{ ft}^2$$

Using ASCE/SEI 7, Equation 4.7-1:

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL} n A_T}}\right) \geq 0.4L_o$$

$$= (80 \text{ psf})\left[0.25 + \frac{15}{\sqrt{(4)(3)(690 \text{ ft}^2)}}\right] > 0.4(80 \text{ psf})$$

$$= 33.2 \text{ psf} > 32.0 \text{ psf}$$

Therefore, use L = 33.2 psf.

Columns: 1B, 1E, 8B, 8E
Exterior column without cantilever slabs
K_{LL} = 4 (ASCE/SEI 7, Table 4.7-1)
L_o = 80 psf
n = 3

$$A_T = (5.00 \text{ ft} + 0.5 \text{ ft})(22.5 \text{ ft})$$

$$= 124 \text{ ft}^2$$

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL} n A_T}}\right) \geq 0.4L_o$$

$$= (80 \text{ psf})\left[0.25 + \frac{15}{\sqrt{(4)(3)(124 \text{ ft}^2)}}\right] > 0.4(80 \text{ psf})$$

$$= 51.1 \text{ psf} > 32.0 \text{ psf}$$

Use L = 51.1 psf.

Columns: 1A, 1F, 8A, 8F
Corner column without cantilever slabs
K_{LL} = 4 (ASCE/SEI 7, Table 4.7-1)
L_o = 80 psf
n = 3

$$A_T = (15.0 \text{ ft} + 0.5 \text{ ft})(22.5 \text{ ft} + 0.5 \text{ ft}) - \left(\frac{124 \text{ ft}^2}{2}\right)$$

$$= 295 \text{ ft}^2$$

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL} n A_T}}\right) \geq 0.4L_o$$

$$= (80 \text{ psf})\left[0.25 + \frac{15}{\sqrt{(4)(3)(295 \text{ ft}^2)}}\right] > 0.4(80 \text{ psf})$$

$$= 40.2 \text{ psf} > 32.0 \text{ psf}$$

Therefore, use L = 40.2 psf.

Columns: 1C, 1D, 8C, 8D
Exterior column without cantilever slabs
K_{LL} = 4 (ASCE/SEI 7, Table 4.7-1)
L_o = 80 psf
n = 3

$$A_T = (15.0\text{ ft} + 0.5\text{ ft})\left(\frac{45\text{ ft} + 30\text{ ft}}{2}\right) - \left(\frac{124\text{ ft}^2}{2}\right)$$
$$= 519\text{ ft}^2$$

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL}nA_T}}\right) \geq 0.4L_o$$
$$= (80\text{ psf})\left[0.25 + \frac{15}{\sqrt{(4)(3)(519\text{ ft}^2)}}\right] > 0.4(80\text{ psf})$$
$$= 35.2\text{ psf} > 32.0\text{ psf}$$

Therefore, use L = 35.2 psf.

Columns: 2C, 2D, 3C, 3D, 4C, 4D, 5C, 5D, 6C, 6D, 7C, 7D
Interior column
K_{LL} = 4 (ASCE/SEI 7, Table 4.7-1)
L_o = 80 psf
n = 3

$$A_T = \left(\frac{45\text{ ft} + 30\text{ ft}}{2}\right)(30\text{ ft})$$
$$= 1{,}125\text{ ft}^2$$

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL}nA_T}}\right) \geq 0.4L_o$$
$$= (80\text{ psf})\left[0.25 + \frac{15}{\sqrt{(4)(3)(1{,}125\text{ ft}^2)}}\right] < 0.4(80\text{ psf})$$
$$= 30.3\text{ psf} < 32.0\text{ psf}$$

Therefore, use L = 32.0 psf.

A summary of the column loads at the floors is given in Table III-3.

Table III-3 Summary of Column Loads at the Floors							
Column	Loading Width, ft	Loading Length, ft	Area, ft^2	DL, kip/ft^2	P_D, kips	LL, kip/ft^2	P_L, kips
2A, 2F, 3A, 3F, 4A, 4F, 5A, 5F, 6A ,6F, 7A, 7F	23.0	30.0	690	0.075	51.8	0.0332	22.9
Exterior wall		30.0		0.503 kip/ft	15.1		
					66.9		**22.9**
1B, 1E, 8B, 8E	5.50	22.5	124	0.075	9.30	0.0511	6.34
Exterior wall		22.5		0.503 kip/ft	11.3		
					20.6		**6.34**
1A, 1F, 8A, 8F	23.0	15.5	$357 - \frac{124\text{ ft}^2}{2} = 295$	0.075	22.1	0.0402	11.9
Exterior wall		27.3		0.503 kip/ft	13.7		
					35.8		**11.9**
1C, 1D, 8C, 8D	37.5	15.5	$581 - \frac{124\text{ ft}^2}{2} = 519$	0.075	38.9	0.0352	18.3
Exterior wall		26.3		0.503 kip/ft	13.2		
					52.1		**18.3**
2C, 2D, 3C, 3D, 4C, 4D, 5C, 5D, 6C, 6D, 7C, 7D	37.5	30.0	1,125	0.075	**84.4**	0.0320	**36.0**

The spandrel panel weight was calculated as 0.503 kip/ft as part of the selection process for the W18×35 edge beams at the north and south ends of the building.

The column loads are summarized in Table III-4.

Table III-4
Summary of Column Loads

Column	Floor	P_D, kips	P_L, kips	P_S, kips
2A, 2F, 3A, 3F, 4A, 4F, 5A, 5F, 6A, 6F, 7A, 7F	Roof	26.2		18.7
	4th	66.9	22.9	
	3rd	66.9	22.9	
	2nd	66.9	22.9	
	Total	**227**	**68.7**	**18.7**
1B, 1E, 8B, 8E	Roof	10.9		2.85
	4th	20.6	6.34	
	3rd	20.6	6.34	
	2nd	20.6	6.34	
	Total	**72.7**	**19.0**	**2.85**
1A, 1F, 8A, 8F	Roof	17.7		9.14
	4th	35.8	11.9	
	3rd	35.8	11.9	
	2nd	35.8	11.9	
	Total	**125**	**35.7**	**9.14**
1C, 1D, 8C, 8D	Roof	21.7		14.6
	4th	52.1	18.3	
	3rd	52.1	18.3	
	2nd	52.1	18.3	
	Total	**178**	**54.9**	**14.6**
2C, 2D, 7C, 7D	Roof	22.5		28.1
	4th	84.4	36.0	
	3rd	84.4	36.0	
	2nd	84.4	36.0	
	Total	**276**	**108**	**28.1**
3C, 3D, 4C, 4D, 5C, 5D, 6C, 6D	Roof	40.5		38.1
	4th	84.4	36.0	
	3rd	84.4	36.0	
	2nd	84.4	36.0	
	Total	**294**	**108**	**38.1**

SELECT TYPICAL INTERIOR LEANING COLUMNS

Columns 3C, 3D, 4C, 4D, 5C, 5D, 6C, 6D

Elevation of second floor slab: 113.5 ft
Elevation of first floor slab: 100 ft
Column unbraced length: $L_x = L_y = 13.5$ ft

Note: $K_x = K_y = 1.0$ for a leaning column when using the effective length method.

$$\begin{aligned} L_{cx} &= K_x L_x \\ &= 1.0(13.5 \text{ ft}) \\ &= 13.5 \text{ ft} \end{aligned}$$

$$\begin{aligned} L_{cy} &= K_y L_y \\ &= 1.0(13.5 \text{ ft}) \\ &= 13.5 \text{ ft} \end{aligned}$$

From ASCE/SEI 7, Chapter 2, the required axial strength is determined using the following controlling load combinations (including the 0.5 live load reduction permitted for LRFD):

LRFD	ASD
$P_u = 1.2(294 \text{ kips}) + 1.6(108 \text{ kips}) + 0.5(38.1 \text{ kips})$ $= 545 \text{ kips}$	$P_a = 294 \text{ kips} + 0.75(108 \text{ kips}) + 0.75(38.1 \text{ kips})$ $= 404 \text{ kips}$

Using AISC *Manual* Table 4-1a, enter with L_c = 14.0 ft (conservative) and proceed across the table until reaching the lightest size that has sufficient available strength at the required unbraced length. Select a W12×65. The available strength in axial compression is:

LRFD	ASD
$\phi_c P_n = 685 \text{ kips} > 545 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 456 \text{ kips} > 404 \text{ kips}$ **o.k.**

Note: A W14×68 would also be an acceptable selection.

Columns 2C, 2D, 7C, 7D

Elevation of second floor slab: 113.5 ft
Elevation of first floor slab: 100 ft
Column unbraced length: $L_x = L_y = 13.5$ ft

Note: $K_x = K_y = 1.0$ for a leaning column when using the effective length method.

$$\begin{aligned} L_{cx} &= K_x L_x \\ &= 1.0(13.5 \text{ ft}) \\ &= 13.5 \text{ ft} \end{aligned}$$

$$\begin{aligned} L_{cy} &= K_y L_y \\ &= 1.0(13.5 \text{ ft}) \\ &= 13.5 \text{ ft} \end{aligned}$$

From ASCE/SEI 7, Chapter 2, the required axial strength is determined using the following controlling load combinations (including the 0.5 live load reduction permitted for LRFD):

LRFD	ASD
$P_u = 1.2(276\text{ kips}) + 1.6(108\text{ kips}) + 0.5(28.1\text{ kips})$ $= 518\text{ kips}$	$P_a = 276\text{ kips} + 0.75(108\text{ kips}) + 0.75(28.1\text{ kips})$ $= 378\text{ kips}$

Using AISC *Manual* Table 4-1a, enter with L_c = 14.0 ft (conservative) and proceed across the table until reaching the lightest size that has sufficient available strength at the required unbraced length. Select a W12×65. The available strength in axial compression is:

LRFD	ASD
$\phi_c P_n = 685\text{ kips} > 518\text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 456\text{ kips} > 378\text{ kips}$ **o.k.**

Note: A W14×61 would also be an acceptable selection. However, W12×65 columns were selected to keep sizes consistent for all interior columns.

SELECT TYPICAL EXTERIOR LEANING COLUMNS

Columns 1B, 1E, 8B, 8E

Elevation of second floor slab: 113.5 ft
Elevation of first floor slab: 100 ft
Column unbraced length: $L_x = L_y = 13.5$ ft

Note: $K_x = K_y = 1.0$ for a leaning column when using the effective length method.

$$\begin{aligned} L_{cx} &= K_x L_x \\ &= 1.0(13.5\text{ ft}) \\ &= 13.5\text{ ft} \end{aligned}$$

$$\begin{aligned} L_{cy} &= K_y L_y \\ &= 1.0(13.5\text{ ft}) \\ &= 13.5\text{ ft} \end{aligned}$$

From ASCE/SEI 7, Chapter 2, the required axial strength is determined using the following controlling load combinations (including the 0.5 live load reduction permitted for LRFD):

LRFD	ASD
$P_u = 1.2(72.7\text{ kips}) + 1.6(19.0\text{ kips}) + 0.5(2.85\text{ kips})$ $= 119\text{ kips}$	$P_a = 72.7\text{ kips} + 0.75(19.0\text{ kips}) + 0.75(2.85\text{ kips})$ $= 89.1\text{ kips}$

Using AISC *Manual* Table 4-1a, enter with L_c = 14.0 ft (conservative) and proceed across the table until reaching the lightest size that has sufficient available strength at the required unbraced length. Select a W12×40. The available strength in axial compression is:

LRFD	ASD
$\phi_c P_n = 304\text{ kips} > 119\text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 202\text{ kips} > 89.1\text{ kips}$ **o.k.**

Note, A W12 column was selected above for ease of erection of framing beams (bolted double-angle connections can be used without bolt staggering). Final column selections at the moment and braced frames are illustrated later in this example.

WIND LOAD DETERMINATION

Use the Envelope Procedure for simple diaphragm buildings from ASCE/SEI 7, Chapter 28, Part 2.

To qualify for the simplified wind load method for low-rise buildings, per ASCE/SEI 7, Section 28.5.2, the following conditions must be met:

1. Simple diaphragm building **o.k.**
2. Low-rise building ≤ 60 ft **o.k.**
3. Enclosed, and conforms to wind borne debris provisions **o.k.**
4. Regular-shaped **o.k.**
5. Not a flexible building **o.k.**
6. Does not have response characteristics requiring special considerations **o.k.**
7. Symmetrical shape with flat or gable roof with θ ≤ 45° **o.k.**
8. Torsional load cases from ASCE/SEI 7, Figure 28.3-1 do not control design of MWFRS **o.k.**

Define input parameters

1. Risk category: II from ASCE/SEI 7, Table 1.5-1
2. Basic wind speed: V = 107 mph (3-s) from ASCE/SEI 7, Figure 26.5-1B
3. Exposure category: C from ASCE/SEI 7, Section 26.7.3
4. Topographic factor: K_{zt} = 1.0 from ASCE/SEI 7, Section 26.8.2
5. Mean roof height: 55.0 ft
6. Height and exposure adjustment: λ = 1.59 from ASCE/SEI 7, Figure 28.5-1
7. Roof angle: θ = 0°

$$p_s = \lambda K_{zt} p_{s30}$$ (ASCE/SEI 7, Eq. 28.5-1)

$$= (1.59)(1.0)(18.2 \text{ psf}) = 28.9 \text{ psf (Horizontal pressure zone A)}$$

$$= (1.59)(1.0)(12.0 \text{ psf}) = 19.1 \text{ psf (Horizontal pressure zone C)}$$

$$= (1.59)(1.0)(-21.9 \text{ psf}) = -34.8 \text{ psf (Vertical pressure zone E)}$$

$$= (1.59)(1.0)(-12.4 \text{ psf}) = -19.7 \text{ psf (Vertical pressure zone F)}$$

$$= (1.59)(1.0)(-15.2 \text{ psf}) = -24.2 \text{ psf (Vertical pressure zone G)}$$

$$= (1.59)(1.0)(-9.59 \text{ psf}) = -15.2 \text{ psf (Vertical pressure zone H)}$$

a = 10% of least horizontal dimension or 0.4h, whichever is smaller, but not less than either 4% of least horizontal dimension or 3 ft

a = the lesser of:
 10% of the least horizontal dimension = 12.3 ft
 40% of the eave height = 22.0 ft
 but not less than 4% of the least horizontal dimension or 3 ft = 4.92 ft

Thus, a = 12.3 ft and 2a = 24.6 ft.

Zone A: End zone of wall (width = 2a)
Zone C: Interior zone of wall
Zone E: End zone of windward roof (width = 2a)
Zone F: End zone of leeward roof (width = 2a)
Zone G: Interior zone of windward roof
Zone H: Interior zone of leeward roof

Calculate load on roof diaphragm

Mechanical screen wall height:	6 ft
Wall height:	0.5[55 ft – 3(13.5 ft)] = 7.25 ft
Parapet wall height:	2 ft
Total wall height at roof at screen wall:	$6\text{ ft}+7.25\text{ ft}=13.3\text{ ft}$
Total wall height at roof at parapet:	$2\text{ ft}+7.25\text{ ft}=9.25\text{ ft}$

$$\begin{aligned}w_{s(A)}&=(28.9\text{ psf})(9.25\text{ ft})\\&=267\text{ plf}\end{aligned}$$

$$\begin{aligned}w_{s(C)}&=(19.1\text{ psf})(9.25\text{ ft})\\&=177\text{ plf (at parapet)}\end{aligned}$$

$$\begin{aligned}w_{s(C)}&=(19.1\text{ psf})(13.3\text{ ft})\\&=254\text{ plf (at screen wall)}\end{aligned}$$

Calculate load on fourth floor diaphragm

Wall height:	$0.5(55.0\text{ ft}-40.5\text{ ft})=7.25\text{ ft}$
	$0.5(40.5\text{ ft}-27.0\text{ ft})=6.75\text{ ft}$
Total wall height at floor:	$6.75\text{ ft}+7.25\text{ ft}=14.0\text{ ft}$

$$\begin{aligned}w_{s(A)}&=(28.9\text{ psf})(14.0\text{ ft})\\&=405\text{ plf}\end{aligned}$$

$$\begin{aligned}w_{s(C)}&=(19.1\text{ psf})(14.0\text{ ft})\\&=267\text{ plf}\end{aligned}$$

Calculate load on third floor diaphragm

Wall height:	$0.5(40.5\text{ ft}-27.0\text{ ft})=6.75\text{ ft}$
	$0.5(27.0\text{ ft}-13.5\text{ ft})=6.75\text{ ft}$
Total wall height at floor:	$6.75\text{ ft}+6.75\text{ ft}=13.5\text{ ft}$

$$w_{s(A)} = (28.9 \text{ psf})(13.5 \text{ ft})$$
$$= 390 \text{ plf}$$

$$w_{s(C)} = (19.1 \text{ psf})(13.5 \text{ ft})$$
$$= 258 \text{ plf}$$

Calculate load on second floor diaphragm

Wall height:	$0.5(27.0 \text{ ft} - 13.5 \text{ ft}) = 6.75 \text{ ft}$
	$0.5(13.5 \text{ ft} - 0 \text{ ft}) = 6.75 \text{ ft}$
Total wall height at floor:	$6.75 \text{ ft} + 6.75 \text{ ft} = 13.5 \text{ ft}$

$$w_{s(A)} = (28.9 \text{ psf})(13.5 \text{ ft})$$
$$= 390 \text{ plf}$$

$$w_{s(C)} = (19.1 \text{ psf})(13.5 \text{ ft})$$
$$= 258 \text{ plf}$$

Determine the wind load on each frame at each level. Conservatively apply the end zone pressures on both ends of the building simultaneously,

where
l = length of structure, ft
b = width of structure, ft
h = height of wall at building element, ft

For wind from a north or south direction:

Total load to each frame:

$$P_{W(N\text{-}S)} = w_{s(A)}(2a) + w_{s(C)}\left(\frac{l}{2} - 2a\right)$$

Shear in diaphragm:

$$v_{(N\text{-}S)} = \frac{P_{W(N\text{-}S)}}{120 \text{ ft}}, \text{ for roof}$$

$$v_{(N\text{-}S)} = \frac{P_{W(N\text{-}S)}}{90 \text{ ft}}, \text{ for floors (deduction for stair openings)}$$

For wind from an east or west direction:

Total load to each frame:

$$P_{W(E\text{-}W)} = w_{s(A)}(2a) + w_{s(C)}\left(\frac{b}{2} - 2a\right)$$

Shear in diaphragm:

$$v_{(E\text{-}W)} = \frac{P_{W(E\text{-}W)}}{210 \text{ ft}}, \text{ for roof and floors}$$

Table III-5 summarizes the total wind load in each direction acting on a steel frame at each level. The wind load at the ground level has not been included in the chart because it does not affect the steel frame.

The roof level dimensions exclude the screen wall area. The floor level dimensions correspond to the outside dimensions of the cladding.

Table III-5
Summary of Wind Loads at Each Level

	l, ft	b, ft	$2a$, ft	h, ft	$p_{s(A)}$, psf	$p_{s(C)}$, psf	$w_{s(A)}$, plf	$w_{s(C)}$, plf	$P_{W(N\text{-}S)}$, kips	$P_{W(E\text{-}W)}$, kips	$v_{(N\text{-}S)}$, plf	$v_{(E\text{-}W)}$, plf
Screen	93.0	33.0	0	13.3	0	19.1	0	254	11.8	4.19	–	–
Roof	120	90.0	24.6	9.25	28.9	19.1	267	177	12.8	10.2	205	68.5
4th	213	123	24.6	14.0	28.9	19.1	405	267	31.8	19.8	353	94.3
3rd	213	123	24.6	13.5	28.9	19.1	390	258	30.7	19.1	341	91.0
2nd	213	123	24.6	13.5	28.9	19.1	390	258	30.7	19.1	341	91.0
Base									118	72.4		

SEISMIC LOAD DETERMINATION

The floor plan area: 120 ft, column center line to column center line, by 210 ft, column centerline to column center line, with the edge of floor slab or roof deck 6 in. beyond the column center line.

$$\begin{aligned}\text{Area} &= (121\text{ ft})(211\text{ ft})\\ &= 25{,}500\text{ ft}^2\end{aligned}$$

The perimeter cladding system length:

$$\begin{aligned}\text{Length} &= (2)(123\text{ ft}) + (2)(213\text{ ft})\\ &= 672\text{ ft}\end{aligned}$$

The perimeter cladding weight at floors:

Brick spandrel panel with metal stud backup:	$(7.50\text{ ft})(0.055\text{ kip/ft}^2)$	$= 0.413$ kip/ft
Window wall system:	$(6.00\text{ ft})(0.015\text{ kip/ft}^2)$	$= 0.090$ kip/ft
Total:		**0.503 kip/ft**

Typical roof dead load (from previous calculations):

Although 40 psf was used to account for the mechanical units and screen wall for the beam and column design, the entire mechanical area will not be uniformly loaded. Use 30% of the uniform 40 psf mechanical area load to determine the total weight of all of the mechanical equipment and screen wall for the seismic load determination.

Roof area:	$(25{,}500\text{ ft}^2)(0.020\text{ kip/ft}^2)$	$= 510$ kips
Wall perimeter:	$(672\text{ ft})(0.413\text{ kip/ft})$	$= 278$ kips
Mechanical area:	$(2{,}700\text{ ft}^2)(0.3)(0.040\text{ kip/ft}^2)$	$= 32.4$ kips
Total:		**820 kips**

Typical third and fourth floor dead load:

Note: An additional 10 psf has been added to the floor dead load to account for partitions per ASCE/SEI 7, Section 12.7.2.

Floor area:	$(25{,}500\text{ ft}^2)(0.085\text{ kip/ft}^2)$	$= 2{,}170$ kips
Wall perimeter:	$(672\text{ ft})(0.503\text{ kip/ft})$	$= 338$ kips
Total:		**2,510 kips**

Second floor dead load (the floor area is reduced because of the open atrium):

Floor area:	$(24{,}700\text{ ft}^2)(0.085\text{ kip/ft}^2)$	$= 2{,}100$ kips
Wall perimeter:	$(672\text{ ft})(0.503\text{ kip/ft})$	$= 338$ kips
Total:		**2,440 kips**

Total dead load of the building:

Roof	820 kips
Fourth floor	2,510 kips
Third floor	2,510 kips
Second floor	2,440 kips
Total	**8,280 kips**

Calculate the seismic forces.

Determine the seismic risk category and importance factors.

Office Building: Risk Category II, from ASCE/SEI 7, Table 1.5-1
Seismic Importance Factor: I_e = 1.00, from ASCE/SEI 7, Table 1.5-2

The site coefficients are given in this example. S_S and S_1 can also be determined from ASCE/SEI 7, Figures 22-1 and 22-2, respectively.

$S_S = 0.121g$
$S_1 = 0.060g$

Soil, Site Class D (given)

F_a @ $S_S \le 0.25$ = 1.6 from ASCE/SEI 7, Table 11.4-1
F_v @ $S_1 \le 0.1$ = 2.4 from ASCE/SEI 7, Table 11.4-2

Determine the maximum considered earthquake accelerations.

From ASCE/SEI 7, Equation 11.4-1:

$$\begin{aligned} S_{MS} &= F_a S_S \\ &= 1.6(0.121g) \\ &= 0.194g \end{aligned}$$

From ASCE/SEI 7, Equation 11.4-2:

$$\begin{aligned} S_{M1} &= F_v S_1 \\ &= 2.4(0.060g) \\ &= 0.144g \end{aligned}$$

Determine the design earthquake accelerations.

From ASCE/SEI 7, Equation 11.4-3:

$$\begin{aligned} S_{DS} &= \tfrac{2}{3} S_{MS} \\ &= \tfrac{2}{3}(0.194g) \\ &= 0.129g \end{aligned}$$

From ASCE/SEI 7, Equation 11.4-4:

$$
\begin{aligned}
S_{D1} &= \tfrac{2}{3}S_{M1} \\
&= \tfrac{2}{3}(0.144g) \\
&= 0.096g
\end{aligned}
$$

Determine the seismic design category from ASCE/SEI 7, Table 11.6-1.

With $S_{DS} < 0.167g$ and Risk Category II, Seismic Design Category A applies.

With $0.067g \le S_{D1} < 0.133g$ and Risk Category II, Seismic Design Category B applies.

Select the seismic force-resisting system from ASCE/SEI 7, Table 12.2-1. For Seismic Design Category B it is permissible to select a structural steel system not specifically detailed for seismic resistance (Item H). The response modification coefficient, R, is 3.

Determine the approximate fundamental period, T_a.

Building height, h_n = 55.0 ft

C_t = 0.02 and x = 0.75 from ASCE/SEI 7, Table 12.8-2 ("All other structural systems")

From ASCE/SEI 7, Equation 12.8-7:

$$
\begin{aligned}
T_a &= C_t h_n^x \\
&= (0.02)(55.0 \text{ ft})^{0.75} \\
&= 0.404 \text{ s}
\end{aligned}
$$
(ASCE/SEI 7, Eq. 12.8-7)

Determine the redundancy factor from ASCE/SEI 7, Section 12.3.4.1.

ρ = 1.0, for Seismic Design Category B

From ASCE/SEI 7, Equation 12.4-4a, determine the vertical seismic effect term:

$$
\begin{aligned}
E_v &= 0.2S_{DS}D \\
&= 0.2(0.129g)D \\
&= 0.0258D
\end{aligned}
$$
(ASCE/SEI 7, Eq. 12.4-4a)

From ASCE/SEI 7, Equation 12.4-3, determine the horizontal seismic effect term:

$$
\begin{aligned}
E_h &= \rho Q_E \\
&= 1.0(Q_E)
\end{aligned}
$$
(ASCE/SEI 7, Eq. 12.4-3)

The following seismic load combinations are as specified in ASCE/SEI 7, Sections 2.3.6 and 2.4.5 as directed by Section 12.4.2. Where the prescribed seismic load effect, $E = f(E_v, E_h)$, is combined with the effects of other loads, the following load combinations apply. Note that $L = 0.5L$ for LRFD per ASCE/SEI 7, Section 2.3.6 Exception 1.

LRFD	ASD
$1.2D + E_v + E_h + L + 0.2S$ $= 1.2D + 0.2S_{DS}D + \rho Q_E + 0.5L + 0.2S$ $= (1.2 + 0.0258)D + 1.0Q_E + 0.5L + 0.2S$ $= 1.23D + 1.0Q_E + 0.5L + 0.2S$	$1.0D + 0.7E_v + 0.7E_h$ $= 1.0D + 0.7(0.2S_{DS}D) + 0.7\rho Q_E$ $= [1.0 + 0.7(0.0258)]D + 0.7(1.0)Q_E$ $= 1.02D + 0.7Q_E$

LRFD	ASD
$0.9D - E_v + E_h$ $= 0.9D - 0.2S_{DS}D + \rho Q_E$ $= (0.9 - 0.0258)D + 1.0Q_E$ $= 0.874D + 1.0Q_E$	$1.0D + 0.525E_v + 0.525E_h + 0.75L + 0.75S$ $= 1.0D + 0.525(0.2S_{DS}D) + 0.525\rho Q_E + 0.75L + 0.75S$ $= [1.0 + 0.525(0.0258)]D + 0.525(1.0)Q_E + 0.75L$ $+ 0.75S$ $= 1.01D + 0.525Q_E + 0.75L + 0.75S$ $0.6D - 0.7E_v + 0.7E_h$ $= 0.6D - 0.7(0.2S_{DS}D) + 0.7\rho Q_E$ $= [0.6 - 0.7(0.0258)]D + 0.7(1.0)Q_E$ $= 0.582D + 0.7Q_E$

Where the prescribed seismic load effect with overstrength, $E = f(E_v, E_{mh})$, is combined with the effects of other loads, the following load combinations apply.

The overstrength factor, Ω_o, is determined from ASCE/SEI 7, Table 12.2-1. $\Omega_o = 3$ for steel systems not specifically detailed for seismic resistance, excluding cantilever column systems.

Determine the horizontal seismic effect term including overstrength.

$$E_{mh} = \Omega_o Q_E \le E_{cl} \qquad \text{(from ASCE/SEI 7, Eq. 12.4-7)}$$
$$= 3(Q_E)$$

where Q_E is the effect from seismic forces from seismic base shear, V, as calculated per ASCE/SEI 7, Section 12.8.1; diaphragm design forces, F_{px}, as calculated per ASCE/SEI 7, Section 12.10; or seismic design force, F_p, as calculated per Section 13.3.1. The capacity-limited horizontal seismic load effect, E_{cl}, is defined in ASCE/SEI 7, Section 11.3.

LRFD	ASD
$1.2D + E_v + E_{mh} + L + 0.2S$ $= 1.2D + 0.2S_{DS}D + \Omega_o Q_E + 0.5L + 0.2S$ $= (1.2 + 0.0258)D + 3Q_E + 0.5L + 0.2S$ $= 1.23D + 3.0Q_E + 0.5L + 0.2S$ $0.9D - E_v + E_{mh}$ $= 0.9D - 0.2S_{DS}D + \Omega_o Q_E$ $= (0.9 - 0.0258)D + 3Q_E$ $= 0.874D + 3.0Q_E$	$1.0D + 0.7E_v + 0.7E_{mh}$ $= 1.0D + 0.7(0.2S_{DS}D) + 0.7\Omega_o Q_E$ $= [1.0 + 0.7(0.0258)]D + 0.7(3)Q_E$ $= 1.02D + 2.1Q_E$ $1.0D + 0.525E_v + 0.525E_{mh} + 0.75L + 0.75S$ $= 1.0D + 0.525(0.2S_{DS}D) + 0.525\Omega_o Q_E + 0.75L + 0.75S$ $= [1.0 + 0.525(0.0258)]D + 0.525(3)Q_E + 0.75L$ $+ 0.75S$ $= 1.01D + 1.58Q_E + 0.75L + 0.75S$ $0.6D - 0.7E_v + 0.7E_{mh}$ $= 0.6D - 0.7(0.2S_{DS}D) + 0.7\Omega_o Q_E$ $= [0.6 - 0.7(0.0258)]D + 0.7(3)Q_E$ $= 0.582D + 2.1Q_E$

Calculate the seismic base shear using ASCE/SEI 7, Section 12.8.1.

Determine the seismic response coefficient, C_s, from ASCE/SEI 7, Equation 12.8-2:

$$\begin{aligned} C_s &= \frac{S_{DS}}{\left(\dfrac{R}{I_e}\right)} \\ &= \frac{0.129}{\left(\dfrac{3}{1.00}\right)} \\ &= 0.0430 \end{aligned}$$

Let $T_a = T$, as is permitted in Section 12.8.2. From ASCE/SEI 7, Figure 22-14, $T_L = 12 > T$ (midwestern city); therefore, use ASCE/SEI 7, Section 12.8.1.1, to determine the upper limit of C_s.

$$\begin{aligned} C_s &= \frac{S_{D1}}{T\left(\dfrac{R}{I_e}\right)} \\ &= \frac{0.096}{0.404\left(\dfrac{3}{1.00}\right)} \\ &= 0.0792 \end{aligned}$$ (ASCE/SEI 7, Eq. 12.8-3)

C_s shall not be taken less than:

$$\begin{aligned} C_s &= 0.044 S_{DS} I_e \geq 0.01 \\ &= 0.044(0.129)(1.00) < 0.01 \\ &= 0.00568 < 0.01 \end{aligned}$$ (ASCE/SEI 7, Eq. 12.8-5)

Therefore, $C_s = 0.0430$.

Calculate the seismic base shear from ASCE/SEI 7, Section 12.8.1:

$$\begin{aligned} V &= C_s W \\ &= 0.0430(8{,}280 \text{ kips}) \\ &= 356 \text{ kips} \end{aligned}$$ (ASCE/SEI 7, Eq. 12.8-1)

Determine vertical distribution of seismic forces from ASCE/SEI 7, Section 12.8.3.

$$\begin{aligned} F_x &= C_{vx} V \\ &= C_{vx}(356 \text{ kips}) \end{aligned}$$ (ASCE/SEI 7, Eq. 12.8-11)

$$C_{vx} = \frac{w_x {h_x}^k}{\sum_{i=1}^{n} w_i {h_i}^k}$$ (ASCE/SEI 7, Eq. 12.8-12)

for structures having a period of 0.5 s or less, $k = 1$.

Determine horizontal shear distribution at each level per ASCE/SEI 7, Section 12.8.4.

$$V_x = \sum_{i=x}^{n} F_i \qquad \text{(ASCE/SEI, Eq. 12.8-13)}$$

Determine the overturning moment at each level per ASCE/SEI 7, Section 12.8.5.

$$M_x = \sum_{i=x}^{n} F_i (h_i - h_x)$$

The seismic forces at each level are summarized in Table III-6.

Table III-6
Summary of Seismic Forces at Each Level

	w_x, kips	h_x^k, ft	$w_x h_x^k$, kip-ft	C_{vx}	F_x, kips	V_x, kips	M_x, kip-ft
Roof	820	55.0	45,100	0.182	64.8	64.8	
4th	2,510	40.5	102,000	0.411	146	211	940
3rd	2,510	27.0	67,800	0.273	97.2	308	3,790
2nd	2,440	13.5	32,900	0.133	47.3	355	7,940
Base	**8,280**		**248,000**		**355**		**12,700**

Calculate strength and determine rigidity of diaphragms.

Determine the diaphragm design forces from ASCE/SEI 7, Section 12.10.1.1.

F_{px} is the largest of:

1. The force F_x at each level determined by the vertical distribution above

2. $F_{px} = \dfrac{\sum_{i=x}^{n} F_i}{\sum_{i=x}^{n} w_i} w_{px} \le 0.4 S_{DS} I_e w_{px}$, from ASCE/SEI 7, Equations 12.10-1 and 12.10-3

$$\le 0.4(0.129)(1.00)w_{px}$$
$$\le 0.0516 w_{px}$$

3. $F_{px} = 0.2 S_{DS} I_e w_{px}$, from ASCE/SEI 7, Equation 12.10-2

$$= 0.2(0.129)(1.00)w_{px}$$
$$= 0.0258 w_{px}$$

The diaphragm shear forces include the effects of openings in the diaphragm (such as stair shafts) and an accidental torsion calculated using an eccentricity of 5% of the building dimension per ASCE/SEI 7, Section 12.8.4. The accidental torsion resulted in a 10% increase in the shear force.

A summary of the diaphragm forces is given in Table III-7,

where

F_{px} = max(A, B, C)

A = force at a level based on the vertical distribution of seismic forces

$$B = F_{px} = \frac{\sum_{i=x}^{n} F_i}{\sum_{i=x}^{n} w_i} w_{px} \leq 0.4 S_{DS} I_e w_{px}$$

C $= 0.2 S_{DS} I_e w_{px}$

L = the length of the frame connected to the diaphragm (in the N-S or E-W direction)

V = shear force in the diaphragm

Table III-7
Summary of Diaphragm Forces

	w_{px}, kips	A, kips	B, kips	C, kips	F_{px}, kips	$L_{(N\text{-}S)}$, ft	$L_{(E\text{-}W)}$, ft	$v_{(N\text{-}S)}$, plf	$v_{(E\text{-}W)}$, plf
Roof	820	64.8	42.3	21.2	64.8	240	420	297	170
4th	2,510	146	130	64.8	146	180	420	892	382
3rd	2,510	97.2	130	64.8	130	180	420	794	340
2nd	2,440	47.3	105	63.0	105	180	420	642	275

Roof

Roof deck: 1½-in.-deep, 22 gage, wide rib
Support fasteners: ⅝-in. puddle welds in 36/5 pattern
Sidelap fasteners: (3) #10 TEK screws
Joist spacing: s = 6.00 ft
Diaphragm length: 210 ft
Diaphragm width: l_v =120 ft

By inspection, the critical condition for the diaphragm is loading from the north or south directions.

LRFD	ASD
From the ASCE/SEI 7 load combinations for strength design, the earthquake load is:	From the ASCE/SEI 7 load combinations for allowable stress design, the earthquake load is:
$v_r = E_h$ $= \rho Q_E$ $= 1.0(0.297 \text{ klf})$ $= 0.297 \text{ klf}$	$v_r = 0.7E_h$ $= 0.7\rho Q_E$ $= 0.7(1.0)(0.297 \text{ klf})$ $= 0.208 \text{ klf}$
The wind load is:	The wind load is:
$v_r = 1.0W$ $= 1.0(0.205 \text{ klf})$ $= 0.205 \text{ klf}$	$v_r = 0.6W$ $= 0.6(0.205 \text{ klf})$ $= 0.123 \text{ klf}$

From the SDI *Diaphragm Design Manual* (SDI, 2015), the available shear strengths are determined as follows:

For panel buckling strength: $v_n = 3.88$ klf
For connection strength: $v_n = 0.815$ klf

LRFD	ASD
Panel buckling strength: $\phi v_n = 0.80(3.88 \text{ klf})$ $= 3.10 \text{ klf} > 0.297 \text{ ksf}$ **o.k.** Connection strength: Earthquake $\phi v_n = 0.55(0.815 \text{ klf})$ $= 0.448 \text{ klf} > 0.297 \text{ ksf}$ **o.k.** Wind $\phi v_n = 0.70(0.815 \text{ klf})$ $= 0.571 \text{ klf} > 0.205 \text{ ksf}$ **o.k.**	Panel buckling strength: $\dfrac{v_n}{\Omega} = \dfrac{3.88 \text{ klf}}{2.00}$ $= 1.94 \text{ klf} > 0.208 \text{ klf}$ **o.k.** Connection strength: Earthquake $\dfrac{v_n}{\Omega} = \dfrac{0.815 \text{ klf}}{3.00}$ $= 0.272 \text{ klf} > 0.208 \text{ ksf}$ **o.k.** Wind $\dfrac{v_n}{\Omega} = \dfrac{0.815 \text{ klf}}{2.35}$ $= 0.347 \text{ klf} > 0.123 \text{ ksf}$ **o.k.**

Check diaphragm flexibility.

From the SDI *Diaphragm Design Manual* (SDI, 2015):

$$
\begin{aligned}
D_{xx} &= 607 \text{ ft} \\
K_1 &= 0.286 \text{ ft}^{-1} \\
K_2 &= 870 \text{ kip/in.} \\
K_4 &= 3.55
\end{aligned}
$$

From SDI *Diaphragm Design Manual*, Section 9:

$$
\begin{aligned}
G' &= \frac{K_2}{K_4 + \dfrac{0.3D_{xx}}{s} + 3K_1 s} \\
&= \frac{870 \text{ kip/in.}}{3.55 + \dfrac{0.3(607 \text{ ft})}{6.00 \text{ ft}} + 3\left(\dfrac{0.286}{\text{ft}}\right)(6.00 \text{ ft})} \\
&= 22.3 \text{ kip/in.}
\end{aligned}
$$

Seismic loading on diaphragm.

$$
\begin{aligned}
w &= \frac{64.8 \text{ kips}}{210 \text{ ft}} \\
&= 0.309 \text{ klf}
\end{aligned}
$$

Calculate the maximum diaphragm deflection.

$$\Delta = \frac{wL^2}{8l_v G'}$$
$$= \frac{(0.309 \text{ klf})(210 \text{ ft})^2}{8(120 \text{ ft})(22.3 \text{ kip/in.})}$$
$$= 0.637 \text{ in.}$$

Story drift = 0.154 in. (from computer output)

The diaphragm deflection exceeds two times the story drift; therefore, the diaphragm may be considered to be flexible in accordance with ASCE/SEI 7, Section 12.3.1.3.

The roof diaphragm is flexible in the N-S direction, but using a rigid diaphragm distribution is more conservative for the analysis of this building. By similar reasoning, the roof diaphragm will also be treated as a rigid diaphragm in the E-W direction.

Third and fourth floors

Floor deck: 3-in.-deep, 22 gage, composite deck with normal weight concrete
Support fasteners: ⅝-in. puddle welds in a 36/4 pattern
Sidelap fasteners: (3) #10 TEK screws
Beam spacing: s = 10 ft
Diaphragm length: 210 ft
Diaphragm width: 120 ft
l_v = 120 ft – 30 ft = 90 ft, to account for the stairwell

By inspection, the critical condition for the diaphragm is loading from the north or south directions.

LRFD	ASD
From the ASCE/SEI 7 load combinations for strength design, the earthquake load for the fourth floor is: $v_r = E_h$ $= \rho Q_E$ $= 1.0(0.892 \text{ klf})$ $= 0.892 \text{ klf}$ For the fourth floor, the wind load is: $v_r = 1.0W$ $= 1.0(0.353 \text{ klf})$ $= 0.353 \text{ klf}$	From the ASCE/SEI 7 load combinations for strength design, the earthquake load for the fourth floor is: $v_r = E_h$ $= 0.7\rho Q_E$ $= 0.7(1.0)(0.892 \text{ klf})$ $= 0.624 \text{ klf}$ For the fourth floor, the wind load is: $v_r = 0.6W$ $= 0.6(0.353 \text{ klf})$ $= 0.212 \text{ klf}$

LRFD	ASD
From the ASCE/SEI 7 load combinations for strength design, the earthquake load for the third floor is: $v_r = E_h = \rho Q_E = 1.0(0.794\text{ klf}) = 0.794\text{ klf}$ For the third floor, the wind load is: $v_r = 1.0W = 1.0(0.341\text{ klf}) = 0.341\text{ klf}$	From the ASCE/SEI 7 load combinations for strength design, the earthquake load for the third floor is: $v_r = E_h = 0.7\rho Q_E = 0.7(1.0)(0.794\text{ klf}) = 0.556\text{ klf}$ For the third floor, the wind load is: $v_r = 0.6W = 0.6(0.341\text{ klf}) = 0.205\text{ klf}$

From the SDI *Diaphragm Design Manual* (SDI, 2015), the nominal connection shear strength is v_n = 5.38 klf.

Calculate the available strengths.

LRFD	ASD
Connection Strength (same for earthquake or wind) (SDI, 2015) $\phi v_n = 0.5(5.38\text{ klf}) = 2.69\text{ klf} > 0.892\text{ klf}$ **o.k.**	Connection Strength (same for earthquake or wind) (SDI, 2015) $\dfrac{v_n}{\Omega} = \dfrac{5.38\text{ klf}}{3.25} = 1.66\text{ klf} > 0.624\text{ klf}$ **o.k.**

Check diaphragm flexibility.

From the SDI *Diaphragm Design Manual* (SDI, 2015):

$$
\begin{aligned}
K_1 &= 0.318\text{ ft}^{-1} \\
K_2 &= 870\text{ kip/in.} \\
K_3 &= 2{,}380\text{ kip/in.} \\
K_4 &= 3.54
\end{aligned}
$$

$$
\begin{aligned}
G' &= \left(\frac{K_2}{K_4 + 3K_1 s}\right) + K_3 \\
&= \left[\frac{870\text{ kip/in.}}{3.54 + 3\left(\dfrac{0.318}{\text{ft}}\right)(10\text{ ft})}\right] + 2{,}380\text{ kip/in.} \\
&= 2{,}450\text{ kip/in.}
\end{aligned}
$$

Fourth floor

Calculate seismic loading on the diaphragm based on the fourth floor seismic load.

$$w = \frac{146 \text{ kips}}{210 \text{ ft}}$$
$$= 0.695 \text{ klf}$$

Calculate the maximum diaphragm deflection on the fourth floor.

$$\Delta = \frac{wL^2}{8l_vG'}$$
$$= \frac{(0.695 \text{ klf})(210 \text{ ft})^2}{8(90 \text{ ft})(2{,}450 \text{ kip/in.})}$$
$$= 0.0174 \text{ in.}$$

Third floor

Calculate seismic loading on the diaphragm based on the third floor seismic load.

$$w = \frac{130 \text{ kips}}{210 \text{ ft}}$$
$$= 0.619 \text{ klf}$$

Calculate the maximum diaphragm deflection on the third floor.

$$\Delta = \frac{wL^2}{8l_vG'}$$
$$= \frac{(0.619 \text{ klf})(210 \text{ ft})^2}{8(90 \text{ ft})(2{,}450 \text{ kip/in.})}$$
$$= 0.0155 \text{ in.}$$

The diaphragm deflection at the third and fourth floors is less than two times the story drift (story drift = 0.268 in. from computer output); therefore, the diaphragm is considered rigid in accordance with ASCE/SEI 7, Section 12.3.1.3. By inspection, the floor diaphragm will also be rigid in the E-W direction.

Second floor

Floor deck:	3-in.-deep, 22 gage, composite deck with normal weight concrete
Support fasteners:	⅝-in. puddle welds in a 36/4 pattern
Sidelap fasteners:	(3) #10 TEK screws
Beam spacing:	s = 10 ft
Diaphragm length:	210 ft
Diaphragm width:	120 ft

Because of the atrium opening in the floor diaphragm, an effective diaphragm depth of 75 ft will be used for the deflection calculations.

By inspection, the critical condition for the diaphragm is loading from the north or south directions.

LRFD	ASD
From the ASCE/SEI 7 load combinations for strength design, the earthquake load is: $v_r = E_h$ $= \rho Q_E$ $= 1.0(0.642\text{ klf})$ $= 0.642\text{ klf}$ The wind load is: $v_r = 1.0W$ $= 1.0(0.341\text{ klf})$ $= 0.341\text{ klf}$	From the ASCE/SEI 7 load combinations for strength design, the earthquake load is: $v_r = E_h$ $= 0.7\rho Q_E$ $= 0.7(1.0)(0.642\text{ klf})$ $= 0.449\text{ klf}$ The wind load is: $v_r = 0.6W$ $= 0.6(0.341\text{ klf})$ $= 0.205\text{ klf}$

From the SDI *Diaphragm Design Manual* (SDI, 2015), the nominal connection shear strength is: $v_n = 5.38$ klf.

Calculate the available strengths.

LRFD	ASD
Connection Strength (same for earthquake or wind) (SDI, 2015) $\phi v_n = 0.50(5.38\text{ klf})$ $= 2.69\text{ klf} > 0.642\text{ klf}$ **o.k.**	Connection Strength (same for earthquake or wind) (SDI, 2015) $\dfrac{v_n}{\Omega} = \dfrac{5.38\text{ klf}}{3.25}$ $= 1.66\text{ klf} > 0.449\text{ klf}$ **o.k.**

Check diaphragm flexibility.

From the SDI *Diaphragm Design Manual* (SDI, 2015):

$$K_1 = 0.318\text{ ft}^{-1}$$
$$K_2 = 870\text{ kip/in.}$$
$$K_3 = 2{,}380\text{ kip/in.}$$
$$K_4 = 3.54$$

$$G' = \left(\frac{K_2}{K_4 + 3K_1 s}\right) + K_3$$
$$= \left[\frac{870\text{ kip/in.}}{3.54 + 3\left(\dfrac{0.318}{\text{ft}}\right)(10\text{ ft})}\right] + 2{,}380\text{ kip/in.}$$
$$= 2{,}450\text{ kip/in.}$$

Calculate seismic loading on the diaphragm.

$$w = \frac{105\text{ kips}}{210\text{ ft}}$$
$$= 0.500\text{ klf}$$

Calculate the maximum diaphragm deflection.

$$\Delta = \frac{wL^2}{8bG'}$$
$$= \frac{(0.500 \text{ klf})(210 \text{ ft})^2}{8(75 \text{ ft})(2{,}450 \text{ kip/in.})}$$
$$= 0.0150 \text{ in.}$$

Story drift = 0.210 in. (from computer output)

The diaphragm deflection is less than two times the story drift; therefore, the diaphragm is considered rigid in accordance with ASCE/SEI 7, Section 12.3.1.3. By inspection, the floor diaphragm will also be rigid in the E-W direction.

Horizontal Shear Distribution and Torsion

The seismic forces to be applied in the frame analysis in each direction, including the effect of accidental torsion, in accordance with ASCE/SEI 7, Section 12.8.4, are shown in Tables III-8 and III-9.

Table III-8
Horizontal Shear Distribution including Accidental Torsion—Grids 1 and 8

	F_y	Load on Frame		Load to Grids 1 and 8 Accidental Torsion		Total
	kips	%	kips	%	kips	kips
Roof	64.8	50	32.4	5	3.24	35.6
4th	146	50	73.0	5	7.30	80.3
3rd	97.2	50	48.6	5	4.86	53.5
2nd	47.3	50	23.7	5	2.37	26.1
Base						196

Table III-9
Horizontal Shear Distribution including Accidental Torsion—Grids A and F

	F_y	Load on Frame		Load to Grids A and F Accidental Torsion		Total
	kips	%	kips	%	kips	kips
Roof	64.8	50	32.4	5	3.24	35.6
4th	146	50	73.0	5	7.30	80.3
3rd	97.2	50	48.6	5	4.86	53.5
2nd	47.3	50.8[1]	24.0	5	2.37	26.4
Base						196

[1] Note: In this example, Grids A and F have both been conservatively designed for the slightly higher load on Grid A due to the atrium opening. The increase in load is calculated Table III-10.

Table III-10

	Area, ft^2	Mass, kips	*y*-dist, ft	M_y, kip-ft
I	25,500	2,170	60.5	131,000
II	841	71.5	90.5	6,470
Base	24,700	2,100		125,000

$$y = \frac{125{,}000 \text{ kip-ft}}{2{,}100 \text{ kips}}$$
$$= 59.5 \text{ ft}$$

$$(100\%)\frac{(121 \text{ ft} - 59.5 \text{ ft})}{121 \text{ ft}} = 50.8\%$$

MOMENT FRAME MODEL

Grids 1 and 8 were modeled in conventional structural analysis software as two-dimensional models. The second-order option in the structural analysis program was not used. Rather, for illustration purposes, second-order effects are calculated separately, using the "Approximate Second-Order Analysis" method described in AISC *Specification* Appendix 8.

The column and beam layouts for the moment frames follow. Although the frames on Grids A and F are the same, slightly heavier seismic loads accumulate on Grid F after accounting for the atrium area and accidental torsion. The models are half-building models. The frame was originally modeled with W14×82 interior columns and W21×44 non-composite beams, but was revised because the beams and columns did not meet the strength requirements. The W14×82 column size was increased to a W14×90 and the W21×44 beams were upsized to W24×55 beams. Minimum composite studs are specified for the beams (corresponding to $\Sigma Q_n = 0.25F_yA_s$). Since the span does not exceed 30 ft, the ductility requirement is met per AISC *Specification* Commentary Section I3.2d.1. The beams were modeled with a stiffness of $I_{eq} = I_s$.

The frame was checked for both wind and seismic story drift limits. Based on the results on the computer analysis, the frame meets the L/400 drift criterion for a 10-year wind (0.7W) indicated in ASCE/SEI 7, Commentary Section CC.2.2. In addition, the frame meets the $0.025h_{sx}$ allowable story drift limit given in ASCE/SEI 7, Table 12.12-1, for Risk Category II.

All of the vertical loads on the frame were modeled as point loads on the frame. The dead load and live load are shown in the load cases that follow. The wind, seismic and notional loads from leaning columns are modeled and distributed 1/14 to exterior columns and 1/7 to the interior columns. This approach minimizes the tendency to accumulate too much load in the lateral system nearest an externally applied load.

Also shown in the following models are the remainder of the half-building model gravity loads from the interior leaning columns accumulated in a single leaning column which was connected to the frame portion of the model with pinned ended links. Because the second-order analyses that follow will use the "Approximate Second-Order Analysis" (amplified first-order) approach given in the AISC *Specification* Appendix 8, the inclusion of the leaning column is unnecessary, but serves to summarize the leaning column loads and illustrate how these might be handled in a full second-order analysis. See "A Practical Approach to the 'Leaning' Column" (Geschwindner, 1994).

There are five lateral load cases. Two are the wind load and seismic load, per the previous discussion. In addition, notional loads of $N_i = 0.002Y_i$ were established. The model layout, nominal dead, live, and snow loads with associated notional loads, wind loads and seismic loads are shown in Figures III-15 through III-23.

The same modeling procedures were used in the braced frame analysis. If column bases are not fixed in construction, they should not be fixed in the analysis.

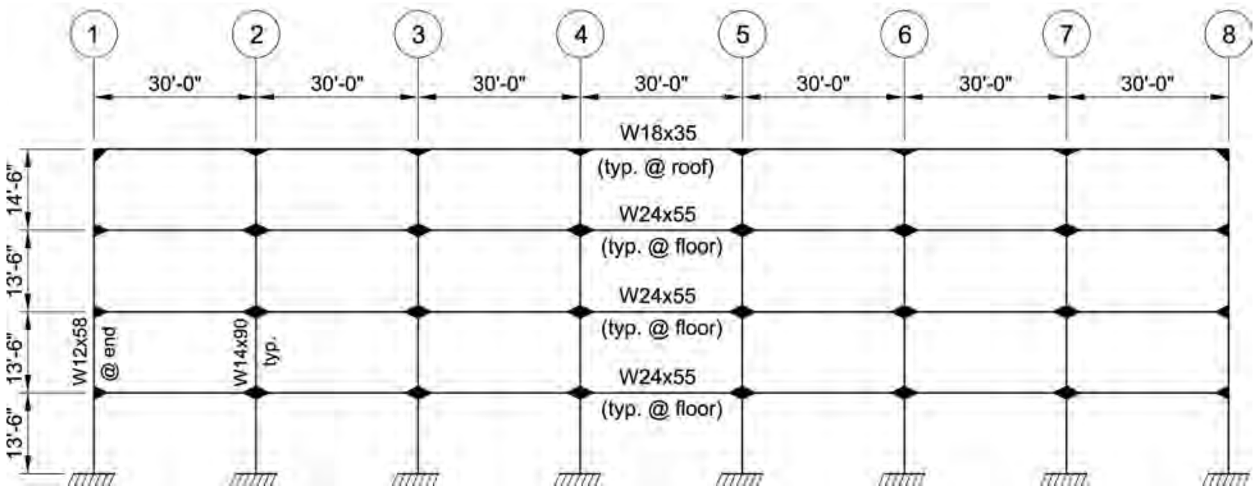


Fig. III-15. Frame layout—Grid A and F.

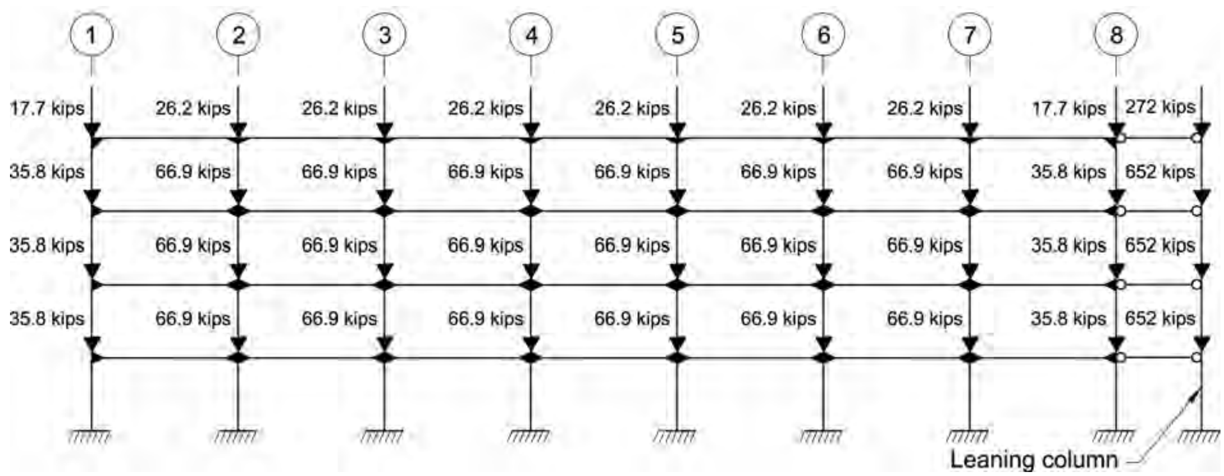


Fig. III-16. Nominal dead loads—Grid A and F.

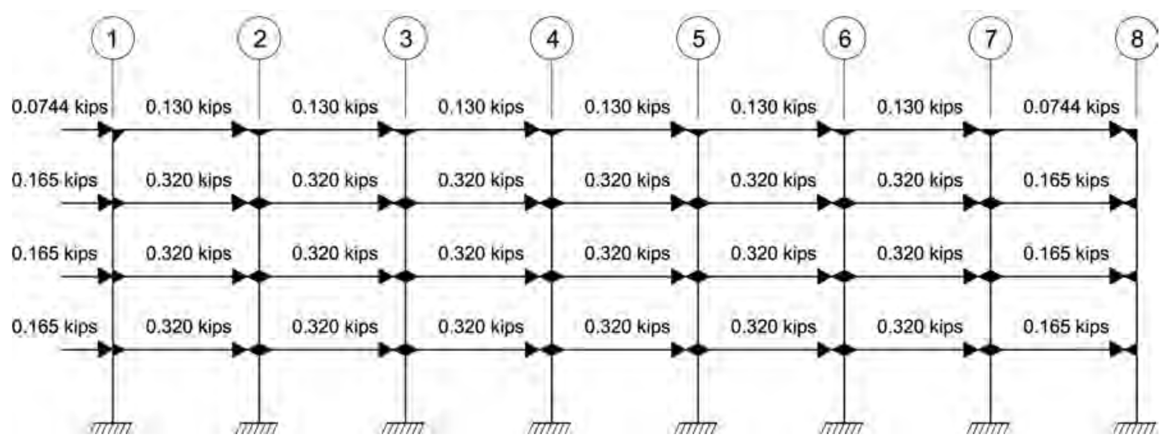


Fig. III-17. Notional dead loads—Grid A and F.

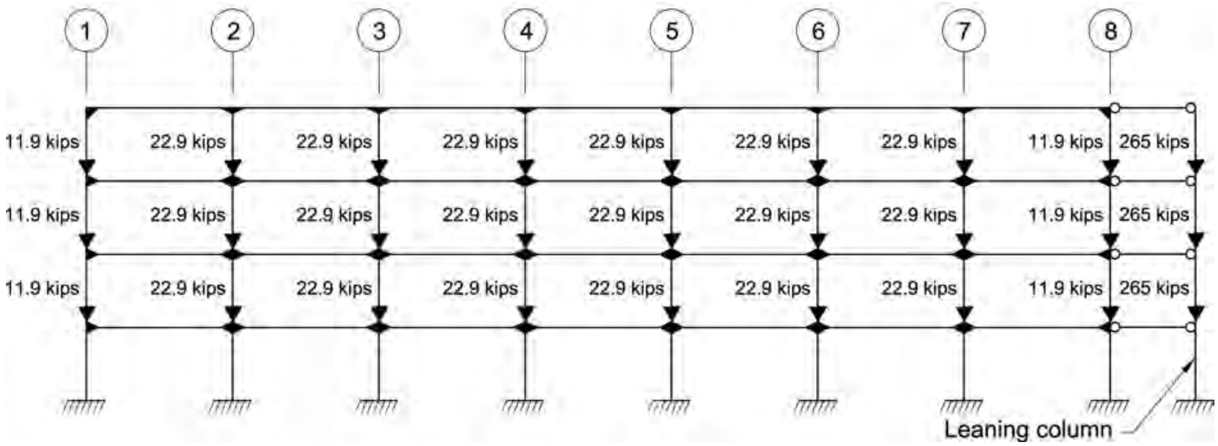


Fig. III-18. Nominal live loads—Grid A and F.

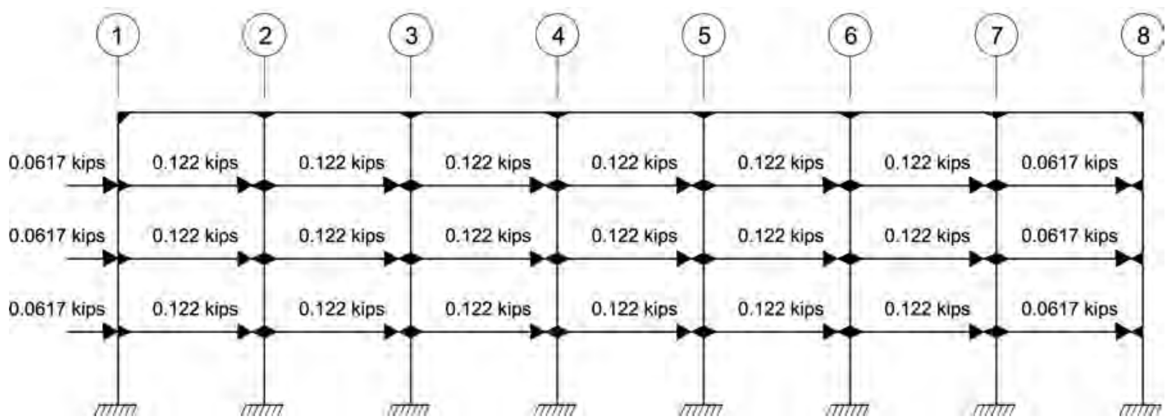


Fig. III-19. Notional live loads—Grid A and F.

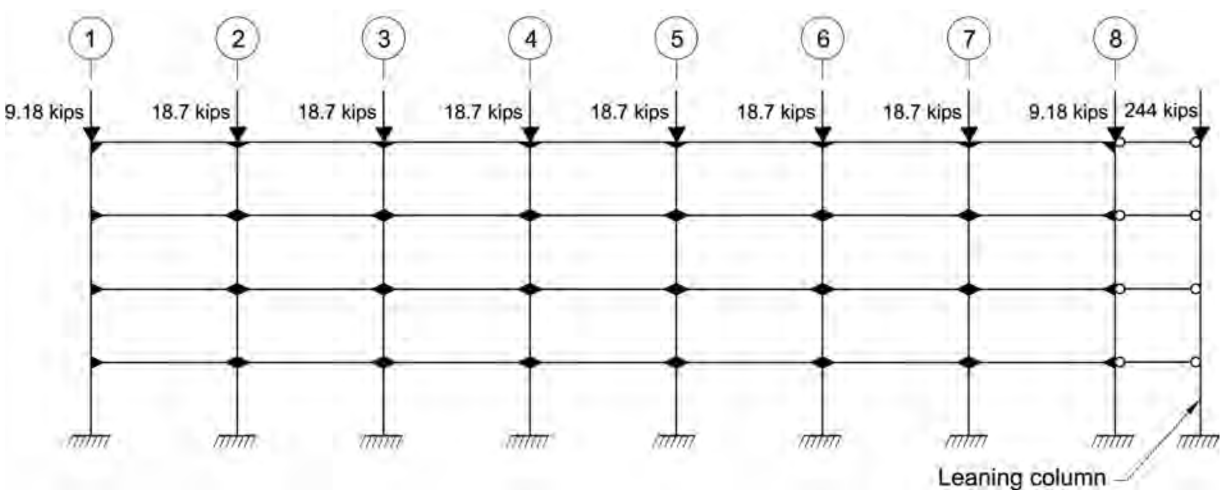


Fig. III-20. Nominal snow loads—Grid A and F.

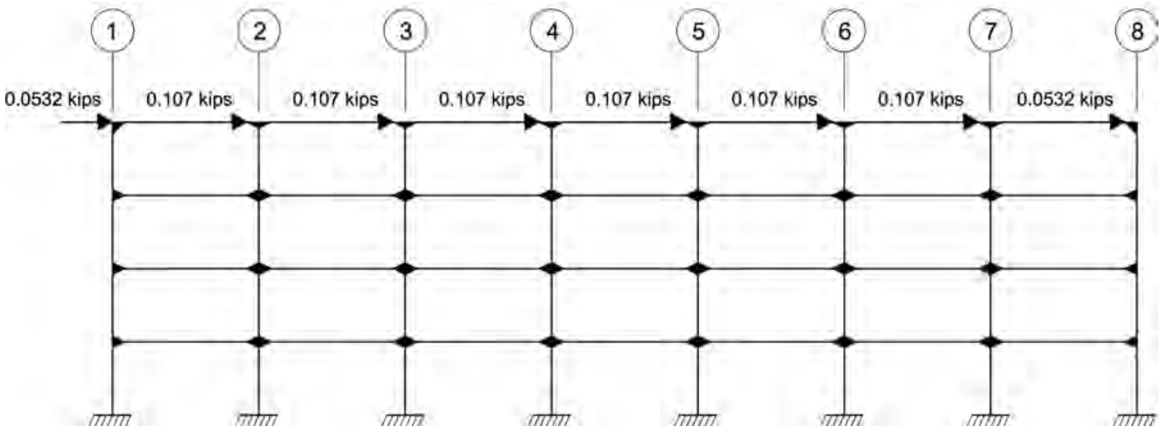


Fig. III-21. Notional snow loads—Grid A and F.

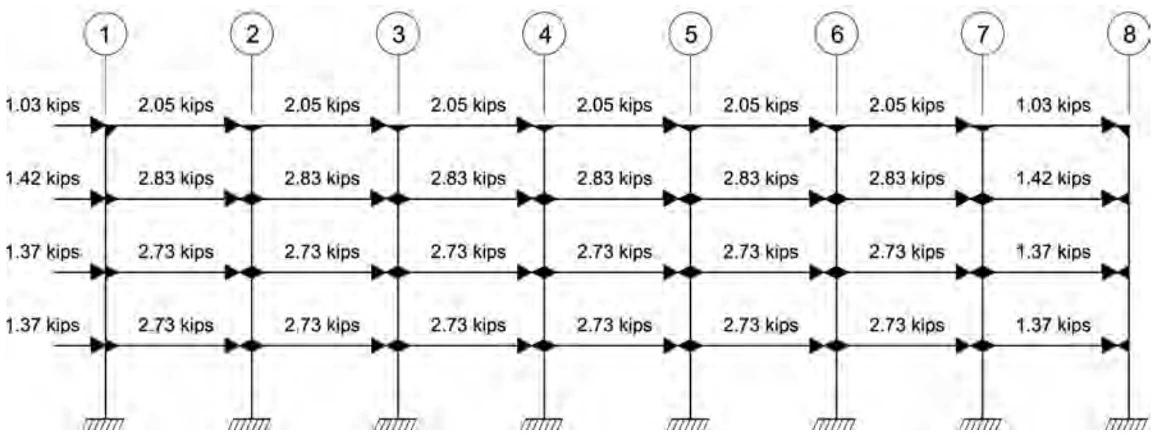


*Fig. III-22. Nominal wind loads (1.0*W*)—Grid A and F.*

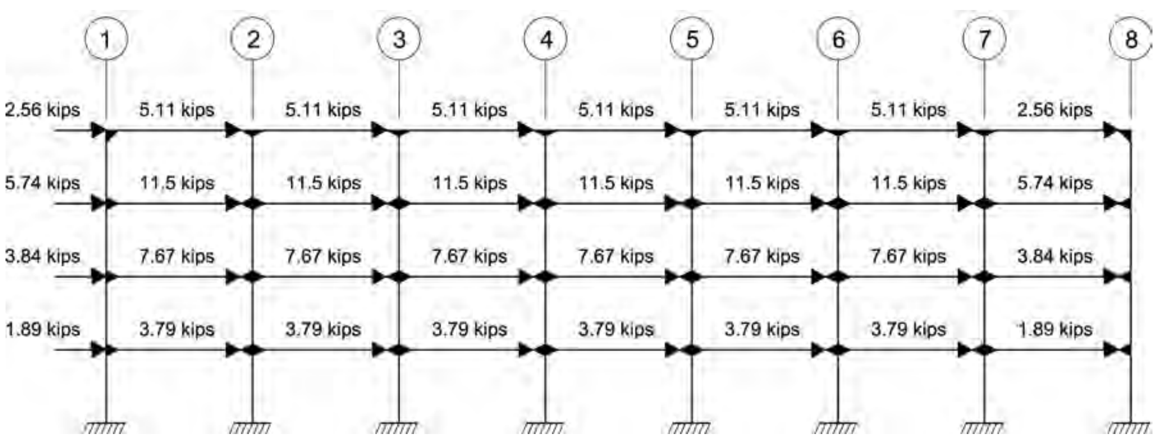


Fig. III-23. Seismic loads (1.0Q_E)—Grid A and F.

CALCULATION OF REQUIRED STRENGTH—THREE METHODS

Three methods for checking one of the typical interior column designs at the base of the building are presented below. All three of the methods presented require a second-order analysis (either direct via computer analysis techniques or by amplifying a first-order analysis). A fourth method called the "First-Order Analysis Method" is also an option. This method does not require a second-order analysis; however, this method is not presented below. For additional guidance on applying any of these methods, see the discussion in AISC *Manual* Part 2 titled Required Strength, Stability, Effective Length, and Second-Order Effects.

GENERAL INFORMATION FOR ALL THREE METHODS

Seismic load combinations controlled over wind load combinations in the direction of the moment frames in the example building. The frame analysis was run for all LRFD and ASD load combinations; however, only the controlling combinations have been illustrated in the following examples. A lateral load of 0.2% of gravity load was included for all gravity-only load combinations per AISC *Manual* Part 2.

The second-order analysis for all of the following examples were carried out by doing a first-order analysis and then amplifying the results to achieve a set of second-order design forces using the approximate second-order analysis procedure from AISC *Specification* Appendix 8.

METHOD 1—DIRECT ANALYSIS METHOD

Design for stability by the direct analysis method is found in AISC *Specification* Chapter C. This method requires that both the flexural and axial stiffness are reduced and that 0.2% notional lateral loads are applied in the analysis to account for geometric imperfections and inelasticity, per AISC *Specification* Section C2.2b(a). Any general second-order analysis method that considers both P-δ and P-Δ effects is permitted. The amplified first-order analysis method of AISC *Specification* Appendix 7 is also permitted provided that the B_1 and B_2 factors are based on the reduced flexural and axial stiffnesses. A summary of the axial loads, moments and first floor drifts from the first-order analysis is shown in the following. The floor diaphragm deflection in the east-west direction was previously determined to be very small and will thus be neglected in these calculations. Second-order member forces are determined using the approximate procedure of AISC *Specification* Appendix 8.

It was assumed, subject to verification, that B_2 is less than 1.7 for each load combination; therefore, per AISC *Specification* Section C2.2b(d), the notional loads were applied to the gravity-only load combinations. The required seismic load combinations, as given in ASCE/SEI 7, Section 12.4, were derived previously.

LRFD	ASD
$1.23D \pm 1.0Q_E + 0.5L + 0.2S$ (Controls columns and beams) From a first-order analysis with notional loads where appropriate and reduced stiffnesses: For interior column design $P_u = 317$ kips $M_{u1} = 148$ kip-ft (from first-order analysis) $M_{u2} = 233$ kip-ft (from first-order analysis) First story drift with reduced stiffnesses = 0.718 in.	$1.01D + 0.525Q_E + 0.75L + 0.75S$ (Controls columns and beams) From a first-order analysis with notional loads where appropriate and reduced stiffnesses: For interior column design $P_a = 295$ kips $M_{a1} = 77.9$ kip-ft $M_{a2} = 122$ kip-ft First story drift with reduced stiffnesses = 0.377 in.

Note: For ASD, ordinarily the second-order analysis must be carried out under 1.6 times the ASD load combinations and the results must be divided by 1.6 to obtain the required strengths. For this example, second-order analysis by the approximate B_1-B_2 analysis method is used. This method incorporates the 1.6 multiplier directly in the B_1 and B_2 amplifiers, such that no other modification is needed.

The required second-order flexural strength, M_r, and required axial strength, P_r, are determined as follows. For typical interior columns, the gravity-load moments are approximately balanced, therefore, M_{nt} = 0 kip-ft.

Calculate the amplified forces and moments in accordance with AISC *Specification* Appendix 8 at the ground floor. The required second-order flexural strength is determined as follows:

$$M_r = B_1 M_{nt} + B_2 M_{lt} \qquad \text{(}Spec.\text{ Eq. A-8-1)}$$

Determine B_1

Per AISC *Specification* Appendix 8, Section 8.2.1, note that for members subject to axial compression, B_1 may be calculated based on the first-order estimate; therefore:

$$P_r = P_{nt} + P_{lt}$$

where
P_r = required second-order axial strength using LRFD or ASD load combinations

From AISC *Specification* Appendix 8, Section 8.2.1, the B_1 multiplier for the W14×90 column is determined as follows:

LRFD	ASD
$B_1 = \dfrac{C_m}{1-\dfrac{\alpha P_r}{P_{e1}}} \geq 1$ (*Spec*. Eq. A-8-3)	$B_1 = \dfrac{C_m}{1-\dfrac{\alpha P_r}{P_{e1}}} \geq 1$ (*Spec*. Eq. A-8-3)
where	where
P_r = 317 kips (from first-order computer analysis)	P_r = 295 kips (from first-order computer analysis)
I_x = 999 in.^{4}	I_x = 999 in.^{4}
τ_b = 1.0 (to be verified per *Spec*. Section C2.3(b))	τ_b = 1.0 (to be verified per *Spec*. Section C2.3(b))
α = 1.0	α = 1.6
As discussed in AISC *Specification* Appendix 8, Section 8.2.1, $EI^* = 0.8\tau_b EI$ when using the direct analysis method.	As discussed in AISC *Specification* Appendix 8, Section 8.2.1, $EI^* = 0.8\tau_b EI$ when using the direct analysis method.
$P_{e1} = \dfrac{\pi^2 EI^*}{(L_{c1})^2}$ (*Spec*. Eq. A-8-5)	$P_{e1} = \dfrac{\pi^2 EI^*}{(L_{c1})^2}$ (*Spec*. Eq. A-8-5)
$= \dfrac{\pi^2(0.8)(1.0)(29{,}000\text{ ksi})(999\text{ in.}^4)}{[(1.0)(13.5\text{ ft})(12\text{ in./ft})]^2}$	$= \dfrac{\pi^2(0.8)(1.0)(29{,}000\text{ ksi})(999\text{ in.}^4)}{[(1.0)(13.5\text{ ft})(12\text{ in./ft})]^2}$
$= 8{,}720$ kips	$= 8{,}720$ kips
$C_m = 0.6 - 0.4(M_1/M_2)$ (*Spec*. Eq. A-8-4)	$C_m = 0.6 - 0.4(M_1/M_2)$ (*Spec*. Eq. A-8-4)
$= 0.6 - 0.4(148\text{ kip-ft}/233\text{ kip-ft})$	$= 0.6 - 0.4(77.9\text{ kip-ft}/122\text{ kip-ft})$
$= 0.346$	$= 0.345$

LRFD	ASD
$B_1 = \dfrac{0.346}{1-\dfrac{1.0(317\text{ kips})}{8{,}720\text{ kips}}} < 1$ $= 0.359 < 1$ Therefore, use $B_1 = 1$	$B_1 = \dfrac{0.345}{1-\dfrac{1.6(295\text{ kips})}{8{,}720\text{ kips}}} < 1$ $= 0.365 < 1$ Therefore, use $B_1 = 1$

Determine B_2

LRFD	ASD
$P_{mf} = 2{,}250$ kips (gravity load in moment frame) $P_{story} = 5{,}440$ kips (from computer output) $\Delta_H = 0.718$ in. (from computer output) $\alpha = 1.0$	$P_{mf} = 2{,}090$ kips (gravity load in moment frame) $P_{story} = 5{,}120$ kips (from computer output) $\Delta_H = 0.377$ in. (from computer output) $\alpha = 1.6$
$R_M = 1-0.15\left(\dfrac{P_{mf}}{P_{story}}\right)$ (*Spec*. Eq. A-8-8) $= 1-0.15\left(\dfrac{2{,}250\text{ kips}}{5{,}440\text{ kips}}\right)$ $= 0.938$	$R_M = 1-0.15\left(\dfrac{P_{mf}}{P_{story}}\right)$ (*Spec*. Eq. A-8-8) $= 1-0.15\left(\dfrac{2{,}090\text{ kips}}{5{,}120\text{ kips}}\right)$ $= 0.939$
From previous seismic force distribution calculations: $H = 1.0Q_E$ (Lateral) $= 1.0(196\text{ kips})$ $= 196$ kips	From previous seismic force distribution calculations: $H = 0.525Q_E$ (Lateral) $= 0.525(196\text{ kips})$ $= 103$ kips
$P_{e\,story} = R_M \dfrac{HL}{\Delta_H}$ (*Spec*. Eq. A-8-7) $= (0.938)\dfrac{(196\text{ kips})(13.5\text{ ft})(12\text{ in./ft})}{0.718\text{ in.}}$ $= 41{,}500$ kips	$P_{e\,story} = R_M \dfrac{HL}{\Delta_H}$ (*Spec*. Eq. A-8-7) $= (0.939)\dfrac{(103\text{ kips})(13.5\text{ ft})(12\text{ in./ft})}{0.377\text{ in.}}$ $= 41{,}600$ kips
$B_2 = \dfrac{1}{1-\dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1$ (*Spec*. Eq. A-8-6) $= \dfrac{1}{1-\dfrac{1.0(5{,}440\text{ kips})}{41{,}500\text{ kips}}} > 1$ $= 1.15 > 1$	$B_2 = \dfrac{1}{1-\dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1$ (*Spec*. Eq. A-8-6) $= \dfrac{1}{1-\dfrac{1.6(5{,}120\text{ kips})}{41{,}600\text{ kips}}} > 1$ $= 1.25 > 1$
Because $B_2 < 1.7$, it is verified that it was unnecessary to add the notional loads to the lateral loads for this load combination.	Because $B_2 < 1.7$, it is verified that it was unnecessary to add the notional loads to the lateral loads for this load combination.

Calculate amplified moment and axial load

From AISC *Specification* Equation A-8-1, the required second-order flexural strength is determined as follows:

LRFD	ASD
$M_r = B_1 M_{nt} + B_2 M_{lt}$ $= (1.0)(0\text{ kip-ft}) + (1.15)(233\text{ kip-ft})$ $= 268\text{ kip-ft}$	$M_r = B_1 M_{nt} + B_2 M_{lt}$ $= (1.0)(0\text{ kip-ft}) + (1.25)(122\text{ kip-ft})$ $= 153\text{ kip-ft}$

The required second-order axial strength is determined using AISC *Specification* Equation A-8-2 as follows. Note, for a long frame, such as this one, the change in load to the interior columns associated with lateral load is negligible.

LRFD	ASD
$P_{nt} = 317$ kips (from computer analysis) $P_r = P_{nt} + B_2 P_{lt}$ $= 317\text{ kips} + (1.15)(0\text{ kips})$ $= 317\text{ kips}$	$P_{nt} = 295$ kips (from computer analysis) $P_r = P_{nt} + B_2 P_{lt}$ $= 295\text{ kips} + (1.25)(0\text{ kips})$ $= 295\text{ kips}$

Note the flexural and axial stiffness of all members in the moment frame were reduced using $0.8E$ in the computer analysis.

Check that the flexural stiffness was adequately reduced for the analysis per AISC *Specification* Section C2.3(b)(1).

LRFD	ASD
$\alpha = 1.0$ $P_r = 317\text{ kips}$ Because the W14×90 column is nonslender: $P_{ns} = F_y A_g$ $= (50\text{ ksi})(26.5\text{ in.}^2)$ $= 1{,}330\text{ kips}$ $\dfrac{\alpha P_r}{P_{ns}} = \dfrac{1.0(317\text{ kips})}{1{,}330\text{ kips}}$ $= 0.238$ Because $\alpha P_r/P_{ns} \le 0.5$: $\tau_b = 1.0$ Therefore, the previous assumption is verified.	$\alpha = 1.6$ $P_r = 295\text{ kips}$ Because the W14×90 column is nonslender: $P_{ns} = F_y A_g$ $= (50\text{ ksi})(26.5\text{ in.}^2)$ $= 1{,}330\text{ kips}$ $\dfrac{\alpha P_r}{P_{ns}} = \dfrac{1.6(295\text{ kips})}{1{,}330\text{ kips}}$ $= 0.355$ Because $\alpha P_r/P_{ns} \le 0.5$: $\tau_b = 1.0$ Therefore, the previous assumption is verified.

Note: By inspection $\tau_b = 1.0$ for all of the beams in the moment frame.

Interaction of Flexure and Axial

From AISC *Specification* Section H1, interaction of flexure and axial are checked as follows. From AISC *Specification* Section C3, K = 1.0 using the direct analysis method, therefore:

$$\begin{aligned} L_c &= KL \\ &= 1.0(13.5 \text{ ft}) \\ &= 13.5 \text{ ft} \end{aligned}$$

LRFD	ASD
From AISC *Manual* Table 6-2, for a W14×90, with L_c = 13.5 ft: $P_c = \phi_c P_n$ $= 1{,}040 \text{ kips}$	From AISC *Manual* Table 6-2, for a W14×90, with L_c = 13.5 ft: $P_c = \dfrac{P_n}{\Omega_b}$ $= 690 \text{ kips}$
From AISC *Manual* Table 6-2, for a W14×90, with L_b = 13.5 ft: $M_{cx} = \phi_b M_{nx}$ $= 574 \text{ kip-ft}$	From AISC *Manual* Table 6-2, for a W14×90, with L_b = 13.5 ft: $M_{cx} = \dfrac{M_{nx}}{\Omega_b}$ $= 382 \text{ kip-ft}$
$\dfrac{P_r}{P_c} = \dfrac{317 \text{ kips}}{1{,}040 \text{ kips}}$ $= 0.305$	$\dfrac{P_r}{P_c} = \dfrac{295 \text{ kips}}{690 \text{ kips}}$ $= 0.428$
Because $\dfrac{P_r}{P_c} \geq 0.2$, use AISC *Specification* Equation H1-1a:	Because $\dfrac{P_r}{P_c} \geq 0.2$, use AISC *Specification* Equation H1-1a:
$\dfrac{P_r}{P_c} + \left(\dfrac{8}{9}\right)\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$ $0.305 + \left(\dfrac{8}{9}\right)\left(\dfrac{268 \text{ kip-ft}}{574 \text{ kip-ft}} + 0\right) < 1.0$ $0.720 < 1.0$ **o.k.**	$\dfrac{P_r}{P_c} + \left(\dfrac{8}{9}\right)\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$ $0.428 + \left(\dfrac{8}{9}\right)\left(\dfrac{153 \text{ kip-ft}}{382 \text{ kip-ft}} + 0\right) < 1.0$ $0.784 < 1.0$ **o.k.**

METHOD 2—EFFECTIVE LENGTH METHOD

Required strengths of frame members must be determined from a second-order analysis. In this example, the second-order analysis is performed by amplifying the axial forces and moments in members and connections from an approximate analysis using the provisions of AISC *Specification* Appendix 8. The available strengths of compression members are calculated using effective length factors computed from a sidesway stability analysis.

A first-order frame analysis is conducted using the load combinations for LRFD or ASD. A minimum lateral load (notional load) equal to 0.2% of the gravity loads is included for any gravity-only load combination as summarized in AISC *Manual* Part 2 titled "Required Strength, Stability, Effective Length, and Second-Order Effects." The required load combinations are given in ASCE/SEI 7.

A summary of the axial loads, moments and 1st floor drifts from the first-order computer analysis is shown below. The floor diaphragm deflection in the east-west direction was previously determined to be very small and will thus be neglected in these calculations.

LRFD	ASD
$1.23D \pm 1.0Q_E + 0.5L + 0.2S$ (Controls columns and beams) For interior column design: $P_u = 317$ kips $M_{u1} = 148$ kip-ft (from first-order analysis) $M_{u2} = 233$ kip-ft (from first-order analysis) First-order story drift = 0.575 in.	$1.01D + 0.525Q_E + 0.75L + 0.75S$ (Controls columns and beams) For interior column design: $P_a = 295$ kips $M_{a1} = 77.9$ kip-ft (from first-order analysis) $M_{a2} = 122$ kip-ft (from first-order analysis) First-order story drift = 0.302 in.

The required second-order flexural strength, M_r, and axial strength, P_r, are calculated as follows. For typical interior columns, the gravity load moments are approximately balanced; therefore, $M_{nt} = 0$ kip-ft.

Calculate the amplified forces and moments in accordance with AISC *Specification* Appendix 8 at the ground floor. The required second-order flexural strength is determined as follows:

$$M_r = B_1 M_{nt} + B_2 M_{lt} \quad \text{(}Spec.\text{ Eq. A-8-1)}$$

Determine B_1

Per AISC *Specification* Appendix 8, Section 8.2.1, note that for members subject to axial compression, B_1 may be calculated based on the first-order estimate; therefore:

$$P_r = P_{nt} + P_{lt}$$

where

P_r = required second-order axial strength using LRFD or ASD load combinations

From AISC *Specification* Appendix 8, Section 8.2.1, the B_1 multiplier for the W14×90 column is determined as follows:

LRFD	ASD
$B_1 = \dfrac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}} \geq 1$ (*Spec*. Eq. A-8-3)	$B_1 = \dfrac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}} \geq 1$ (*Spec*. Eq. A-8-3)
LRFD	ASD

LRFD	ASD
where $P_r = 317$ kips (from first-order computer analysis) $I_x = 999\text{ in.}^4$ $\tau_b = 1.0$ (to be verified per *Spec.* Section C2.3(b)) $\alpha = 1.0$ $P_{e1} = \dfrac{\pi^2 EI^*}{(L_{c1})^2}$ (*Spec.* Eq. A-8-5) $= \dfrac{\pi^2(29{,}000\text{ ksi})(999\text{ in.}^4)}{[(1.0)(13.5\text{ ft})(12\text{ in./ft})]^2}$ $= 10{,}900$ kips $C_m = 0.6 - 0.4(M_1/M_2)$ (*Spec.* Eq. A-8-4) $= 0.6 - 0.4(148\text{ kip-ft}/233\text{ kip-ft})$ $= 0.346$ $B_1 = \dfrac{0.346}{1 - \dfrac{1.0(317\text{ kips})}{10{,}900\text{ kips}}} < 1$ $= 0.356 < 1$ Therefore, use $B_1 = 1$	where $P_r = 295$ kips (from first-order computer analysis) $I_x = 999\text{ in.}^4$ $\tau_b = 1.0$ (to be verified per *Spec.* Section C2.3(b)) $\alpha = 1.6$ $P_{e1} = \dfrac{\pi^2 EI^*}{(L_{c1})^2}$ (*Spec.* Eq. A-8-5) $= \dfrac{\pi^2(29{,}000\text{ ksi})(999\text{ in.}^4)}{[(1.0)(13.5\text{ ft})(12\text{ in./ft})]^2}$ $= 10{,}900$ kips $C_m = 0.6 - 0.4(M_1/M_2)$ (*Spec.* Eq. A-8-4) $= 0.6 - 0.4(77.9\text{ kip-ft}/122\text{ kip-ft})$ $= 0.345$ $B_1 = \dfrac{0.345}{1 - \dfrac{1.6(295\text{ kips})}{10{,}900\text{ kips}}} < 1$ $= 0.361 < 1$ Therefore, use $B_1 = 1$

Determine B_2

LRFD	ASD
$P_{mf} = 2{,}250$ kips (gravity load in moment frame) $P_{story} = 5{,}440$ kips (from computer output) $\Delta_H = 0.575$ in. (from computer output) $\alpha = 1.0$ $R_M = 1 - 0.15\left(\dfrac{P_{mf}}{P_{story}}\right)$ (*Spec.* Eq. A-8-8) $= 1 - 0.15\left(\dfrac{2{,}250\text{ kips}}{5{,}440\text{ kips}}\right)$ $= 0.938$ From previous seismic force distribution calculations: $H = 1.0Q_E$ (Lateral) $= 1.0(196\text{ kips})$ $= 196$ kips	$P_{mf} = 2{,}090$ kips (gravity load in moment frame) $P_{story} = 5{,}120$ kips (from computer output) $\Delta_H = 0.302$ in. (from computer output) $\alpha = 1.6$ $R_M = 1 - 0.15\left(\dfrac{P_{mf}}{P_{story}}\right)$ (*Spec.* Eq. A-8-8) $= 1 - 0.15\left(\dfrac{2{,}090\text{ kips}}{5{,}120\text{ kips}}\right)$ $= 0.939$ From previous seismic force distribution calculations: $H = 0.525Q_E$ (Lateral) $= 0.525(196\text{ kips})$ $= 103$ kips

LRFD	ASD

$P_{e\,story} = R_M \dfrac{HL}{\Delta_H}$ (*Spec*. Eq. A-8-7) $= 0.938 \dfrac{(196 \text{ kips})(13.5 \text{ ft})(12 \text{ in./ft})}{0.575 \text{ in.}}$ $= 51{,}800 \text{ kips}$ $B_2 = \dfrac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1$ (*Spec*. Eq. A-8-6) $= \dfrac{1}{1 - \dfrac{1.0(5{,}440 \text{ kips})}{51{,}800 \text{ kips}}} > 1$ $= 1.12 > 1$ Note, B_2 < 1.5, therefore use of the effective length method is acceptable per AISC *Specification* Appendix 7, Section 7.2.1(b).	$P_{e\,story} = R_M \dfrac{HL}{\Delta_H}$ (*Spec*. Eq. A-8-7) $= 0.939 \dfrac{(103 \text{ kips})(13.5 \text{ ft})(12 \text{ in./ft})}{0.302 \text{ in.}}$ $= 51{,}900 \text{ kips}$ $B_2 = \dfrac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1$ (*Spec*. Eq. A-8-6) $= \dfrac{1}{1 - \dfrac{1.6(5{,}120 \text{ kips})}{51{,}900 \text{ kips}}} > 1$ $= 1.19 > 1$ Note, B_2 < 1.5, therefore use of the effective length method is acceptable per AISC *Specification* Appendix 7, Section 7.2.1(b).

Calculate amplified moment and axial load

From AISC *Specification* Equation A-8-1, the required second-order flexural strength is determined as follows:

LRFD	ASD
$M_r = B_1 M_{nt} + B_2 M_{lt}$ $= (1)(0 \text{ kip-ft}) + (1.12)(233 \text{ kip-ft})$ $= 261 \text{ kip-ft}$	$M_r = B_1 M_{nt} + B_2 M_{lt}$ $= (1)(0 \text{ kip-ft}) + (1.19)(122 \text{ kip-ft})$ $= 145 \text{ kip-ft}$

The required second-order axial strength is determined using AISC *Specification* Equation A-8-2 as follows. Note, for a long frame, such as this one, the change in load to the interior columns associated with lateral load is negligible.

LRFD	ASD
$P_{nt} = 317 \text{ kips}$ (from computer analysis) $P_r = P_{nt} + B_2 P_{lt}$ $= 317 \text{ kips} + (1.12)(0 \text{ kips})$ $= 317 \text{ kips}$	$P_{nt} = 295 \text{ kips}$ (from computer analysis) $P_r = P_{nt} + B_2 P_{lt}$ $= 295 \text{ kips} + (1.19)(0 \text{ kips})$ $= 295 \text{ kips}$

Determine the Controlling Effective Length

For out-of-plane buckling in the moment frame, K_y = 1.0; therefore:

$$K_y L_y = 1.0(13.5 \text{ ft})$$
$$= 13.5 \text{ ft}$$

For in-plane buckling in the moment frame, use the story stiffness procedure from AISC *Specification* Commentary Appendix 7 to determine K_x.

$$K_2 = \sqrt{\left(\frac{P_{story}}{R_M P_r}\right)\left(\frac{\pi^2 EI}{L^2}\right)\left(\frac{\Delta_H}{HL}\right)} \geq \sqrt{\left(\frac{\pi^2 EI}{L^2}\right)\left(\frac{\Delta_H}{1.7 H_{col} L}\right)} \qquad \text{(\textit{Spec}. Eq. C-A-7-5)}$$

Simplifying and substituting terms previously calculated results in:

$$K_x = \sqrt{\left(\frac{P_{story}}{R_M}\right)\left(\frac{P_e}{P_r}\right)\left(\frac{ratio}{H}\right)} \geq \sqrt{P_e\left(\frac{ratio}{1.7H}\right)}$$

where

$$P_e = P_{e1}$$

$$ratio = \frac{\Delta_H}{L}$$

LRFD	ASD
$P_e = P_{e1}$ $= 10{,}900 \text{ kips}$ $ratio = \dfrac{\Delta_H}{L}$ $= \dfrac{0.575 \text{ in.}}{(13.5 \text{ ft})(12 \text{ in./ft})}$ $= 0.00355$ $K_x = \sqrt{\left(\dfrac{5{,}440 \text{ kips}}{0.938}\right)\left(\dfrac{10{,}900 \text{ kips}}{317 \text{ kips}}\right)\left(\dfrac{0.00355}{196 \text{ kips}}\right)} \geq$ $\sqrt{(10{,}900 \text{ kips})\left[\dfrac{0.00355}{1.7(196 \text{ kips})}\right]}$ $= 1.90 > 0.341$ Therefore, use $K_x = 1.90$. From AISC *Manual* Table 4-1a, for a W14×90: $r_x/r_y = 1.66$ $L_{cy\ eq} = \dfrac{KL_x}{r_x/r_y}$ (from *Manual* Eq. 4-1) $= \dfrac{1.90(13.5 \text{ ft})}{1.66}$ $= 15.5 \text{ ft}$ Because $L_{cy\ eq} > L_{cy}$, use $L_c = 15.5$ ft.	$P_e = P_{e1}$ $= 10{,}900 \text{ kips}$ $ratio = \dfrac{\Delta_H}{L}$ $= \dfrac{0.302 \text{ in.}}{(13.5 \text{ ft})(12 \text{ in./ft})}$ $= 0.00186$ $K_x = \sqrt{\left(\dfrac{5{,}120 \text{ kips}}{0.939}\right)\left(\dfrac{10{,}900 \text{ kips}}{295 \text{ kips}}\right)\left(\dfrac{0.00186}{103 \text{ kips}}\right)} \geq$ $\sqrt{(10{,}900 \text{ kips})\left[\dfrac{0.00186}{1.7(103 \text{ kips})}\right]}$ $= 1.91 > 0.340$ Therefore, use $K_x = 1.91$. From AISC *Manual* Table 4-1a, for a W14×90: $r_x/r_y = 1.66$ $L_{cy\ eq} = \dfrac{KL_x}{r_x/r_y}$ (from *Manual* Eq. 4-1) $= \dfrac{1.91(13.5 \text{ ft})}{1.66}$ $= 15.5 \text{ ft}$ Because $L_{cy\ eq} > L_{cy}$, use $L_c = 15.5$ ft.

Interaction of Flexure and Axial

From AISC *Specification* Section H1, interaction of flexure and axial are checked as follows:

LRFD	ASD
From AISC *Manual* Table 6-2, for a W14×90, with L_c = 15.5 ft:	From AISC *Manual* Table 6-2, for a W14×90, with L_c = 15.5 ft:
$P_c = \phi_c P_n$ $= 990 \text{ kips}$	$P_c = \dfrac{P_n}{\Omega_c}$ $= 660 \text{ kips}$
From AISC *Manual* Table 6-2, for a W14×90, with L_b = 13.5 ft:	From AISC *Manual* Table 6-2, for a W14×90, with L_b = 13.5 ft:
$M_{cx} = \phi_b M_{nx}$ $= 574 \text{ kip-ft}$	$M_{cx} = \dfrac{M_{nx}}{\Omega_b}$ $= 382 \text{ kip-ft}$
$\dfrac{P_r}{P_c} = \dfrac{317 \text{ kips}}{990 \text{ kips}}$ $= 0.320$	$\dfrac{P_r}{P_c} = \dfrac{295 \text{ kips}}{660 \text{ kips}}$ $= 0.447$
Because $\dfrac{P_r}{P_c} \geq 0.2$, use AISC *Specification* Equation H1-1a:	Because $\dfrac{P_r}{P_c} \geq 0.2$, use AISC *Specification* Equation H1-1a:
$\dfrac{P_r}{P_c} + \left(\dfrac{8}{9}\right)\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$ $0.320 + \left(\dfrac{8}{9}\right)\left(\dfrac{261 \text{ kip-ft}}{574 \text{ kip-ft}}\right) < 1.0$ $0.724 < 1.0$ **o.k.**	$\dfrac{P_r}{P_c} + \left(\dfrac{8}{9}\right)\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$ $0.447 + \left(\dfrac{8}{9}\right)\left(\dfrac{145 \text{ kip-ft}}{382 \text{ kip-ft}}\right) < 1.0$ $0.784 < 1.0$ **o.k.**

METHOD 3—SIMPLIFIED EFFECTIVE LENGTH METHOD

A simplification of the effective length method using a method of second-order analysis based upon drift limits and other assumptions is described in Part 2 of the AISC *Manual* titled "Simplified Determination of Required Strength." A first-order frame analysis is conducted using the load combinations for LRFD or ASD. A minimum lateral load (notional load) equal to 0.2% of the gravity loads is included for all gravity-only load combinations. The floor diaphragm deflection in the east-west direction was previously determined to be very small and will thus be neglected in these calculations.

LRFD	ASD
$1.23D \pm 1.0Q_E + 0.5L + 0.2S$ (Controls columns and beams) For interior column design: $P_u = 317$ kips $M_{u1} = 148$ kip-ft (from first-order analysis) $M_{u2} = 233$ kip-ft (from first-order analysis) First-order first story drift = 0.575 in.	$1.01D + 0.525Q_E + 0.75L + 0.75S$ (Controls columns and beams) For interior column design: $P_a = 295$ kips $M_{a1} = 77.9$ kip-ft (from first-order analysis) $M_{a2} = 122$ kip-ft (from first-order analysis) First-order first story drift = 0.302 in.

Calculate the amplified forces and moments in accordance with AISC *Manual* Part 2 at the ground floor. The following steps are executed.

LRFD	ASD
Step 1: Lateral load = 196 kips Deflection due to first-order elastic analysis Δ = 0.575 in., between first and second floor Floor height = 13.5 ft $\text{Drift ratio} = \dfrac{(13.5\text{ ft})(12\text{ in./ft})}{0.575\text{ in.}}$ $= 282$ *Step 2:* Design story drift limit = H/400 $\text{Adjusted lateral load} = \left(\dfrac{282}{400}\right)(196\text{ kips})$ $= 138\text{ kips}$	*Step 1:* Lateral load = 103 kips Deflection due to first-order elastic analysis Δ = 0.302 in., between first and second floor Floor height = 13.5 ft $\text{Drift ratio} = \dfrac{(13.5\text{ ft})(12\text{ in./ft})}{0.302\text{ in.}}$ $= 536$ *Step 2:* Design story drift limit = H/400 $\text{Adjusted lateral load} = \left(\dfrac{536}{400}\right)(103\text{ kips})$ $= 138\text{ kips}$

LRFD	ASD
Step 3: $$\text{Load ratio} = (1.0)\left(\frac{\text{total story load}}{\text{lateral load}}\right)$$ $$= (1.0)\left(\frac{5{,}440 \text{ kips}}{138 \text{ kips}}\right)$$ $$= 39.4$$ From AISC *Manual* Table 2-1: $B_2 = 1.1$ Which matches the value obtained in Method 2 to the two significant figures of the table	*Step 3:* (for an ASD design the ratio must be multiplied by 1.6) $$\text{Load ratio} = (1.6)\left(\frac{\text{total story load}}{\text{lateral load}}\right)$$ $$= (1.6)\left(\frac{5{,}120 \text{ kips}}{138 \text{ kips}}\right)$$ $$= 59.4$$ From AISC *Manual* Table 2-1: $B_2 = 1.2$ Which matches the value obtained in Method 2 to the two significant figures of the table

Note: Intermediate values are not interpolated from the table because the precision of the table is two significant digits. Additionally, the design story drift limit used in Step 2 need not be the same as other strength or serviceability drift limits used during the analysis and design of the structure.

Step 4:

Multiply all the forces and moment from the first-order analysis by the value of B_2 obtained from the table. This presumes that B_1 is less than or equal to B_2, which is usually the case for members without transverse loading between their ends.

LRFD	ASD
Step 5: Since the selection is in the shaded area of the chart, ($B_2 \le 1.1$), use $K = 1.0$. Multiply both sway and nonsway moments by B_2. $$M_r = B_2\left(M_{nt} + M_{lt}\right)$$ $$= 1.1(0 \text{ kip-ft} + 233 \text{ kip-ft})$$ $$= 256 \text{ kip-ft}$$ $$P_r = B_2\left(P_{nt} + P_{lt}\right)$$ $$= 1.1(317 \text{ kips} + 0 \text{ kips})$$ $$= 349 \text{ kips}$$ From AISC *Manual* Table 6-2, for a W14×90, with $L_c = 13.5$ ft: $$P_c = \phi_c P_n$$ $$= 1{,}040 \text{ kips}$$	*Step 5*: Since the selection is in the unshaded area of the chart ($B_2 > 1.1$), the effective length factor, K, must be determined through analysis. From previous analysis, use an effective length of 15.5 ft. Multiply both sway and nonsway moments by B_2. $$M_r = B_2\left(M_{nt} + M_{lt}\right)$$ $$= 1.2(0 \text{ kip-ft} + 122 \text{ kip-ft})$$ $$= 146 \text{ kip-ft}$$ $$P_r = B_2\left(P_{nt} + P_{lt}\right)$$ $$= 1.2(295 \text{ kips} + 0 \text{ kips})$$ $$= 354 \text{ kips}$$ From AISC *Manual* Table 6-2, for a W14×90, with $L_c = 15.5$ ft: $$P_c = \frac{P_n}{\Omega_c}$$ $$= 660 \text{ kips}$$

LRFD	ASD
From AISC *Manual* Table 6-2, for a W14×90, with L_b = 13.5 ft:	From AISC *Manual* Table 6-2, for a W14×90, with L_b = 13.5 ft:
$M_{cx} = \phi_b M_{nx}$ $= 574 \text{ kip-ft}$	$M_{cx} = \dfrac{M_{nx}}{\Omega_b}$ $= 382 \text{ kip-ft}$
$\dfrac{P_r}{P_c} = \dfrac{349 \text{ kips}}{1{,}040 \text{ kips}}$ $= 0.336$	$\dfrac{P_r}{P_c} = \dfrac{354 \text{ kips}}{660 \text{ kips}}$ $= 0.536$
Because $\dfrac{P_r}{P_c} \geq 0.2$, use AISC *Specification* Equation H1-1a:	Because $\dfrac{P_r}{P_c} \geq 0.2$, use AISC *Specification* Equation H1-1a:
$\dfrac{P_r}{P_c} + \left(\dfrac{8}{9}\right)\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$	$\dfrac{P_r}{P_c} + \left(\dfrac{8}{9}\right)\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$
$0.336 + \left(\dfrac{8}{9}\right)\left(\dfrac{256 \text{ kip-ft}}{574 \text{ kip-ft}} + 0\right) < 1.0$	$0.536 + \left(\dfrac{8}{9}\right)\left(\dfrac{146 \text{ kip-ft}}{382 \text{ kip-ft}} + 0\right) < 1.0$
$0.732 < 1.0$ **o.k.**	$0.876 < 1.0$ **o.k.**

BEAM ANALYSIS IN THE MOMENT FRAME

The controlling load combinations for the beams in the moment frames are shown in Tables III-11 and III-12, and evaluated for the second floor beam. The dead load, live load and seismic moments were taken from a computer analysis. These tables summarizes the calculation of B_2 for the stories above and below the second floor.

Table III-11
Summary of B_2 Calculation for Controlling Load Combination—First to Second Floor

1st – 2nd	LRFD Combination	ASD Combination 1	ASD Combination 2
	$1.23D + 1.0Q_E + 0.5L + 0.2S$	$1.02D + 0.7Q_E$	$1.01D + 0.525Q_E + 0.75L + 0.75S$
H	196 kips	137 kips	103 kips
L	13.5 ft	13.5 ft	13.5 ft
Δ_H	0.575 in.	0.402 in.	0.302 in.
P_{mf}	2,250 kips	1,640 kips	2,090 kips
R_M	0.938	0.937	0.939
$P_{e\ story}$	51,800 kips	51,700 kips	51,900 kips
P_{story}	5,440 kips	3,920 kips	5,120 kips
B_2	1.12	1.14	1.19

Table III-12
Summary of B_2 Calculation for Controlling Load Combination—Second to Third Floor

2nd – 3rd	LRFD Combination	ASD Combination 1	ASD Combination 2
	$1.23D + 1.0Q_E + 0.5L + 0.2S$	$1.02D + 0.7Q_E$	$1.01D + 0.525Q_E + 0.75L + 0.75S$
H	170 kips	119 kips	89.3 kips
L	13.5 ft	13.5 ft	13.5 ft
Δ_H	0.728 in.	0.509 in.	0.382 in.
P_{mf}	1,590 kips	1,160 kips	1,490 kips
R_M	0.938	0.937	0.939
$P_{e\ story}$	35,500 kips	35,500 kips	35,600 kips
P_{story}	3,840 kips	2,770 kips	3,660 kips
B_2	1.12	1.14	1.20

For beam members, the larger of the B_2 values from the story above or below is used.

From computer output at the controlling beam:

$$M_{dead} = 153 \text{ kip-ft}$$
$$M_{live} = 80.6 \text{ kip-ft}$$
$$M_{snow} = 0 \text{ kip-ft}$$
$$M_{earthquake} = 154 \text{ kip-ft}$$

LRFD	ASD
$B_2M_{lt} = 1.12(154 \text{ kip-ft})$ $= 172 \text{ kip-ft}$ $M_u = \begin{bmatrix} 1.23(153 \text{ kip-ft}) + 1.0(172 \text{ kip-ft}) \\ +0.5(80.6 \text{ kip-ft}) \end{bmatrix}$ $= 400 \text{ kip-ft}$	Combination 1: $B_2M_{lt} = 1.14(154 \text{ kip-ft})$ $= 176 \text{ kip-ft}$ $M_a = 1.02(153 \text{ kip-ft}) + 0.7(176 \text{ kip-ft})$ $= 279 \text{ kip-ft}$

LRFD	ASD
	Combination 2: $B_2 M_{lt} = 1.20(154 \text{ kip-ft})$ $= 185 \text{ kip-ft}$ $M_a = \begin{bmatrix} 1.01(153 \text{ kip-ft}) + 0.525(185 \text{ kip-ft}) \\ +0.75(80.6 \text{ kip-ft}) \end{bmatrix}$ $= 312 \text{ kip-ft}$

Calculate C_b for W24×55 beam with compression in the bottom flange braced at 10 ft on center.

LRFD	ASD
For load combination $1.23D + 1.0Q_E + 0.5L + 0.2S$:	For load combination $1.02D + 0.7Q_E$:
From AISC *Manual* Table 6-2 with $L_b = 0$ ft (fully braced):	From AISC *Manual* Table 6-2 with $L_b = 0$ ft (fully braced):
$\phi_b M_n = 503 \text{ kip-ft}$	$\dfrac{M_n}{\Omega_b} = 334 \text{ kip-ft}$
$C_b = 1.86$ (from computer output)	$C_b = 1.86$ (from computer output)
From AISC *Manual* Table 6-2 with $L_b = 10$ ft:	From AISC *Manual* Table 6-2 with $L_b = 10$ ft:
$\phi_b M_n C_b \leq \phi_b M_p$ $(386 \text{ kip-ft})(1.86) = 718 \text{ kip-ft} > 503 \text{ kip-ft}$	$\dfrac{M_n}{\Omega_b} C_b \leq \dfrac{M_p}{\Omega_b}$ $(257 \text{ kip-ft})(1.86) = 478 \text{ kip-ft} > 334 \text{ kip-ft}$
Therefore:	Therefore:
$\phi M_n = 503 \text{ kip-ft} > 400 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega} = 334 \text{ kip-ft} > 279 \text{ kip-ft}$ **o.k.**

LRFD	ASD
	For load combination $1.01D + 0.525Q_E + 0.75L$: From AISC *Manual* Table 6-2 with L_b = 0 ft (fully braced): $\frac{M_n}{\Omega_b} = 334$ kip-ft C_b = 2.01 (from computer output) From AISC *Manual* Table 6-2 with L_b = 10 ft : $\frac{M_n}{\Omega_b} C_b \leq \frac{M_p}{\Omega_b}$ $(257 \text{ kip-ft})(2.01) = 517 \text{ kip-ft} > 334 \text{ kip-ft}$ Therefore: $\frac{M_n}{\Omega} = 334 \text{ kip-ft} > 312 \text{ kip-ft}$ **o.k.**
From AISC *Manual* Table 6-2, a W24×55 has a design shear strength of 252 kips and an I_x of 1,350 in.^{4}	From AISC *Manual* Table 6-2, a W24×55 has an allowable shear strength of 167 kips and an I_x of 1,350 in.^{4}

The moments and shears on the roof beams due to the lateral loads were also checked but do not control the design.

The connections of these beams can be designed by one of the techniques illustrated in the Chapter IIB of the design examples.

TOC Back

BRACED FRAME ANALYSIS

The braced frames at Grids 1 and 8 were analyzed for the required load combinations. The stability design requirements from Chapter C were applied to this system.

The model layout is shown in Figure III-24. The nominal dead, live, and snow loads with associated notional loads, wind loads and seismic loads are shown in Figures III-25 and III-26.

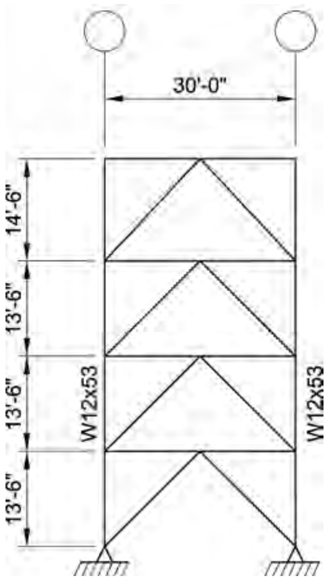
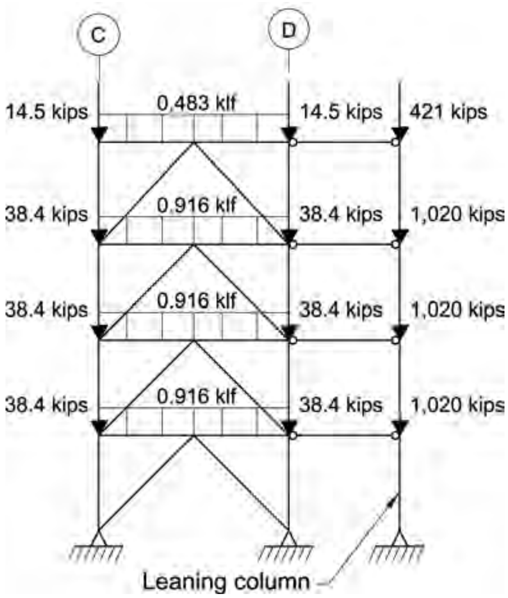
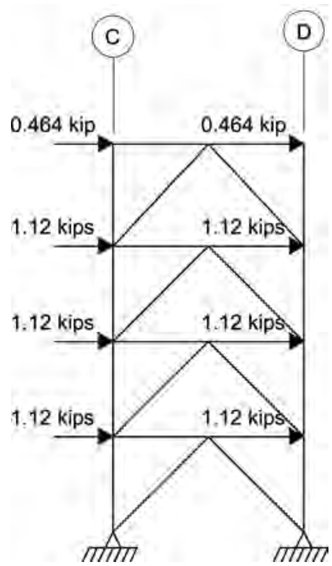


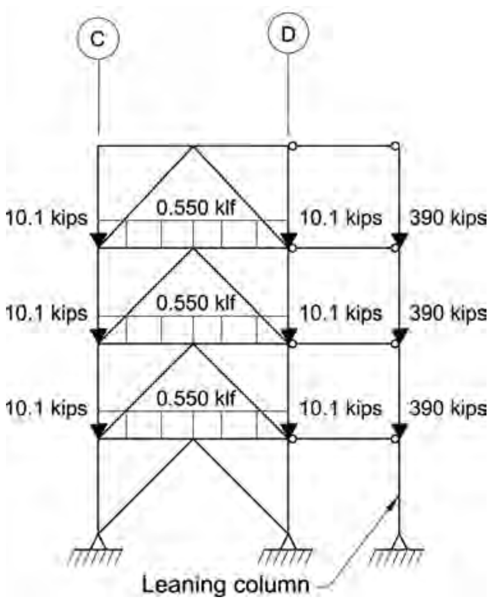
Fig. III-24. Braced frame layout—Grid 1 and 8.



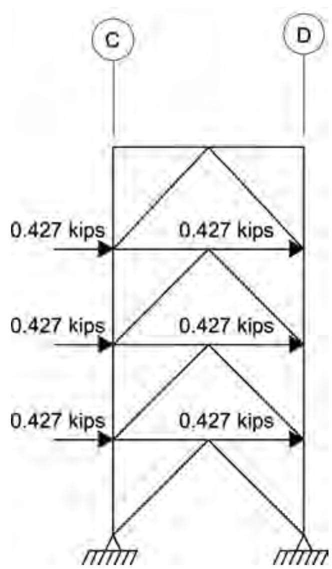
(a) Nominal dead loads



(b) Notional dead loads

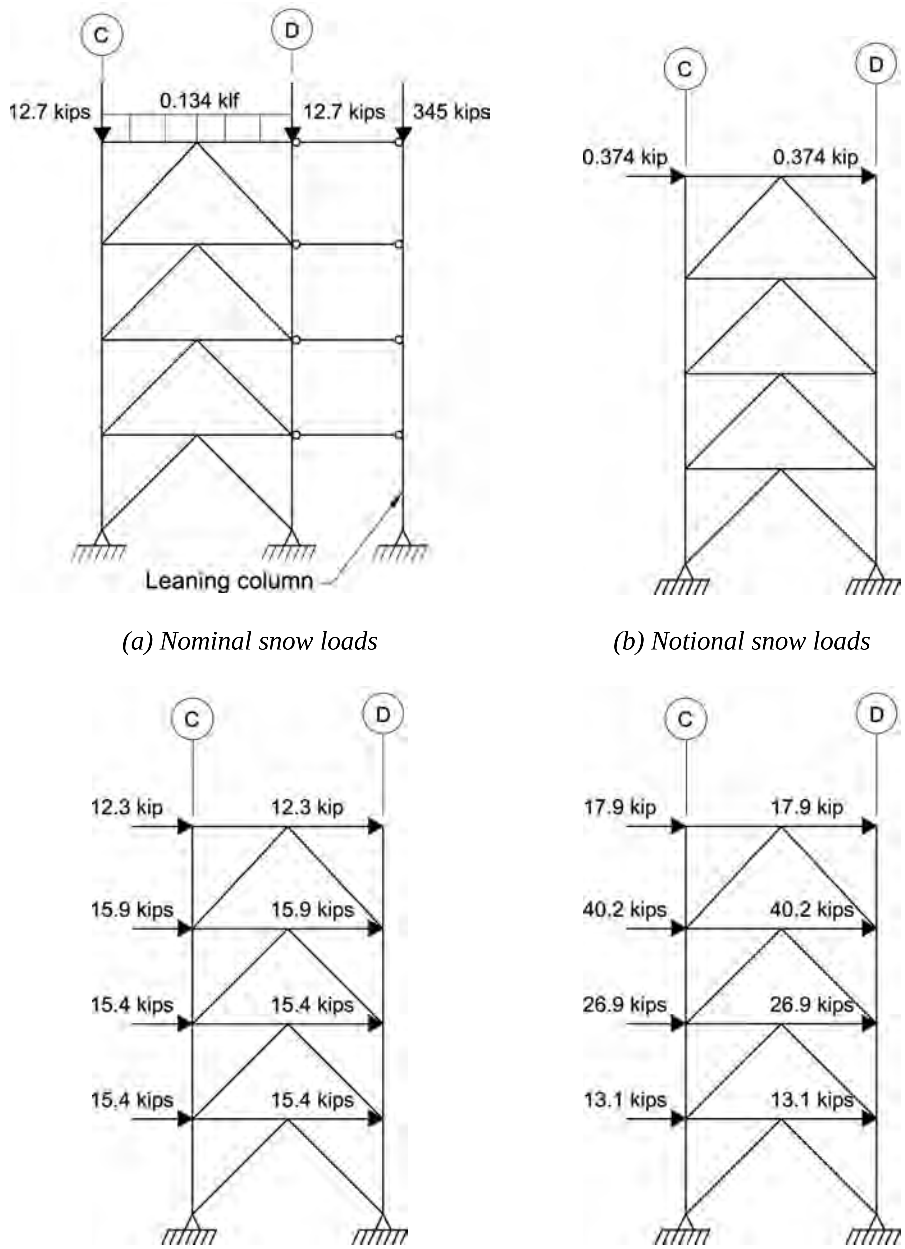


(c) Nominal live loads



(d) Notional live loads

Fig. III-25. Dead and live loads.



(a) Nominal snow loads

(b) Notional snow loads

(c) Wind loads (1.0W)

(d) Seismic loads (1.0Q_E)

Fig. III-26. Snow, wind and seismic loads.

Second-order analysis by amplified first-order analysis

In the following, the approximate second-order analysis method from AISC *Specification* Appendix 8 is used to account for second-order effects in the braced frames by amplifying the axial forces in members and connections from a first-order analysis.

A first-order frame analysis is conducted using the load combinations for LRFD and ASD. From this analysis the critical axial loads, moments and deflections are obtained.

A summary of the axial loads and first floor drifts from the first-order computer analysis is shown below. The floor diaphragm deflection in the north-south direction was previously determined to be very small and will thus be neglected in these calculations.

The required seismic load combinations, as given in ASCE/SEI 7, Section 12.4, were derived previously.

LRFD	ASD
$1.23D \pm 1.0Q_E + 0.5L + 0.2S$ (Controls columns and beams) From first-order analysis. For interior column design: $P_{nt} = 236$ kips $P_{lt} = 146$ kips The moments are negligible. First-order first story drift = 0.211 in.	$1.01D + 0.525Q_E + 0.75L + 0.75S$ (Controls columns and beams) From first-order analysis. For interior column design: $P_{nt} = 219$ kips $P_{lt} = 76.6$ kips The moments are negligible. First-order first story drift = 0.111 in.

The required second-order axial strength, P_r, is computed as follows:

LRFD	ASD
$P_r = P_{nt} + B_2 P_{lt}$ (*Spec*. Eq. A-8-2) Determine B_2. $B_2 = \dfrac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1$ (*Spec*. Eq. A-8-6) $P_{story} = 5,440$ kips (previously calculated) $P_{e\,story} = R_M \dfrac{HL}{\Delta_H}$ (*Spec*. Eq. A-8-7) where $H = 196$ kips (from previous calculations) $\Delta_H = 0.211$ in. (from computer output) $R_M = 1.0$ for braced frames	$P_r = P_{nt} + B_2 P_{lt}$ (*Spec*. Eq. A-8-2) Determine B_2. $B_2 = \dfrac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1$ (*Spec*. Eq. A-8-6) $P_{story} = 5,120$ kips (previously calculated) $P_{e\,story} = R_M \dfrac{HL}{\Delta_H}$ (*Spec*. Eq. A-8-7) where $H = 103$ kips (from previous calculations) $\Delta_H = 0.111$ in. (from computer output) $R_M = 1.0$ for braced frames

LRFD	ASD
$P_{e\,story} = (1.0)\left[\dfrac{(196\text{ kips})(13.5\text{ ft})(12\text{ in./ft})}{0.211\text{ in.}}\right]$ $= 150{,}000\text{ kips}$ $B_2 = \dfrac{1}{1-\dfrac{1.0(5{,}440\text{ kips})}{150{,}000\text{ kips}}} > 1$ $= 1.04 > 1$ Therefore, use B_2 = 1.04. $P_r = P_{nt} + B_2 P_{lt}$ (*Spec*. Eq. A-8-2) $= 236\text{ kips} + (1.04)(146\text{ kips})$ $= 388\text{ kips}$ From AISC *Manual* Table 6-2 for a W12×53 with L_c = 13.5 ft: $P_c = \phi_c P_n$ $= 514\text{ kips}$ From AISC *Specification* Equation H1-1a: $\dfrac{P_r}{P_c} = \dfrac{388\text{ kips}}{514\text{ kips}} \leq 1.0$ $= 0.755 < 1.0$ **o.k.**	$P_{e\,story} = (1.0)\left[\dfrac{(103\text{ kips})(13.5\text{ ft})(12\text{ in./ft})}{0.111\text{ in.}}\right]$ $= 150{,}000\text{ kips}$ $B_2 = \dfrac{1}{1-\dfrac{1.6(5{,}120\text{ kips})}{150{,}000\text{ kips}}} > 1$ $= 1.06 > 1$ Therefore, use B_2 = 1.06. $P_r = P_{nt} + B_2 P_{lt}$ (*Spec*. Eq. A-8-2) $= 219\text{ kips} + (1.06)(76.6\text{ kips})$ $= 300\text{ kips}$ From AISC *Manual* Table 6-2 for a W12×53 with L_c = 13.5 ft: $P_c = \dfrac{P_n}{\Omega_c}$ $= 342\text{ kips}$ From AISC *Specification* Equation H1-1a: $\dfrac{P_r}{P_c} = \dfrac{300\text{ kips}}{342\text{ kips}} \leq 1.0$ $= 0.877 < 1.0$ **o.k.**

Note: Notice that the lower sidesway displacements of the braced frame produce much lower values of B_2 than those of the moment frame. Similar results could be expected for the other two methods of analysis.

Although not presented here, second-order effects should be accounted for in the design of the beams and diagonal braces in the braced frames at Grids 1 and 8.

ANALYSIS OF DRAG STRUTS

The fourth floor delivers the highest diaphragm force to the braced frames at the ends of the building: Q_E = 80.3 kips (from previous calculations). This force is transferred to the braced frame through axial loading of the W18×35 beams at the end of the building.

The gravity dead loads for the edge beams are the floor loading of 75 psf (5.50 ft) plus the exterior wall loading of 0.503 kip/ft, giving a total dead load of 0.916 kip/ft. The gravity live load for these beams is the floor loading of 80 psf (5.50 ft) = 0.440 kip/ft. The resulting midspan moments are M_D = 58.0 kip-ft and M_L = 27.8 kip-ft.

The required seismic load combinations, as given in ASCE/SEI 7, Section 12.4, were derived previously. The controlling load combination for LRFD is 1.23D + 1.0Q_E + 0.5L. The controlling load combinations for ASD are 1.01D + 0.525Q_E + 0.75L or 1.02D + 0.7Q_E.

LRFD	ASD
$M_u = 1.23M_D + 0.5M_L$ $= 1.23(58.0 \text{ kip-ft}) + 0.5(27.8 \text{ kip-ft})$ $= 85.2 \text{ kip-ft}$	$M_a = 1.01M_D + 0.75M_L$ $= 1.01(58.0 \text{ kip-ft}) + 0.75(27.8 \text{ kip-ft})$ $= 79.4 \text{ kip-ft}$ or $M_a = 1.02M_D$ $= 1.02(58.0 \text{ kip-ft})$ $= 59.2 \text{ kip-ft}$
Load from the diaphragm shear due to earthquake loading $F_p = 1.0Q_E$ $= 1.0(80.3 \text{ kips})$ $= 80.3 \text{ kips}$	Load from the diaphragm shear due to earthquake loading $F_p = 0.525Q_E$ $= 0.525(80.3 \text{ kips})$ $= 42.2 \text{ kips}$ or $F_p = 0.7Q_E$ $= 0.7(80.3 \text{ kips})$ $= 56.2 \text{ kips}$

Only the two 45-ft-long segments on either side of the brace can transfer load into the brace, because the stair opening is in front of the brace.

Use AISC *Specification* Section H2 to check the combined bending and axial stresses.

LRFD	ASD
$V = \dfrac{80.3\text{ kips}}{2(45\text{ ft})}$ $= 0.892\text{ kip/ft}$	$V = \dfrac{42.2\text{ kips}}{2(45\text{ ft})}$ $= 0.469\text{ kip/ft}$ or $V = \dfrac{56.2\text{ kips}}{2(45\text{ ft})}$ $= 0.624\text{ kip/ft}$

From AISC *Manual* Table 1-1, for a W18×35:

$$S_x = 57.6\text{ in.}^3$$

LRFD	ASD
The top flange bending stress is: $f_{rbw} = \dfrac{M_u}{S_x}$ $= \dfrac{(85.2\text{ kip-ft})(12\text{ in./ft})}{57.6\text{ in.}^3}$ $= 17.8\text{ ksi}$	The top flange bending stress is: $f_{rbw} = \dfrac{M_a}{S_x}$ $= \dfrac{(79.4\text{ kip-ft})(12\text{ in./ft})}{57.6\text{ in.}^3}$ $= 16.5\text{ ksi}$ or $f_{rbw} = \dfrac{M_a}{S_x}$ $= \dfrac{(59.2\text{ kip-ft})(12\text{ in./ft})}{57.6\text{ in.}^3}$ $= 12.3\text{ ksi}$

Note: It is often possible to resist the drag strut force using the slab directly. For illustration purposes, this solution will instead use the beam to resist the force independently of the slab. The full cross section can be used to resist the force if the member is designed as a column braced at one flange only (plus any other intermediate bracing present, such as from filler beams). Alternatively, a reduced cross section consisting of the top flange plus a portion of the web can be used. Arbitrarily use the top flange and 8 times an area of the web equal to its thickness times a depth equal to its thickness, as an area to carry the drag strut component.

$$\begin{aligned}\text{Area} &= b_f t_f + 8(t_w)^2 \\ &= (6.00\text{ in.})(0.425\text{ in.}) + 8(0.300\text{ in.})^2 \\ &= 3.27\text{ in.}^2\end{aligned}$$

Ignoring the small segment of the beam between Grid C and D, the axial stress due to the drag strut force is:

LRFD	ASD
$f_{ra} = \dfrac{80.3\text{ kips}}{2\left(3.27\text{ in.}^2\right)}$ $= 12.3\text{ ksi}$	$f_{ra} = \dfrac{42.2\text{ kips}}{2\left(3.27\text{ in.}^2\right)}$ $= 6.45\text{ ksi}$ or $f_{ra} = \dfrac{56.2\text{ kips}}{2\left(3.27\text{ in.}^2\right)}$ $= 8.59\text{ ksi}$

LRFD	ASD
Using AISC *Specification* Section H2, assuming the top flange is continuously braced: $F_{ca} = \phi_c F_y$ $= 0.90(50\text{ ksi})$ $= 45.0\text{ ksi}$ $F_{cbw} = \phi_b F_y$ $= 0.90(50\text{ ksi})$ $= 45.0\text{ ksi}$ $\dfrac{f_{ra}}{F_{ca}} + \dfrac{f_{rbw}}{F_{cbw}} \le 1.0$ (from *Spec*. Eq. H2-1) $\dfrac{12.3\text{ ksi}}{45.0\text{ ksi}} + \dfrac{17.8\text{ ksi}}{45.0\text{ ksi}} = 0.669 < 1.0$ **o.k.**	From AISC *Specification* Section H2, assuming the top flange is continuously braced: $F_{ca} = F_y / \Omega_c$ $= 50\text{ ksi}/1.67$ $= 29.9\text{ ksi}$ $F_{cbw} = \dfrac{F_y}{\Omega_b}$ $= 50\text{ ksi}/1.67$ $= 29.9\text{ ksi}$ $\dfrac{f_{ra}}{F_{ca}} + \dfrac{f_{rbw}}{F_{cbw}} \le 1.0$ (from *Spec*. Eq. H2-1) Load Combination 1: $\dfrac{6.45\text{ ksi}}{29.9\text{ ksi}} + \dfrac{16.5\text{ ksi}}{29.9\text{ ksi}} = 0.768 < 1.0$ **o.k.** Load Combination 2: $\dfrac{8.59\text{ ksi}}{29.9\text{ ksi}} + \dfrac{12.3\text{ ksi}}{29.9\text{ ksi}} = 0.699 < 1.0$ **o.k.**

Note: Because the drag strut load is a horizontal load, the method of transfer into the strut, and the extra horizontal load that must be accommodated by the beam end connections should be indicated on the drawings.

PART III EXAMPLE REFERENCES

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SDI (2015), *Diaphragm Design Manual,* 4th Ed., Steel Deck Institute, Glenshaw, PA.

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West, M.A. and Fisher, J.M. (2003), *Serviceability Design Considerations for Steel Buildings,* Design Guide 3, 2nd Ed., AISC, Chicago, IL.


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2ND FLOOR FRAMING PLAN
S2.1
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MOMENT FRAME (TYP)
3" DECK w/ 3" NORMAL WEIGHT CONCRETE T.O. SLAB ELEV = 113'-6"
CHEVRON BRACE
STAIR OPENING
ELEVATOR OPENING
29'-0" OPENING
W24x55 [10,4,10]
W21x50 [44] C=1.5"
46K W21x68 [12,4,12] C=0.75" 46K
W18x35
HSS8x8x¾
W12x58
W12x40
W12x53
W12x65
W14x90
30'-0"
22'-6"
45'-0"
BEAM SIZE
POSITIVE CAMBER
xxK W8x10 [50] (+1") xxK
ASD END REACTION (KIPS) TYP
OF 3/4"ø x 5 1/2" LONG HEADED STUDS
BEAM LEGEND
2ND FLOOR FRAMING PLAN
1/8" = 1'-0"
NORTH

1
2
3
4
5
6
7
8
30'-0"
A
B
C
D
E
F
45'-0"
30'-0"
22'-6"
MOMENT FRAME (TYP)
W12x58
W14x90
W12x40
W12x53
W12x65
W24x55
38K
31K
16K
22K
46K
W18x35
W21x50 [44] C=1.5"
W21x68 [12,4,12] C=0.75"
HSS8x8x¼
CHEVRON BRACE
STAIR OPENING
ELEVATOR OPENING
3" DECK w/ 3"
NORMAL WEIGHT CONCRETE
T.O. 3RD FLR SLAB
ELEV = 127'-0"
T.O. 4TH FLR SLAB
ELEV = 140'-6"
S4.1
S5.1
BEAM SIZE
POSITIVE CAMBER
xxK W8x10 [50] (+1") xxK
ASD END REACTION (KIPS) TYP
OF 3/4"ø x 5 1/2" LONG HEADED STUDS
BEAM LEGEND
3RD & 4TH FLOOR FRAMING PLAN
1/8" = 1'-0"
1
S2.2
NORTH
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3RD & 4TH FLOOR FRAMING PLAN
S2.2
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ROOF FRAMING PLAN
1/8" = 1'-0"
1 S2.3
NORTH
T.O. DECK ELEV = 155'-0"
T.O. DECK ELEV = 154'-0"
MOMENT FRAME (TYP)
1 1/2" 22GA WIDE RIB DECK T.O. DECK ELEV = 155'-0"
SCREEN WALL OR MECHANICAL PENTHOUSE
ELEVATOR OPENING
TAPERED INSULATION THIS AREA
CHEVRON BRACE
BEAM DRAG STRUT
24KCS4
18K5
18K7
W12x58
W12x40
W12x53
W14x90
W14x65
W18x35
W21x44
W21x55
W16x26 (+2 1/2")
W14x22 (+2 1/2")
30'-0"
22'-6"
45'-0"
3'-0"
BEAM SIZE
W8x10
xxK
ASD END REACTION (KIPS) TYP
BEAM LEGEND
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ROOF FRAMING PLAN
S2.3
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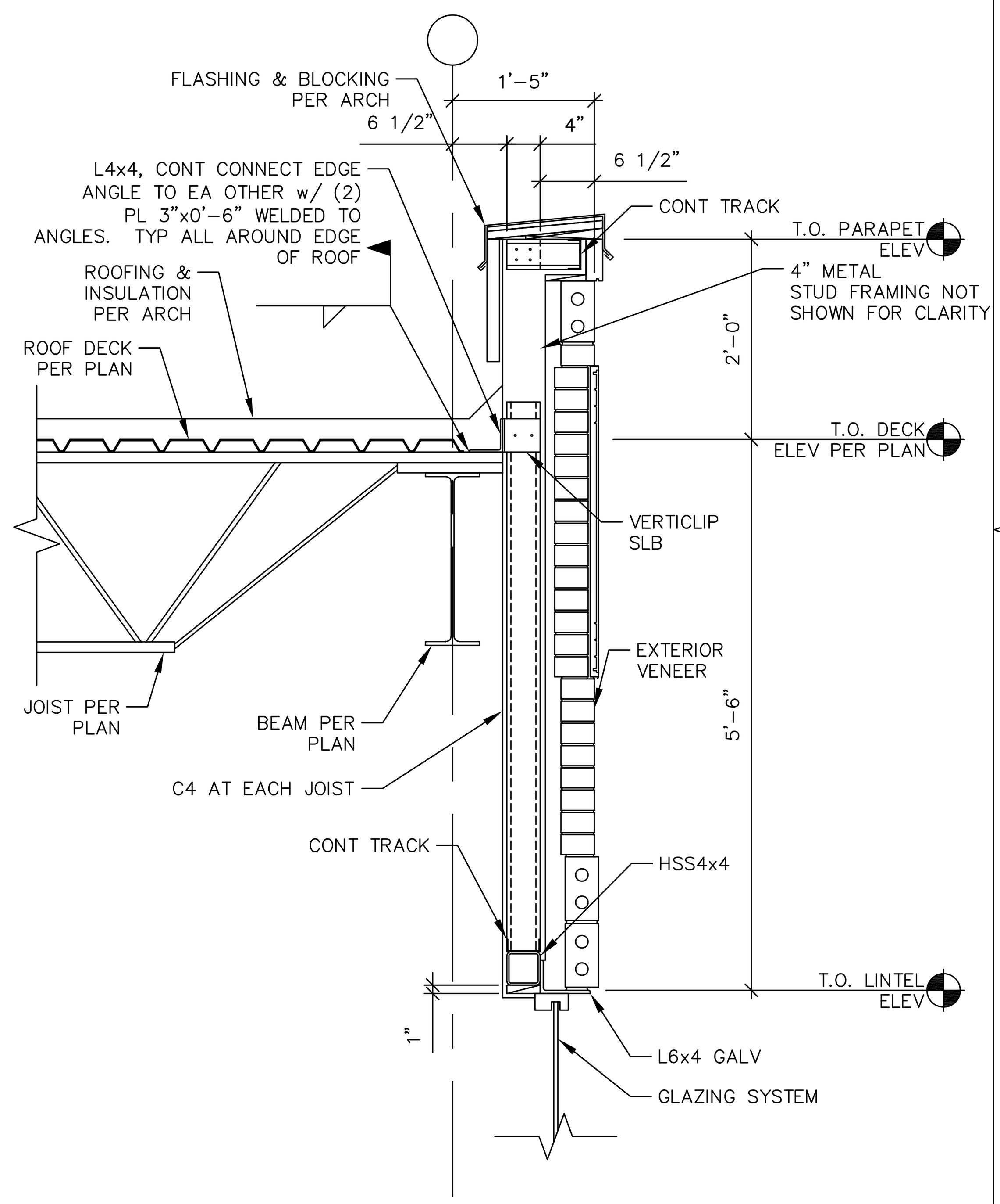
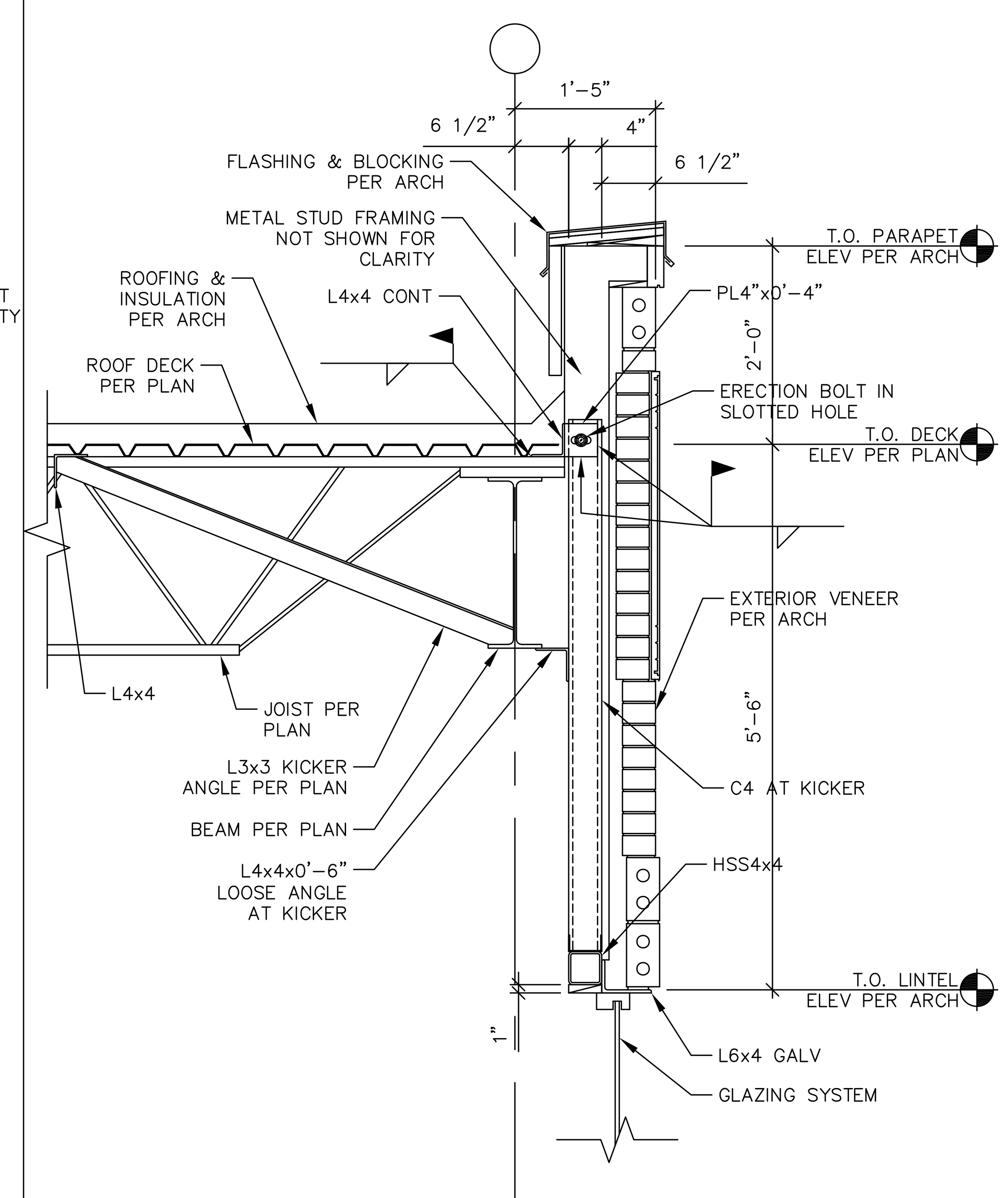


SECTION 1 / S4.1
3/4" = 1'-0"



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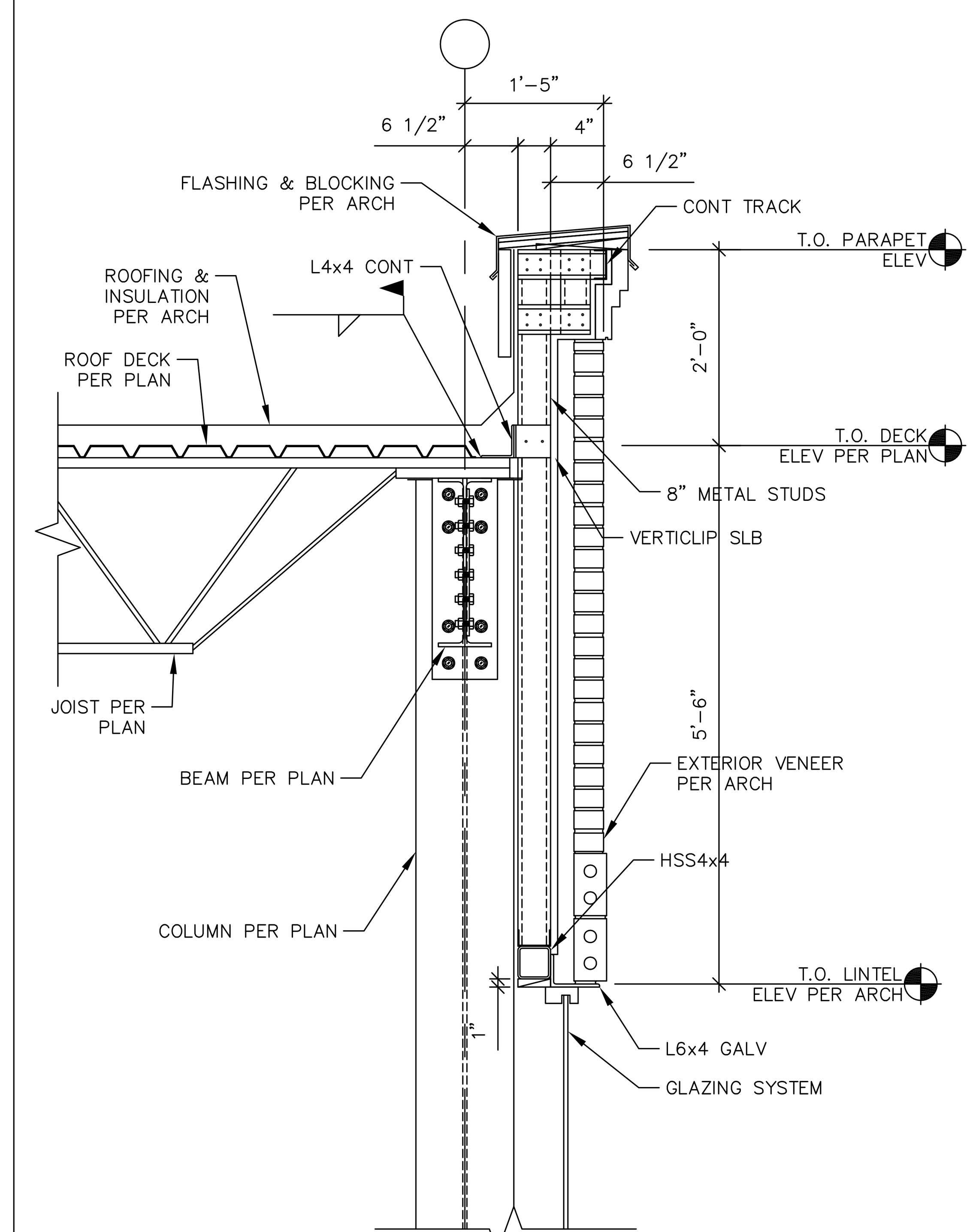
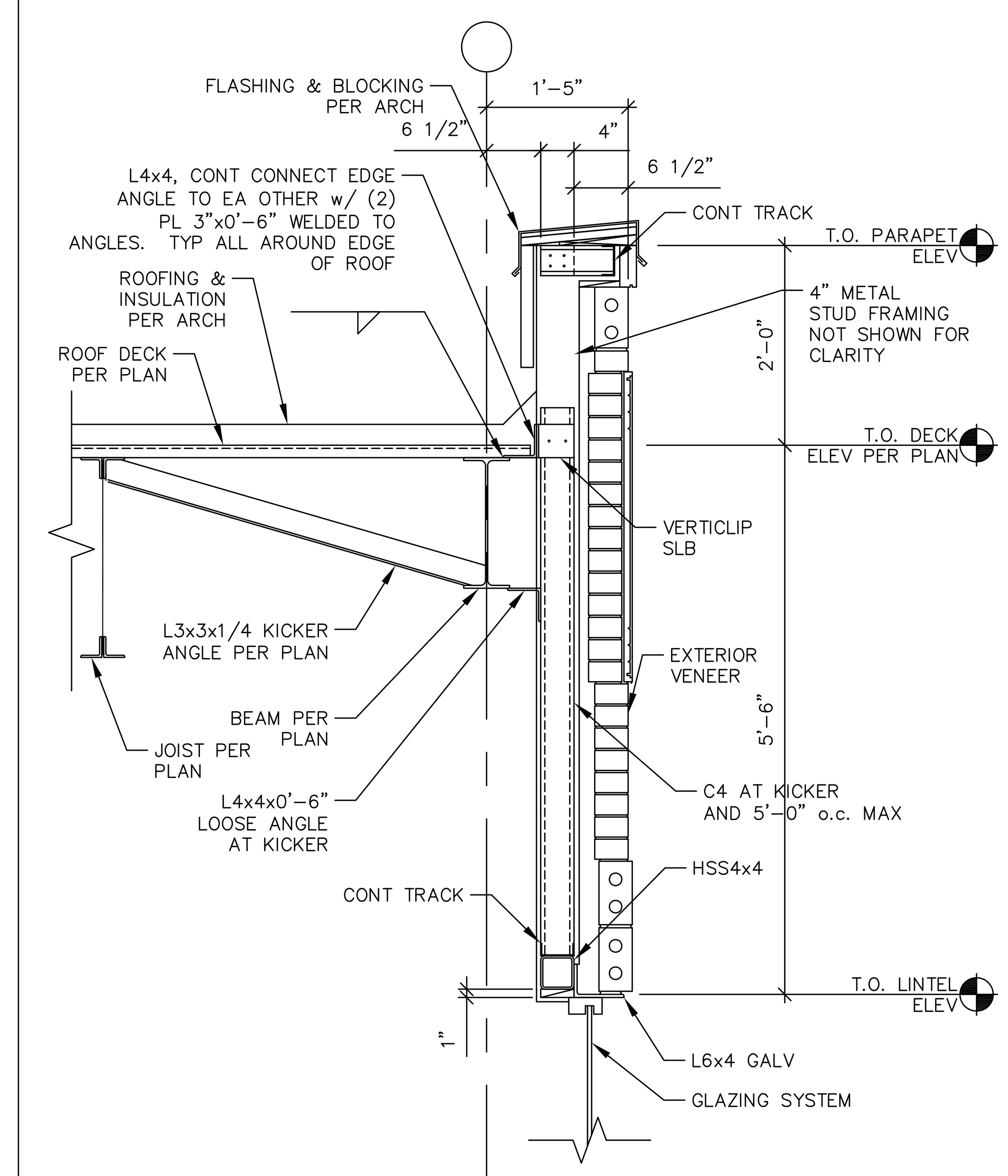


SECTION 3 / S4.1
3/4" = 1'-0"



SECTION 4 / S4.1
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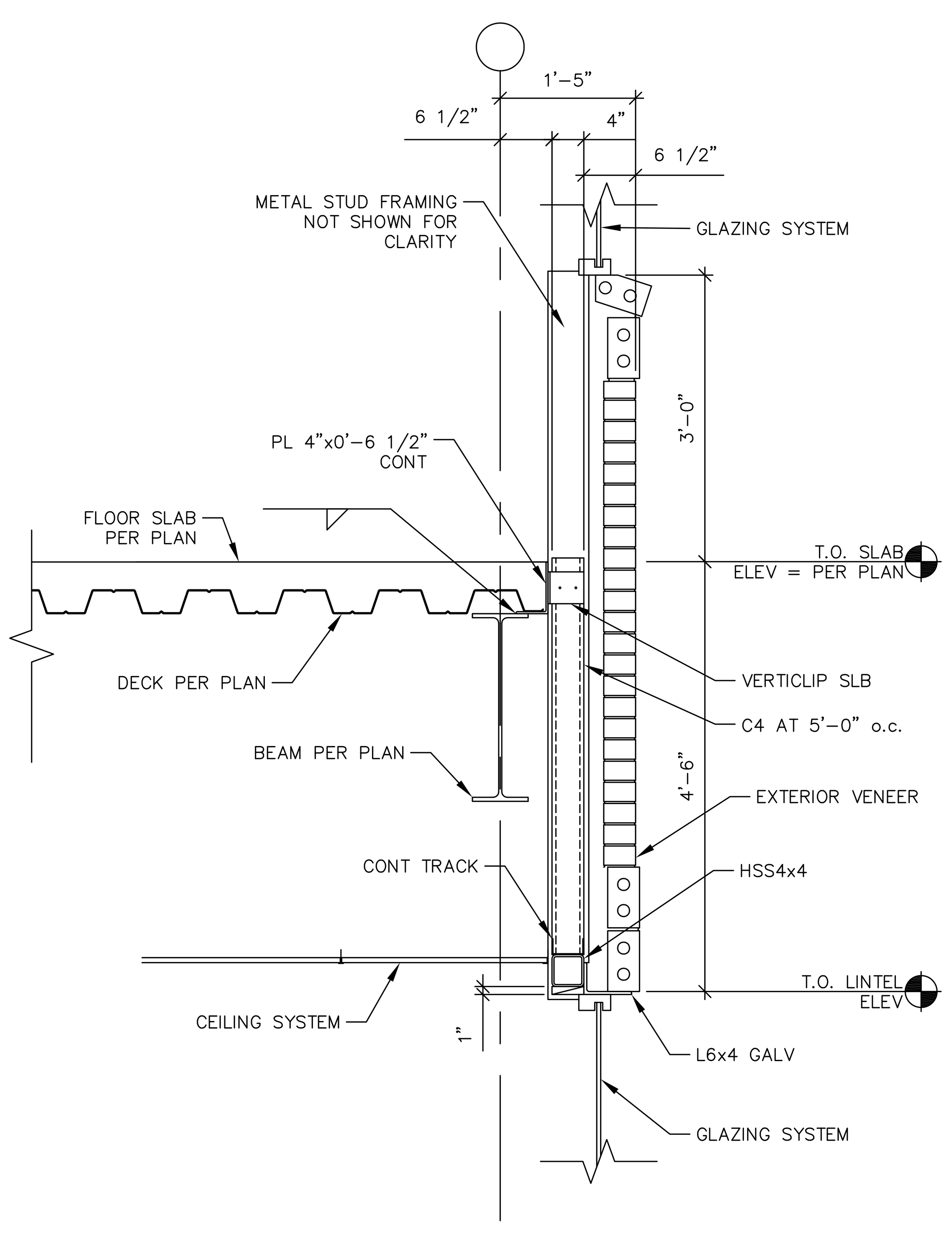
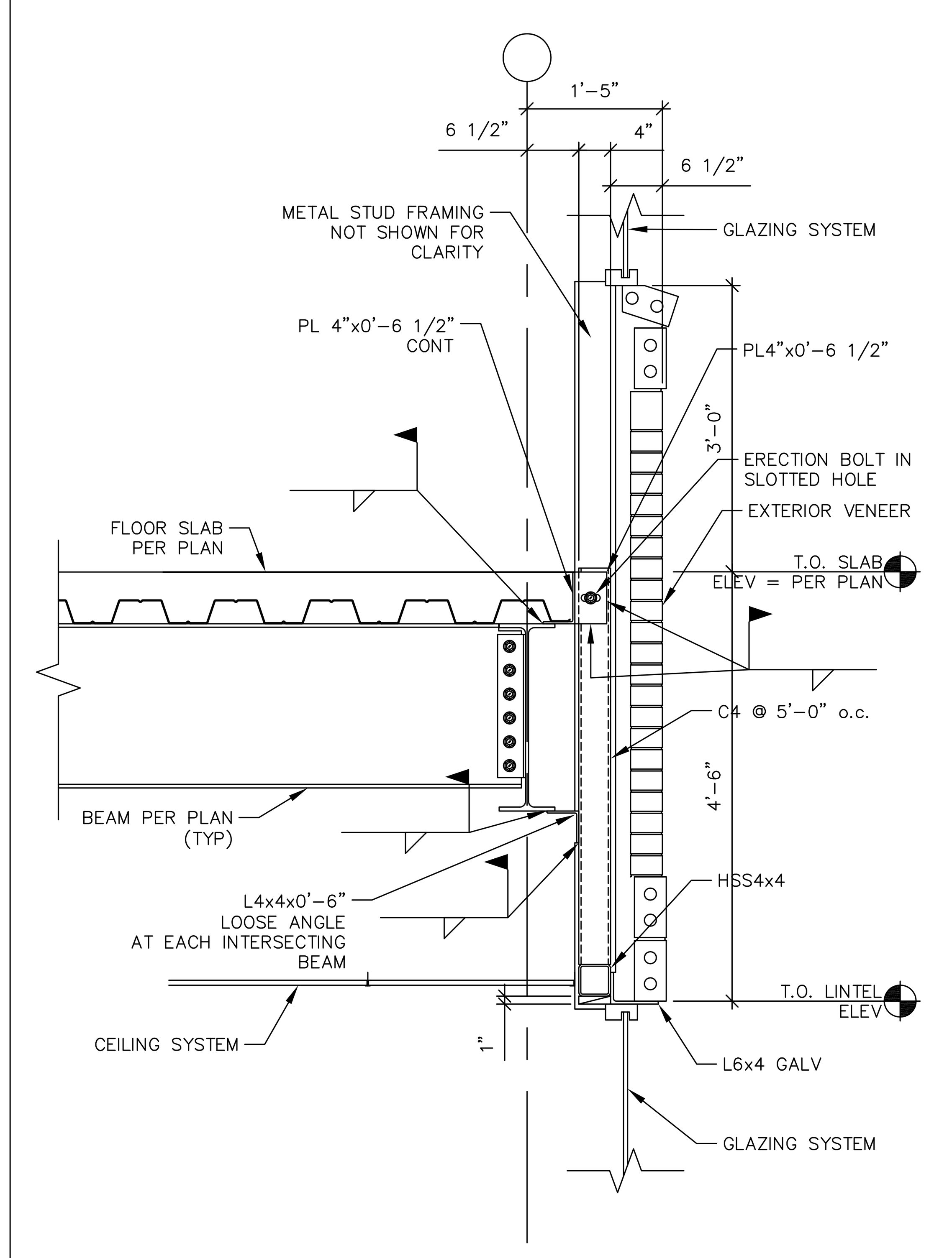
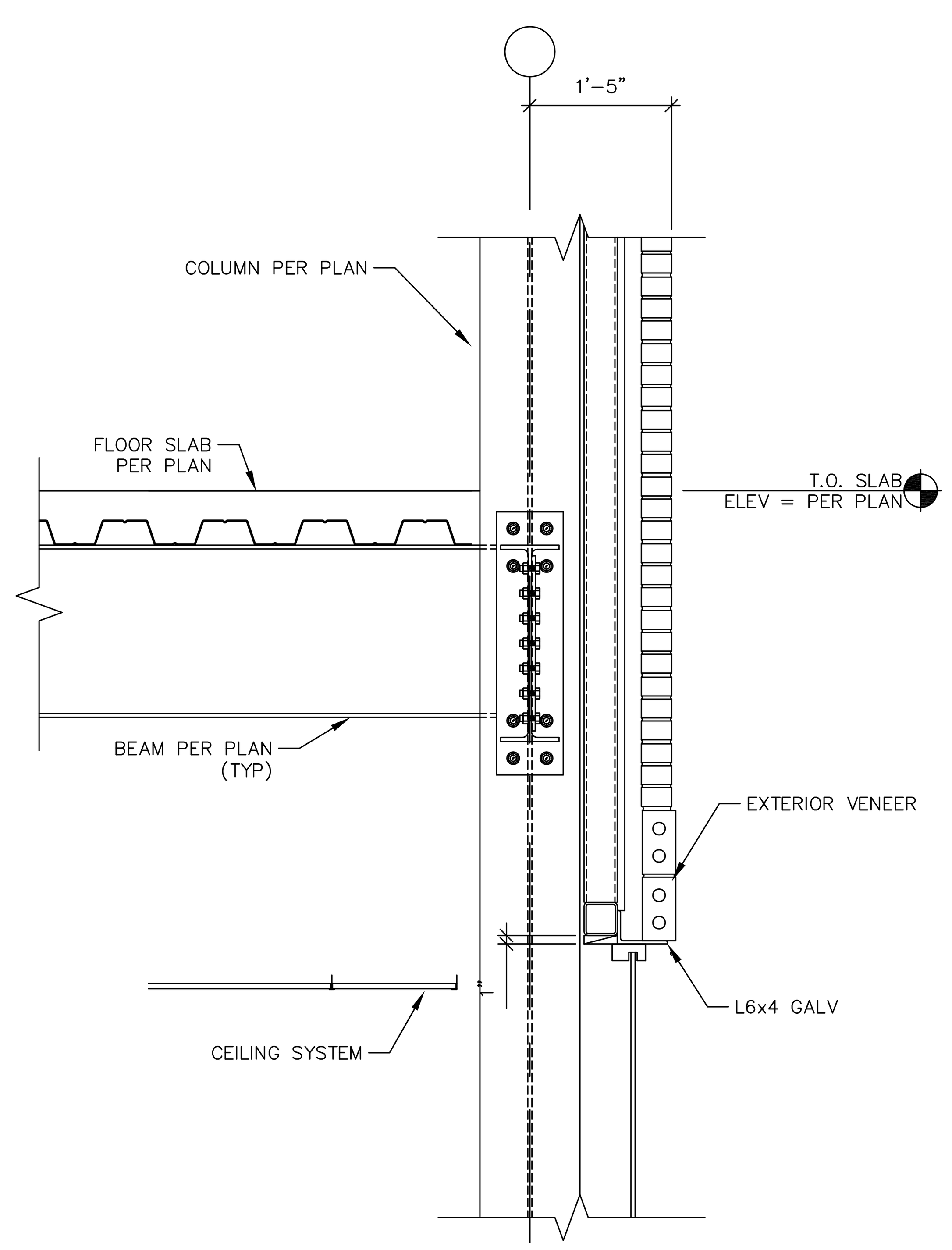


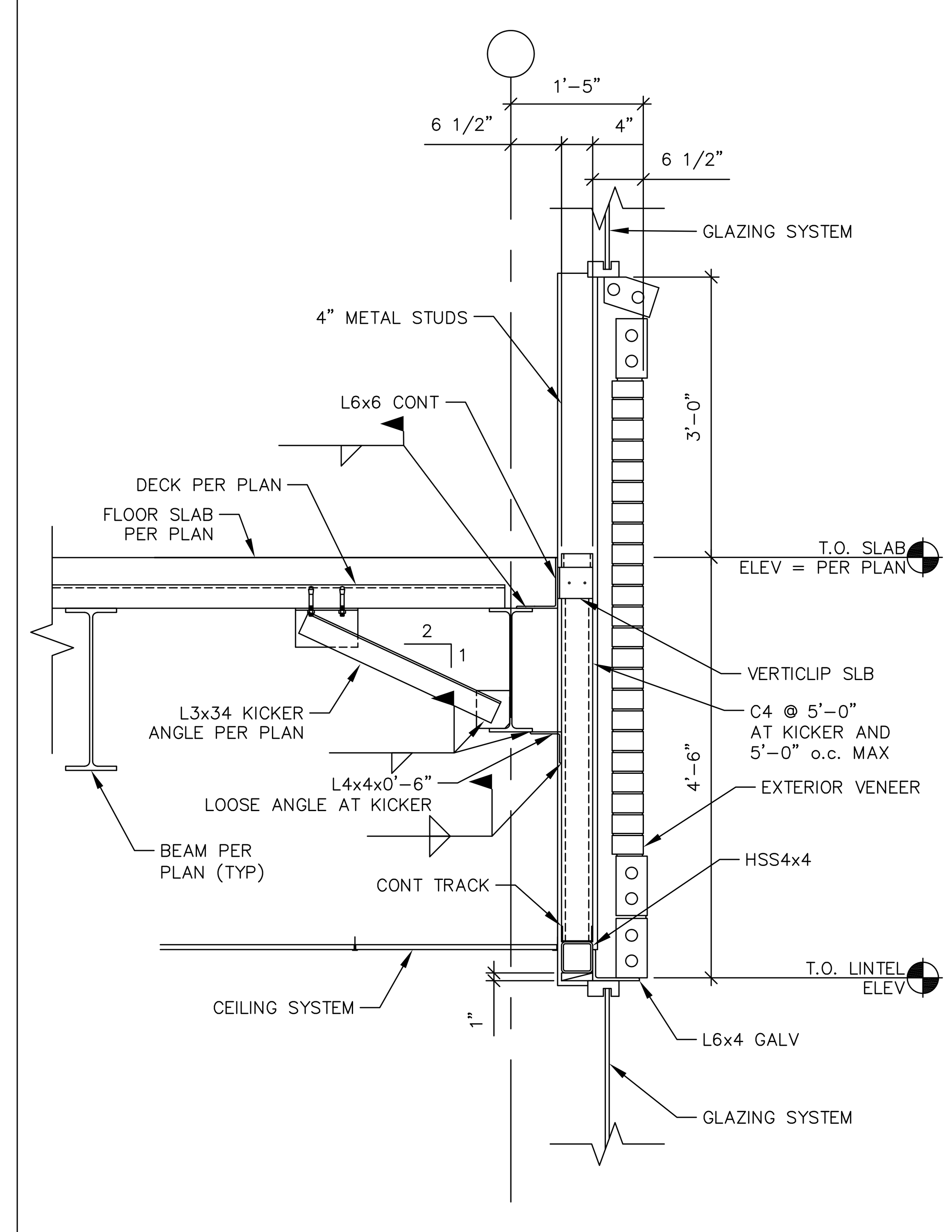
SECTION 5 / S4.1
3/4" = 1'-0"



SECTION 6 / S4.1
3/4" = 1'-0"



SECTION 7 / S4.1
3/4" = 1'-0"



SECTION 8 / S4.1
3/4" = 1'-0"

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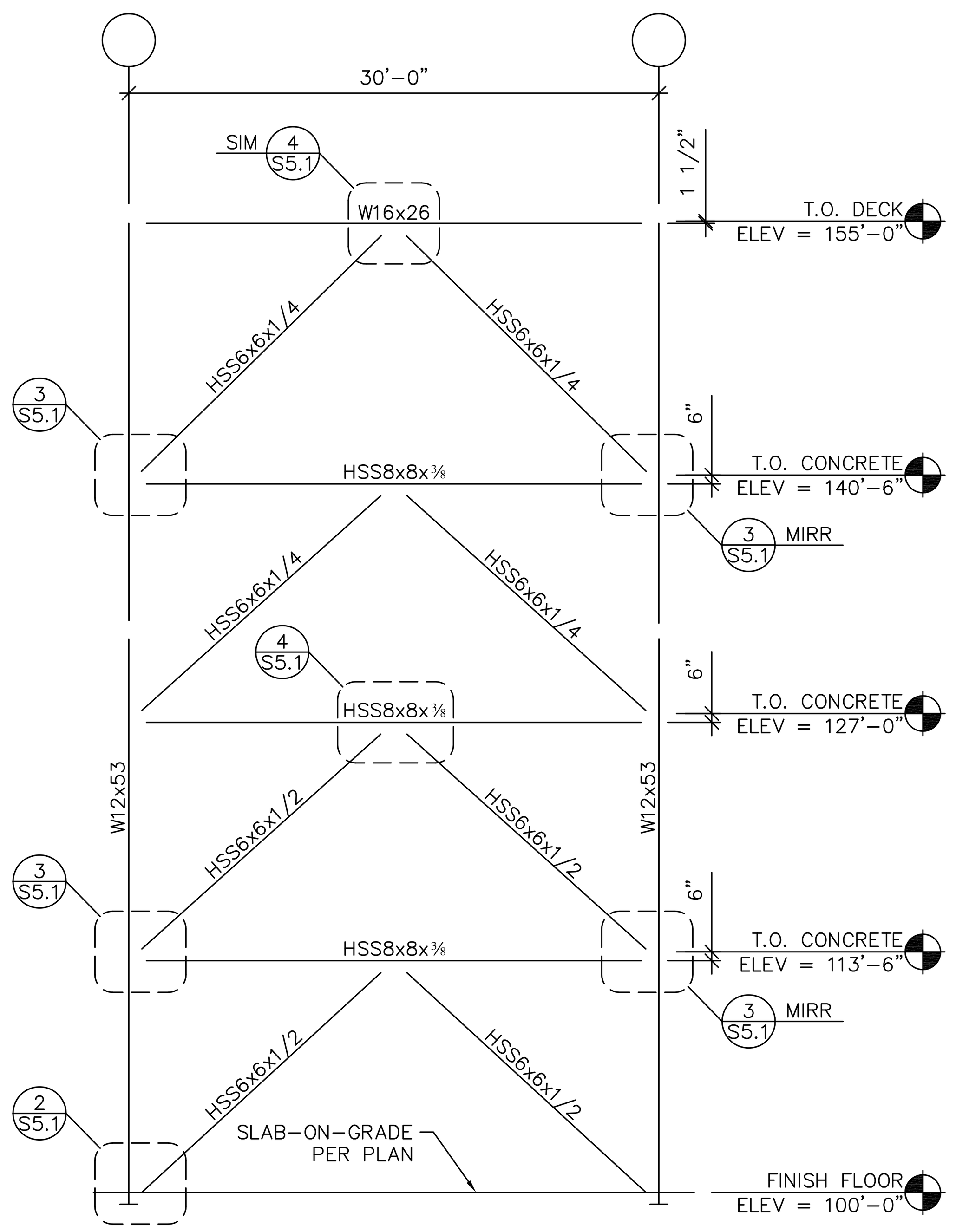
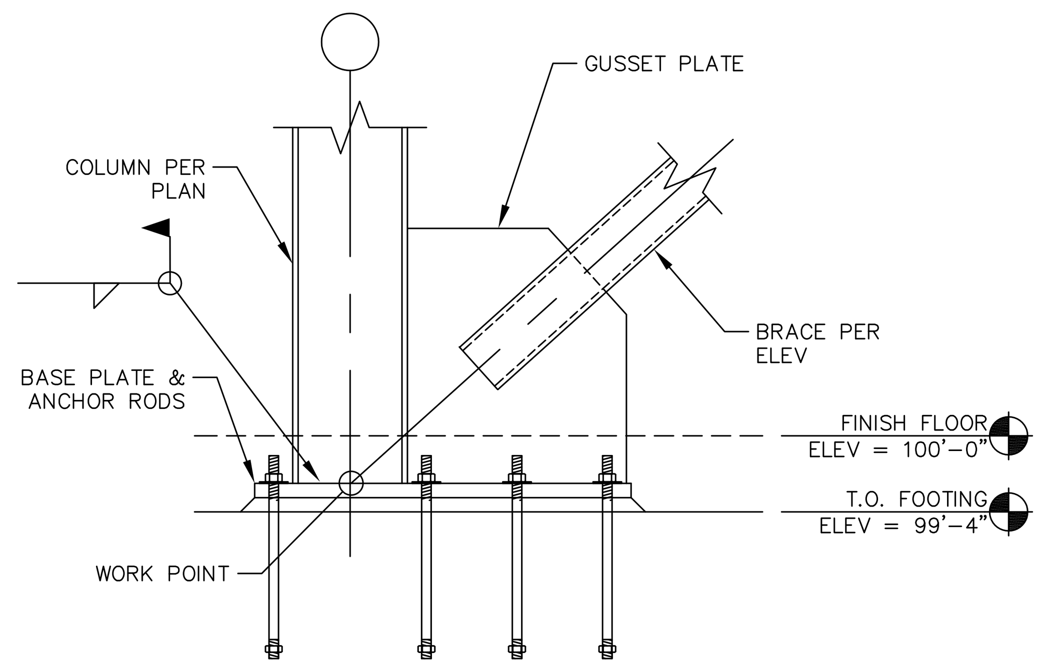
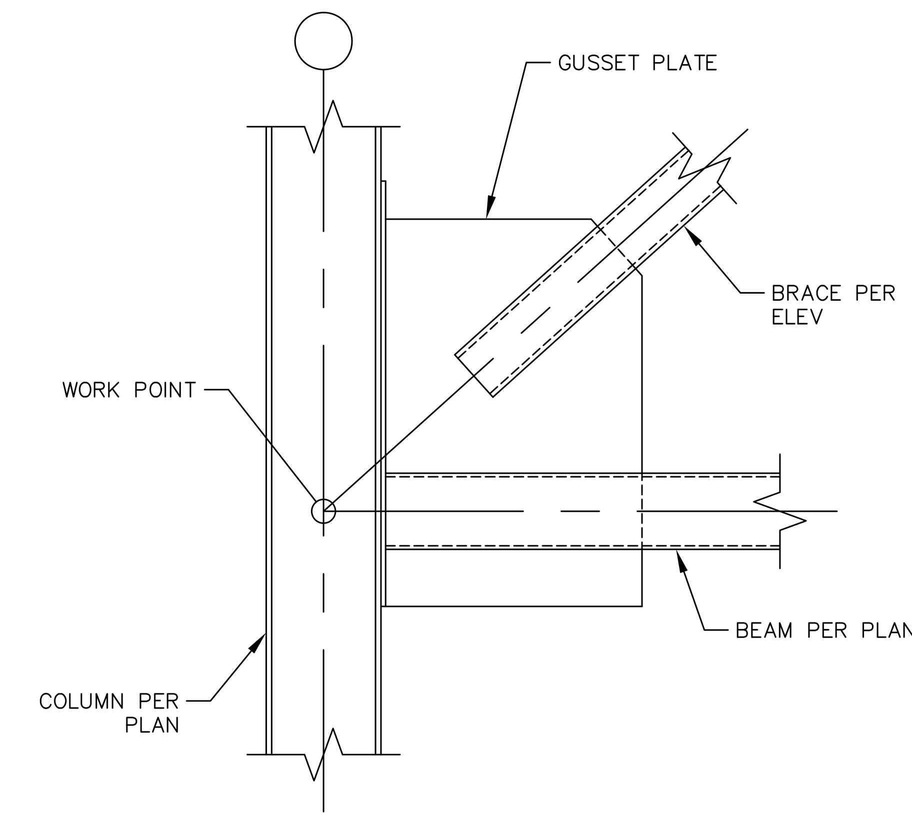
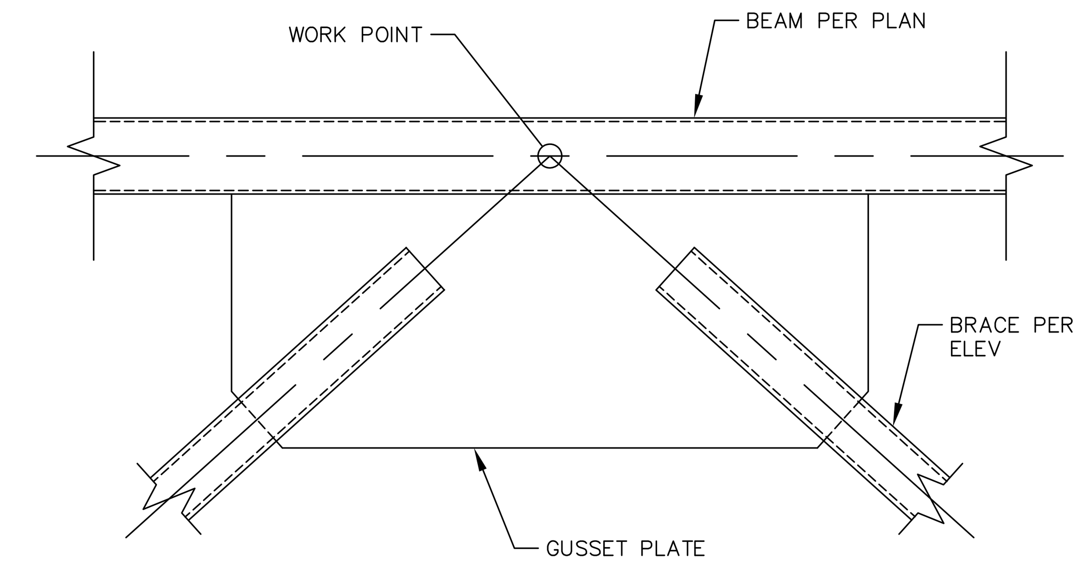
ROOF FRAMING SECTIONS

S4.1









CHEVRON BRACE ELEVATION 1 S5.1
N.T.S.

DETAIL 2 S5.1
1 1/2" = 1'-0"

DETAIL 3 S5.1
1 1/2" = 1'-0"

DETAIL 4 S5.1
1 1/2" = 1'-0"

NOT USED 5 S5.1

NOT USED 6 S5.1

NOT USED 7 S5.1

NOT USED 8 S5.1

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CHEVRON BRACE ELEVATION & DETAILS

S5.1